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Article

# Sustainable Integral Optimization of Service Queues: A Human-Centered Approach Using IAM and SMAA

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## Abstract

This paper contributes to queueing systems by introducing the concept of sustainable integral optimization, which integrates both quantitative and qualitative criteria, such as comfort, anxiety, and perceived service quality, into traditional performance metrics. We extend Kendall's notation with a new element (G), focused on subjective sustainability indicators. The optimization process is conducted via the Integral Analysis Method (IAM), complemented by the SMAA framework for multi-criteria decision-making. To demonstrate the applicability of the proposed approach, a case study in a high-end car dealership is developed. The goal is to address a persistent deficiency in queueing theory literature by incorporating human-centered sustainability principles, seeking a systemic balance between operational efficiency and user-oriented effectiveness in service delivery.

**Keywords:** queueing theory; IAM; SMAA; sustainable service systems; integral optimization; multicriteria analysis; user-centered sustainability; perceived service quality

## 1. Introduction

This study addresses the sustainability of service systems by integrating qualitative criteria, such as: comfort, competition, and perceived service quality, into queueing theory models. These criteria have a direct impact on the user experience, resource efficiency, and service equity, all of which are central to the concept of sustainable management. By applying the Integral Analysis Method (IAM), we aim to support decision-making processes that balance operational performance with human-centered values. In this way, the study contributes to the sustainability of service delivery systems in sectors where user satisfaction and systemic resilience are key concerns (Mardani et al., 2015)

This study emphasizes the critical need to incorporate qualitative criteria—such as customer comfort, anxiety, and perception—into queueing analysis. Through the Integral Analysis Method (IAM), we provide a structured integration of these qualitative aspects into the traditional quantitative framework, offering a more comprehensive and context-sensitive approach to decision-making. Additionally, we propose a formal extension of Kendall's classical notation by introducing a fourth element 'G', which captures the qualitative dimension of service quality. While this notation is not the core contribution of the study, it serves as a formal tool to support the integration of subjective variables within a standardized modeling paradigm.

Queueing Theory seeks to improve the effectiveness of a system by balancing Effectiveness (i.e., providing the desired service level) and Efficiency (minimizing resource utilization and maximizing results). In 1953, Kendall defined the (A/B/C) notation to classify waiting systems, wherein A is the type of time probability distribution among consecutive client arrivals; B is the type of output time (also known as service or attention time) probability distribution; and C is the number of servers operating in parallel at the workstation. In 1966, Lee added two symbols to this notation: D and E; and in 1968, Taha added F as a sixth symbol. D is the discipline or priority policy that defines

customer service, which determines various aspects of service quality.  $E$  is the total capacity of the system (finite or infinite), i.e., the maximum allowable number of customers who are either queuing or being served, with  $E \geq C$ . Finally,  $F$  is the size of the population or source from which customers are coming. Kendall's notation has been repeatedly modified by many authors according to their needs (Tao and Templeton, 1987; Kalashnikov, 1994). Along these lines, the present work involves the inclusion of a new element ( $Q$ ) in Kendall's notation, associated to classical queueing systems, but applicable to general queueing systems.

Despite important theoretical progress achieved over the decades, there is an unspoken deficiency in the literature, regarding the lack of attention to the inclusion of qualitative aspects that are relevant to the analysis and performance of the system. One of the difficulties that are inherent to optimization contexts is the integration of qualitative and quantitative aspects. In this sense, the Integral Analysis Method - IAM (García-Cáceres et al., 2009) was presented as a pioneering work to overcome this problem by means of a technique known as integral optimization. It is worthwhile noting that the non-inclusion of qualitative aspects is not a particular condition of waiting line systems, but an unspoken deficiency of optimization.

The current document is organized as follows: After a literature review, the relevant qualitative aspects of the queueing theory are highlighted. Subsequently, the application of integral optimization to the queueing theory is presented and then illustrated by means of a case study. Finally, conclusions and research perspectives are put forward.

## 2. Literature Review

The foundations of the Queueing Theory were set at the beginning of the 20th century, when its basic concepts and some practical applications were deployed, e.g., to satisfy the demand uncertainty that could be observed in telephone traffic systems (Saaty, 1983). In the 1970s, the theory focused on obtaining exact analytical expressions of indicators that described the performance of systems (Cohen, 1982). In the 1980s, a wave of solutions was generated, all of which contemplated time spent in the system as a strategic factor for companies, thus resulting in an increased number of applications (Stalk, 1988).

The design and management of a system that is characterized by waiting lines or queues poses permanent challenges for any modern organization. This is so because this feature affects the costs and the service level, which, in turn, impacts competitiveness and business sustainability (Beamon, 1998; Ballou, 2004; Manish, 1999). Applications based on the waiting line theory have been extended to a good number of industries and systems such as those aimed at maintenance service, quality inspections, service facilities for employees, computer centers, machinery installations (Gross et al., 2008; Kulkarni et al., 1997; Prabhu, 2002), telephone communication (Saaty, 1983; Afolalu et al., 2021), vehicle traffic and machine breakdowns (Manish, 1999; Zhang et al., 2021), applications to judicial and legislative systems, hospital emergency rooms, real estate purchase subsidy allocation systems (Artalejo, 1995; 2010), car wash services (Kondrashova, 2021), waste management (Zhang and Ahmed, 2022), and airport terminals (Alnowibet et al., 2022), among others (Cárdenas-Barrón et al., 2021).

Queues typically generate economic losses and social welfare detriment, thus threatening competitive priorities, among which quality and costs certainly stand out (Winser and Fawcett, 1991). From a broad economic perspective, it can be said that waiting lines do not generate value and should, therefore, be reduced to their minimum possible dimensions (Coase, 1937, Manish, 1999). However, they are actually unavoidable due to the capacity constraints of the systems and, therefore, they must be properly managed to mitigate their negative effect on business performance. The theory of waiting lines constitutes a remarkable development in this sense, since it focuses on system performance (Manish, 1999). Waiting line management is mainly aimed at service quality and customer satisfaction, which have been gaining importance in the current economic context, to the point of being often considered more important than costs (Ballou, 2004; Winser and Fawcett, 1991).

From the literature review, it can be deduced that research on waiting lines has triggered multiple applications and application perspectives that seek to increase productivity by minimizing costs, maximizing service level, and optimizing queue size, service times, service utilization requests and integrity in each one of the systems under study. It further shows that the applied techniques are particularly related to stochastic calculus and stochastic processes, which fundamentally link quantitative (Artalejo 1995; 2010; Manish, 1999; Chia-Huang and Dong-Yuh, 2021; Singh et al., 2021a; Singh et al., 2021b) and qualitative (Itoh and Konno, 1992; Khan, 2005; Ullah, 2014; Salehin, 2020) considerations.

Qualitative approaches to the Queuing theory have focused on aspects such as leisure (Doshi, 1986), new trial (Tao and Templeton, 1987; Falin, 1990), customer type (positive and negative) (Artalejo, 2010a), computer attacks (Khan and Traore, 2005; Akhlaghiet al., 2008), service priorities (Lin et al., 2008), location of internet qualitative states (Salehim et al., 2020) and banking entities (Ullah et al., 2014), all of them treated quantitatively in the sense that expert preference information is not used. There is also evidence of the use of MCDM (Multi Criteria Decision Making) techniques for the inclusion of quantitative and qualitative aspects in decision problems (Khan, 2005). Nonetheless, the present work is the first one to treat preference information through a specialized method for the inclusion of qualitative and quantitative aspects, as is the case of IAM.

Recent contributions to queue optimization reinforce the importance of hybrid methods that integrate qualitative and quantitative indicators. Freire et al. (2022) propose a queue-priority optimized algorithm for runtime systems, aligned with the IAM's goal to manage multivariable constraints in real-time service environments. Similarly, Huang et al. (2019) analyze queueing delay optimization for service-oriented networks, directly resonating with IAM's integration of waiting time and server idleness. Additionally, Alhulayil et al. (2025) introduce a fairness-based waiting time model, which supports the inclusion of qualitative fairness and equity indicators within queue system designs.

### 3. Description of the Problem

Integral optimization is supported on the Integral Analysis Method (IAM), see Appendix A, proposed by García-Cáceres et al. (2009), which consists of three mathematical steps: (i) Cardinal analysis, which in the present case resorts to the Queueing Theory to provide the solution method; (ii) Qualitative analysis, where the alternatives resulting from the quantitative analysis are evaluated through ordinal criteria by means of Stochastic Multi-criteria Acceptability Analysis with Ordinal data (SMAA-O) (Lahdelma et al., 2003); and (iii) Integration analysis, where the alternatives are jointly analyzed using the deterministic version of SMAA (Lahdelma et al., 2000; Lahdelma and Salminen, 2001) and probability elements. It is carried out using parameter ( $X$ ), expected number of idle servers, as central performance measure. The input of this integration analysis are the results of the cardinal and ordinal analyses (Sustainability, 2023). The integral optimization of the queueing system discussed here seeks to determine the optimal number of servers in a queueing line, considering not only the typical quantitative aspects, but also some other relevant qualitative variables.

To facilitate understanding for readers who may not be deeply familiar with queueing theory, the following table summarizes the key variables and parameters used throughout the manuscript. This also supports consistency in terminology and enhances the clarity of the mathematical formulation.

**Table 1.** Summary of Variables Used in the Queuing Model.

Symbol	Description	Units
$\lambda$	Arrival rate	customers/hour
$\mu$	Service rate per server	customers/hour
$c$	Number of servers	count

$L$	Expected number of users	customers
$Lq$	Expected number in the queue	customers
$W$	Expected time in the system	minutes
$Wq$	Expected time in queue	minutes
$X$	% of server inactivity	percentage
$\alpha, \beta$	Aspiration thresholds ( $W$ and $X$ respectively)	minutes / percentage

### 3.1. Cardinal Analysis

The Queuing theory is the mathematical study of waiting lines within a given system, which, in turn, conditions the model developed to study it. Mathematical models usually employ notation systems that include parameters and variables which, in the present case, correspond to  $\lambda$  = rate of arrivals to the system and  $\mu$  = service rate of each service channel. On the other hand, the variables of the system are:  $L$  = expected number of users in the system,  $Lq$  = expected number of users on the queue,  $W$  = expected time spent in the system by each user, and  $Wq$  = expected time spent by each user on the queue. These are the main performance indicators, which can be compared to industry parameters or to those resulting from the system's own policies to assess its operation (Gross et al., 2008; Kulkarni et al., 1997; Prabhu, 2002).

Little's Law, which relates queue size to waiting time, is described as follows:  $L = \lambda W$ ;  $Lq = \lambda Wq$ ;  $W = Wq + 1/\mu$ , where  $L = \sum_{n=0}^{\infty} nP_n$ , and  $P_n$  is the probability that there be  $n$  customers in the system (Kulkarni et al., 1997; Ritzman et al., 2002; Ross, 2007). The utilization factors are defined by  $\rho = \lambda / (c\mu)$ , where  $c$  is the number of s.

A classical queueing model ( $M/M/c : GD/\infty/\infty/Q$ ) was applied to provide a clear example of the integral optimization process in queueing systems. This model characterizes the functioning of a dealership that sells high-end vehicles. It is assumed that the system is not capacity constrained, as there is no evidence that customers refuse to come into the shop. The Poisson arrival processes range from 5 to 15 average customers per hour, depending on the day of the month. Exponentially distributed average service time was estimated to be 20 min. The service covers the following activities: Identification of customer needs, orientation about possible vehicles to be purchased, showing the vehicles on display and, finally, quoting and simulation of vehicle cost and delivery times.

The cardinal analysis focuses on the problem of minimizing the total cost, as indicated by the following objective function:

$$\min CT(c) = cC_c + C_w L(c) \quad (1)$$

where  $CT(c)$  is the total cost, as a function of the number of servers;  $C_c$  is the cost per server;  $C_w$  is the cost of making a client wait;  $L(c) = L$  is the number of people waiting in the system as a function of the number of servers.

Although car dealers estimate that servers imply an elevated cost, the varying level of commercial competition usually observed in this trade has led them to favor commercial strategies that promote customer service quality (Carolyn, 2001). In this case, the way the products are displayed is very important, together with an attentive attitude expressed in the way of being, having and doing, always aimed at customer need satisfaction (Maslow, 1943).

As a tool to support the decision process at this stage, we employed the parameter "aspiration level", which resorts to the following variables of analysis: Average time spent in the system ( $W$ ) and the percentage of inactivity of the servers ( $X$ ), both of them as functions of the number of customers. Said percentage is calculated as the ratio between the average number of idle servers ( $\bar{c}$ ) and the total number of servers, as shown in the following expression.

$$X = \frac{c - \bar{c}}{c} = \left(1 - \frac{\lambda_{ef}}{c\mu}\right) \quad (2)$$

In order to determine an acceptance range that facilitates decision making on the optimal number of servers, acceptance levels are defined for the variables Time spent in the system ( $\alpha$ ) and Percentage of server inactivity ( $\beta$ ), considering:

$$Ws \leq \alpha \wedge X \leq \beta \quad (3)$$

The aspiration level model requires information about the acceptable levels of the two performance variables that express customer aspiration. Since they usually have antagonistic behaviors with respect to the number of servers ( $c$ ), the latter has to be set at a value that optimizes both variables. Therefore, in the current case study,  $c$  is the decision variable of the cardinal analysis.

For the case study, the value of  $\alpha$  was set at 25 min ( $\alpha = 25$  min), so that customers do not have to wait to be served for more than 5 minutes ( $Wq$ ). This parameter was defined taking into account that customers are usually sensitive to waiting time (Levine, 2006). In this competitive segment, the customer values exclusivity and product differentiation, and the service must be characterized by its high quality and immediacy, because the risk of customer loss is particularly high when dealing with high-end vehicles. On the other hand,  $1 - \beta = 68\%$  was defined as the maximum allowed level of idleness. This definition was based on the notion of "work pace valuation", proposed by Niebel (2001), in addition to the previously expressed quality considerations.

The following table presents the results of the simulation of the queueing system, which was carried out to determine the optimal number of servers ( $c$ ) that satisfies the aspiration levels. This includes the typical performance variables of a queueing system, together with those contemplated in the aspiration function, in order to clearly determine the performance of the system. The data, rounded to two decimal places, include the following variables, in addition to those mentioned above: ( $\lambda_{eff}$ ) Effective arrival rate, ( $p_n$ ) Probability of the instance, and ( $p_i$ ) Probability of the alternative. After obtaining the indicators, the Percentage of server inactivity ( $X$ ) was calculated through Equation (3).

The case study assumes moderate homogeneity among customers in terms of perception of service-related qualitative criteria. However, heterogeneity may be addressed in future research. The Likert-scale values for qualitative criteria were obtained through expert elicitation involving service managers and operational staff at the dealership. This approach is justified by the need for informed judgments when direct customer surveys are unavailable.

Based on historical data, 10 scenarios were evaluated in the 1 to 8 servers range, the Poisson arrival rate varying from 6 to 15 customers per hour on average. Table 2 summarizes the results of the inspected scenarios. Table 2 presents the fundamental parameters of the M/M/ $c$  system used in this study, including:  $L_s$  (expected number of users in the system),  $W_s$  (expected time in system),  $p_n$  (probability of having  $n$  customers in the system), and  $p_i$  (probability that the system is idle). These variables form the quantitative basis for assessing queue performance.

**Table 2.** Expected number of selected servers.

$n$	$\lambda$ (Arrival rate)	$\mu$ (Service rate)	$c$ (number of servers)	$\lambda_{eff}$	$L_s$	$W_s$ (min)	$L_q$	$Wq$ (min)	100- $X$ (%)	$p_n$	$p_i$
1	6	3	4	6	2.17	21.73	0.17	1.74	50.00	0.1	
2	7	3	4	7	2.70	23.19	0.37	3.19	58.33	0.1	$i = 1,$ 0.3
3	8	3	4	8	3.42	25.67	0.75	5.67	66.67	0.1	
4	9	3	5	9	3.35	22.36	0.35	2.36	60.00	0.1	$i = 2,$
5	10	3	5	10	3.98	23.92	0.65	3.91	66.67	0.1	0.2
6	11	3	6	11	3.99	21.79	0.32	1.80	61.11	0.1	$i = 3,$
7	12	3	6	12	4.56	22.84	0.56	2.85	66.67	0.1	0.2
8	13	3	7	13	4.63	21.4	0.30	1.40	61.90	0.1	$i = 4,$
9	14	3	7	14	5.16	22.14	0.50	2.14	66.67	0.1	0.2
10	15	3	8	15	5.27	21.11	0.27	1.11	62.5	0.1	$i = 5, 0.1$

Note: The values of  $L_q$  and  $Wq$  were estimated based on standard approximations under the assumed arrival and service conditions. Slight deviations from Little's Law may occur due to rounding and the integration of multicriteria adjustments within the IAM framework.

The results presented in the Table above show that the studied range of servers conforms to the aspirational peaks. Probability  $p_i$ , which constitutes the outcome of IAM's Cardinal Analysis, is the Marginal Cardinal Probability Distribution of variable  $i$ , which is discriminated by variable  $c$ , as it can be seen in Table 1. In sum,  $p_i$  expresses the probability of the optimal values of the number of servers ( $c$ ) resulting from the scenarios of the problem. In the present case, the optimal relation between them indicates that  $c = i + 3$ .

### 3.2. Ordinal Analysis

The qualitative factors that affect service queues were defined in such a way that they support decision making by estimating indicators that help to measure the performance of the system, see Appendix B, (e.g., the performance of the installed capacity, or the percentage of idle servers) and those of interest to its customers (e.g., waiting time). These models also allow enhancing service quality by estimating and informing the customer how long they have to wait to be served, among other improvements (Singer, 2008). This stage is essential for capturing sustainability-related subjective factors, such as user equity, perceived comfort, and service accessibility. These qualitative dimensions are critical for promoting human-centered sustainability in service systems, ensuring that operational improvements are aligned with user well-being and fairness.

Some qualitative aspects were included in the present study, so as to provide an adequate example of the application of IAM to this type of information, within the necessary decision making process of queueing systems: a) Quality level, understood as the standard of a product or service, and related to the level of customer satisfaction; b) Comfort level, which is the wellbeing perceived by the interaction of the customer with the service environment, c) Competition, which is the level of monopoly that exists in the market wherein the company providing the service performs its activity (Chan, 2003; Manish et al., 2000).

Each of the alternatives is evaluated qualitatively. In this illustrative example, the additional element (G) is associated with four qualitative criteria that are evaluated through the Likert table shown in Table 3, which presents 5 ordinal categories.

**Table 3.** Likert Scale.

Range	Level
1	Very high
2	High
3	Medium
4	Low
5	Very low

**Consumer Anxiety:** This criterion is measured according to the Cognitive-Somatic Anxiety Questionnaire (CSAQ) (1978), the Likert scale of which discriminates the level of customer anxiety. In the present case, it was determined that the higher the number of servers, the lower the level of anxiety within the optimal 3-8 servers range, because the service time is correspondingly shorter. However, as the number of people in the system increase and the place gets crowded to its full capacity, density saturation is likely to generate more anxiety, this time not derived from waiting time, but from the vital space decrease and the system's entropy increase. The Likert scale, defined in the 1 to 5 range, identifies 1 as a very good level of anxiety, and 5 as a very bad level.

**Noise:** This criterion is governed by Colombian regulations. Resolution 8321 of August 4, 1983, issued by the Ministry of Health, defines the maximum permissible levels of noise exposure, which depend on sound intensity and time spent under its influence. The Likert scale shown in Table 4 illustrates the impact of noise on human health, where the greater the number of servers, the higher the service and noise levels.

**Table 4.** Results for the criterion Noise.

dB	Likert level	Level
>=80	5	Very bad
[70-80)	4	Bad
[60-70)	3	Medium
(50-60)	2	Good
50<=	1	Very good

**Thermal load:** This criterion is proportionally related to the number of people, and its evaluation depends on the Wet-Bulb Globe Temperature (WBGT) index. The American Conference of Governmental Industrial Hygienists (ACGIH) defines its maximum allowed level at 38°C. The following Likert scale is used for its estimation.

**Table 5.** Results for the variable Thermal Load.

WBGT (°C)	Likert level	Level
>38 o <5	5	Very bad
[32-38) o [5-10)	4	Bad
[26-32) o [10-15)	3	Medium
[23-26)	2	Good
[15- 23)	1	Very good

Although the temperature can be managed through air conditioning, the application of this resource involves cultural, economic and environmental considerations. Therefore, the decision to use it requires analysis. In the case in Bogota, it is rarely employed.

**Competition:** Lerner's Index, which ranges between 0 and 1, allows estimating competition, as expressed in Table 6.

**Table 6.** Results of the variable Competition.

Lerner's index	Likert level	Level
[0-0.2)	1	Very good
[0.2-0.4)	2	Good
[0.4-0.6)	3	Medium
[0.6-0.8)	4	Bad
[0.8-1]	5	Very bad

It is commonly assumed that the higher the level of competition, the larger the number of servers and the concomitant investment. This analysis should take into account that the number of employees should not exceed the capacity of the facilities of the company.

According to IAM (García-Cáceres et al., 2009), the Ordinal analysis supported by SMAA-O (Lahdelma et al., 2003) is restricted to ordinal criteria. The following table rates the 4 ordinal criteria considered for the alternatives under study.

**Table 7.** Ordinal Analysis.

$i$	$c$	Frequency	$p_i$	$j:1$ Consumer's anxiety	$j:2$ Noise	$j:3$ Thermal load	$j:4$ Competition	$b_1^i$
1	4	3	0.3	5	1	1	5	0.25
2	5	2	0.2	4	2	2	4	1/6
3	6	2	0.2	3	3	3	3	1/6
4	7	2	0.2	2	4	4	2	1/6

5	8	1	0.1	1	5	5	1	0.25
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The ordinal stage of IAM makes it possible to identify the set of favourable weights that support each of the alternatives in a particular ranking, according to the ordinal evaluation criteria. The most important indicator of the analysis is the Ordinal Acceptability Index ( $b_r^i$ ), which defines the probability of acceptance of each alternative and determines the ordinal ranking ( $r$ ). The qualitative stage is supported by another indicator, the central weight vector, which represents the typical combination of weights that favours a given alternative in ordinal ranking 1. This vector represents the centre of mass of the set of favourable weights. Table 12 presents the ordinal analysis for each one of the alternatives in question.

Ordinal ranking 1 favors alternatives 1 and 5,  $b_1^1=b_5^1=1/4$ , which are supported by a higher volume of feasible weights. These alternatives not only attained the best and worse scores in 2 of the 4 criteria, but also exhibit opposite ratings: Criteria 2 and 3 support Alternative 1, while criteria 1 and 4 support Alternative 5. Alternatives 3 to 5 are also well rated, with  $b_1^2=b_3^3=b_5^4=1/6$ . Alternative 2 is supported by criteria 2 and 3, while Alternative 4 is supported by criteria 1 and 4. For its part, Alternative 3 is supported by an intermediate rating in all 4 criteria.

### 3.3. Integration Analysis

In the context of IAM, the integration analysis is supported by the deterministic version of SMAA (Lahdelma et al., 2001). Using as input the 2 output variables resulting from the prior cardinal and ordinal analyses (which are assumed to be independent), the integration analysis calculates the Joint Integral Index  $p(e^i)$  y el integration acceptability index  $a_1^i$ , which assesses each alternative's joint probability - both cardinal and ordinal - of obtaining a specific ranking. This integrative step embodies the principles of sustainability by balancing quantitative efficiency (e.g., productivity and waiting time reduction) with qualitative well-being (e.g., user comfort and anxiety mitigation). The ability to synthesize these dimensions within a unified decision-making framework reflects a commitment to sustainable service design, where operational goals do not compromise human-centered values.

$$p(e^i) = p_i b_r^i \quad (4)$$

Although the evaluated server counts range from  $c = 4$  to  $c = 8$ , the analysis was extended to  $c = 9$  based on SMAA recommendations. Thus,  $c^* = 9$  is reported as the optimal configuration considering both cardinal and ordinal criteria.

Table 8 presents the integration analysis. The two holistic indexes mentioned above show that Alternative 1, with 4 servers, is the most supported one. Consequently, it is the one that integrally optimizes the problem in question.

**Table 8.** Results of the integration analysis.

$i$	1	2	3	4	5
$c^*$ (optimal number of servers)	4	5	6	7	9
$p_i$	0.30	0.20	0.20	0.20	0.10
$b_1^i$	1/4	1/6	1/6	1/6	1/4
$p(e^i)$	0.075	0.033	0.033	0.033	0.025
$a_1^i$	1	0	0	0	0

The optimization strategy used in this study is based on a discrete evaluation of server quantity scenarios ( $c$  values). For each scenario, performance metrics are calculated and then integrated using the SMAA multicriteria analysis. This enumeration-based approach enables practical decision-making without the use of continuous optimization or heuristic algorithms.

#### 4. Conclusions and Future Perspectives

The present paper incorporates a new element (G) to the notation developed by Kendall (A/B/C), Lee (D/E) and Taha (F), among others. This novel factor represents the qualitative aspects of the problem, which are the relevant criteria for the study of a queueing service system. This addition reflects a more realistic treatment of service environments where subjective perceptions (e.g., comfort, competition, anxiety) are key.

The use of a qualitative approach in the conformation of models allows obtaining holistic solutions which are likely to come closer to reality. Including this type of aspect is an approach to the integral optimization of the queueing problem, which can contribute significantly to its theoretical and practical development.

In conclusion, the results demonstrate a well-structured and practical approach, particularly through the case application to high-end vehicle service management. The study effectively bridges queueing theory with multi-criteria decision-making tools such as SMAA, offering a robust framework for incorporating both quantitative and qualitative dimensions in service system optimization.

In particular, future research could explore the integration of broader sustainability criteria within queueing models, such as environmental impacts of waiting environments, energy efficiency in service provision, and social equity in customer prioritization. These directions would further consolidate the role of queueing theory as a tool for achieving the sustainable development goals (SDGs) in service contexts. Through the application of IAM, this paper presents a comprehensive optimization approach to queueing theory. Future research directions include not only extending the current study to other classical queueing systems but also applying the methodology to more complex queueing network systems. The work is well aligned with recent studies that address fairness, qualitative variables, and hybrid queueing models.

#### Appendix A. Integral Analysis Method (IAM) - Mathematics Stages.

The cardinal analysis is explicitly presented in the main body of the manuscript.

##### Ordinal Analysis - SMAA-O

The stage restricts the SMAA-O to ordinal criteria. Each of the original variables of an optimization problem can have several ordinal variables associated that are defined by the decision makers. These values will be used to characterize the ordinal analysis, but, before explaining the method it is necessary to define the concept of class as a set of alternatives which have identical utilities for all ordinal variables associated. As a result of the analysis a group of indicators is obtained that help to determine the ordinal acceptability of each studied alternative.

Now the SMAA-O used by IAM is explained in more detail

##### **Constants, indexes and sets:**

$m$ : Number of possibilities or alternatives

$n$ : Number of ordinal variables

$s$ : Number of class, where  $s \leq m$

$r$ : Ordinal ranking, where:  $r \in \{1, 2, \dots, N(r)\}$  and  $N(r) \leq m$ .

$A$ : Classes, where:  $a \in \{1, 2, \dots, s\}$

$F(a)$ : Alternatives concerning class ( $a$ )

$C$ : Ordinal ranking

$C_i$ : Ordinal ranking for alternative ( $i$ )

Other Symbols:

$X_{aj}$ : Ordinal criteria values of quantitative aspect or ordinal variable ( $j$ ) for class ( $a$ )

$w_j$ : Weights vector of ordinal variable ( $j$ )

$u_j(X_{aj})$ : Mapping from ordinal criteria values of ordinal variable ( $j$ ) for class ( $a$ )

$\eta_r$ : Random number from the uniform distribution  $[0,1]$

$\varepsilon_{aj}$  : Stochastic cardinal criteria values of ordinal variable ( $j$ ) for class ( $a$ )

$W_a^r$  : Favourable ordinal vector ranking weights for class ( $a$ )

$W$ : Set of non-negative normalized weights

$b_a^r$  : Ordinal acceptability index for class ( $a$ )

$b_i^r$  : Ordinal acceptability index for alternative ( $i$ )

$c^i$ : Number of ordinal ranking for alternative ( $i$ )

$R^i$  : Central ordinal value for alternative ( $i$ )

$V^i$  : Dispersion ordinal value for alternative ( $i$ )

The method is based on the idea of determining the utility values that support each class. The utility function can be additive, as in the following case:

$$u(X_a, w) = \sum_{j=1}^n w_j u_j(X_{aj})$$

The process starts with the generation of random numbers from a uniform distribution in the interval  $[0, 1]$  and sorting these numbers along with 1 and 0 in decreasing order, as shown below:

$$\gamma_{j1} > \gamma_{j2} > \dots > \gamma_{jj^{\max}}$$

$$\gamma_{j1} = 1, \gamma_{jj^{\max}} = 0, \gamma_{jk} \leftarrow U(0,1), \quad k \neq 1 \wedge j^{\max}$$

This procedure ranks these random numbers just as in [Miettinen et.al. 1999] and [Lahdelma et al., 2002]. The distinctness of  $(\gamma_{jr})$  can be ensured by rejecting sets containing identical values. On the other hand ( $j^{\max}$ ) refers to the maximum number of different rank categories of possible solutions for each one of the selected variables. These numbers are used as a sample of stochastic cardinal criteria values ( $\varepsilon_{aj}$ ) so that for each class ( $a$ ), this is set equal to  $(\gamma_{jr})$ ; thus:

$$u_j(X_{aj}) = \varepsilon_{aj}$$

In which the elements of  $\varepsilon \in X$  follow some distribution  $f(\varepsilon)$ :  $X = \{f(\varepsilon)\}$ , where  $(X)$  is the set of stochastic cardinal utilities. All stochastic cardinal assignments must comply with the following requirements in order to find the weight sets:

$$\varepsilon_{aj} = \gamma_{jr}, \quad r = X_{aj}$$

Thus, the stochastic cardinal values and the convex weights accepted will be those that conform to the ordinal ranking of the alternative. In order to ensure that the adequate ranking for each alternative is assigned, the utility functions of each alternative are compared by means of the following function

$$\text{rank}(\varepsilon_a, w) = 1 + \sum_{k=1}^s \rho(\varepsilon_k, w > \varepsilon_a, w), \text{ where: } \rho(\text{true}) = 1, \rho(\text{false}) = 0$$

where each alternative is ranked from 1 to  $m$ . The ranking function's objective is to obtain the set of feasible variable weights associated to each ranking, by assigning cardinal utilities. The mathematical representation is the following:

$$W_a^r(\varepsilon) = \{w \in W : \text{rank}(\varepsilon_a, w) = r\}$$

$$W = \left\{ w \in R^n : w \geq 0, \sum_{j=1}^n w_j = 1 \right\}$$

Finally, the ordinal acceptability index is found; it measure the weighs volume under the ranking ( $r$ ) of each class ( $a$ ). The index is calculated through a multidimensional integral on the distribution of stochastic cardinal utility and favourable weights:

$$b_r^a = \int_X f(\varepsilon) \int_{w_a^r(\varepsilon)} f(w) d_w d_\varepsilon$$

The ordinal ranking for each alternative can be easily obtained by assigning to it the ordinal ranking corresponding to the class to which it belongs, so that:

$$b_r^i = b_r^a \Leftrightarrow i \in F(a)$$

The decision making in the ordinal context is based on an agreement between the characteristics of feasible weights and acceptability indexes; these last ones, support the ordinal ranking of each alternative.

In summary, as a result of the cardinal modeling, a solution set ( $F$ ) of optimal possibilities for the combinatorial problem is obtained, which is made up of ( $m$ ) alternatives that are the input for ordinal analysis. In the process, as a result of the ordinal analysis, a set of ordinal rankings associated with each alternative ( $C_i$ ) is obtained:

$$C_i \in \{1, 2, \dots, m\}, \text{ where: } N(C_i) = c_i \leq m$$

The number of ordinal rankings for each alternative is lower or equal to ( $m$ ), because there may be some alternatives with zero ordinal acceptability indexes. Finally, the number of ordinal rankings for the set of alternatives( $C$ ) is given by:

$$N(C) = \sum_{i=1}^m c_i \leq m^2$$

The central ordinal ranking value and the associated dispersed ranking value for each alternative is given by:

$$R^i = \sum_{r=1}^m r b_r^i$$

$$V^i = \sum_{r=1}^m (r - R^i)^2 b_r^i$$

It's relevant to point out that these two values correspond to the expected ordinal ranking value and the variance ordinal ranking value for the class to which the alternative belongs.

### **Integration Analysis - SMAA**

The stage restricts the SMAA, deterministic version, to two variables.

#### ***Indexes and sets***

$O_r^i$  Integral ranking in ordinal ranking  $r$  for alternative  $i$ , where  $i \in \{1, 2, \dots, m\}$ , and  $r \in \{1, 2, \dots, m\}$

$O_r$  Integral ranking in ordinal ranking  $r$ , where  $r \in \{1, 2, \dots, m\}$

#### ***Other symbols***

$u^{i,r}$  Overall utility in ordinal ranking  $r$  for alternative  $i$

$p(e^i)$  Cardinal utility result variable for alternative  $i$

$b_r^i$  Utility of ordinal result variable in ranking  $r$  for alternative  $i$

$W_r$  Set of feasible weight vectors  $[w_p^r, w_r^r]$  in ordinal ranking  $r$

$w_r^j$  Weight of result variable  $j$  in ranking  $r$

$w_r^{i,b}$  Basic favourable weight vectors in ordinal ranking  $r$  for alternative  $i$

$w_o^{j,r,c}$  Central weight vector of integral ranking  $o$  in ordinal ranking  $r$  for alternative  $i$

$W_r^i$  Set of favorable weight vectors in ordinal ranking  $r$  for alternative  $i$

$d_o^{i,r}$  Integral acceptability index  $o$  in ordinal ranking  $r$  for alternative  $i$

$o_r^i$  Number of integral states in ordinal ranking  $r$  for alternative  $i$

$p_r(e^i)$  Joint integral index in ordinal ranking  $r$  for alternative  $i$

The overall utility of each alternative is based on the typical relative values of its result (that is, the utility of cardinal and ordinal result variables). An additive utility function is used, so that the overall utility of each alternative is the result of summing the products of the feasible convex weights

of its variables and their associated utilities. The following is the corresponding mathematical description:

$$u_r^i = w_r^p p(e^i) + w_r^q b_r^i, w \in W, r \in R,$$

$$W_r = \{w_r \in R^2, w_r \geq 0, w_r^p + w_r^q = 1\}$$

The set of favorable weight vectors  $W_r$  is a one-dimensional simplex in the bi-dimensional weight space. An alternative is dominant when its utility surpasses those of all other alternatives. The problem of finding a set of basic favorable weights for integral ranking  $o$  is solved for each ordinal ranking  $r$  by means of the following LP:

Max 0

Subjected to:  $w_r^p p(e^i) + w_r^q b_r^i \geq w_r^p p(e^h) + w_r^q b_r^h,$

$$h = 1, 2, \dots, m, h \neq i,$$

$$w_r^p + w_r^q = 1,$$

$$w_r^j \geq 0, j \in \{p, q\}$$

The set of basic favorable weights  $w_r^j$  is a convex polytope that can be represented as a convex combination of its vertices:

$$w_r^i = \{w_r \in R^+ : w_r = \sum_b \alpha_r^b w_r^{i,b}, \sum_b \alpha_r^b = 1, \alpha_r^b \geq 0\}$$

The integral acceptability index of each alternative in ordinal ranking  $r$  is defined as the ratio between the alternative's weight volume and its feasible weight volume, both in that same ranking; which gives the alternative an integration ranking  $o$ .

$$vol(W_o^{i,r}) = \int_{w_r^i} dw_r,$$

A low integral acceptability value  $o$  (close to zero) for an alternative in any ordinal ranking  $r$  implies a low number of favorable weight combinations, which makes this (alternative) the dominant one, as presented below:

$$d_o^{i,r} = \frac{vol(W_o^{i,r})}{vol(W_r^i)}, |vol(W_r^i)| > 0.$$

Finally, it can be stated that without any prior knowledge of the decision-makers' expertise, the central weight vector is the best representation of a typically non-biased decision-maker. The central weight vector in ordinal ranking  $r$  for integral ranking  $o$  is defined as the center of gravity of the polytope

$$w_o^{i,r,c} = \frac{\int_{w_r^i} w dw}{\int_{w_r^i} dw}$$

The set of integral rankings in ordinal ranking  $r$  for each alternative  $i$  ( $O_r^i$ ) is given by the following states:  $O_r^i \in \{1, 2, \dots, m\}$ ; which is made up of  $O_r^i$  states

$$N(O_r^i) = O_r^i \leq m.$$

## Appendix B. Qualitative Aspects in Queuing Theory

### Appendix B.1. Service Quality

This aspect concerns a comprehensive estimation of customer satisfaction with the waiting line service. For Hellriegel and Slocum (2004), perceived waiting time is as, or more, important than actual waiting time. Customer service is increasingly seen as the top business priority. It is predicted that in the future, most customers will not accept or tolerate inferior products or services (Maister, 1985).

The first research works on service quality attempted to provide conceptual models of this notion, analyzing what it is and how it can be measured. But this endeavour slowly evolved towards more complex models in which the main objective was not only associated with the conceptualization and measurement of service quality, but also with the analysis of how it is related to other concepts such as the satisfaction of the customer and their future behavioral intentions (Pamies, 2004).

According to Robbins (1996), perception is the sensory process by which subjects organize and interpret their physical impressions, thus giving meaning to the environment around them. For Gordon (1997), perception is “the active process of perceiving reality and organizing it into sensible interpretations or visions”.

In psychological terms, the perception of time is subject to the “Pygmalion effect” or “self-fulfilling prophecy” (Kierein, 2000), through which previous expectations regarding the occurrence of a certain phenomenon are likely to affect perception, because even the behavior of the perceiver contributes to the expected result. The present work considers the following facets of time perception: 1) Anxiety of the consumer, 2) Importance of the service for the client, 3) Service attentiveness.

#### Appendix B.1.1. Anxiety of the Consumer

The effect of anxiety on the perception of time (defined in terms of expected time and perceived time) in a given situation is a clear indicator of the impact of this factor on the tolerance of users to impatience in a waiting line (Dube-Rioux et al., 1989). Maister (1985), who is considered the father of psychological perception in queues, describes that the perception of time can generate user anxiety in the queue, giving the impression that the queue is longer than it actually is. This author mentions that the greater the anxiety, the greater the intolerance.

In order to measure the level of service anxiety, the following methodologies can be used:

- The Magellan Anxiety Scale (EMANS) is a questionnaire containing 15 statements describing physiological sensations and involuntary movements related to tension, discomfort and overwhelm, among others. The person being evaluated reports the frequency with which each of these sensations or movements have been experienced during the last two months (Magaz, 1998, cited in Aparicio, 2009).

- Cognitive-Somatic Anxiety Questionnaire (CSAQ) by Schwartz (1978), consists of 14 items, 7 of which are cognitive (cognitive subscale), while 7 are somatic (somatic subscale). When they feel nervous or anxious, subjects must answer the different items on a Likert-type scale graduated from 1 to 5, according to how they typically experience each of the symptoms (Chorot, 1998, cited in Aparicio, 2009).

#### Appendix B.1.2. Importance of Customer Service

Maister (1985) defines attention as a relevant aspect of service quality perception. It is in this context that the client estimates the value of the service, which to a large extent defines the time they are willing to wait (Maister, 1985). This factor can be evaluated through a 1 to 5 Likert scale in the population that frequents the type of service to be analyzed. Likewise, the impact of the service with regard to the five hierarchical levels of basic human needs (Maslow, 1943) is evaluated.

#### Appendix B.1.3. Service in Face of Customer's Attention

Attention has often been conceived as an attribute of perception, through which we more effectively select the information that is relevant to us. The act of waiting (and expectation in general) focuses on the passing of time, which is therefore perceived as longer than usual. Therefore, in case it is not possible to reduce service time, it is important to create an environment that distracts people's attention in order to improve the service experience (Piaget, 1959; Fraisse, 1973). These authors refer to the subjective perception of time in terms of attention given to it.

Attention can be measured by means of psychomotor reaction tests such as the Toulouse and Pieron Test or Attention Test, which handles the variables time, hits and errors (Dallenbach, 1928).

#### Appendix B.2. Comfort Level

The evaluation of time during service depends on the interaction of several factors such as environmental comfort or discomfort (Gododdy, 1995), the individual characteristics of the subject, the internal attributes of the time period, or the cognitive tasks it requires, plus the time it actually

takes for the person to carry out the activities imposed by the experimental or environmental requirements (Block, 1994). Baker and Cameron (1996) point out that when the spatial distribution of a service environment promotes the perception of social justice, subjects express more positive evaluations of service provision.

A service provider must offer minimum environmental comfort conditions to their customers while they wait in the queue. These include temperature, lighting, seating space and noise levels (Davis, 1995; Heineke et al., 1997). The following is a description of each of the components to be taken into account when assessing this factor.

#### Appendix B.2.1. Lighting

Its main purpose is to facilitate an adequate visualization of the workplace, so that the labor can be carried out in acceptable conditions of efficiency, comfort and safety. This has a favourable impact on people, reducing fatigue and contributing to worker performance and work quality (MAPFRE, 2003).

A synthesis of studies aimed at analysing user comfort during service has been presented by Davis and Heineke (2007). The input data for calculating the lighting required in an area are: 1) Type of activity to be carried out and 2) Dimensions and characteristics of the enclosure to be illuminated. This information influences the choice of lamp types, which depend on chromatic reproduction needs, lighting levels and other conditions. To obtain it, measurements are taken with a luxmeter and then the corresponding calculations are carried out to determine the necessary luminous flux, the power of the lamps and their number and distribution (MAPFRE, 2003). According to MAPFRE (2003), the following are the necessary indicators for the basic calculation of a lighting system:

**Table A1.** Lighting indicators.

Indicator	Calculation formula
Total necessary luminous flux	$\Phi_t = \frac{E_m \cdot S}{\eta \cdot f_c}$ , where $\Phi_t$ = Total necessary luminous flux (lumens) $E_m$ = Average illuminance (lux) $S$ = Area to be illuminated (m <sup>2</sup> ) $\eta$ = Lighting performance $f_c$ = Maintenance factor of the lighting system
Average illuminance $E_m$	It is set according to the visual requirements of the tasks to be carried out, which are specified in the corresponding technical standards, such as article 28 of Colombia's General Ordinance on Safety and Hygiene at Work ( <i>Ordenanza General de Seguridad e Higiene en el Trabajo - OGSHT</i> ).
Lighting performance ( $\eta$ )	$(\eta) = \eta_R \cdot \eta_L$ , where $\eta_R$ = Performance of the room $\eta_L$ = Luminaire performance
Maintenance factor of the lighting system ( $f_c$ )	<p>This factor ranges from 0.5 to 0.8. 0.5 corresponds to dusty rooms with poorly maintained lighting systems. 0.8 corresponds to lighting systems located in clean places, equipped with enclosed luminaires and low luminous depreciation lamps, where frequent cleaning and total or partial lamp replacements are systematically carried out.</p> <p>This factor is determined by loss of luminous flux, loss of reflection or transmission of the lamps due to natural aging or dirt that is deposited on them.</p>

Number of light points (N)	$N = \frac{\Phi_t}{\Phi_n}$ , where $\Phi_t$ = Total necessary luminous flux $\Phi_n$ = Nominal luminous flux of the lamps contained in a luminaire If luminaires with high luminous flux are used, the same total flux is achieved with fewer light points (with a lower total cost of the system), but uniformity is directly affected because the space between luminaires is larger, which gives rise to intermediate zones with less illumination.																		
Average uniformity ( $f_{um}$ )	$f_{um} = \frac{E_{med}}{E_{min}}$																		
Height of luminaires above the working plane (h)	In order to achieve acceptable average uniformity and glare risk levels, the luminaires must be distributed at a certain height (h) above the working plane and a corresponding distance (d) between them Minimum height: $h = \frac{2}{3}d$ Advisable height: $h = \frac{3}{4}d$ Optimum height: $h = \frac{4}{5}d$ In the case of indirect and semi-direct lighting, the optimum height must not be exceeded.																		
Distance between luminaires (d)	It is a function of (h) and the beam opening angle of the luminaire. <table border="1" data-bbox="754 1059 1137 1238" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>Type of luminaire</th> <th>Distance</th> </tr> </thead> <tbody> <tr> <td>Intensive</td> <td><math>D \leq 1.2 h</math></td> </tr> <tr> <td>Semi-intensive</td> <td><math>D \leq 1.5 h</math></td> </tr> <tr> <td>Extensive</td> <td><math>D \leq 1.6 h</math></td> </tr> </tbody> </table> Selection of luminaire type as a function of (h) <table border="1" data-bbox="730 1368 1225 1590" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>Height of the room</th> <th>Type of luminaire</th> </tr> </thead> <tbody> <tr> <td>Up to 4 meters</td> <td>Extensive</td> </tr> <tr> <td>From 4 to 6 meters</td> <td>Semi-extensive</td> </tr> <tr> <td>From 6 to 10 meters</td> <td>Semi-intensive</td> </tr> <tr> <td>More than 10 meters</td> <td>Intensive</td> </tr> </tbody> </table>	Type of luminaire	Distance	Intensive	$D \leq 1.2 h$	Semi-intensive	$D \leq 1.5 h$	Extensive	$D \leq 1.6 h$	Height of the room	Type of luminaire	Up to 4 meters	Extensive	From 4 to 6 meters	Semi-extensive	From 6 to 10 meters	Semi-intensive	More than 10 meters	Intensive
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### Appendix B.2.2. Noise

Noise is an acoustic phenomenon that produces unpleasant auditory sensations, interferes with or impedes some human activity. In the most unfavourable cases, it can lead to the appearance of significant psychological disabilities or limitations (MAPFRE, 2003). As presented by MAPFRE (2003), the achievement of adequate sound levels is an issue that should be taken into account in the project phase of a new premises or enclosure. When this is not done, subsequent efforts are always more expensive and laborious and, sometimes, simply impossible.

The existing noise inside a room has two components: The noise received directly (direct acoustic wave) and the noise reflected on the different surfaces (reflected acoustic wave). The following are the necessary indicators to calculate the interior acoustic adequation of premises according to MAPFRE (2003):

**Table A2.** Noise indicators.

Indicator	Calculation formula
Critical distance ( $r$ ):	$r=0.14\sqrt{R\cdot Q}$ , where: $r$ : Critical distance in meters (within this distance the acoustic conditioning of the walls is not appreciable, because of the dominance of direct waves). $R$ : Constant of the room, in square meters $Q$ : Directivity coefficient.
Absorption ( $A$ )	$A_f=\alpha_f S$ , where: $A$ : Absorption of frequency $f$ in $m^2$ . It quantifies the energy extracted from the acoustic field when the sound wave passes through a given medium or collides with the boundary surfaces of the enclosure. $A_m$ : Average absorption in meters $\alpha_f$ : Absorption coefficient of the material $S$ : Surface of the material in $m^2$
Reverberation time ( $T$ )	$T=0.163\frac{V}{A}$ , where: $V$ : Volume of the premises in $m^3$ $A$ : Absorption of the premises in $m^2$

### Appendix B.2.3. Thermal Load

It refers to the sum of the environmental thermal load, resulting from heat generated in metabolic processes. Its measurement consists in determining the Wet Bulb Globe Temperature (WBGT) Index (MAPFRE, 2003). According to (MAPFRE, 2003), the following are the necessary indicators to calculate the indoor thermal load conditioning of premises:

**Table A3.** Thermal load indicators.

Indicator	Calculation formula	
Wet-Bulb Globe Temperature (WBGT)	The WBGT index consists in the fractional weighing of wet, balloon and sometimes dry temperatures.	
	(WBGT) outdoors (sun exposure)	(WBGT) indoors (in the shade)
	$WBGT = 0.7 Tw + 0.2 Tg + 0.1 Ta$	$WBGT = 0.7 Tw + 0.3 Tg$
	Where: $Tw$ : Natural temperature of wet bulb $Tg$ : Globe temperature (measured through radiation load on a thermometer inside a 6-inch diameter black copper sphere). $Ta$ : Dry bulb temperature (basic ambient temperature; shaded thermometer shielded from radiation).	

### Appendix B.3. Marketing Factors

Marketing is the human activity directed at satisfying needs and wants through exchange (Kotler, 1976). It is also defined as the process of conceiving, planning, executing, pricing, promoting and distributing ideas, goods or services to create exchanges that satisfy the objectives of individuals and organizations" (American Marketing Association - A.M.A., 1985). The most important marketing factors are Price and Competition.

#### Appendix B.4. Transaction Costs

The cost of a service is not only its monetary value, but all its associated costs. The time that users lose when they are waiting in a queue could be interpreted as an additional cost associated with the transaction. In economic theory this is part of the transaction costs, which are necessary to carry out a given economic pursue, but do not add value and should therefore be minimized (Coase, 1937; Williamson, 1975; 1991). The analysis of these costs is based on the assessment of their dimensions, namely asset specificity, transaction measurement uncertainty, and transaction measurement difficulty. As these dimensions rise, it is suggested to implement higher levels of integration between the agents carrying out the economic exchange. Since these dimensions cannot be directly measured, they have to be estimated through Likert tables.

#### Appendix B.5. Competition Level

When several companies deliver equal or substitute services to the market, their price tends to stabilize at a point where no competitor can reduce it any more, since customers will always seek the lowest price, coupled to some quality standards, though. On the contrary, in a monopoly environment where a single company is the provider of a good or service, the price tends to rise at the will of the monopolist, a situation which can only be regulated by the State (Pindyck, 2005). The following is an index of the level of monopoly in a market:

**Table A4.** Competition Indicators.

Indicator	Calculation formula
Lerner's index ( $L$ )	<p>In a market with perfect competition, the market price (<math>P</math>) would be equal to the marginal cost of production (<math>MC</math>). Based on this premise, the Lerner index (<math>L</math>) is defined by the difference between those parameters, divided by the market price (<math>P</math>), in order to establish a fractional measure. <math>L</math> represents the power of a monopoly in the market.</p> $L = \frac{P - MC}{P}$ <p>This index ranges from 0 to 1. Higher values indicate greater market</p>

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