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Posted Date: 30 May 2024

doi: 10.20944/preprints202405.2020.v1

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Article

Safe Robust Adaptive Motion Control for Underactuated Marine Robots

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Abstract: This article presents an innovative approach to the design of a safe adaptive backstepping control system. Tailored specifically for underactuated marine robots, the system utilizes simple sensors for enhanced practicality and efficiency. Given their operation in diverse oceanic environments fraught with various sources of uncertainties, ensuring the system's safe and robust behavior holds paramount importance in the control literature. To address this concern, this paper introduces a control strategy designed to ensure robustness at both the kinematic and dynamic levels. By emphasizing the compensation for the system uncertainties, the design integrates a straightforward fuzzy system structure. To further ensure the system's safety, a funnel surface is defined, followed by the design of a suitable nonlinear sliding surface as a function of the funnel and tracking error. Specifically leveraging the Lyapunov function, the study formally establishes the Semi-globally Practically Finite-time Stability of the closed-loop system, validated through simulations conducted on underactuated marine robots.

Keywords: marine robots; funnel control; backstepping control; fuzzy systems; finite-time stability

1. Introduction

Autonomous and self-mission robots, particularly those designated for unmanned operations in the ocean such as marine vehicles, accompanied by suitable control schemes, have long stood as compelling research subjects, captivating the attention of numerous researchers. The resulting advancements have led to significant progress, providing valuable insights into the field. These insights could find numerous applications, such as marine research, subsea pipeline monitoring, lifesaving operations, oil and gas exploration, and many other missions [1–3]. However, despite the considerable strides made, the complexities surrounding underwater robotics persist. Challenges such as external disturbances, the unpredictable nature of fluid dynamics, navigation, energy management, environmental hazards and obstacles, underwater communication, autonomous decision-making and adaptation, as well as data processing, remain formidable obstacles. While considerable research has been dedicated to addressing the aforementioned issues, numerous challenges persist in the shadows. Therefore, additional research and discourse are imperative to develop robust and reliable algorithms capable of meeting the requisite standards and illuminating the intricacies of these challenges. In addition, autonomous marine vehicles encounter challenges categorized as underactuated, wherein the number of available actuators is fewer than the degrees of freedom. This circumstance adds further complexity to the design of control systems [3,4].

To meet these goals, schemes are needed to be implemented that are stable and safe. There are different ways to ensure the stability and safety of the system. One strong way to ensure formal stability is to use the Control Lyapunov functions [1–3]. These functions can provide a rigorous mathematical foundation for stability analysis. But this technique cannot alone ensure safety. To address safety, Control Barrier Functions (CBF) [5,6] and Funnel Control (FC) [7,8] have been proposed. One difficulty revolving around CBF is finding a suitable barrier function. Another issue regarding CBF is that it involves solving optimization problems as part of the control design process [5,6], which may be time-consuming and could be problematic in those applications that a fast and online reaction concerns.

Conversely, in FC [7,8], the defined boundary variable can easily mixed with nonlinear sliding surfaces and be used in Lyapunov functions in an outlier supervisory-like loop for monitoring some signals or confining some internal signals to achieve the requirement of safety and stability, without having to solve the optimization problems.

To address different uncertainties a strong approximation may be used. Among different approaches, Fuzzy Systems due to their special mathematical structure can be flexible enough to attract great attention to providing a strategy that could generally work for nonlinear systems and handling complex systems while being computationally efficient [2,8–10]. To overcome unmodelled dynamics, parametric uncertainties, and external disturbances, Sliding Mode Control (SMC) [1,11] proved to be effective enough to address these difficulties. However, from the practical point of view, the convergence time always matters, and thereby with ordinary SMC, designers may have less control over the rate of convergence. To achieve this objective, Terminal Sliding Mode Control (TSMC) has been developed, capitalizing on the robustness inherent in the original SMC-based methodologies, to ensure convergence within a finite time frame [3]. In this reference, a TSMC approach was devised for the trajectory tracking of underactuated underwater robots. Additionally, in [12], an innovative study introduced an adaptive finite-time impedance backstepping control scheme tailored for robot manipulators operating within unfamiliar environments. In practical terms, the inclusion of certain types of functions within the nonlinear sliding surface, such as the utilization of a sign function [1], can potentially lead to chattering in certain scenarios. To mitigate these challenges, the TSMC approach, as outlined in [3], has replaced the sign function with a saturation function, effectively resolving the potential issue of chattering. Another challenge associated with SMC pertains to the development of methodologies that do not impose restrictions on the rotational motion of the robot. For instance, this limitation is evident in the research cited in [13,14], as reported in [3].

Unlike many proposed control approaches like what was conducted on motion tracking [13,14] which have the limitation of being only valid for tracking special cases of reference trajectories, our proposed method can be used for a wide range of trajectories regardless of their shape. Moreover, compared to the safety methods developed in [5,6], in our case, we do not have to solve possible time-consuming optimizations since we introduce a novel boundary layer that utilizes a new soft funnel shape, contrasting with the approach in [7]. Additionally, unlike the methods discussed in [1,12], our proposed boundary layer does not incorporate a sign function within its structure, ensuring the robot's signals remain within the safe set. In terms of feasibility and robustness, our proposed scheme surpasses much of the above research by considering a broader range of uncertainties through the utilization of a more straightforward fuzzy estimator compared to [2,8]. Moreover, the developed method does not use the direct feedback of the lateral velocity, eliminating this way the need for a costly lateral velocity sensor and thereby enhancing its economic viability. The primary contributions of this article, as an extended version of [15], are outlined as follows:

- Trajectory tracking of an underactuated marine vehicle is addressed amidst significant sources of uncertainties, encompassing time-varying external disturbances at both the Kinematics and Dynamics levels, unmodeled dynamics, time-varying water current speeds, lateral velocities, and estimator errors;
- Robust funnel control is proposed for trajectory tracking with formal stability guarantees, ensuring finite-time stability and safety;
- Control design eliminates the necessity for exact knowledge of the robot dynamics, enhancing design simplicity and feasibility through the utilization of straightforward feedback sensors. For instance, there is no requirement for lateral sensors or other complex sensors such as acceleration feedback;
- Leveraging developed mathematics and formulations, along with a TSMC and novel funnel boundary shape, designers can tailor to diverse requirements and uphold various safety measures across different initial conditions;
- Implementation of a fuzzy estimator facilitates the estimation of unmodeled dynamics in the robot, employing a simple design and straightforward inputs;

- Compensation for the imprecision of fuzzy systems is achieved through the integration of a robust controller.

The remainder of this article is organized as follows: Section 2 delves into the Kinematics and Dynamics of marine vehicles within a two-dimensional framework. In Section 3, the architecture of the proposed control technique is detailed, followed by a discussion on stability analysis in Section 4. Section 5 presents simulation results conducted on challenging scenarios, while Section 6 concludes with final remarks.

2. Robot Kinematics and Dynamics

By employing a two-dimensional representation of the robot's movement path on a plane, the Kinematics of the marine vehicle can be expressed through the following equation (see e.g., [16,17]):

$$\dot{\mathbf{X}} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_r + u_c \\ v_r + v_c \\ r \end{bmatrix} + \begin{bmatrix} \tau_{d_{kx}} \\ \tau_{d_{ky}} \\ \tau_{d_{k\psi}} \end{bmatrix} \quad (1)$$

where x , y , and ψ denote the displacement along the x and y axes and the heading angle of the vehicle, respectively. Other signals including u_r , v_r , and r represent the surge, sway, and rotational speeds of the vehicle, while u_c and v_c denote the water current along surge and sway, respectively. Let us consider the vector τ_{d_K} representing the external disturbances exerted in displacements along the x and y axes as $\tau_{d_K} = [\tau_{d_{kx}} \quad \tau_{d_{ky}}]^\top$. The dynamics equation of the robot, which encapsulates motion along the surge, sway, and rotational axes, can be formulated as follows [16,17]:

$$m_u \dot{u}_r - m_v v_r r + d_{u_r} u_r - \tau_{d_{u_r}} = \tau_u, \quad (2a)$$

$$m_v \dot{v}_r + m_u u_r r + d_{v_r} v_r - \tau_{d_{v_r}} = 0, \quad (2b)$$

$$m_r \dot{r} - (m_u - m_v) u_r v_r + d_r r - \tau_{d_r} = \tau_r \quad (2c)$$

Here, m_u , m_v , and m_r are terms of the inertia matrix including hydrodynamic added mass information, while d_{u_r} , d_{v_r} , and d_r characterize the hydrodynamic damping effects. Other signals, such as $\tau_{d_{u_r}}$, $\tau_{d_{v_r}}$, and τ_{d_r} , represent external disturbances along the surge, lateral, and rotational axes, respectively. The signals τ_u and τ_r denote the control signals applied in the surge and rotational directions, respectively.

By combining equations (1) and (2), and considering the position vector $\mathbf{p} = [x \quad y]^\top$, the following is derived:

$$D\mathbf{R}^\top \ddot{\mathbf{p}} + (D\dot{\mathbf{R}}^\top + C\mathbf{R}^\top) \dot{\mathbf{p}} + D\dot{\boldsymbol{\lambda}} + C\boldsymbol{\lambda} - \tau_{d_D} - (D\mathbf{R}^\top \boldsymbol{\tau}_{d_K} + (D\dot{\mathbf{R}}^\top + C\mathbf{R}^\top) \tau_{d_K}) = \boldsymbol{\tau} \quad (3)$$

in which the matrices D , C , $\boldsymbol{\tau}$, and τ_{d_D} are defined as $D = \begin{bmatrix} m_u & 0 \\ 0 & m_r \end{bmatrix}$,

$C = \begin{bmatrix} d_{u_r} & -m_v v_r \\ -(m_u - m_v) v_r & d_r \end{bmatrix}$, $\boldsymbol{\tau} = [\tau_{u_r} \quad \tau_r]^\top$, and $\tau_{d_D} = [\tau_{d_{u_r}} \quad \tau_{d_r}]^\top$, respectively. In (3),

R is the rotation matrix defined as $R = \begin{bmatrix} \cos(\psi) & -\sin(\psi) \\ \sin(\psi) & \cos(\psi) \end{bmatrix}$ and $\boldsymbol{\lambda}$ is formulated as:

$$\boldsymbol{\lambda} = - \begin{bmatrix} u_c & -r + v_r + v_c \end{bmatrix}^\top \quad (4)$$

Next, we introduce a new variable $\mathbf{q} = \mathbf{R}^\top \mathbf{p}$. In doing so, equation (3) can be reformulated as follows:

$$\ddot{\mathbf{q}} = \hat{D}^{-1} \boldsymbol{\tau} + \boldsymbol{\mu} \quad (5)$$

in which μ is given by

$$\mu = -\hat{D}^{-1} \left(D\dot{\lambda} + C\lambda - \tau_{d_D} + \eta + \left(DR^T \ddot{R} + (D\dot{R}^T + CR^T) \dot{R} \right) q + (D - \hat{D}) \dot{q} \right) - \hat{D}^{-1} \left(2DR^T \dot{R} + D\dot{R}^T R + CR^T R \right) \dot{q} \quad (6)$$

where η has the following components:

$$\eta = -DR^T \dot{\tau}_{d_K} - \left(D\dot{R}^T + CR^T \right) \tau_{d_K} \quad (7)$$

3. Proposed Controller

Before designing the Backstepping controller, it is imperative to establish a tracking error, defined as the disparity between the actual rotated path and its desired trajectory. Let us define the tracking error as

$$\xi = R^T (p - p_d), \quad (8)$$

where the vector ξ is composed of two elements, and each ξ_i for $i = 1$ to 2 is correlated to every task space variable, represented as: $\xi = \begin{bmatrix} \xi_1 & \xi_2 \end{bmatrix}^T$. In consideration of the controller's safety measures, the following funnel variable is employed:

$$\chi_i = \xi_i^2 (\nu_i^2 - \xi_i^2)^{-1} \quad (9)$$

In this context, ν_i represents a function with a funnel shape that delineates a safe permitted area for the tracing error over time and in this paper is proposed as follows:

$$\nu_i = (\varrho_0 - \varrho_\infty)(1 + \beta t)^{-1} + \varrho_\infty \quad (10)$$

Here, ϱ_0 and ϱ_∞ represent the initial and steady values of ν_i , respectively, while β is a positive constant chosen by the designer to control the funnel boundary. The following nonlinear sliding surface is proposed as follows:

$$S_i = \xi_i + \zeta_i \int \tanh(\chi_i^2) \xi_i d\tau \quad (11)$$

where $\xi_i \neq 0$ and $\zeta_i > 0$. In the proposed control method, the nonlinear sliding surface $x_{1i} = s_i$ and its time derivative $x_{2i} = \dot{s}_i$ are considered as the system variables. These variables are later demonstrated to serve as inputs for the fuzzy estimator. To calculate the time derivatives of x_{1i} and x_{2i} , it would be straightforward if we first compute the time derivative of χ_i . To this end, the time derivative of χ is given as the following equation:

$$\dot{\chi}_i = (2\nu_i^2 \xi_i \dot{\xi}_i - 2\nu_i \xi_i^2 \dot{\nu}_i) (\nu_i^2 - \xi_i^2)^{-2} \quad (12)$$

Let us assign A_i and B_i as $A_i = (2\nu_i^2 \xi_i) (\nu_i^2 - \xi_i^2)^{-2}$ and $B_i = (2\nu_i \xi_i^2 \dot{\nu}_i) (\nu_i^2 - \xi_i^2)^{-2}$, then one can rewrite equation (12) as follows:

$$\dot{\chi}_i = A_i \dot{\xi}_i - B_i \quad (13)$$

Taking the time derivative of this equation yields the following equality:

$$\ddot{\chi}_i = A_i \ddot{\xi}_i + \dot{A}_i \dot{\xi}_i - \dot{B}_i \quad (14)$$

The state-space representation can be expressed as follows:

$$\dot{x}_{1i} = x_{2i}, \quad (15a)$$

$$\dot{x}_{2i} = \ddot{\xi}_i + 2\zeta_i \dot{\chi}_i \operatorname{sech}^2(\chi_i^2) \xi_i + \zeta_i \tanh(\chi_i^2) \dot{\xi}_i \quad (15b)$$

By defining $\ddot{\xi}_i$ as $\ddot{\xi}_i = \ddot{q}_i - \ddot{q}_{di}$ and utilizing equation (5), equation (15) can be rewritten as follows:

$$\dot{x}_{1i} = x_{2i}, \quad (16a)$$

$$\dot{x}_{2i} = \hat{D}_i^{-1} \tau_i + \bar{\mu}_i \quad (16b)$$

in which $\bar{\mu}_i$ is given by

$$\bar{\mu}_i = \mu_i - \ddot{q}_{di} + 2\zeta_i \dot{\chi}_i \operatorname{sech}^2(\chi_i^2) \xi_i + \zeta_i \tanh(\chi_i^2) \dot{\xi}_i \quad (17)$$

The state representation, as formulated in (16) with the uncertainty function (17), will be employed in designing the proposed control technique. This paper introduces the following control law:

$$\tau_i = -\hat{D}[x_{1i} + K_{1i}(2l-1)x_{2i}x_{1i}^{2l-2} + K_{2i}(x_{2i} + k_{1i}x_{1i}^{2l-1})^{2l-1}] + \hat{D}[\tau_{ri} - \hat{\theta}_i^\top \gamma_i] \quad (18)$$

Here, K_{1i} and K_{2i} are the control design parameters, which must be positive, l is a natural number, and τ_{ri} represents the robust term formulated as follows:

$$\tau_{ri} = -\left(x_{2i} + k_{1i}x_{1i}^{2l-1}\right)\hat{\rho}^2\left(|x_{2i} + k_{1i}x_{1i}^{2l-1}|\hat{\rho} + \delta_{ri}e^{-\sigma_{ri}t}\right)^{-1} \quad (19)$$

Here, $\hat{\rho}$ represents the upper estimated bound of the modeling uncertainty using the fuzzy system. To prevent the denominator of equation (19) from being zero, as evident in the formula, a small positive value $\delta_{ri}e^{-\sigma_{ri}t}$ is added to the denominator, and σ_{ri} is another positive constant selected by designers. The adaptation procedure for the signals $\hat{\theta}_i$ and $\hat{\rho}_i$ in this paper is outlined by the following two equations:

$$\dot{\hat{\theta}}_i = \beta_i \left(x_{2i} + k_{1i}x_{1i}^{2l-1}\right) \gamma_i - \sigma_{\theta i} \hat{\theta}_i \quad (20)$$

$$\dot{\hat{\rho}}_i = \alpha_i |x_{2i} + k_{1i}x_{1i}^{2l-1}| - \sigma_{\rho i} \hat{\rho}_i \quad (21)$$

where β_i and α_i are the learning rates, and $\sigma_{\theta i}$ and $\sigma_{\rho i}$ are constant parameters chosen by designers. In equation (18), $\hat{\theta}_i^\top \gamma_i$ represents the output of the fuzzy system corresponding to the i -th estimator, which is intended to estimate the uncertainty. The fuzzy estimator structure in this article employs a product inference engine, singleton fuzzifier, center-average defuzzifier, and Gaussian membership functions [18]. Its architecture is as follows:

$$\hat{\mu}_i = \hat{\theta}_i^\top \gamma_i = \frac{\sum_{i=1}^{M_i} \bar{\theta}_i^{i_i} \mu_{A_1}^{i_i}(x_1) \mu_{A_2}^{i_i}(x_2)}{\sum_{i=1}^{M_i} \mu_{A_1}^{i_i}(x_1) \mu_{A_2}^{i_i}(x_2)} \quad (22)$$

In this context, M_i denotes the number of fuzzy rules corresponding to each task-space variable, respectively, while $\mu_{A_j}^{i_i}(x_j)$ represents the membership function defined over the input range. In the realm of fuzzy systems, the degree of membership signifies the level to which an element is associated with a specific fuzzy set.

Considering the problem of two-dimensional motion control as outlined in the dynamic equation in this paper, it is necessary to employ two fuzzy estimators: one dedicated to compensating for uncer-

tainties along the x-axis and the other along the y-axis. Each estimator is designed to accommodate two inputs. Based on these two fuzzy inputs, which are S_i and $\dot{S}_i$, one can rewrite (22) as follows:

$$\hat{\mu}_i = \frac{\sum_{i=1}^{M_i} \bar{\theta}_i^{l_i} \mu_{A_1}^{l_i}(\xi_i + \zeta_i \int \tanh(\chi_i^2) \xi_i d\tau) \mu_{A_2}^{l_i}(\dot{\xi}_i + \zeta_i \tanh(\chi_i^2) \dot{\xi}_i)}{\sum_{i=1}^{M_i} \mu_{A_1}^{l_i}(\xi_i + \zeta_i \int \tanh(\chi_i^2) \xi_i d\tau) \mu_{A_2}^{l_i}(\dot{\xi}_i + \zeta_i \tanh(\chi_i^2) \dot{\xi}_i)} \quad (23)$$

Simplicity dictates that three membership functions, namely N , Z , and P as depicted in Figure 1, are assigned with the following mathematical equations for each fuzzy estimator's input:

$$\mu_N(x_i) = \begin{cases} 1 & \text{if } x_i < -1, \\ e^{\frac{-(x_i+1)^2}{2 \times 0.5^2}} & \text{if } x_i \geq -1. \end{cases} \quad (24)$$

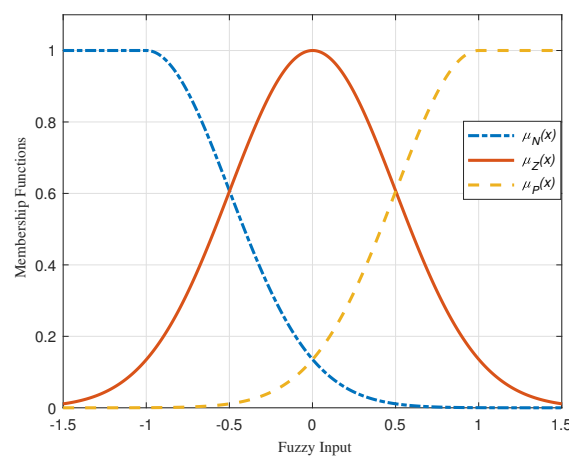


Figure 1. Membership functions of fuzzy systems.

$$\mu_Z(x_i) = e^{\frac{-x_i^2}{2 \times 0.5^2}} \quad (25)$$

$$\mu_P(x_i) = \begin{cases} e^{\frac{-(x_i-1)^2}{2 \times 0.5^2}} & \text{if } x_i < 1, \\ 1 & \text{if } x_i \geq 1. \end{cases} \quad (26)$$

To achieve a comprehensive understanding of the developed controller, the overall block diagram of the proposed method and the coordinate frames of the robot's motion are illustrated in Figure 2, parts (a) and (b) respectively.

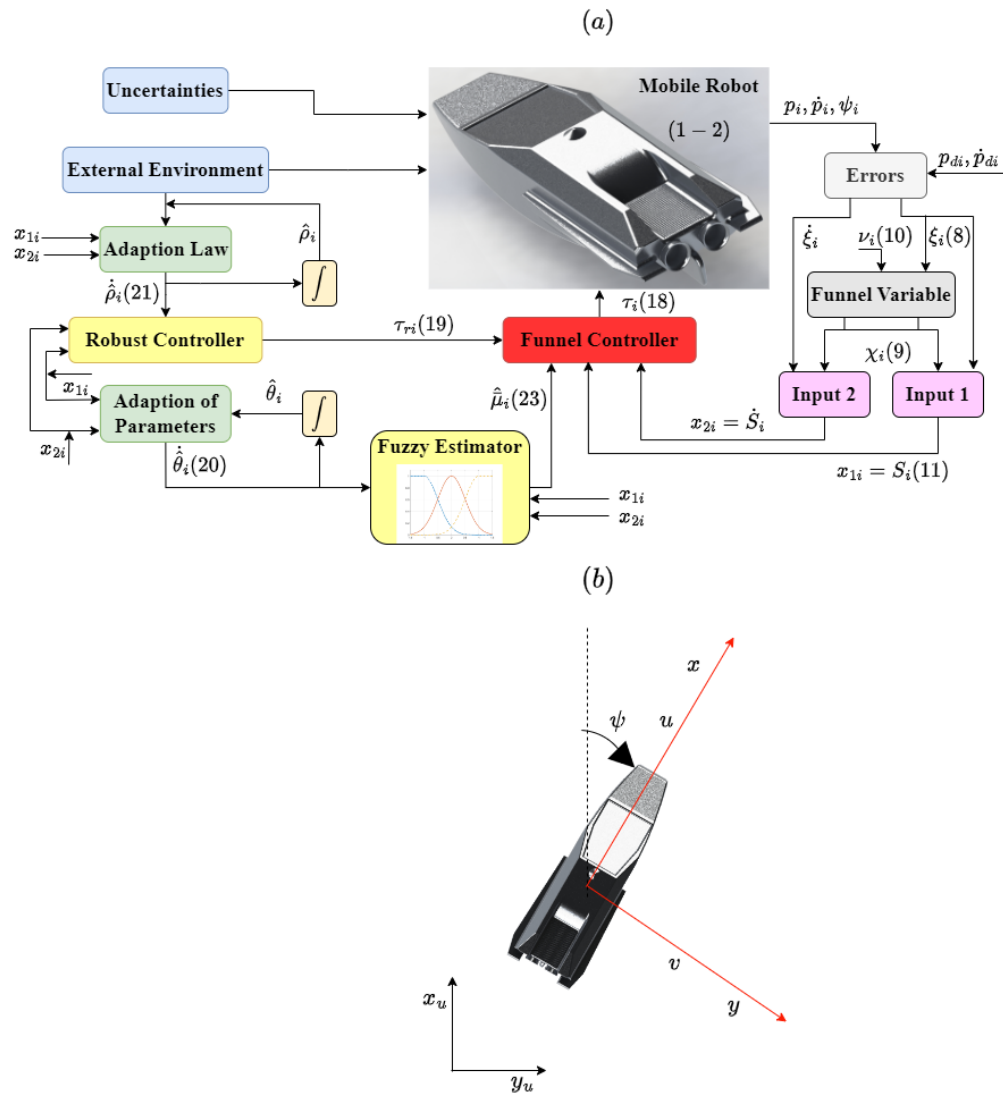


Figure 2. Block diagram illustrating the developed scheme and the robot's task space.

For designing the fuzzy system let us consider the following definitions for the membership functions and the set of rules:

Definition 1. In the context of fuzzy systems, a membership function is considered normal if its maximum height value is one, as defined by [18], that is:

$$\mu_{A_i}^{l_i}(x_i) \leq 1 \quad (27)$$

Here, x_i represents the i -th input for $i = 1$ to n_i , where n_i is the total number of inputs. l_i denotes the rule index for the membership function associated with x_i , where $l_i = 1$ to M_i , and M_i is the total number of rules for the fuzzy set A_i . $\mu_{A_i}^{l_i}(x_i)$ stands for the membership function of input element x_i in the fuzzy set A_i .

Definition 2. A collection of fuzzy IF-THEN rules is considered complete provided that for every $x_i \in U_i$, there is at least one rule in the fuzzy rule base, for instance, ι_i , denoted as $\mu_{A_i}^{\iota_i}(x_i)$ in the format of (24)-(26), where:

$$\mu_{A_i}^{\iota_i}(x_i) \neq 0 \quad (28)$$

In other words, a membership function is considered complete if it covers the entire input range with appropriate overlap, ensuring that every point in the domain has at least a membership in one fuzzy set [18].

According to Definition 1, Equations (24) to (26) are normal. Furthermore, designing based on Definition 2 implies that the denominator of Equation (23) is not zero.

4. Analyzing Control System Stability

4.1. Overall Stability of the System

This section aims to analyze the stability of the overall system, which consists of two subsystems based on the backstepping principles. To achieve this goal, the following information needs to be considered:

Assumption 1. The external disturbances influencing the Kinematics, and Dynamics of the robots are considered to be bounded, that is,

$$\| \tau_{d_{kx}}, \tau_{d_{ky}}, \tau_{d_{k\psi}} \| \leq \epsilon_K \quad (29)$$

$$\| \tau_{d_{ur}}, \tau_{d_{vr}}, \tau_{d_r} \| \leq \epsilon_D \quad (30)$$

where ϵ_K and ϵ_D are positive constants.

Assumption 2. The desired trajectory, p_d , is sufficiently smooth, and its time derivative is assumed to be readily available up to the required order [10].

Assumption 3. It is assumed that the water current velocity signals in the surge and sway directions are bounded.

$$|u_c| \leq \epsilon_{u_c} \quad (31)$$

$$|v_c| \leq \epsilon_{v_c} \quad (32)$$

Definition 3 ([7]). To establish a particular stability criterion, we initially examine a nonlinear system subject to the following conditions

$$\dot{z} = N(z, u), \quad N(0, 0) = 0, \quad z \in \mathbb{R}^n, \quad u \in \mathbb{R}^m \quad (33)$$

in which z signifies the state vector and u denotes the control input. The function N , which maps the domain from $\mathbb{R}^{n+m}$ to $\mathbb{R}^n$, is assumed to be continuous and defined within an open region around the origin. If there exists a neighborhood $\mathbf{U}$ encompassing the equilibrium point, and for each initial condition $z_0 \in \mathbf{U}$, there exists a positive constant ϵ along with a time-varying yet bounded parameter $\delta(\epsilon, z_0)$ such that $\|z(t)\| < \epsilon$ for all $t > t_0 + \delta$, then the system described by (33) is termed to have Semi-Globally Practically Finite-Time Stability (SGPFS).

Proposition 1. Assuming that the vector z operates within the input universe denoted as $\mathbf{U}$, a compact set within $\mathbb{R}^n$, let's consider any real continuous function $\bar{\mu}_i(z)$ defined over this set and an arbitrary $\epsilon_i > 0$. Under these conditions, it can be asserted that there exists a fuzzy system, denoted as $\hat{\mu}_i(z)$, structured with a product inference engine, singleton fuzzifier, center-average defuzzifier, and Gaussian membership functions. This system is designed such that the following relation holds:

$$\sup_{z \in A} |\hat{\mu}_i(z) - \bar{\mu}_i(z)| \leq \epsilon_i \quad (34)$$

This inequality suggests that the fuzzy system mentioned serves as a universal approximation [18]. In this context, $\hat{\mu}_i$ represents the output of the fuzzy system, and ϵ_i denotes both the precision of the fuzzy system and the upper bound on unmodeled dynamics, as established by the universal approximation.

Proposition 2. Assuming the presence of a positively definite function $V(z)$ established within a vicinity surrounding zero for the nonlinear system (5), if $V(0) = 0$ and its time-derivative adheres to the subsequent inequality:

$$\dot{V} + \iota V^l - \varrho \leq 0 \quad (35)$$

where $0 < l < 1$, ι , and ϱ are fixed positive values, then the system exhibits SGPFs behavior.

Theorem 1. Under Assumptions 1–3 and Proposition 1, the funnel backstepping control strategy, as presented in equation (18) with its robustifying term specified in (19) and estimator given in (23), coupled with the adaptation laws (20) and (21), guarantees the SGPFs of the system (5), while also ensuring that the tracking error (8) remains within the safe set.

Proof of Theorem 1. To derive a control law that can guarantee the stability of the first subsystem, the following Lyapunov function will be introduced as a potential candidate:

$$V_1 = \frac{1}{2} \sum_{i=1}^{i=n} x_{1i}^2 \quad (36)$$

One way to stabilize the first subsystem is to propose the virtual input as $\dot{x}_{1i} = -k_{1i}x_{1i}^{2l-1}$. By implementing this virtual input, the time derivative of the Lyapunov function given in (36) will be as the following form:

$$\dot{V}_1 = - \sum_{i=1}^{i=n} k_{1i} x_{1i}^{2l} \leq 0 \quad (37)$$

Equation (37) implies that the first subsystem is semi-definite stable. In terms of the main subsystem, the strategy will be to first define a new variable as

$$z_i = x_{2i} + k_{1i}x_{1i}^{2l-1} \quad (38)$$

and then, based on the newly formed state space representation, formulate the Lyapunov candidate for the second subsystem as:

$$V_2 = \sum_{i=1}^{i=n} \left[V_{1i} + 0.5(x_{2i} + k_{1i}x_{1i}^{2l-1})^2 + (0.5/\alpha_i)\tilde{\rho}_i^2 + (0.5/\beta_i)\tilde{\theta}_i^\top \tilde{\theta}_i \right] \quad (39)$$

in which $\tilde{\rho}_i$ and $\tilde{\theta}_i$ are defined as $\tilde{\rho}_i = \rho_i - \hat{\rho}_i$ and $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$, respectively. Here, ρ_i denotes the actual parameter representing the upper bound of the unmodeled dynamics, while $\hat{\rho}_i$ represents the estimated value of this upper bound. Similarly, θ_i and $\hat{\theta}_i$ refer to the actual parameters and their estimations related to the modeled uncertain dynamics.

To analyze the rate of energy changes within the Lyapunov function, one can take the time derivative of Equation (39) to obtain:

$$\dot{V}_2 = \sum_{i=1}^{i=n} \left[x_{1i}x_{2i} + \left(x_{2i} + k_{1i}x_{1i}^{2l-1} \right) \left(\dot{x}_{2i} + K_{1i}(2l-1)x_{2i}x_{1i}^{2l-2} \right) - (1/\alpha_i)\tilde{\rho}_i\dot{\hat{\rho}}_i - (1/\beta_i)\tilde{\theta}_i^\top \dot{\hat{\theta}}_i \right] \quad (40)$$

Subsequently, the dynamics of the robot, as described in (16), must be integrated into (40) to produce:

$$\dot{V}_{2i} = \sum_{i=1}^{i=n} \left[x_{1i}x_{2i} + \left(x_{2i} + k_{1i}x_{1i}^{2l-1} \right) \left(\hat{D}_i^{-1}\tau_i + \bar{\mu}_i + K_{1i}(2l-1)x_{2i}x_{1i}^{2l-2} \right) \right] - \sum_{i=1}^{i=n} \left[(1/\alpha_i)\tilde{\rho}_i\dot{\hat{\rho}}_i + (1/\beta_i)\tilde{\theta}_i^\top \dot{\hat{\theta}}_i \right] \quad (41)$$

Here, $\bar{\mu}_i$ represents the lumped uncertainty and can be modeled by the following equation:

$$\bar{\mu}_i = \theta_i^\top \gamma_i + \epsilon_i \quad (42)$$

In equation (42), θ_i , γ_i , and ϵ_i represent the uncertainty parameters, regressors, and unmodeled dynamics, respectively. To address this uncertainty, the approach involves utilizing a fuzzy system for estimation. In essence, the fuzzy system will be designed with an appropriate structure and inputs so that its output provides a suitable estimate of the uncertainty.

By applying the control law τ_i defined in equation (18) along with the robustifying term τ_{ri} as formulated in equation (19), and through subsequent manipulation, the following components will be incorporated:

$$\dot{V}_{2i} = \sum_{i=1}^{i=n} \left[-k_{1i}x_{1i}^{2l} - k_{2i} \left(x_{2i} + k_{1i}x_{1i}^{2l-1} \right)^{2l} - \left(x_{2i} + k_{1i}x_{1i}^{2l-1} \right)^2 \hat{\rho}^2 \left(|x_{2i} + k_{1i}x_{1i}^{2l-1}| \hat{\rho} + \delta_{ri}e^{-\sigma_{ri}t} \right)^{-1} \right] \\ \sum_{i=1}^{i=n} \left[\left(x_{2i} + k_{1i}x_{1i}^{2l-1} \right) \left(\tilde{\theta}_i^\top \gamma_i + \epsilon_i \right) - (1/\alpha_i)\tilde{\rho}_i\dot{\hat{\rho}}_i - (1/\beta_i)\tilde{\theta}_i^\top \dot{\hat{\theta}}_i \right] \quad (43)$$

Lemma 1. For any non-negative variables a , b , and δ , the following inequality holds:

$$a^2b^2(ab + \delta)^{-1} \geq ab - \delta \quad (44)$$

By using the inequality given in Lemma 1, the following can be concluded:

$$- \left(x_{2i} + k_{1i}x_{1i}^{2l-1} \right)^2 \hat{\rho}^2 \left(|x_{2i} + k_{1i}x_{1i}^{2l-1}| \hat{\rho} + \delta_{ri}e^{-\sigma_{ri}t} \right)^{-1} \leq -|x_{2i} + k_{1i}x_{1i}^{2l-1}| \hat{\rho} + \delta_{ri}e^{-\sigma_{ri}t} \quad (45)$$

Proposition 1 establishes that the unmodeled dynamics ϵ_i can be bounded provided that the fuzzy system is appropriately structured. This conclusion, along with considering ρ_i as an upper bound for ϵ_i , facilitates the derivation of the following inequality for all x_1 and x_2 :

$$\left(x_{2i} + k_{1i}x_{1i}^{2l-1} \right) \epsilon_i \leq |x_{2i} + k_{1i}x_{1i}^{2l-1}| \rho_i \quad (46)$$

By considering the worst-case scenario modeled with the upper bound defined in equations (45) and (46), the exchange energy rate in the Lyapunov function, as defined in equation (43), can be rewritten as follows:

$$\dot{V}_{2i} \leq \sum_{i=1}^{i=n} \left[-k_{1i}x_{1i}^{2l} - k_{2i} \left(x_{2i} + k_{1i}x_{1i}^{2l-1} \right)^{2l} + \delta_{ri}e^{-\sigma_{ri}t} + \left(x_{2i} + k_{1i}x_{1i}^{2l-1} \right) \tilde{\theta}_i^\top \gamma_i + |x_{2i} + k_{1i}x_{1i}^{2l-1}| \tilde{\rho}_i \right] \\ - \sum_{i=1}^{i=n} \left[(1/\alpha_i)\tilde{\rho}_i\dot{\hat{\rho}}_i + (1/\beta_i)\tilde{\theta}_i^\top \dot{\hat{\theta}}_i \right] \quad (47)$$

By substituting (20) and (21) into (47) and performing some manipulation and simplification, the following can be achieved:

$$\dot{V}_{2i} \leq \sum_{i=1}^{i=n} \left[-k_{1i}x_{1i}^{2l} - k_{2i} \left(x_{2i} + k_{1i}x_{1i}^{2l-1} \right)^{2l} + \delta_{ri}e^{-\sigma_{ri}t} + (\sigma_{\theta i}/\beta_i)\tilde{\theta}_i^\top \tilde{\theta}_i + (\sigma_{\rho i}/\alpha_i)\tilde{\rho}_i\hat{\rho}_i \right] \quad (48)$$

Lemma 2 ([19]). For positive constants a , b , and c , and for any real variables z_1 and z_2 the subsequent inequality holds:

$$|z_1|^a |z_2|^b \leq \frac{a}{a+b} c |z_1|^{a+b} + \frac{b}{a+b} c^{-a/b} |z_2|^{a+b} \quad (49)$$

Lemma 3 ([20]). Supposing $c_1, c_2, \dots, c_n$ are all positive numbers and $0 < l < 1$, the following inequality remains valid:

$$(c_1 + c_2 + \dots + c_n)^l \leq c_1^l + c_2^l + \dots + c_n^l \quad (50)$$

In accordance with the worst-case scenario for the exchange energy change of the Lyapunov function, let's consider the following equality:

$$\sum_{j=1}^{j=M} \tilde{\theta}_{ij} \hat{\theta}_{ij} = \sum_{j=1}^{j=M} \tilde{\theta}_{ij} \theta_{ij} - \sum_{j=1}^{j=M} \tilde{\theta}_{ij}^2 \quad (51)$$

On the other hand, the following inequality holds:

$$-0.5(\tilde{\theta}_{ij}^2 + \theta_{ij}^2) \leq \tilde{\theta}_{ij} \theta_{ij} \leq 0.5(\tilde{\theta}_{ij}^2 + \theta_{ij}^2) \quad (52)$$

By using (52), the left hand side of equation (51) can be written as:

$$\sum_{j=1}^{j=M} \tilde{\theta}_{ij} \hat{\theta}_{ij} \leq 0.5 \sum_{j=1}^{j=M} \theta_{ij}^2 - 0.5 \sum_{j=1}^{j=M} \tilde{\theta}_{ij}^2 \quad (53)$$

Similarly, the following inequality can be concluded:

$$\tilde{\rho}_i \hat{\rho}_i \leq 0.5 \rho_i^2 - 0.5 \tilde{\rho}_i^2 \quad (54)$$

Upon substituting (53) and (54) into (48), the following is obtained:

$$\begin{aligned} \dot{V}_{2i} \leq \sum_{i=1}^{i=n} \left[-k_{1i}x_{1i}^{2l} - k_{2i} \left(x_{2i} + k_{1i}x_{1i}^{2l-1} \right)^{2l} - (\sigma_{\theta i}/2\beta_i)\tilde{\theta}_i^\top \tilde{\theta}_i - (\sigma_{\rho i}/2\alpha_i)\tilde{\rho}_i^2 \right] \\ + \sum_{i=1}^{i=n} \left[(\sigma_{\theta i}/2\beta_i)\theta_i^\top \theta_i + (\sigma_{\rho i}/2\alpha_i)\rho_i^2 + \delta_{ri}e^{-\sigma_{ri}t} \right] \end{aligned} \quad (55)$$

By using Lemma 2, and defining $0 < l < 1$, $a = 1 - l$, $b = l$, $z_1 = 1$, $z_2 = \tilde{\theta}_{ij}^2$, $c_1 = l^{\frac{l}{1-l}}$, one can conclude that:

$$(\tilde{\theta}_{ij}^2)^l \leq (1-l)c_1 + \tilde{\theta}_{ij}^2 \quad (56)$$

Then the following can be concluded:

$$-\tilde{\theta}_i^\top \tilde{\theta}_i \leq -(\tilde{\theta}_i^\top \tilde{\theta}_i)^l + (1-l)c \quad (57)$$

Similarly, one can reach to the following inequality:

$$-\tilde{\rho}_i^2 \leq -(\tilde{\rho}_i^2)^l + (1-l)c \quad (58)$$

Substituting (57) and (58) into (55) yields:

$$\begin{aligned} \dot{V}_{2i} \leq & \sum_{i=1}^{i=n} \left[-k_{1i}x_{1i}^{2l} - k_{2i} \left(x_{2i} + k_{1i}x_{1i}^{2l-1} \right)^{2l} - (\sigma_{\theta i}/2\beta_i)(\tilde{\theta}_i^\top \tilde{\theta}_i)^l - (\sigma_{\rho i}/2\alpha_i)(\tilde{\rho}_i^2)^l \right] \\ & + \sum_{i=1}^{i=n} \left[(\sigma_{\theta i}/2\beta_i)\theta_i^\top \theta_i + (\sigma_{\rho i}/2\alpha_i)\rho_i^2 + \delta_{ri}e^{-\sigma_{ri}t} + (1-l)c(\sigma_{\theta i}/2\beta_i + \sigma_{\rho i}/2\alpha_i) \right] \end{aligned} \quad (59)$$

Using Lemma 3 in (59) implies that

$$\begin{aligned} \dot{V}_{2i} \leq & \sum_{i=1}^{i=n} \left[- \left(k_{1i}x_{1i}^2 + k_{2i} \left(x_{2i} + k_{1i}x_{1i}^{2l-1} \right)^2 + (\sigma_{\theta i}/2\beta_i)(\tilde{\theta}_i^\top \tilde{\theta}_i) + (\sigma_{\rho i}/2\alpha_i)(\tilde{\rho}_i^2) \right)^l \right] \\ & + \sum_{i=1}^{i=n} \left[(\sigma_{\theta i}/2\beta_i)\theta_i^\top \theta_i + (\sigma_{\rho i}/2\alpha_i)\rho_i^2 + \delta_{ri}e^{-\sigma_{ri}t} + (1-l)c(\sigma_{\theta i}/2\beta_i + \sigma_{\rho i}/2\alpha_i) \right] \end{aligned} \quad (60)$$

To simplify the equation further, let's designate ϖ as the minimum among the four coefficients below, following the strategy of designing for the worst-case scenario.

$$\iota = \min \{ 2k_{1i}, 2k_{2i}, \sigma_{\theta i}, \sigma_{\rho i} \} \quad (61)$$

Regarding the remaining components within (61), let's consider them collectively as follows:

$$\varrho = \sum_{i=1}^{i=n} \left[(\sigma_{\theta i}/2\beta_i)\theta_i^\top \theta_i + (\sigma_{\rho i}/2\alpha_i)\rho_i^2 + \delta_{ri}e^{-\sigma_{ri}t} + (1-l)c(\sigma_{\theta i}/2\beta_i + \sigma_{\rho i}/2\alpha_i) \right] \quad (62)$$

By employing equations (61) and (62), the final expression for the Lyapunov exchange energy can be represented by the following mathematical equation:

$$\dot{V}_{2i} + \iota^l V_{2i}^l - \varrho \leq 0 \quad (63)$$

By comparing this to Proposition 2, the SGPFs of the main subsystem will be achieved. $\square$

4.2. Internal Behavior of the System

In the preceding subsection, the establishment of SGPFs was discussed. This establishment, as delineated by (9), ensures that if the initial condition falls within the task space funnel, the system's energy undergoes decay and can be confined within a predefined funnel-shaped energy boundary. Consequently, equation (63) asserts the boundedness of the components of the Lyapunov function, namely x_{1i} , x_{2i} , $\tilde{\theta}_i$, and $\tilde{\rho}_i$. This boundedness extends to the sliding surface S_i and its time derivative $\dot{S}_i$. The consequent boundedness of S_i and $\dot{S}_i$ implies the boundedness of ξ_i and $\dot{\xi}_i$. By selecting positive values for β_i , α_i , $\sigma_{\theta i}$, and $\sigma_{\rho i}$, it is inferred that $\hat{\theta}_i$ and $\hat{\rho}_i$ are also bounded in accordance with (20) and (21). Additionally, the fuzzy regressor γ_i , which is a function of x_{1i} and x_{2i} , is bounded when considering the normalized membership function condition alongside the completeness of the fuzzy system. Consequently, all signals on the right-hand side of (23) exhibit bounded behavior, thereby implying the boundedness of $\hat{\mu}_i$.

By designing the robot's desired trajectory to circumvent singularity points (i.e., ensuring the existence of the inverse of the Jacobian matrix), it can be demonstrated that the robot variable q_i and its time derivative also remain bounded.

Likewise, the boundedness of all components on the right-hand side of equations (18) and (19) indicates the boundedness of the robust controller τ_{ri} and the main controller τ_i . Utilizing Proposition 1 and substituting the bounded (18) and (19) into (5), it can be concluded that the acceleration signal $\ddot{q}_i$ is also bounded.

The boundedness of q_i implies the boundedness of p_i , considering the upper bound of the rotation matrix. This boundedness of p_i extends to the boundedness of x , y , and ψ of the robot. Similarly, it can be inferred that $\dot{p}_i$ is also bounded. Referring to (1), the boundedness of $\dot{p}_i$, along with the Assumptions 1 and 3, and considering the upper-boundedness of external disturbances in Kinematics, leads to the conclusion that the internal velocity signals u_r and r are also bounded. Referring to (2), the boundedness of $\dot{u}_r$ and $\dot{r}$ can be deduced under the assumption of upper-bounded external disturbances in Dynamics and lateral velocity, as well as other upper bounded signals in (2).

5. Simulation Results and Discussions

For the effectiveness assessment of the proposed method and comparison, MATLAB was employed.

5.1. Simulation A: Proposed Method

Here the developed scheme is simulated on the robot in the presence of important uncertainties. The dynamics of an under-actuated marine vehicle navigating in two dimensions were analyzed using the following parametric table [21]:

Table 1. Dynamic parameters of the mobile robot.

m_u (kg)	m_v (kg)	m_r (kg.m ²)	d_{ur} (kg/s)	d_{vr} (kg/s)	d_r (kg.m ² /s)
47.52	104.05	13.38	13.5	44.96	27.2

In the MATLAB setup, the sampling time was configured as $T_s = 0.001$ seconds, and the program was executed for $t_f = 1800$ seconds. The intended trajectory for the robot within the two-dimensional plane is expressed through the following set of equations:

$$x_d(t) = \begin{cases} 0.5t & \text{if } t < T_{c1}, \\ (Ra - Ts/60) \cos(\pi(t - T_{c1})/Wc) & \text{if } T_{c1} \leq t < T_{c2}, \\ -80 & \text{if } t \geq T_{c2}. \end{cases} \tag{64}$$

$$y_d(t) = \begin{cases} 0 & \text{if } t < T_{c1}, \\ (Ra - Ts/60) \sin(\pi(t - T_{c1})/Wc) & \text{if } T_{c1} \leq t < T_{c2}, \\ -0.4(t - T_{c2}) & \text{if } t \geq T_{c2}. \end{cases} \tag{65}$$

where T_{c1} , T_{c2} and Wc are defined as $T_{c1} = 200$, $T_{c2} = 1400$, $Wc = 400$. The trajectory consists of three distinct sub-trajectories: the first being a linear segment for $t < T_{c1}$, followed by a rotational trajectory with a time-varying radius for $T_{c1} \leq t < T_{c2}$, and concluding with another linear segment for $t \geq T_{c2}$. The parameters of the proposed control are illustrated in Table 2.

Table 2. Proposed control method specifications.

q_0	q_∞	ζ_i	β	k_{1i}	k_{2i}	δ_{ri}	σ_{ri}	β_1	β_2	$\sigma_{\theta i}$	α_i	$\sigma_{\rho i}$
25	0.25	0.8	0.06	8	39.5	0.1	0.2	20	8	0.0005	0.1	0.1

The transition between linear and rotational trajectories is deliberately designed to occur abruptly, resulting in a discontinuity where the first-time derivative does not exist. This intentional design choice serves to simulate scenarios where significant challenges arise, allowing for the effective evaluation of the system’s performance and stability. An instance of such a scenario could occur in the presence of underwater turbulence or during obstacle avoidance maneuvers.

When confronted with the erratic and irregular movement of water or the need to avoid collisions with the environment, the system must respond promptly based on data collected by safety sensors.

One approach to mitigating the adverse effects of sudden changes in trajectory is to rapidly adjust the desired trajectory. However, this adjustment may sacrifice smoothness, potentially necessitating points of non-differentiability at transition points, denoted as Tc_1 and Tc_2 . The primary controller must be adequately equipped to address this challenge and undergo thorough evaluation and testing to ensure its effectiveness.

The robot experiences external disturbances affecting its Kinematics, with the components represented by $\tau_{d_{kx}} = \tau_{d_{ky}} = \tau_{d_{kr}} = 0.05 \sin(0.1t - 10)$. In terms of dynamic uncertainties, signals for $\tau_{d_{ur}}$, $\tau_{d_{vr}}$, and τ_{d_r} are expressed as $\tau_{d_{ur}} = 10 \sin(0.1t - 10)$ and $\tau_{d_{vr}} = \tau_{d_r} = 0.2 \sin(0.1t - 10)$. The desired and actual trajectories of the robot, along with a schematic representation of the robot indicating its orientation, are depicted in Figure 3. According to this plot, the robot initially starts with a heading angle of $\psi_0 = \pi/6$, and its starting position is $p_0 = \begin{bmatrix} -1.2 & -8 \end{bmatrix}^T m$. The heading angle is separately depicted in Figure 4. The change in the heading angle and its oscillatory behavior enable the robot to generate the necessary force for the engine to execute the maneuvers required to track the trajectory, even as unwanted disturbances affect the robot, attempting to divert it from the desired path. The robot successfully tracks the desired trajectory while reducing its error within a finite time frame. The robot's task space comprises four regions: Z_1 , Z_2 , Z_3 , and Z_4 , representing scenarios with varying water current speeds that may endanger system stability. The water current speed in each zone is determined by the following equations:

- $Z_1 : v_{c1} = \begin{bmatrix} -0.1 \cos(0.1t) & +0.2 \sin(0.1t) \end{bmatrix}^T$;
- $Z_2 : v_{c2} = \begin{bmatrix} +0.2 \cos(0.1t) & -0.1 \sin(0.1t) \end{bmatrix}^T$;
- $Z_3 : v_{c3} = \begin{bmatrix} +0.4 \cos(0.1t) & +0.2 \sin(0.1t) \end{bmatrix}^T$;
- $Z_4 : v_{c4} = \begin{bmatrix} +0.3 \cos(0.1t) & -0.1 \sin(0.1t) \end{bmatrix}^T$.

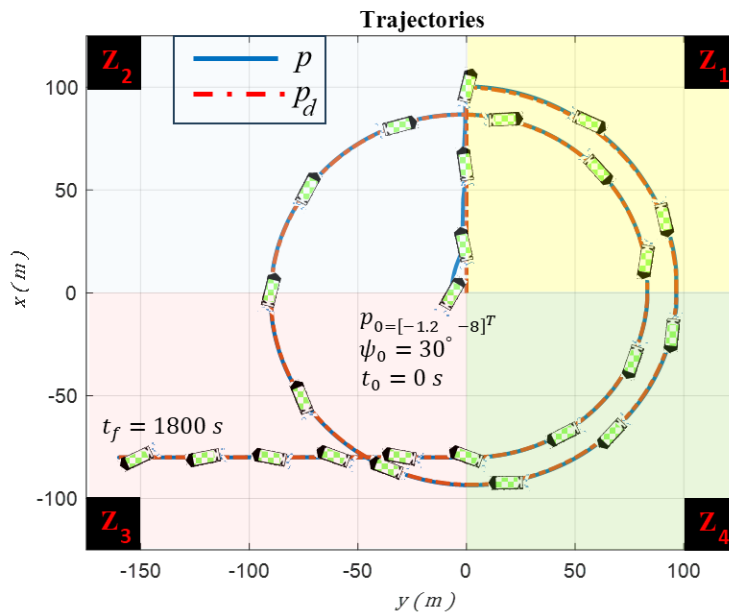


Figure 3. Trajectories of the mobile robot.

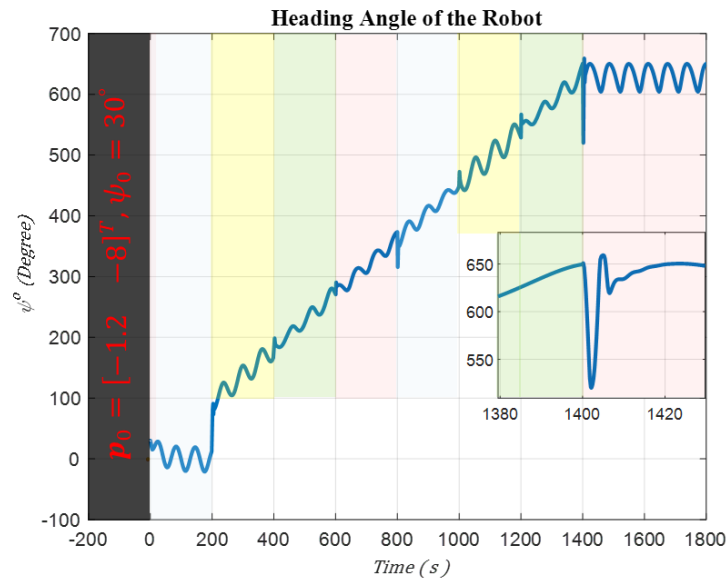


Figure 4. Heading angle of the mobile robot.

For better clarity, Figures 3 and 4 are color-coded based on entry into each zone, making it easier to observe the behavior of the system. The tracking errors of the robot along the x and y directions, as well as the safe area, are illustrated in Figure 5.

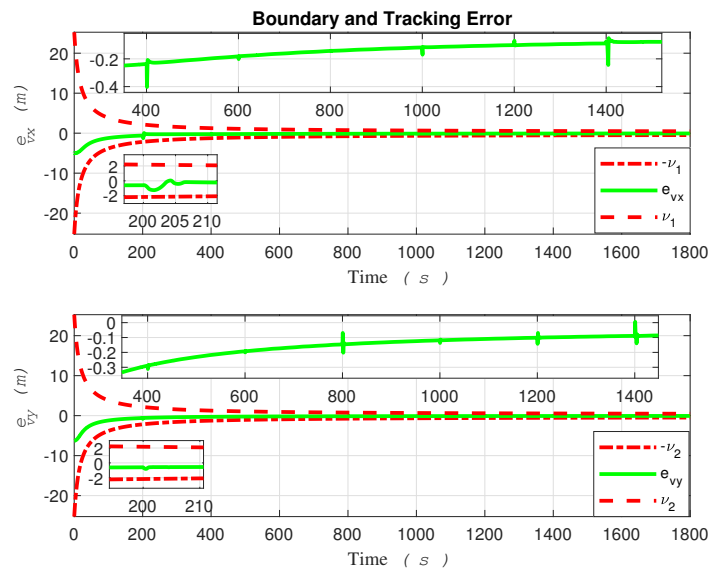


Figure 5. Error evolution in the proposed funnel control.

Thanks to the robust funnel control strategy proposed in this paper, the tracking error consistently stays within the boundaries of the funnel shape. However, the robot must commence its movement within the funnel for this strategy to take effect.

Despite encountering abrupt changes in the desired path at times $t = 200$ and $t = 1400$ seconds, as well as sudden alterations in the amplitude and direction of the water current speeds at the boundaries, which attempted to push the error beyond the funnel boundaries, the controller successfully coerced the system back inside the safe set, as demonstrated in the Figure 5. Ensuring that the tracking error remains within the funnel boundary is crucial for guaranteeing the safety of the controller. By

observing the performance of the robot as depicted in Figures 3–5, one can conclude that the controller effectively compensates for external disturbances affecting the robot, as well as various changes in water current speed that pose a threat to the robot's stability.

The control signal τ_i is plotted in Figure 6 and has the potential to be directly applied to the robot's actuators. Plotting signals u_r and r can significantly aid in understanding and ensuring the internal stability of the closed-loop system. These two signals are displayed in Figure 7. They demonstrate the stable behavior of the internal dynamics of the robot, even in the presence of significant sources of uncertainty.

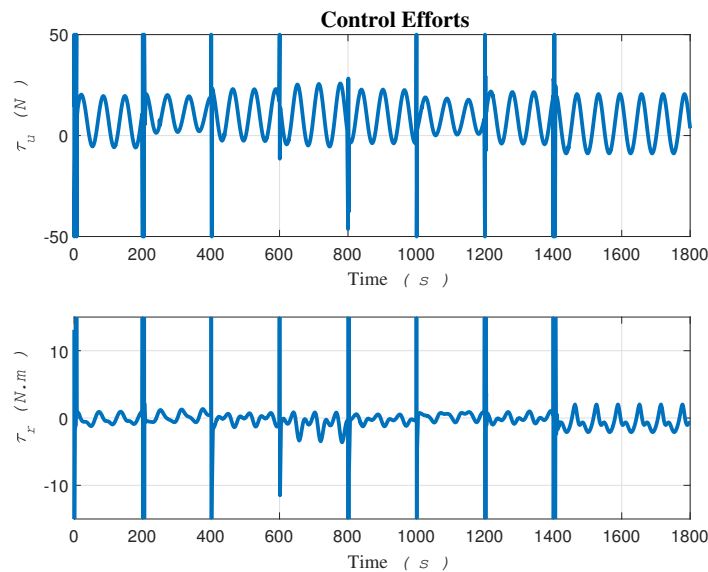


Figure 6. Control signals exerting to robot's actuators.

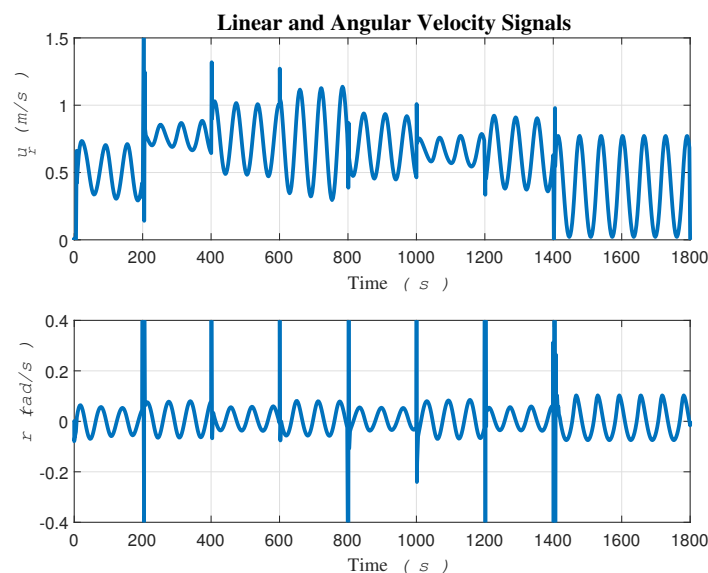


Figure 7. Internal behavior of the system.

The adaptation of fuzzy parameter estimation $\hat{\theta}_{ij}$ is illustrated in Figure 8.

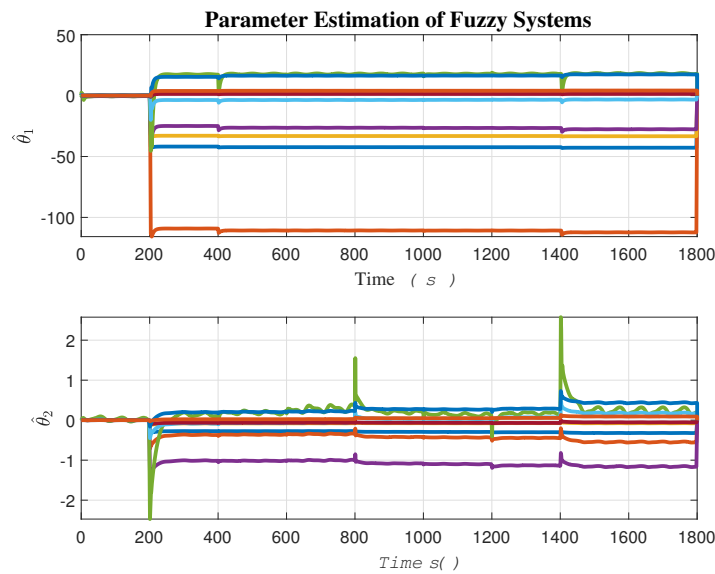


Figure 8. Adaption parameters of the fuzzy estimators.

Considering three membership functions for each of the two fuzzy inputs, x_1 and x_2 , nine fuzzy rules, which correspond to the number of parameters, form the rule base of the estimator along each task space direction, x and y . Two fuzzy estimators are required: one for compensating uncertainties along the x direction and the other along the y direction. The uncertainty upper bound estimation, represented as $\hat{\rho}_i$, is depicted in Figure 9.

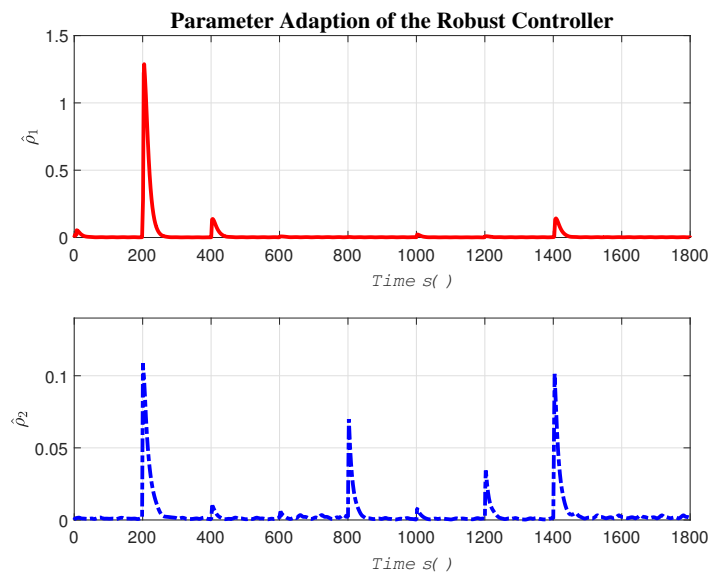


Figure 9. Adaption parameter of the robust control.

The rapid dynamical behavior of certain fuzzy parameters $\hat{\theta}_i$ and the robust term $\hat{\rho}_i$ enables the controller to react quickly and effectively to external disturbances at both the Kinematics and Dynamics levels.

5.2. Simulation B: Gradient-Descent Based Control

For comparison purposes, the methodology described in [15] has been implemented under identical simulation conditions. This facilitates a clearer demonstration of the improvement and

novelty of the proposed method as an extended version of the previous work. For further information regarding the controller and its structure, please refer to [15].

The tracking errors, control efforts exerted on robot actuators, and velocity control laws provided by the outer loop are illustrated in Figures 10, 11, and 12, respectively.

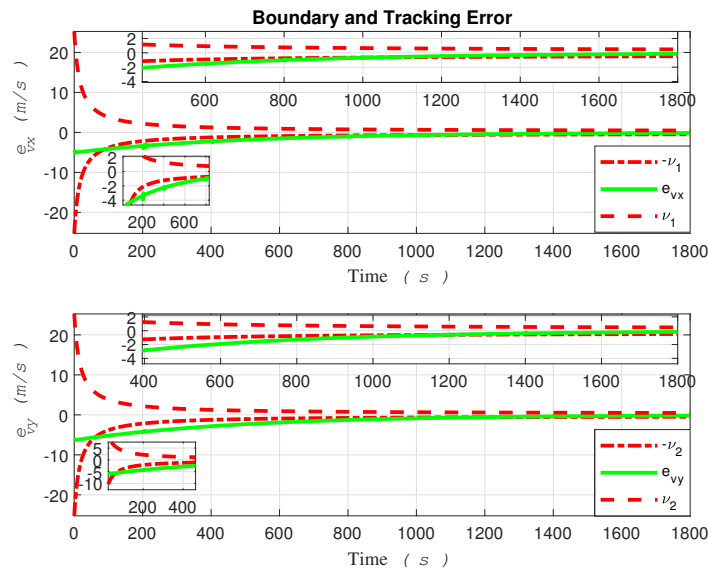


Figure 10. Error evolution in the gradient-descent control [15].

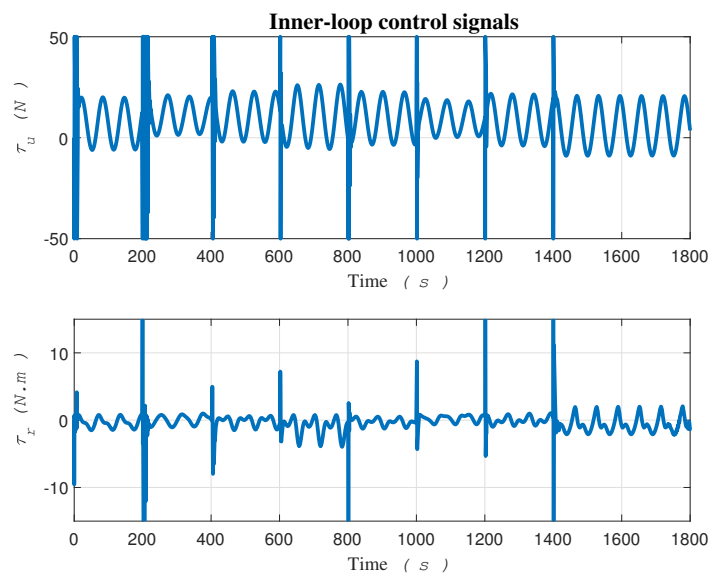


Figure 11. Inner-loop control signals in the gradient-descent approach [15].

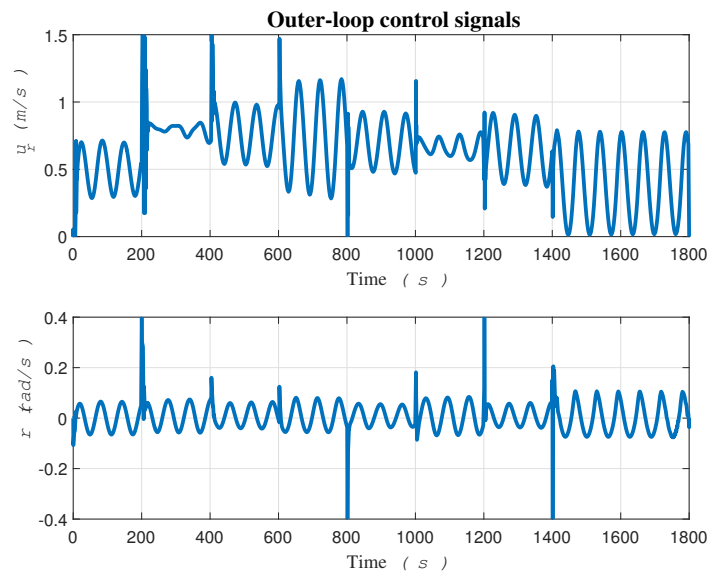


Figure 12. Outer-loop control signals in the gradient-descent approach [15].

To facilitate comparison, Figure 13 presents the tracking error norms for both the proposed method and the gradient descent-based controller.

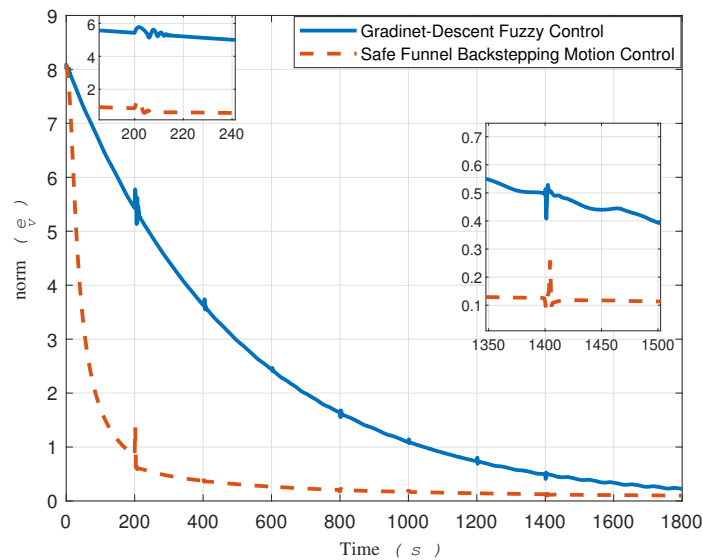


Figure 13. Comparison of the two norm errors of the proposed and gradient-descent based control.

Upon analyzing this signal, it becomes evident that the proposed method significantly outperforms the gradient descent fuzzy estimator method. According to Figure 10, it is evident that the tracking error may not necessarily have remained within the safe area. Consequently, it cannot be deemed as a safe controller.

6. Conclusions

In this paper, we introduced a novel funnel backstepping controller designed to ensure the safe navigation of an underactuated marine robot, with formal stability guarantees incorporated into its design. We have tackled major challenges from ocean conditions, directly affecting the robot's stability. The controller's structure, coupled with its fuzzy estimator, is simplistic and integrates straightforward

sensory data feedback mechanisms. These attributes render it highly suitable for practical implementation in real-world settings. Our simulation results and comparative analyses demonstrate the effectiveness, robustness, and safety enhancements achieved by the robot, as evidenced by the visualization of internal signal plots. These simulations encompassed a spectrum of challenging scenarios, including time-varying external disturbances, unmodeled dynamics, fluctuating water currents, and lateral velocities. The proposed safe backstepping control mechanism demonstrated robustness in navigating intricate marine conditions, supported by comprehensive comparative analysis.

Acknowledgments: The authors acknowledge the financial support of the Foundation for Science and Technology (FCT/MCTES) in the framework of the Associated Laboratory - Advanced Production and Intelligent Systems (AL ARISE, ref. LA/P/0112/2020), the R&D Unit SYSTEC (Base UIDB/00147/2020 and Programmatic UIDP/00147/2020 funds), and project RELIABLE - Advances in control design methodologies for safety critical systems applied to robotics (ref. PTDC/EEI-AUT/3522/2020) both funded by national funds through the FCT/MCTES (PIDDAC). The first author was supported by a Ph.D. Scholarship, grant 2022.11470.BD from FCT, Portugal.

Abbreviations

The following abbreviations are used in this manuscript:

SMC	Sliding Mode Control
TSMC	Terminal Sliding Mode Control
FC	Funnel Control
CBF	Control Barrier Functions
SGPFS	Semi-Globally Practically Finite-Time Stability

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