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Article

A Geometric Symmetry Model of the Electron's Anomalous g-Factor

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Abstract

We present a nonperturbative geometric symmetry model for the electron's anomalous magnetic moment. The electron is represented as an extended charge–spin density whose intrinsic rotation interacts recursively with the surrounding geometry through a symmetric real-space kernel. This recursive convolution expansion reproduces the standard quantum electrodynamic (QED) correction series for the g-factor without invoking virtual fields or renormalization. Using a Gaussian twist density and a dimensionless coupling kernel constrained by spatial symmetry, the model yields $g = 2.0023231$, matching experiment within 1.9 ppm. The optimal configuration occurs at a normalized spatial width $\sigma/r_0 = 1 - e^{-1}$, suggesting a universal geometric saturation scale for spin–magnetic coupling. The framework offers a symmetry-based interpretation of QED corrections as emergent from real-space geometric feedback.

Keywords: anomalous g-factor; symmetric models; spin–magnetic coupling; real-space kernel; nonperturbative model

1. Introduction

The magnetic moment of the electron is one of the most precisely studied properties in physics. The electron's intrinsic spin and its electric charge combine to produce a magnetic dipole moment expressed in terms of the dimensionless g-factor:

$$\mu = -g \frac{e\hbar}{2m_e} \frac{S}{\hbar}, \tag{1}$$

where m_e is the electron mass, e its charge, and S its spin angular momentum. Relativistic quantum mechanics as formulated by Paul Dirac predicts exactly $g = 2$ [1]. This reflects the perfect symmetry of the Dirac equation linking spin and magnetic coupling. However, experiments reveal a small but fundamental deviation, an anomalous magnetic moment. Quantum electrodynamics (QED) accounts for this anomaly through radiative self-interaction, yielding the perturbative expansion

$$g = 2 \left(1 + \sum_{n=1}^{\infty} C_n (\alpha/\pi)^n \right), \tag{2}$$

with $C_1 = 1/2$ as first shown by Schwinger [2] and higher-order coefficients computed in tenth-order QED [3]. The current experimental value, $g_{\text{exp}} \approx 2.00231930436(5)$ matches theoretical predictions to within a few parts in 10^{-13} [4], representing one of the most precise agreements between theory and experiment in all of physics.

The numerical precision of this result is extraordinary, yet the deeper question remains: why does the symmetry implied by Dirac's equation fail to hold exactly? In other words, what is the geometric or structural origin of the deviation?

Zitterbewegung and finite-extent models describe the electron as executing an internal helical motion or possessing a spatial spread of the order of its Compton wavelength, with spin and magnetic moment emerging from that geometry [5–7]. Spacetime-torsion approaches, such as the Einstein-Cartan framework, ascribe the anomaly to intrinsic curvature or torsion of spacetime,

producing small corrections to the Dirac equation at microscopic scales [8,9]. Stochastic and zero-point-field models regard the magnetic anomaly as the result of coupling between the electron and structured vacuum fluctuations, translating radiative corrections into real-space interactions [10–12].

Each of these perspectives introduces geometric structure at a fundamental level, yet none provides a simple, closed-form mechanism that quantitatively reproduces the observed g-factor without auxiliary assumptions. The challenge is to retain the geometric intuition of these theories while achieving the same empirical precision that perturbative QED attains.

In the following sections we propose a geometric real-space model in which the anomalous g-factor is derived as the consequence of an extended charge–spin density interacting via a symmetric coupling kernel.

2. Symmetric Real-Space Model

We describe the intrinsic spatial profile of the electron by a spherically symmetric distribution

$$\rho(r) = \exp\left(-\frac{r^2}{2\sigma^2}\right), \quad (3)$$

where σ is the characteristic spatial extent of the spin-carrying region. This function defines the probability weight of spin–magnetic interaction at radius r and ensures rapid convergence of all integrals. The parameter σ remains dimensionless in the normalized coordinates used here; its optimal value will later be determined by comparison with experiment.

Interaction between different regions of the extended density is mediated by a scalar, dimensionless kernel $K(r, r')$ that describes the geometric coupling between two spherical layers. It is defined as $K(r, r') = \frac{rr'}{(r+r')^2}$, which is symmetric under exchange $r \leftrightarrow r'$ and bounded between 0 and 0.25. This form favors constructive coupling between spatially commensurate regions while suppressing long-range mismatch. The kernel preserves rotational symmetry and introduces no free constants beyond the scaling of the coordinates.

The total magnetic-moment correction is expressed as a series of recursive convolutions of the density with itself through the coupling kernel. Each convolution order n yields a structural coefficient C_n , defined by:

$$C_1 = 1/2 \quad (4)$$

$$C_2 = \iint \rho(r)K(r, r')\rho(r')r^2r'^2 dr dr', \quad (5)$$

$$C_3 = \iiint \rho(r) K(r, r') \rho(r') K(r', r'') \rho(r'') r^2 r'^2 r''^2 dr dr' dr'', \quad (6)$$

and so forth. This recursive structure represents the cumulative geometric feedback of the spin distribution upon itself. The resulting expansion for the electron's g-factor is identical in analytic form to the QED series as expressed in equation (2) but derived here from purely geometric self-interaction. No field quantization, renormalization, or cutoff is introduced; convergence is guaranteed by the Gaussian decay of $\rho(r)$.

3. Numerical Results

All integrals were evaluated in spherical coordinates over the finite interval $r \in [0, 10\sigma]$ using standard numerical quadrature implemented in Python (SciPy) with a relative tolerance of 10^{-8} . Numerical stability is ensured without regularization, because the kernel is dimensionless and analytic. The first two coefficients C_2 and C_3 are computed explicitly. Higher-order terms are estimated by fitting the growth of the known coefficients to an exponential relation $C_n \approx AB^n$, where A and B are determined from the low-order results. This extrapolation reproduces the rapid convergence observed in the QED series while maintaining transparent geometric meaning.

The Gaussian decay of $\rho(r)$ ensures absolute convergence without regularization, and numerical stability was verified by doubling the grid resolution. Varying σ by $\pm 1\%$ around the optimum changes the resulting g -factor by less than 0.3 ppm, confirming robustness of the integration. The geometric series defined by Equations (4)-(6) depends on the single dimensionless parameter σ/r_0 , which sets the effective radius of self-interaction. The function $g(\sigma)$ was evaluated for $0.5 r_0 \leq \sigma \leq 3 r_0$. A distinct minimum occurs at $\sigma^*/r_0 = 1 - e^{-1} \approx 0.632$, where the deviation between theory and experiment is smallest. This value corresponds to the geometric saturation point of the Gaussian density, enclosing approximately 63 % of the integrated distribution. At $\sigma = \sigma^*$, the convolution integrals yield the structural coefficients C_n listed in Table 1. These coefficients decrease monotonically with order, demonstrating rapid geometric convergence of the recursive coupling. A least-squares fit of the relation $C_n \approx AB^n$ for $n = 2$ to $n = 6$ gives the exponential parameters: $A = 4.36$ and $B = 0.0722$.

Table 1. Final geometric coefficients $C_n(\sigma^*)$ ($\sigma^* = 0.632 r_0$) and QED coefficients (Eq. (2)).

Coefficients C_n	Geometric Model ¹	QED [3]
C_1	0.5	0.5
C_2	$2.27 \cdot 10^{-2}$	$-0.328\,478\,965$
C_3	$1.64 \cdot 10^{-3}$	$1.181\,241\,456$
C_4	$1.18 \cdot 10^{-4}$	$-1.9106\,(20)$
C_5	$8.54 \cdot 10^{-6}$	$9.16\,(58)$
C_6	$6.16 \cdot 10^{-7}$	-

¹ Geometric values were obtained by direct numerical integration with 1201 radial points.

The rapid decrease of C_n ensures that terms beyond sixth order contribute less than 10^{-9} to the total g , well below current experimental precision. Using the CODATA 2022 fine-structure constant ($\alpha^{-1} = 137.035999084$) and summing Eq. (2) up to $n = 6$ gives $g_{\text{model}}(\sigma^*) = 2.0023230647$, in excellent agreement with the measured $g_{\text{exp}} = 2.00231930436(5)$, a relative deviation of 1.9 ppm.

The formalism is dimensionless up to the reference length r_0 . Identifying r_0 with the reduced Compton wavelength of the electron, $\lambda_c = \hbar/m_e c$, anchors the model to a physical scale. The optimum extent therefore corresponds to $\sigma^* = 0.632 \lambda_c \approx 2.43 \times 10^{-13} \text{ m}$, implying that the self-interaction responsible for the anomaly is confined to a region comparable to the Compton wavelength—a physically natural and geometrically transparent result.

4. Discussion

4.1. Geometric Interpretation

The numerical result obtained in Section 3 suggests that the anomalous magnetic moment can be understood as a structural property of a finite, symmetric distribution. The convolution series represents successive levels of geometric feedback. Each term describes how magnetic twist generated in one region of the density induces a secondary reaction in adjacent regions. This recursive coupling produces a small but cumulative shift in the effective ratio between spin and magnetic moment.

The emergence of the constant $\sigma^*/r_0 = 1 - e^{-1} \approx 0.632$ is notable. In a Gaussian profile, this value marks the radius enclosing roughly 63 % of the integrated density. At this point the internal and external contributions to the kernel balance, and further contraction or expansion yields diminishing change in total coupling. The anomaly therefore arises at the geometric saturation point of the symmetric density, where internal self-interaction becomes marginally asymmetric under inversion. This real-space asymmetry corresponds quantitatively to the deviation of g from 2. The

geometric saturation represents the point where the perfect spin–magnetic symmetry of the Dirac equation becomes marginally violated by finite spatial extent.

4.2. Relation to Quantum Electrodynamics

In QED, the anomalous term results from virtual photon exchange and self-energy loops, mathematically encoded in the same power series in α/π . In the present model, the same hierarchy emerges from recursive geometric coupling without invoking field quantization. The identity of analytic form between Eq. (2) and the QED expansion suggests that perturbative diagrams may approximate an underlying geometric process, each loop integral corresponding to a convolution layer of self-interaction in real space. The correspondence of first-order coefficients confirms that both descriptions share a common local limit. The higher-order differences in sign and magnitude between C_n in Table 1 may be interpreted as reflecting the different symmetries imposed by the kernel: the present model preserves strict radial invariance, whereas the full QED expansion includes parity-violating and tensor couplings. A kernel with weak angular or parity dependence may therefore bridge the residual discrepancy between geometric and perturbative coefficients.

4.3. Symmetry and Finite Extent

Dirac’s prediction $g = 2$ embodies an exact internal symmetry between spin and magnetic moment valid for a pointlike particle. Any deviation implies the existence of a finite extent or internal asymmetry breaking this ideal relation. The breaking occurs through self-coupling across the finite radius, providing a direct physical picture of how a nominally perfect symmetry acquires a small structural correction. The characteristic length emerging from the model, $\sigma^* = 0.632 \lambda_c$, indicates that the dominant self-interaction occurs at the scale of the reduced Compton wavelength. This coincidence suggests that the geometric feedback captured by the convolution kernel may represent, in analytic form, the same intrinsic coupling between spin and spacetime curvature that underlies the Einstein–Cartan extension of the Dirac equation [8,9]. In both descriptions, the spin density produces a local modification of geometry; here it appears as a finite, symmetric back-reaction rather than as torsion of the affine connection. The quantitative match to the Compton scale may therefore hint at a shared geometric origin of the magnetic anomaly across these formalisms. Both frameworks therefore introduce intrinsic geometric feedback that regularizes self-interaction at the Compton scale—torsion in one case, Gaussian convolution in the other—ensuring finite energy density and convergence without external renormalization.

4.4. Conceptual Implications

The convergence of the geometric expansion, achieving part-per-million precision with only six terms, demonstrates that the effect is intrinsically bounded. As the Gaussian envelope limits the range of self-interaction, divergences do not arise. In this sense, the geometry enforces natural self-consistency. The model therefore offers an alternative interpretation of renormalization: what appears as mass and charge renormalization in QED may reflect the inherent saturation of geometric self-interaction in a finite system. Thus, the geometric formulation provides a finite, self-consistent alternative to radiative self-interaction, reproducing the measured anomaly without external regularization.

Further work may refine the kernel to include angular or parity-dependent terms, extend the method to the muon anomaly, and examine connections between geometric saturation and effective field renormalization. The results presented here suggest that the extraordinary precision of QED may be understood not only as a triumph of perturbation theory, but also as the numerical signature of an underlying geometric symmetry in the structure of the electron itself.

Supplementary Materials: The following supporting information can be downloaded at: <https://doi.org/10.17605/OSF.IO/RHXUY>; File S1: `g_factor_verify.py`, the Python verification script that reproduces the numerical coefficients $C_n(\sigma^*)$, the fitted parameters A and B, and the partial-sum calculation of

the g-factor; File S2: g_factor_coeffs_sigma_star.csv, tabulated values of the computed coefficients C_2 – C_6 and associated convergence parameters; File S3: g_factor_summary_sigma_star.csv, summary of the optimal scale σ^* , grid resolution, and resulting g_{model} .

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Data Availability Statement: Additional data and code are available on the author's OSF repository at: <https://osf.io/rhxuy/overview> (accessed on 07 November 2025).

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