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Article

The Muon $g-2$ in a Regularized Electrodynamics

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Abstract: The present paper reports exact results for the muon self-energy and anomalous g-factor. A unique to the muon cut-off terms that remove the radial singularity in the system and regularize the corresponding electrodynamics are presented. Equations of motion and a transcendental equation satisfied by the muon anomalous g-factor are derived, with solution $a_\mu = 0.0011659201231(18)$. The obtained value matches the latest experimental one found in the literature to about 0.46 ppb ruling out a possible tension between theory and experiment.

Keywords: muon; self-energy; anomalous magnetic dipole moment; regularization; electrodynamics

1. Introduction

The study of the electron's anomalous magnetic moment have played an essential role in the development of quantum theory [1–4]. Its successful theoretical description led to the expectation that the followed calculation of the anomalous component in the muon's intrinsic magnetic moment will strengthen and crystallize the established knowledge [5–10]. Over the decades, however, with the improvement of the experimental setup and the consequent collection of more data from highly precise measurements the gap between the measured and calculated values not only remained but thickened [11–16]. The difference between the most recent experimental average [17,18] and the most recent consensus on the average theoretical value [19] is about 2.49 ppb, which is significantly larger than the relevant uncertainty. It is expected that the analysis of more experimental data will only result in a negligible variation in the measured average value reaffirming the obtained gap. As a result, a variety of theoretical approaches to the calculation of the corresponding anomalous component were proposed, see Ref. [20] and the references therein.

In the light of the apparent discrepancy many efforts to revise and improve the hadronic vacuum polarization corrections [21–25] and the hadronic light-by-light scattering one [24–28] have been considered, see also the revision of Dyson-Schwinger approach [29]. Despite the increasing number of fitting parameters and relevant ambiguity in the choice of observables needed to calculate these contributions, it is believed that the used quantum field theory approaches have the prospect to reduce the obtained tension.

On the other hand, the utility of refining the regularization technique is also considered. Any progress in the regularization of electromagnetic self-interaction is expected to have a significant contribution to the resolution of named tension. Over the years the application of regularization method in electrodynamics and field theory in general has demonstrated great effectiveness [30–40]. The regularization of self-interaction implies minimal number of effective parameters and leads to high precision results, with Yukawa cut-offs [41,42] being the most prominent tools for removing radial singularities at a microscopic scale in both classical and quantum field approaches. Recently a direct regularization scheme with Yukawa terms describing a massive off shell photons and no free parameters was successfully applied to quantify the electron $g-2$ value and the associated self-interaction [40]. To the best of our knowledge calculations of the muon $g-2$ value based on the direct regularization of its electromagnetic self-interaction via Yukawa coupling have not been undertaken yet.

The present paper introduce a study that implements the regularization technique proposed in Ref. [40] with the aim to quantify the self-energy and anomalous component in the muon's magnetic

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moment. The applied approach represents a regularized electrodynamics of non-composite particles and conjugate to the quantum theory beyond the corresponding principle. Accordingly, exact results for the muon's self-energy, anomalous g-factor and all intrinsic characteristics underlying the dynamics of its self-interaction are reported. Improved accuracy in the calculation of the muon anomalous g-factor is obtained (0.46 ppb), overcoming the existing gap between the average value predicted by the quantum theory [19] and the measured one [17,18]. An essential outcome of the obtained accuracy is an exact bound on the sum of the muon and electron neutrinos rest masses.

The rest of the report is structured as follows. Section 2 briefly introduce the used method starting with the notation and definition of all relevant physical quantities. Section 3 presents the obtained results with all derived equations and their solutions. Section 4 discusses and summarizes the obtained results.

2. Theoretical Framework

In this section we set-out the mathematical notation of all needful physical quantities and ab-initio relations. We find it convenient to restrict all representations within the mathematical framework of three-dimensional vector formalism.

2.1. Generalities

Consider a free muon with rest mass and electric charge denoted by m_μ and $\bar{e} = -e$, respectively, where e is the elementary charge. Let $\mathbf{R}$ be the muon's rest frame of reference and $r_{c\mu} = \alpha\bar{\lambda}_{c\mu}$ be its electromagnetic radius in $\mathbf{R}$, where α and $\bar{\lambda}_{c\mu}$ are the fine structure constant and associated reduced Compton wavelength, respectively. Let $\mathbf{r}_\mu$ be the intrinsic field vector associated to the muon, with magnitude r_μ, and $\tilde{u}_\mu$ be the magnitude of the tangential velocity $\tilde{\mathbf{u}}_\mu$ related to its rotation about the origin of $\mathbf{R}$ in the plane perpendicular to the muon's relative velocity $\mathbf{u}_\mu = u_\mu\boldsymbol{\kappa}$ defined with respect to an observer with frame of reference $\mathbf{O}$, where $\boldsymbol{\kappa}$ is the respective unit vector (see Figure 1). Since the system is closed, we have the constraint $\mathbf{u}_\mu = \tilde{\mathbf{u}}_\mu$. Furthermore, the oscillation of $\mathbf{r}_\mu$ is characterized by an angular velocity $\boldsymbol{\omega}_\mu$, with magnitude $\omega_\mu = \tilde{u}_\mu r_\mu^{-1}$ representing the angular frequency of the corresponding circularly polarized field. For comparison with the classical representation of the electron's intrinsic dynamics the reader may consult Ref. [40]. Within the applied approach the quantities r_μ and $\tilde{u}_\mu$ are conjugate and satisfy

$$r_\mu\tilde{u}_\mu = \bar{\lambda}_{c\mu}c, \tag{1}$$

where c is the light speed in vacuum.

The charge ρ_e and mass ρ_{m_μ} densities satisfying $\rho_e\rho_{m_\mu}^{-1} = em_\mu^{-1}$ are defined within the spherically symmetric spatial domain $\Omega_{c\mu} \in \mathbb{R}^3$, with radius $r_{c\mu}$, boundary $\partial\Omega_{c\mu}$ and volume $V_{c\mu}$. Moreover, ρ_{M_μ} is the muon's effective mass density and $M_\mu = m_\mu(1 + a_\mu)$ is the corresponding effective rest mass defined in $\Omega_{c\mu}$, where a_μ is the muon's anomalous g-factor. Here, $\rho_{M_\mu} = \rho_{M_\mu}(r)$ is a smooth function of the radial parameter r, with $r \in (0, +\infty)$ and $\rho_{M_\mu} > \rho_{m_\mu}$ for all r. Note that if $\nexists\, e$, then $\rho_{M_\mu} = \rho_{m_\mu}$.

The inherent dynamics of $\mathbf{r}_\mu$ underpin the occurrence of intrinsic magnetic moment $\boldsymbol{\mu}_\mu = -\frac{1}{2}g_\mu\mu_\mu\boldsymbol{\kappa}$, where $g_\mu = 2(1 + a_\mu)$ is the g-factor of the muon and $\mu_\mu = e\hbar(2m_\mu)^{-1}$ is the corresponding magneton. We further have

$$g_\mu = \frac{2}{V_{c\mu}}\int_{\Omega_{c\mu}} G_\mu \mathrm{d}v, \qquad G_\mu = \frac{e\rho_{M_\mu}}{m_\mu\rho_e}. \tag{2}$$

In the case shown in Figure 1 the field associated to the muon reads $\mathbf{\Phi}_\mu(\mathrm{x}) = A_\mu\Phi_\mu(\mathrm{x})\mathbf{n}_\mu$, where the amplitude $A_\mu \equiv r_\mu$, $\mathbf{n}_\mu \in \mathbb{C}^3$ is the field's unit vector and the phase factor $\Phi_\mu(\mathrm{x})$ satisfies the Klein-Gordon equation, with $\mathrm{x} \in \mathbb{R}^{1,3}$ denoting the four-vector of the origin of $\mathbf{R}$. In particular, we have $\mathbf{r}_\mu = \sqrt{2}\mathrm{Re}\{\mathbf{\Phi}_\mu(\mathrm{x} = ct)\}$, where $\mathbf{n}_\mu = (\frac{1}{\sqrt{2}}, \pm\frac{i}{\sqrt{2}}, 0)$.

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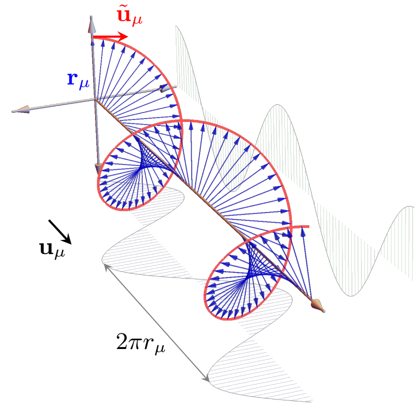

Figure 1. Sketch of a left circularly polarized spatial wave representing a free self-interacting muon. The corresponding field vector $\mathbf{r}_\mu$ (blue arrows) depicts a helix as the particle moves relative to an observer with velocity $\mathbf{u}_\mu$.

2.2. Electromagnetic Field Regularization

Within the considered approach the classical representation of the electromagnetic field coupled to the muon is time independent. Accordingly, for an observer in **O** the electromagnetic field potentials are not retarded and the Lorenz gauge is trivially satisfied. Moreover, the radial singularity in the electromagnetic field is removed via a regularization defined by two cut-off Yukawa terms, with screening constants defined in accordance to the most probable decay of the free muon.

In particular, with respect to an observer in **O** the electromagnetic field potentials inherent to the muon read

$$\varphi_\mu(r) = \gamma_\mu \eta_\mu \psi_\mu(r), \qquad \mathbf{A}_\mu(r) = 2\gamma_\mu \frac{\mathbf{u}_\mu}{c^2} \psi_\mu(r), \tag{3}$$

where γ_μ is the Lorentz factor, $\eta_\mu = 1 + u_\mu^2 c^{-2}$ and

$$\psi_\mu(r) = \frac{\bar{e}}{4\pi\varepsilon_o r}\left(2 - e^{-\chi_\mu r} - e^{-\tilde{\chi}_\mu r}\right), \tag{4}$$

ε_o is the electric constant. The screening constant $\tilde{\chi}_\mu$ is defined in accordance to the most probable decay of a self-interacting muon $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$. We have

$$\chi_\mu = \frac{\gamma_\mu m_\mu c}{(1 + a_e)h}, \qquad \tilde{\chi}_\mu = \frac{\gamma_\mu (m_e + m_\nu)c}{(1 + a_\mu)h}, \tag{5}$$

where m_e and a_e denote the electron's rest mass and anomalous g-factor, respectively. Here, h is Planck's constant, $m_\nu = m_{\nu_\mu} + m_{\bar{\nu}_e}$, where m_{ν_μ} is the muon neutrino rest mass and $m_{\bar{\nu}_e}$ is the electron anti-neutrino rest mass. Therefore, we have $m_\nu > 0$ for all $r \in (0, +\infty)$. The exact value of respective neutrinos' rest masses is unknown leaving the mass term m_ν as a model parameter. Nevertheless, the value of m_ν is bound by the value of a_μ and therefore it is unique within the used approach.

The classical field equation satisfied by the function in Equation (4) reads

$$\Delta_r \psi_\mu(r) - \chi_\mu^2 \phi_\mu(r) - \tilde{\chi}_\mu^2 \tilde{\phi}_\mu(r) = 0, \tag{6}$$

where Δ_r represents the radial Laplace operator in spherical symmetry, ϕ_μ and $\tilde{\phi}_\mu$ are the cut-off terms implying the following boundary conditions

$$\psi_\mu(r) = \begin{cases} 0, & r \to \infty, \quad u_\mu < c, \\ (\chi_\mu + \tilde{\chi}_\mu)\frac{\bar{e}}{4\pi\varepsilon_o}, & r \to 0, \quad u_\mu < c. \end{cases}$$

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We would like to point out, furthermore, that the two Yukawa terms in Equation (4) do not represent an on shell massive photons [43]. For all $r \in (0, +\infty)$, both χ_μ and $\tilde{\chi}_\mu$ represent the wave numbers of effectively massive off shell photons coupled to the muon, with total effective mass $m_\mu(1 + a_e)^{-1}$ and $(m_e + m_\nu)(1 + a_\mu)^{-1}$, respectively. Both massless photon terms with Yukawa potentials reduced to Coulomb ones (see eq. (4)) are implicitly accounted for in Equation (6).

2.3. Electromagnetic Field Energy Regularization

The energy of the electromagnetic field in the considered system, $W_\mu(r)$, is also regularized. In general, integrating the corresponding energy density $\varepsilon_o|\nabla\varphi_\mu(r)|^2$ over $\mathbb{R}^3$, we obtain

$$\begin{aligned} W_\mu(r) = C_\mu \frac{r_{c\mu}}{2r}\Bigg(& 8\big(e^{2(\chi_\mu+\tilde{\chi}_\mu)r} - e^{(2\chi_\mu+\tilde{\chi}_\mu)r} - e^{(\chi_\mu+2\tilde{\chi}_\mu)r}\big) \\ & + (2+\chi_\mu r)e^{2\tilde{\chi}_\mu r} + (2+\tilde{\chi}_\mu r)e^{2\chi_\mu r} \\ & + 4\frac{(\chi_\mu+\tilde{\chi}_\mu+\chi_\mu\tilde{\chi}_\mu r)}{\chi_\mu+\tilde{\chi}_\mu}\Bigg)e^{-2(\chi_\mu+\tilde{\chi}_\mu)r}, \end{aligned}$$

where $C_\mu = \gamma_\mu^2\eta_\mu^2 m_\mu c^2$. At the origin of $\mathbf{R}$ and for $u_\mu < c$ the electromagnetic field energy in the considered system is finite. Thus, we have

$$\lim_{r\to 0} W_\mu(r) = C_\mu r_{c\mu}\frac{\chi_\mu^2 + 6\chi_\mu\tilde{\chi}_\mu + \tilde{\chi}_\mu^2}{2(\chi_\mu + \tilde{\chi}_\mu)}. \tag{7}$$

In contrast to the non-regularized electrodynamics, here for $r \to 0$ the discussed electromagnetic field energy vanish when the particle's rest mass is negligible. In other words, the charge screening (Yukawa cloud) is nearly complete making the electrically charged particle to appear as electrically neutral.

3. Results

3.1. Self-Energy

By analogy to the case of self-interacting electron (see Ref. [40]), the classical Hamiltonian describing a self-interacting muon do not depend explicitly on time. We have

$$H_\mu = \gamma_\mu m_\mu c^2 + \Sigma_\mu, \tag{8}$$

where Σ_μ is the self-energy term. The latter is not a potential energy of a gradient field and equals the spatial average over the domain $\Omega_{c\mu}$ of the interaction energy $\bar{e}\varphi_\mu$. In particular, with respect to Equation (3), we have the representation

$$\Sigma_\mu = \gamma_\mu c^2 \int_{\Omega_{c\mu}} (\rho_{M_\mu} - \rho_{m_\mu})\mathrm{dv}, \tag{9}$$

where the effective mass density reads

$$\begin{aligned} \rho_{M_\mu} &= \rho_{m_\mu}\left(1 + \eta_\mu\frac{r_{c\mu}}{r}\left(2 - e^{-\chi_\mu r} - e^{-\tilde{\chi}_\mu r}\right)\right) \\ &= \rho_{m_\mu}\left(1 + \frac{\eta_\mu \bar{e}}{m_\mu c^2}\psi_\mu\right). \end{aligned} \tag{10}$$

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Table 1. Theoretical and experimental (EXP) data for the muon's anomalous g-factor (second column). The regularized electrodynamics (RED) and quantum field theory (QFT) results are given in the second and third rows, respectively. Fourth row represents the most recent experimental average value. The value of mass ratio ξ_μ follows from Equation (14). For additional details see Figure 2.

	a_μ	ξ_μ	Ref.
RED	0.0011659201231(18)	0.0048363318829445(30)	Equation (13)
QFT	0.00116591810(43)	–	[19]
EXP	0.00116592059(22)	–	[17,18]

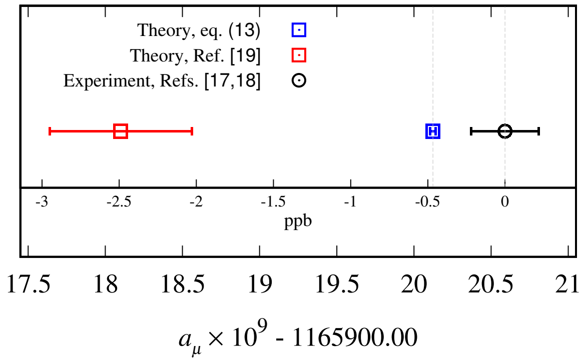

Figure 2. Comparison between the most recent experimental result (black circle) for the muon's anomalous g-factor and its value obtained from Equation (13) (blue square). In addition the latest result predicted by the quantum theory (red square) is also shown. The depicted data is further provided in Table 1.

3.2. The Hamiltonian Density

The information about the muon's intrinsic dynamics is embedded in the Hamiltonian density $\mathcal{H}_\mu$ associated to Equation (8). Taking into account Equation (1) for $r \equiv r_\mu$ we get

$$\mathcal{H}_\mu = c^2 \rho_{m_\mu} \left(\gamma_\mu + \frac{\alpha}{m_\mu c} \mathcal{P}_\mu \right),$$

where

$$\mathcal{P}_\mu = \gamma_\mu \eta_\mu m_\mu \tilde{u}_\mu \left(2 - e^{-\chi_\mu r_\mu} - e^{-\tilde{\chi}_\mu r_\mu}\right)$$

is the corresponding generalized momentum. Accordingly, we have the equations of motion

$$\tilde{u}_\mu = \int_{\Omega_{c\mu}} \frac{\partial \mathcal{H}_\mu}{\partial \mathcal{P}_\mu} \mathrm{dv}, \qquad \dot{\mathcal{P}}_\mu = 0$$

and subsequently the exact values

$$\tilde{u}_\mu = \alpha c, \quad \eta_\mu = 1 + \alpha^2, \quad \gamma_\mu^{-1} = \sqrt{1 - \alpha^2}. \tag{11}$$

In contrast to the relative velocity, from Equation (11) there follows that the magnitude of the tangential velocity is scale invariant.

3.3. Effective Mass-Energy Equivalence

Taking into account Equation (2), from Equations (9) and (10) we obtain $\Sigma_\mu = a_\mu \gamma_\mu m_\mu c^2$. As a result, from Equation (8) we get the effective relativistic energy

$$\mathcal{E}_\mu = \gamma_\mu M_\mu c^2, \qquad \text{with} \qquad M_\mu = \frac{1}{2} g_\mu m_\mu. \tag{12}$$

Therefore, as a result of the self-interaction the total energy of a free muon is $\gamma_\mu(1 + a_\mu)$ times higher than its rest energy and the system's total mass will be $(1 + a_\mu)$ times higher than the muon's rest mass.

3.4. The Anomalous g-Factor

In addition to the Standard Model calculations [14,19], the implemented regularized electrodynamics yields a single-parametric transcendental equation for the calculation of the muon's anomalous g-factor. Accounting for Equations (2), (10) and (11), we obtain

$$a_\mu = a_e + 3\eta_\mu \left(\frac{1}{2} - \left(\frac{1 - e^{-\frac{\alpha \gamma_\mu \xi_\mu}{2\pi(1+a_\mu)}} \left(1 + \frac{\alpha \gamma_\mu \xi_\mu}{2\pi(1+a_\mu)}\right)}{\left(\frac{\alpha \gamma_\mu \xi_\mu}{2\pi(1+a_\mu)}\right)^2} \right) \right), \tag{13}$$

where

$$\xi_\mu = \frac{m_e + m_\nu}{m_\mu}. \tag{14}$$

The value of a_μ calculated from Equation (13) is given in the second row of Table 1, where the values of a_e and α are taken from Ref. [40]. The obtained accuracy with respect to the most recent experimental measurements [17] is about 0.46 ppb, see Figure 2. On the same figure, a comparison with the most recent agreement on the value of a_μ predicted by the Standard Model is also depicted. The value of mass ratio in Equation (14) is given in the third column in Table 1. We have $m_\nu = 3.58686 \times 10^{-38}$ kg, where the values of electron's and muon's rest masses are taken from NIST [44]. This result suggest that the sum of the muon neutrino and electron anti-neutrino rest energies is approximately 0.02012 eV, which is consistent with the reported upper bound on the sum of the three flavor neutrino rest energies of about 0.120 eV (see Ref. [45]). Moreover, it suggests that the electron anti-neutrino mass satisfies the inequality $m_{\bar{\nu}_e} < 3.58686 \times 10^{-38}$ kg. This bound is approximately 40 times lower than the one set by KATRIN collaboration [46], see also Ref. [47].

3.5. Electromagnetic Field Energy

The total energy of the muon given in Equation (12) is only a fraction of the considered system's energy that accounts for the electromagnetic field energy associated to the muon.

In the non-regularized electrodynamics the electromagnetic field energy associated to the three flavors of charged leptons does not depend on their rest mass and is not defined at the origin of **R**. Consequently the corresponding energy density is quantitatively indistinguishable with respect to the flavor state of these particles.

Here, as a result of the applied regularization the electromagnetic field energy and its density are flavor dependent. The larger the lepton's rest mass the higher corresponding electromagnetic field energy. In the considered case, substituting the obtained from Equation (13) values of m_ν and a_μ in Equation (7), we obtain $W_\mu(r \to 0) = 4.673 \times 10^{21}$ eV, which is 43.783×10^3 times the value of electromagnetic field energy associated to the electron at the same limit (see Ref. [40]) and about 4.42274×10^{13} times the muon's rest energy.

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4. Summary and Conclusions

The present paper reports on the most recent progress in the application of the regularization technique in electrodynamics of electrically charged non-composite particles thus uncovering key aspects from the elusive interrelationship between the classical and quantum theory on a microscopic level. The effectiveness of regularization method is demonstrated by calculating exactly the self-energy and anomalous g-factor of the muon. The used regularization implies two Yukawa cut-off terms with unique to the muon decay screening constants (see Equation (5)) that remove the radial singularity in the classical representation of the off shell electromagnetic field governing the muon's self-interaction. Accordingly, exact solutions to the system's equations of motion are derived.

In particular, the muon's self-energy is calculated exactly showing the genuine contribution of the corresponding electromagnetic self-interaction into the muon's total mass and energy, see Equation (12). In addition, the muon's anomalous g-factor is calculated with high accuracy, improving the one obtained from the latest quantum theory calculations, see Figure 2 and Table 1. The obtained accuracy implies that the electromagnetic field contribution into the muon anomalous magnetic moment is significantly larger than previously evaluated. The main contribution results from massive off shell photons with wave vectors given in Equation (5), propagators corresponding to both Yukawa terms given in Equation (4) and interacting with the muon as given in Equation (9). The effective mass of these photons further implies that the muon and electron neutrinos have a rest mass, with upper bound on their sum equal to 3.58686×10^{-38} kg, see Equations (13) and (14). Moreover, it points out that the contribution of both neutrinos' rest masses into the electron-muon mass ratio should be taken by the quantum theory approach in order to improve the relevant result.

The used regularization technique can be applied to quantify the intrinsic dynamics of the tau lepton and to fix the range of values of the corresponding anomalous g-factor determined by the multiplicity of branching fractions. It is, furthermore, applicable to non-composite particles with constant electric charge. Equation (12) points out that the effective rest energy of a self-interacting proton should be at least 2.79 times its rest energy. Essentially, it shows that the total rest energy of a multi-particle system is always greater than the anticipated value obtained by accounting for only the rest masses of constituent particles.

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References

1. Vogel, M. The anomalous magnetic moment of the electron. *Contemp. Phys.* **2009**, *50*, 437–452. https://doi.org/10.1080/00107510902765239.
2. Laporta, S.; Remiddi, E. Analytic QED Calculations of the Anomalous Magnetic Moment of the Electron. In *Advanced Series on Directions in High Energy Physics*; WORLD SCIENTIFIC, 2009; Vol. 20, pp. 119–156. https://doi.org/10.1142/9789814271844_0004.
3. Aoyama, T.; Kinoshita, T.; Nio, M. Theory of the Anomalous Magnetic Moment of the Electron. *Atoms* **2019**, *7*, 28. https://doi.org/10.3390/atoms7010028.
4. Fan, X.; Myers, T.; Sukra, B.; Gabrielse, G. Measurement of the Electron Magnetic Moment. *Phys. Rev. Lett.* **2023**, *130*, 071801. https://doi.org/10.1103/PhysRevLett.130.071801.
5. Farley, F. The 47 years of muon g-2. *Prog. Part. Nucl. Phys.* **2004**, *52*, 1–83. https://doi.org/10.1016/j.ppnp.2003.09.004.

6. Czarnecki, A.; Jentschura, U.D.; Pachucki, K.; Yerokhin, V.A. Anomalous magnetic moments of free and bound leptons. *Can. J. Phys.* **2006**, *84*, 453–462. https://doi.org/10.1139/p06-021.
7. Miller, J.P.; De Rafael, E.; Lee Roberts, B. Muon (g-2): experiment and theory. *Rep. Prog. Phys.* **2007**, *70*, R03. https://doi.org/10.1088/0034-4885/70/5/R03.
8. Jegerlehner, F.; Nyffeler, A. The muon g-2. *Phys. Rep.* **2009**, *477*, 1–110. https://doi.org/10.1016/j.physrep.2009.04.003.
9. Jegerlehner, F. *The Anomalous Magnetic Moment of the Muon*; Vol. 274, *Springer Tracts in Modern Physics*, Springer International Publishing: Cham, 2017. https://doi.org/10.1007/978-3-319-63577-4.
10. Solovtsova, O.P.; Lashkevich, V.I.; Kaptari, L.P. Lepton anomaly from QED diagrams with vacuum polarization insertions within the Mellin-Barnes representation. *Eur. Phys. J. Plus* **2023**, *138*, 212. https://doi.org/10.1140/epjp/s13360-023-03834-4.
11. Marciano, W.J. Anomalous Magnetic Moments. *Int. J. Mod. Phys. A* **2004**, *19*, 77–87. https://doi.org/10.1142/S0217751X04018609.
12. De Conto, G.; Pleitez, V. Electron and muon anomalous magnetic dipole moment in a 3-3-1 model. *J. High Energ. Phys.* **2017**, *2017*, 104. https://doi.org/10.1007/JHEP05(2017)104.
13. Logashenko, I.B.; Eidelman, S.I. Anomalous magnetic moment of the muon. *Phys.-Usp.* **2018**, *61*, 480–510. https://doi.org/10.3367/UFNe.2018.02.038312.
14. Keshavarzi, A.; Khaw, K.S.; Yoshioka, T. Muon $g-2$: A review. *Nucl. Phys. B* **2022**, *975*, 115675. https://doi.org/10.1016/j.nuclphysb.2022.115675.
15. Dinh, D. Muon anomalous magnetic dipole moment in a low scale type I see-saw model. *Nucl. Phys. B* **2023**, *994*, 116306. https://doi.org/10.1016/j.nuclphysb.2023.116306.
16. Winter, P. Status of the Muon g - 2 experiment. *EPJ Web of Conf.* **2023**, *289*, 01001. https://doi.org/10.1051/epjconf/202328901001.
17. Aguillard, D.P.; Albahri, T.; Allspach, D.; Anisenkov, A.; Badgley, K.; Baeßler, S.; Bailey, I.; Bailey, L.; Baranov, V.A.; Barlas-Yucel, E.; et al. Measurement of the Positive Muon Anomalous Magnetic Moment to 0.20 ppm. *Phys. Rev. Lett.* **2023**, *131*, 161802. https://doi.org/10.1103/PhysRevLett.131.161802.
18. Aguillard, D.; Albahri, T.; Allspach, D.; Anisenkov, A.; Badgley, K.; Baeßler, S.; Bailey, I.; Bailey, L.; Baranov, V.; Barlas-Yucel, E.; et al. Detailed report on the measurement of the positive muon anomalous magnetic moment to 0.20 ppm. *Phys. Rev. D* **2024**, *110*, 032009. https://doi.org/10.1103/PhysRevD.110.032009.
19. Aoyama, T.; Asmussen, N.; Benayoun, M.; Bijnens, J.; Blum, T.; Bruno, M.; Caprini, I.; Carloni Calame, C.; Cé, M.; Colangelo, G.; et al. The anomalous magnetic moment of the muon in the Standard Model. *Phys. Rep.* **2020**, *887*, 1–166. https://doi.org/10.1016/j.physrep.2020.07.006.
20. Li, H.; Wang, P. Nonlocal QED and lepton g-2 anomalies. *Eur. Phys. J. C* **2024**, *84*, 654. https://doi.org/10.1140/epjc/s10052-024-13013-z.
21. Morte, M.D.; Francis, A.; Gülpers, V.; Herdoíza, G.; Von Hippel, G.; Horch, H.; Jäger, B.; Meyer, H.B.; Nyffeler, A.; Wittig, H. The hadronic vacuum polarization contribution to the muon $g-2$ from lattice QCD. *J. High Energ. Phys.* **2017**, *2017*, 20. https://doi.org/10.1007/JHEP10(2017)020.
22. Westin, A.; Kamleh, W.; Young, R.; Zanotti, J.; Horsley, R.; Nakamura, Y.; Perlt, H.; Rakow, P.; Schierholz, G.; Stüben, H. Anomalous magnetic moment of the muon with dynamical QCD+QED. *EPJ Web Conf.* **2020**, *245*, 06035. https://doi.org/10.1051/epjconf/202024506035.
23. Borsanyi, S.; Fodor, Z.; Guenther, J.N.; Hoelbling, C.; Katz, S.D.; Lellouch, L.; Lippert, T.; Miura, K.; Parato, L.; Szabo, K.K.; et al. Leading hadronic contribution to the muon magnetic moment from lattice QCD. *Nature* **2021**, *593*, 51–55. https://doi.org/10.1038/s41586-021-03418-1.
24. Gérardin, A. The anomalous magnetic moment of the muon: status of lattice QCD calculations. *Eur. Phys. J. A* **2021**, *57*, 116. https://doi.org/10.1140/epja/s10050-021-00426-7.
25. Nedelko, S.; Nikolskii, A.; Voronin, V. Soft gluon fields and anomalous magnetic moment of muon. *J. Phys. G: Nucl. Part. Phys.* **2022**, *49*, 035003. https://doi.org/10.1088/1361-6471/ac4a82.
26. Blum, T.; Christ, N.; Hayakawa, M.; Izubuchi, T.; Jin, L.; Jung, C.; Lehner, C. Hadronic Light-by-Light Scattering Contribution to the Muon Anomalous Magnetic Moment from Lattice QCD. *Phys. Rev. Lett.* **2020**, *124*, 132002. https://doi.org/10.1103/PhysRevLett.124.132002.
27. Melo, D.; Reyes, E.; Fazio, R. Hadronic Light-by-Light Corrections to the Muon Anomalous Magnetic Moment. *Particles* **2024**, *7*, 327–381. https://doi.org/10.3390/particles7020020.

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28. Leutgeb, J.; Mager, J.; Rebhan, A. Holographic QCD and the muon anomalous magnetic moment. *Eur. Phys. J. C* **2021**, *81*, 1008. https://doi.org/10.1140/epjc/s10052-021-09780-8.
29. Raya, K.; Bashir, A.; Miramontes, A.S.; Roig, P. Dyson-Schwinger equations and the muon g-2. *Supl. Rev. Mex. Fis.* **2022**, *3*, 020709. https://doi.org/10.31349/SuplRevMexFis.3.020709.
30. Groenewold, H.J. Regularized models in electrodynamics. *Physica* **1962**, *28*, 1265–1293. https://doi.org/10.1016/0031-8914(62)90137-4.
31. Barut, A.O.; Van Huele, J.F. Quantum electrodynamics based on self-energy: Lamb shift and spontaneous emission without field quantization. *Phys. Rev. A* **1985**, *32*, 3187–3195. https://doi.org/10.1103/PhysRevA.32.3187.
32. Ibison, M. Some Properties of a Regularized Classical Electromagnetic Self-Interaction. In *Causality and Locality in Modern Physics*; Hunter, G.; Jeffers, S.; Vigier, J.P., Eds.; Springer Netherlands: Dordrecht, 1998; pp. 477–489. https://doi.org/10.1007/978-94-017-0990-3_55.
33. Blinder, S.M. Structure and self-energy of the electron. *Int. J. Quantum Chem.* **2002**, *90*, 144–147. https://doi.org/10.1002/qua.1806.
34. Yaremko, Y. On the regularization procedure in classical electrodynamics. *J. Phys. A: Math. Gen.* **2003**, *36*, 5149–5156. https://doi.org/10.1088/0305-4470/36/18/318.
35. Galakhov, D. Self-interaction and regularization of classical electrodynamics in higher dimensions. *JETP Letters* **2008**, *87*, 452–458. https://doi.org/10.1134/S0021364008080134.
36. Perlick, V. On the Self-force in Electrodynamics and Implications for Gravity. In *Equations of Motion in Relativistic Gravity*; Puetzfeld, D.; Lämmerzahl, C.; Schutz, B., Eds.; Springer International Publishing: Cham, 2015; Vol. 179, pp. 523–542. https://doi.org/10.1007/978-3-319-18335-0_15.
37. Arias-Perdomo, D.C.; Cherchiglia, A.; Hiller, B.; Sampaio, M. A Brief Review of Implicit Regularization and Its Connection with the BPHZ Theorem. *Symmetry* **2021**, *13*, 956. https://doi.org/10.3390/sym13060956.
38. Hale, T.; Kubizňák, D.; Svítek, O.; Tahamtan, T. Solutions and basic properties of regularized Maxwell theory. *Phys. Rev. D* **2023**, *107*, 124031. https://doi.org/10.1103/PhysRevD.107.124031.
39. Prabhu, N. A new method that automatically regularizes scattering amplitudes. *J. Phys. Commun.* **2023**, *7*, 115002. https://doi.org/10.1088/2399-6528/ad0649.
40. Georgiev, M. Exact classical approach to the electron's self-energy and anomalous *g*-factor. *Europhys. Lett.* **2024**, *147*, 20001. https://doi.org/10.1209/0295-5075/ad5469.
41. Ji, X.H.; He, Y.Y.; Jiao, L.G.; Liu, A.; Ho, Y.K. Probability density of the Yukawa particle at the origin: Numerical calculation and analytical solution. *Phys. Lett. B* **2021**, *823*, 136718. https://doi.org/10.1016/j.physletb.2021.136718.
42. Bray, A.; Chabysheva, S.; Hiller, J. Effective potential between static sources in quenched light-front Yukawa theory. *Phys. Rev. D* **2024**, *109*, 096010. https://doi.org/10.1103/PhysRevD.109.096010.
43. Tu, L.C.; Luo, J.; Gillies, G.T. The mass of the photon. *Rep. Prog. Phys.* **2005**, *68*, 77–130. https://doi.org/10.1088/0034-4885/68/1/R02.
44. 2018 CODATA Value: electron mass and muon mass. The NIST Reference on Constants, Units, and Uncertainty. *NIST* **2019**.
45. Planck Collaboration.; Aghanim, N.; Akrami, Y.; Ashdown, M.; Aumont, J.; Baccigalupi, C.; Ballardini, M.; Banday, A.J.; Barreiro, R.B.; Bartolo, N.; et al. *Planck* 2018 results: VI. Cosmological parameters. *A & A* **2020**, *641*, A6. https://doi.org/10.1051/0004-6361/201833910.
46. The KATRIN Collaboration.; Aker, M.; Beglarian, A.; Behrens, J.; Berlev, A.; Besserer, U.; Bieringer, B.; Block, F.; Bobien, S.; Böttcher, M.; et al. Direct neutrino-mass measurement with sub-electronvolt sensitivity. *Nat. Phys.* **2022**, *18*, 160–166. https://doi.org/10.1038/s41567-021-01463-1.
47. Schweiger, C.; Braß, M.; Debierre, V.; Door, M.; Dorrer, H.; Düllmann, C.E.; Enss, C.; Filianin, P.; Gastaldo, L.; Harman, Z.; et al. Penning-trap measurement of the Q value of electron capture in 163Ho for the determination of the electron neutrino mass. *Nat. Phys.* **2024**, *20*, 921–927. https://doi.org/10.1038/s41567-024-02461-9.