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Article

Black Holes as Landauer-Saturating Erasure Channels: Horizon Diagnostics, Area Quantization, and the Measurement Complement

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Abstract

We develop a quantitative framework in which black holes function as erasure channels for exterior classical records at the horizon-side Landauer bound. At asymptotic infinity, greybody scattering turns the full radiation channel into a dissipative filter whose entropy efficiency $\eta_\infty \equiv |dS_{BH}|/dS_{\text{rad}}$ is field-content dependent; for the spin-2 (graviton) channel in Schwarzschild evaporation, $\eta_\infty \approx 0.74$ (Page [17]). Starting from the Cortès–Liddle result that Hawking evaporation saturates the Landauer principle, we make three contributions. First, we define a *Landauer saturation ratio* \mathcal{R}_L as a bookkeeping diagnostic for horizon thermodynamics: Schwarzschild black holes yield $\mathcal{R}_L = 1$ exactly, while the cosmological apparent horizon yields $\mathcal{R}_L = 1/2$ in Trivedi's quasi-local energy accounting. Second, we show that within the standard Bekenstein–Hawking area law and the discrete transition model of Bagchi, Ghosh, and Sen, one-step Landauer-saturating area transitions select the Bekenstein–Mukhanov spacing $\Delta A = 4 \ln 2 l_p^2$; this discrete compatibility result complements, rather than derives, the continuous holographic scaling $S \propto M^2$. Third, we argue that the black hole scrambling time $t_* \sim (\hbar/2\pi k_B T_H) \ln(S_{BH}/k_B)$ provides a partial gravitational analogue of the reversibility time τ_c in quantum measurement: for an old black hole it sets the delay after which newly injected information can begin to reappear in Hawking radiation. We formalize the horizon as an effective coarse-grained erasure channel within fixed-charge sectors via a semiclassical proposition that combines a strict exterior coarse-graining definition with GSL-compatible entropy bookkeeping and horizon-side first-law accounting. We check the supporting identities numerically across the Schwarzschild, Kerr, and Reissner–Nordström parameter spaces, and analyze robustness to the memory burden effect. The framework positions black holes as the thermodynamic complement to quantum measurement: measurement creates classical records by paying Landauer costs; horizons erase exterior access to those records at the quasi-static Landauer limit, while the asymptotic Hawking channel is greybody-dissipative.

Keywords: black hole thermodynamics; Landauer principle; Hawking radiation; erasure channel; Bekenstein–Hawking entropy; area quantization; greybody factors; generalized second law; information paradox; scrambling time; memory burden; cosmological horizon; quantum measurement

1. Introduction

Black hole thermodynamics connects gravity, quantum mechanics, and information theory through a chain of identities that has been progressively tightened over five decades. The Bekenstein–Hawking entropy [1,2]

$$S_{BH} = \frac{k_B c^3 A}{4 \hbar G} \quad (1)$$

assigns an information content proportional to the horizon area. The Smarr relation [3] organizes the mass–energy into thermodynamic potentials. Cortès and Liddle [4] then showed that Hawking evaporation saturates the Landauer bound [33]: each bit of coarse-grained horizon entropy lost releases energy $k_B T_H \ln 2$, the thermodynamic minimum for erasing one bit at temperature T_H .

These results are individually well established. What has been missing is a framework that:

1. elevates Landauer saturation from an observation about one class of horizons to a *diagnostic* for comparing different horizon bookkeeping schemes;
2. connects the continuous (Smarr) and discrete (area-quantized) regimes through a controlled compatibility statement;
3. provides a precise, quantitative bridge between the thermodynamics of *creating* classical records (quantum measurement [31]) and *erasing* them (black hole absorption and evaporation);
4. formalizes “black holes erase classical records” as a limited semiclassical proposition rather than a narrative analogy.

In that comparison, Ref. [31] supplies the measurement-side operational vocabulary: boundary event, record formation, and reversibility time τ_c . Ref. [32] sharpens what is meant by a classical record, namely a stable phase-insensitive register accessible to observers. Neither paper is used as a premise for black hole mechanics; the gravitational claims below rest on the first law, Smarr scaling, and the generalized second law. Taken together, the three papers play distinct roles: Paper A treats record formation, Paper B probability assignment under record-formation constraints, and the present work gravitational coarse-graining of classical records.

This paper provides that framework. We proceed as follows. Section 2 establishes the entropy dictionary separating coarse-grained and fine-grained quantities. Section 3 states the coarse-grained erasure proposition using the generalized second law and relative entropy monotonicity. Section 4 reviews the Smarr identity and Landauer saturation. Section 5 introduces the Landauer saturation ratio \mathcal{R}_L and classifies horizons. Section 6 analyzes the area-quantization compatibility condition. Section 7 examines scrambling time as a partial analogue of τ_c . Sections 8 and 9 extend to Kerr–Newman and analyze memory burden robustness. Section 10 addresses compatibility with unitarity. Section 11 discusses analog gravity tests. Section 13 concludes.

1.1. Relation to Other Approaches

Jacobson [24] derives Einstein’s equations from $\delta Q = T dS$ on local Rindler horizons. Verlinde [25] proposes gravity as an entropic force. Song [26] formalizes a Planck-scale ledger balancing geometric entropy against modular energy flow and Landauer cost. Our framework does not modify the Einstein equations or derive new dynamics. It provides a diagnostic and classificatory scheme built on Landauer saturation, connecting flat-spacetime record formation (Refs. [31,32]) to gravitational horizon thermodynamics. The flat-spacetime measurement bound $\langle Q_{\text{rec}} \rangle \geq k_B T \ln 2 \cdot I(X; Y)$ of Ref. [31] and the phase-insensitive record language of Ref. [32] provide the non-gravitational side of the comparison. The present work does not import their conclusions into gravity; it uses standard black hole mechanics to organize the gravitational side. Song’s causal-horizon ledger [26] writes a local, dimensionless balance for each modular 2π -interval:

$$\frac{\delta A_H}{4G} = -\delta \langle K_\sigma \rangle + \ln 2 \delta N_c, \quad (2)$$

where K_σ is the modular Hamiltonian of the exterior state relative to a reference σ , and N_c counts logically irreversible classical-record updates on a holographic screen. The first term on the right tracks reversible modular-energy flow; the second is the Landauer cost of irreversible bit fixation. We do not assume Eq. (2) below, but the structural parallel with the trilogy is worth noting: Paper A’s calorimetric bound governs the flat-spacetime record-formation cost that populates δN_c , while the present work studies the integrated gravitational counterpart (the Smarr identity $T_H S_{BH} = \frac{1}{2} M c^2$ as the total erasure budget of a Schwarzschild horizon).

2. Entropy Dictionary

To avoid conflation between physically distinct quantities, we define:

- $S_{BH} = k_B c^3 A / (4\hbar G)$: the **Bekenstein–Hawking entropy**, a coarse-grained thermodynamic entropy counting macroscopically indistinguishable microstates consistent with the global charges (M, J, Q) .
- $S(\rho_{\text{rad}})$: the **fine-grained von Neumann entropy** of the Hawking radiation state, tracked by the Page curve [5] and computed by the island formula [6].
- $\chi(\mathcal{E})$: the **Holevo information** of an ensemble \mathcal{E} of exterior classical records, quantifying the maximum classical information accessible to an exterior observer through optimal measurements on the exterior state.
- $N_{\text{bits}} = S_{BH} / (k_B \ln 2)$: the **coarse-grained bit count** of the horizon.

Our accounting uses S_{BH} (coarse-grained). The horizon-side Landauer saturation result of Cort es and Liddle [4] relates dE to dS_{BH} . We do not make claims about $S(\rho_{\text{rad}})$ or its unitarity properties; the Page curve concerns a different quantity.

3. Coarse-Grained Record Erasure

3.1. The Exterior Coarse-Graining Map

Consider an ensemble of exterior classical records $\{p_i, \rho_i\}$ carried by matter outside a black hole, where ρ_i are macroscopically distinguishable states encoding distinct classical bit patterns, and p_i are their prior probabilities. Here “classical record” is used in the same operational sense as in Refs. [31,32]: a stable, observer-accessible, effectively phase-insensitive register, not a fine-grained pure-state label. When this matter falls into the black hole:

1. The system is absorbed; the black hole mass increases by the total energy of the infalling matter.
2. The exterior observer traces over interior degrees of freedom.
3. After equilibration, the exterior macrostate is labeled only by the updated conserved charges (M', J', Q') , regardless of which record ρ_i fell in.

Restrict attention to an ensemble of records that deposits the same asymptotic charge increments $(\Delta M, \Delta J, \Delta Q)$, so that the final macrostate labels are fixed. Define the effective exterior map

$$\mathcal{E}_{BH} : \rho_{\text{ext}}^{(i)} \mapsto \rho'_{\text{ext}} \quad \forall i, \quad (3)$$

where the map includes absorption, interior tracing, and equilibration. This is a many-to-one coarse-graining: distinct initial records produce the same exterior thermodynamic state.

3.2. Main Proposition

Proposition 1 (Semiclassical coarse-grained record erasure). *For an exterior semiclassical observer, black hole absorption defines an effective coarse-graining channel on classically accessible record information. Under this description:*

1. **Classical distinguishability is erased within a fixed-charge sector.** Under the strict thermodynamic coarse-graining defined above, all records in the sector map to the same exterior macrostate, so

$$\chi(\mathcal{E}_{BH}(\mathcal{E})) = 0. \quad (4)$$

More generally, if ensembles with different injected charges are compared, data processing gives $\chi(\mathcal{E}_{BH}(\mathcal{E})) \leq \chi(\mathcal{E})$.

2. **The entropy balance is governed by the generalized second law.** The generalized entropy $S_{\text{gen}} = S_{BH} + S_{\text{outside}}$ satisfies $\delta S_{\text{gen}} \geq 0$ for semiclassical matter across causal horizons [10,11].
3. **Quasi-static Schwarzschild evaporation saturates the horizon-side Landauer ratio.** For a quasi-static Schwarzschild evaporation step, the first law $dE = T_H dS_{BH}$ is exact, so the energy cost per lost coarse-grained bit is

$$dE_{\text{bit}} = k_B T_H \ln 2, \quad (5)$$

the Landauer minimum [4]. This equality is a statement about the horizon-side bookkeeping, not about the entropy of radiation received at infinity.

Proof sketch.

Part 1. Within a fixed-charge sector, absorption, interior tracing, and equilibration map every member of the ensemble to the same exterior macrostate ρ'_{ext} . The output ensemble is therefore trivial, so its Holevo information vanishes: $\chi(\mathcal{E}_{\text{BH}}(\mathcal{E})) = 0$. If one relaxes the fixed-charge restriction and compares sectors with different $(\Delta M, \Delta J, \Delta Q)$, the data-processing inequality [13] still guarantees $\chi(\mathcal{E}_{\text{BH}}(\mathcal{E})) \leq \chi(\mathcal{E})$.

Part 2. Wall [10] supplies the GSL input used here: $\delta S_{\text{gen}} \geq 0$ for semiclassical quantum fields across causal horizons. Casini [12] is cited only as related background showing how relative-entropy positivity bounds exterior matter entropy by modular energy; it is not a second proof of the GSL. The role of Part 2 is therefore simply that the entropy exported to the horizon (coarse-grained) plus the entropy remaining outside (fine-grained) does not decrease.

Part 3. In the semiclassical quasi-static regime $M \gg M_{\text{P}}$, the first law for Schwarzschild evaporation gives $dE/dS_{\text{BH}} = T_H$. Converting to bits: $dE/dN_{\text{bits}} = k_B T_H \ln 2$. This shows horizon-side Landauer saturation in the quasi-static limit. It does not, by itself, prove that the full radiation channel to infinity is thermodynamically reversible. \square

Scope note. This proposition operates at the semiclassical level. It does not require or invoke operator-algebraic constructions of the exterior algebra (cf. Refs. [14,15]), though it is compatible with them. The many-to-one character of \mathcal{E}_{BH} is what makes the process analogous to Landauer erasure, not merely cryptographic inaccessibility: within each fixed-charge sector, distinct initial records produce the same exterior thermodynamic state, reducing the number of exterior-distinguishable states to a single macrostate.

4. The Smarr Identity and Landauer Saturation

4.1. The Factor of $\frac{1}{2}$

For a Schwarzschild black hole of mass M :

$$T_H = \frac{\hbar c^3}{8\pi G M k_B}, \quad (6)$$

$$S_{\text{BH}} = \frac{4\pi k_B G M^2}{\hbar c}. \quad (7)$$

Multiplying:

$$T_H S_{\text{BH}} = \frac{1}{2} M c^2. \quad (8)$$

This is the Smarr relation [3], equivalent to $E = 2T_H S_{\text{BH}}$.

4.2. Origin from Euler Scaling

Since $S_{\text{BH}} \propto E^2$ (holographic scaling), $E(S_{\text{BH}})$ is homogeneous of degree $1/2$ in S_{BH} , and Euler's theorem gives $E = 2T_H S_{\text{BH}}$. If entropy scaled as E^k for some other power k , the relation would be $E = kT_H S_{\text{BH}}$ and the entropy-energy fraction would be $1/k$:

Scaling	Smarr relation	Fraction
$S \propto E^1$ (linear)	$E = T_H S_{\text{BH}}$	1
$S \propto E^2$ (area/holographic)	$E = 2T_H S_{\text{BH}}$	$\frac{1}{2}$
$S \propto E^3$ (volume)	$E = 3T_H S_{\text{BH}}$	$\frac{1}{3}$

The $\frac{1}{2}$ is not a “fundamental symmetry”; it is forced by the area law.

4.3. Landauer Saturation as a Consequence of the First Law

The first law $dE = T_H dS_{BH}$ gives, per bit of coarse-grained entropy:

$$dE_{\text{bit}} = k_B T_H \ln 2. \quad (9)$$

This is the Landauer minimum. Cortès and Liddle [4] established that horizon-side Hawking evaporation saturates this bound. Saturation is exact because the first law is an equality for quasi-static processes.

Greybody factors: source vs. channel.

Greybody factors $\Gamma(\omega)$ modify the Hawking spectrum by suppressing low-frequency modes. This raises the question of whether Landauer saturation holds at asymptotic infinity. At the horizon, the first law $dE = T_H dS_{BH}$ is exact. Greybody factors do not modify this source-side identity; they modify the mapping between the horizon entropy decrease and the entropy carried by radiation that reaches asymptotic infinity.

However, for the *radiation* entropy S_{rad} received at infinity, greybody scattering produces excess entropy: $dS_{\text{rad}} \approx 1.35 |dS_{BH}|$ for the spin-2 (graviton) channel in Schwarzschild evaporation, while other spins yield larger factors (e.g. ≈ 1.50 for spin-1, ≈ 1.64 for spin- $\frac{1}{2}$; computed from the greybody factors of Page [16–18]). Equivalently, for the spin-2 channel the asymptotic entropy efficiency is

$$\eta_\infty \equiv \frac{|dS_{BH}|}{dS_{\text{rad}}} \approx \frac{1}{1.35} \approx 0.74. \quad (10)$$

The energy per bit of *radiation* entropy is therefore $dE/dS_{\text{rad,bit}} \approx 0.74 k_B T_H \ln 2$, which is below the Landauer floor. This is not a violation: the Landauer bound applies to the *source* process (horizon entropy change), not the channel output (radiation entropy). If one judges the process by the asymptotic output alone, the full black-hole-to-infinity channel is not a perfect Landauer saturator. The precise statement is instead that the horizon mechanism saturates Landauer at the source, while the greybody-filtered asymptotic channel is dissipative.

4.4. Integrated Erasure Budget from Landauer Saturation

For Schwarzschild evaporation, the total coarse-grained erasure budget can be written directly as a mass integral:

$$\begin{aligned} N_{\text{bits}}^{\text{tot}} &= \int_0^{M_{\text{init}}} \frac{c^2 dM}{k_B T_H(M) \ln 2} \\ &= \int_0^{M_{\text{init}}} \frac{8\pi G M}{\hbar c \ln 2} dM \\ &= \frac{4\pi G M_{\text{init}}^2}{\hbar c \ln 2} = \frac{S_{BH,\text{init}}}{k_B \ln 2}. \end{aligned} \quad (11)$$

Using the first law, this is equivalently $N_{\text{bits}}^{\text{tot}} = \int dS_{BH}/(k_B \ln 2)$. The result is therefore best read as an integrated erasure-budget identity, not an independent derivation of the area law from Landauer alone. Rather, it shows that once black hole mechanics and the Cortès–Liddle saturation statement are granted, the total horizon-side erasure budget equals the usual Bekenstein–Hawking information content. Kim, Lee, and Lee [19] instead took maximal erasure efficiency as a starting assumption.

For a Schwarzschild black hole ($R = 2GM/c^2$), the same substitution matches the standard Bekenstein expression:

$$I_{\text{max}} = \frac{2\pi R E}{\hbar c \ln 2} = \frac{S_{BH}}{k_B \ln 2}. \quad (12)$$

5. Landauer Diagnostic Across Horizon Types

5.1. The Landauer Saturation Ratio

Not all horizons are thermodynamically equivalent. We define the *Landauer saturation ratio* for any horizon bookkeeping scheme:

$$\mathcal{R}_L \equiv \frac{\delta Q_{\text{hor}}/dN_{\text{bits}}}{k_B T \ln 2} \quad (13)$$

where δQ_{hor} is the heat-like contribution conjugate to horizon entropy in the relevant first law and T is the horizon temperature. For Schwarzschild, $\delta Q_{\text{hor}} = dE$; for Kerr–Newman, $\delta Q_{\text{hor}} = dE - \Omega_H dJ - \Phi_H dQ$. When no exact first-law splitting is available, we state explicitly which quasi-local energy notion is being used. The ratio $\mathcal{R}_L = 1$ indicates exact source-side Landauer saturation in the adopted bookkeeping.

5.2. Black Hole Horizons: $\mathcal{R}_L = 1$

For black holes, the relevant quantity is the entropy-conjugate heat-like term in the first law. Therefore:

$$\frac{\delta Q_{\text{hor}}}{dS_{\text{BH}}} = T_H \implies \mathcal{R}_L = 1 \quad (\text{exact}). \quad (14)$$

For Schwarzschild, $\delta Q_{\text{hor}} = dE$. For Kerr and Reissner–Nordström, δQ_{hor} is obtained after subtracting the rotational and electrostatic work terms. This holds across the parameter space, including near-extremal limits where $T_H \rightarrow 0$: the cost per bit decreases but the source-side ratio remains unity.

5.3. Cosmological Apparent Horizon: $\mathcal{R}_L = 1/2$

Trivedi [7] showed that the cosmological apparent horizon in an expanding FLRW universe does *not* satisfy the Landauer principle. For the Gibbons–Hawking temperature T_{GH} and the Misner–Sharp energy E_{MS} :

$$\frac{dE_{\text{MS}}}{dN_{\text{bits}}} = \frac{1}{2} k_B T_{\text{GH}} \ln 2 \implies \mathcal{R}_L = \frac{1}{2}. \quad (15)$$

The factor of $1/2$ traces to the fact that the Misner–Sharp energy is a quasi-local concept whose derivative $dE_{\text{MS}}/dS_A = T_{\text{GH}}/2$ does not obey an exact first law of the same form as black hole mechanics. In this sense the cosmological result is best read as a warning that the choice of energy notion matters essentially for any Landauer-style horizon bookkeeping.

5.4. Horizon Classification

Within the bookkeeping adopted above, \mathcal{R}_L is a useful diagnostic: $\mathcal{R}_L = 1$ marks horizons whose entropy-conjugate heat term obeys an exact first-law relation, while $\mathcal{R}_L < 1$ flags either an incomplete energy notion or a genuinely non-saturating source/channel description. This transforms Landauer saturation from a property of Schwarzschild evaporation into a comparative tool for horizon thermodynamics, without claiming a universal if-and-only-if theorem for all possible horizons.

For the cosmological case, the conservative interpretation is the energy-bookkeeping one: Trivedi's $\mathcal{R}_L = 1/2$ shows that Misner–Sharp accounting does not furnish the same entropy-conjugate heat variable as black hole mechanics. Whether cosmological horizons are genuinely sub-saturating or whether the discrepancy is entirely prescription-driven is left open here.

Table 1. Landauer saturation ratio \mathcal{R}_L across horizon types in the energy bookkeeping specified for each case. $\mathcal{R}_L = 1$ marks exact saturation of the entropy-conjugate heat term; $\mathcal{R}_L < 1$ signals either a quasi-local energy mismatch or a non-saturating source/channel description.

Horizon type	\mathcal{R}_L	Status
Schwarzschild BH	1 (exact)	Ref. [4]
Kerr BH	1 (exact)	From generalized first law From generalized
Reissner–Nordström BH	1 (exact)	first law Verified with
Cosmological apparent horizon	1/2	Misner–Sharp energy [7] Clausius relation
Rindler horizon	1 (exact)	with boost energy Open; T_{GH} unambiguous but energy choice
de Sitter static patch	Prescription- dependent	(Komar vs. Misner–Sharp) matters
BEC analog horizon	Measurable	Proposed

5.5. Physical Interpretation

The distinction between $\mathcal{R}_L = 1$ and $\mathcal{R}_L < 1$ connects to the long-standing question of whether cosmological horizons are “truly” thermodynamic objects in the same sense as black horizons [27]. Our diagnostic does not resolve that question in full, but it gives a precise bookkeeping criterion: source-side Landauer saturation is natural when the entropy-conjugate energy term is identified cleanly, and it can fail when the adopted quasi-local energy does not supply an exact first-law analogue. The cosmological horizon falls short because the Misner–Sharp quasi-local energy does not capture the full gravitational energy content in the same way that the ADM mass does for asymptotically flat black holes.

6. Area Quantization and the Landauer Spectrum

6.1. Discrete Landauer Constraint

If Hawking evaporation proceeds through discrete area transitions, as suggested by quantum gravity, the Landauer bound constrains which area spectra are physically consistent.

Bagchi, Ghosh, and Sen [8] consider a Schwarzschild black hole with quantized area spectrum $\Delta A = \alpha l_P^2$. For a transition from level $|n+1\rangle$ to $|n\rangle$, the energy released is:

$$\Delta E = T_H \cdot \frac{k_B \alpha}{4}. \quad (16)$$

Landauer saturation requires $\Delta E = k_B T_H \ln 2$, giving:

$$\boxed{\alpha = 4 \ln 2}. \quad (17)$$

This is the Bekenstein–Mukhanov spectrum [9], for which the number of microstates at level n scales as $\Omega(n) \approx 2^n$.

6.2. Joint Constraint: Continuous and Discrete

The counterfactual analysis of Section 4 shows that holographic scaling ($S \propto M^2$, i.e., $k = 2$) produces the 1/2 Smarr fraction and continuous Landauer saturation. The area quantization result adds a discrete counterpart: the specific spacing $\alpha = 4 \ln 2$ makes each single-level transition an optimal one-bit erasure.

Proposition 2 (Conditional compatibility of continuous and discrete saturation). *Within the standard Schwarzschild area law $S_{BH} = k_B A / (4l_P^2)$ and the one-step area-transition model of Bagchi, Ghosh, and Sen, discrete Landauer-saturating transitions select the Bekenstein–Mukhanov spacing $\alpha = 4 \ln 2$. This discrete*

result is compatible with, but does not by itself derive, the continuous holographic scaling $S \propto M^2$ that underlies the Schwarzschild Smarr relation.

Proof. Equation (17) gives $\alpha = 4 \ln 2$ when a single transition is required to saturate the Landauer cost. Under the standard area law, this implies $\Delta S_{BH} = k_B \alpha / 4 = k_B \ln 2$, i.e. one coarse-grained bit per level. Independently, the continuous Schwarzschild Smarr relation relies on the usual holographic scaling $S \propto M^2$. Thus the discrete and continuous pictures are mutually compatible once the standard black hole entropy law is assumed, but the discrete statement does not by itself prove the continuous one. \square

6.3. Connection to Loop Quantum Gravity

The Bekenstein–Mukhanov spectrum also connects to the Immirzi parameter in Loop Quantum Gravity. The standard value $\gamma = \ln 2 / (\pi \sqrt{3})$ can be derived by applying Landauer’s principle to the Boltzmann–Gibbs entropy framework [23]. Barrow entropy and Kaniadakis entropy give modified Immirzi parameters, all derivable from the same Landauer constraint applied to their respective frameworks.

7. Scrambling Time as a Partial Analogue of τ_c

7.1. The Measurement Boundary Event

In the record-formation framework of Ref. [31], τ_c is the *reversibility time*: the temporal boundary between Stage 1 (reversible premeasurement correlation) and Stage 2 (irreversible record formation). Before τ_c , the system–pointer correlation can be reversed; after τ_c , environmental decoherence has stabilized the record and reversal requires microscopic control of the bath. In the circuit-QED implementation, $\tau_c \approx 8 \mu\text{s}$ (set by qubit T_2). Ref. [32] uses the same record-formation setting to motivate phase-insensitive Born probabilities; here we borrow only the timing and stabilization vocabulary, not the Born-rule theorem itself.

7.2. The Gravitational Analogue

For a black hole, the closest analogue of τ_c is the *scrambling time* [20,21]:

$$t_* \sim \frac{\tau_H}{2\pi} \ln\left(\frac{S_{BH}}{k_B}\right) = \frac{\hbar}{2\pi k_B T_H} \ln\left(\frac{S_{BH}}{k_B}\right), \quad (18)$$

where $\tau_H \equiv \hbar / (k_B T_H)$ is the thermal time. This timescale does not play exactly the same role as τ_c in a lab measurement. Rather, it characterizes how quickly newly injected information is delocalized across the black hole degrees of freedom.

For an *old* black hole with access to early radiation and an ideal decoder, Hayden–Preskill suggests the following interpretation:

- **Before t_* :** Newly injected information has not yet been fully mixed into the horizon degrees of freedom, so one should not expect Hayden–Preskill-style recovery from the outgoing radiation.
- **After t_* :** For an old black hole, the injected information can begin in principle to reappear in the Hawking output, provided the observer also has the early radiation and the required decoding power.

This makes the analogy to τ_c suggestive rather than exact. The shared idea is a boundary between reversible/localized bookkeeping and delocalized information processing. In that restricted sense, t_* plays the role of a local irreversibility boundary: before t_* the injected perturbation has not been fully scrambled, whereas after t_* the record is delocalized over horizon degrees of freedom and no simple local reversal analogue remains. Unlike τ_c , however, t_* does not by itself define an observer-independent irreversibility threshold for all black holes; Page-time and decoding assumptions matter essentially.

7.3. Quantitative Correspondence

The key distinction: measurement operates *above* the Landauer bound (the deep-quantum regime has $\Delta Q \sim h\nu \gg k_B T \ln 2$), while Schwarzschild evaporation *saturates* it at the horizon-side bookkeeping level. The analogy is therefore about coarse-grained thermodynamic accounting, not about a literal identity of microscopic mechanisms.

Table 2. Stage correspondence between quantum measurement (Paper A [31]) and black hole absorption (this work).

Concept	Measurement	Black Hole
System	Qubit	Infalling matter Horizon degrees
Record carrier	Resonator photon	of freedom / outgoing modes Exterior quantum
Environment	Absorber ($T = 10$ mK)	fields ($T = T_H$)
Characteristic time	$\tau_c \sim 8 \mu\text{s}$ (T_2)	$t_* \sim (\tau_H/2\pi)$ $\times \ln(S_{BH}/k_B)$ $= k_B T_H \ln 2$
Cost per coarse-grained bit	$\geq k_B T \ln 2$	(horizon-side) Hayden–Preskill
Recovered by	OFF-branch echo Absorber	decoding (old BH) Horizon macrostate
Record after coarse-graining	microstate	(M, J, Q)

8. Extensions to Kerr and Reissner–Nordström

8.1. Generalized Smarr Relation

The generalized Smarr relation [3] for Kerr–Newman black holes is

$$Mc^2 = 2 T_H S_{BH} + 2 \Omega_H J + \Phi_H Q, \quad (19)$$

where Ω_H is the angular velocity and Φ_H is the electrostatic potential at the horizon.

8.2. Erasure with Work Terms

For Kerr and Reissner–Nordström, the first law becomes

$$dE = T_H dS_{BH} + \Omega_H dJ + \Phi_H dQ. \quad (20)$$

The Landauer-relevant piece is the *heat-like term*:

$$\delta Q_H \equiv dE - \Omega_H dJ - \Phi_H dQ = T_H dS_{BH}. \quad (21)$$

This is the energy cost per unit entropy that governs the erasure process. The remaining terms ($\Omega_H dJ, \Phi_H dQ$) are *work terms*: they change conserved charges but do not contribute to the information-erasure cost.

- **Schwarzschild** ($J = Q = 0$): Pure erasure. $T_H S_{BH} = \frac{1}{2} Mc^2$. Landauer saturation exact. $\mathcal{R}_L = 1$.
- **Kerr**: Erasure plus rotational work. $T_H S_{BH} = \frac{1}{2} (Mc^2 - 2\Omega_H J)$. The rotational energy acts as a work reservoir that reduces the entropy-energy fraction. $\mathcal{R}_L = 1$ (first law still exact per unit dS_{BH}).
- **Reissner–Nordström**: Erasure plus electrostatic work. $T_H S_{BH} = \frac{1}{2} (Mc^2 - \Phi_H Q)$. $\mathcal{R}_L = 1$.
- **Extremal limits**: As $T_H \rightarrow 0$, the per-bit cost $k_B T_H \ln 2 \rightarrow 0$, but \mathcal{R}_L remains unity within the formal first-law bookkeeping. The accounting is self-consistent: fewer bits are radiated, each at lower cost, and the total matches the entropy decrease.

The separation $\delta Q_H = T_H dS_{BH}$ (erasure heat) versus $\Omega_H dJ + \Phi_H dQ$ (work) generalizes naturally to extended thermodynamics [28] where $V dP$ terms appear in the first law for AdS black holes.

9. Robustness: Memory Burden and Semiclassical Limits

9.1. The Memory Burden Effect

The memory burden effect [22] argues that after a black hole has evaporated approximately half its mass, quantum backreaction suppresses the evaporation rate. As a representative phenomenological model of this suppression, we adopt

$$\dot{M}_{\text{MB}} = \frac{\dot{M}_{\text{Hawking}}}{[S(M)]^k}, \quad (22)$$

where $k > 0$ is model-dependent. This is not derived from the memory burden literature as a universal rate law, but captures the qualitative feature of entropy-dependent suppression. The question is whether Landauer saturation survives under such a rate modification.

9.2. Per-step Saturation Is Robust

There are two logical possibilities:

1. **Rate-limited but bit-efficient:** The evaporation rate drops, but each emitted quantum still carries $k_B T_H \ln 2$ per bit. The first law $dE = T_H dS_{\text{BH}}$ holds instantaneously; only dS_{BH}/dt is suppressed.
2. **Effective temperature modified:** If the memory burden modifies the effective temperature (e.g., the spectrum deviates from thermal), the per-bit cost changes and \mathcal{R}_L shifts from unity.

Standard memory burden models suggest option (1) at leading order: the suppression enters the rate equation but not the energy-per-bit ratio, because the first law is a state equation valid for each infinitesimal step regardless of process speed. In the explicit $k = 1$ model, once the entropy has decreased to about half its initial value near the Page-time regime, the instantaneous rate is suppressed by an order- $2/S_{\text{init}}$ factor relative to the Hawking value, while the per-step ratio $dE/(T_H dS_{\text{BH}})$ remains unity so long as the emitted spectrum stays thermal up to greybody corrections.

9.3. \mathcal{R}_L as a Probe of Semiclassical Validity

This gives \mathcal{R}_L a new role beyond horizon classification: $\mathcal{R}_L \neq 1$ could signal a departure from the semiclassical source-side first-law description. Deviations from thermality (whether from memory burden, coherence effects in late-stage radiation, or nonperturbative quantum gravity corrections) would shift \mathcal{R}_L away from unity, making the diagnostic a consistency probe for the semiclassical regime.

10. Compatibility with Unitarity

10.1. What Is Lost and What Is Preserved

Our framework distinguishes two quantities (Section 2):

- **Classical records** (coarse-grained): operationally inaccessible to exterior observers after absorption. The many-to-one map into (M, J, Q) is irreversible in the thermodynamic sense. Landauer saturation applies here.
- **Quantum information** (fine-grained): may be preserved in correlations within the Hawking radiation. The global quantum state of (black hole + radiation) may remain pure, consistent with unitarity and the Page curve [5,6].

These are not contradictory. Landauer saturation applies to the coarse-grained classical sector, not necessarily to the fine-grained unitary sector. The “information paradox” arises from expecting both descriptions to track the same quantity.

10.2. Connection to the Page Curve

The Page curve tracks $S(\rho_{\text{rad}})$ (fine-grained). Before the Page time, it tracks the coarse-grained radiation entropy; after the Page time, island contributions cause it to decrease toward zero (unitarity).

Our accounting tracks S_{BH} (coarse-grained, monotonically decreasing during evaporation). The two quantities agree before the Page time and diverge after. The divergence is precisely where

the classical/quantum distinction becomes physically sharp: our framework says “classical records continue to become inaccessible at the Landauer rate”; the Page curve says “quantum information is becoming accessible in radiation correlations.” Both are correct; they track different quantities.

11. Analog Gravity Tests

11.1. The Landauer Ratio Estimator

We define a proposed experimental estimator for analog systems:

$$\widehat{\mathcal{R}}_L = \frac{\int P_{\text{rad}}(t) dt}{k_B \ln 2 \cdot \int T_{\text{eff}}(t) \dot{N}_{\text{bits}}(t) dt} \quad (23)$$

where P_{rad} is the energy flux of entangled Hawking modes and \dot{N}_{bits} is the rate at which entropy-carrying degrees of freedom generate classical bits. $\widehat{\mathcal{R}}_L = 1$ indicates source-side Landauer saturation; $\widehat{\mathcal{R}}_L > 1$ indicates dissipation above the bound. If \dot{N}_{bits} is inferred from detector-side output entropy rather than source-side entropy production, then $\widehat{\mathcal{R}}_L < 1$ can arise from channel entropy inflation or mode conversion, just as greybody scattering can lower the energy per detected radiation bit without violating the source-side Landauer statement.

11.2. BEC Acoustic Horizons

Steinhauer’s experiments [29] on supersonic BEC flow ($T_H \approx 1.2$ nK, entanglement detected at 6σ) provide partial inputs relevant to estimating $\widehat{\mathcal{R}}_L$. In BEC analogs, exact Lorentz invariance breaks down at the healing length ζ , leading to Bogoliubov dispersion. The key experimental question is whether $\widehat{\mathcal{R}}_L$ remains near unity when the Hawking spectrum deviates from exact thermality near the dispersive cutoff.

11.3. Optical Analogues

Fiber-optic horizon simulations [30] through refractive index variations offer a second platform. Whether the effective energy–entropy ratio in such systems matches a Landauer-type bound remains an open experimental question.

12. Scope: What This Paper Does Not Claim

To prevent misreading:

- We do not modify the Einstein equations or derive new gravitational dynamics.
- We do not claim that fine-grained quantum information is destroyed. Our erasure proposition concerns coarse-grained classical records only.
- We do not derive the island formula or make claims about the Page curve beyond noting compatibility.
- We do not claim that analog systems test the full gravitational Smarr identity. The proposed $\widehat{\mathcal{R}}_L$ estimator tests Landauer-scale energy–entropy coupling in systems with effective horizons.
- Papers A and B are used only to motivate the measurement-side comparison and define the notion of a classical record. The black hole claims in this paper do not depend on the Born-rule theorem or the circuit-QED calorimetry proposal.
- The “information and energy accounting” language is an interpretive bookkeeping device, not an ontological claim about localizable gravitational energy density.
- The de Sitter static-patch entry in Table 1 is prescription-dependent and left open. The Rindler entry follows from the Clausius relation with boost energy but is not derived in detail here.

Derived or standard (citable, not novel): first law of BH mechanics, Smarr relation, Landauer saturation of Hawking evaporation [4], Page curve / island formula, GSL [10,11].

New contributions of this paper:

1. The \mathcal{R}_L diagnostic and horizon classification (Table 1)

2. The conditional compatibility statement linking discrete Landauer-saturating area transitions to the standard Schwarzschild area law (Proposition 2)
3. The scrambling time as a partial gravitational analogue of τ_c and the quantitative measurement–BH stage correspondence (Table 2)
4. The coarse-grained erasure proposition (Proposition 1)
5. The integrated horizon erasure-budget identity (Eq. 11)
6. Memory burden robustness analysis and \mathcal{R}_L as a semiclassical validity probe

13. Conclusion

We have developed a framework in which black holes function as horizon-side Landauer-saturating erasure channels for exterior classical records within fixed conserved-charge sectors. At asymptotic infinity, greybody scattering makes the full Hawking channel dissipative rather than perfectly saturating. The framework provides:

1. A **diagnostic tool**: the Landauer saturation ratio \mathcal{R}_L compares the entropy-conjugate heat term across horizon bookkeeping schemes. Black hole horizons yield $\mathcal{R}_L = 1$ when the generalized first law is written in heat-plus-work form, whereas Trivedi’s cosmological apparent-horizon bookkeeping yields $\mathcal{R}_L = 1/2$.
2. A **discrete–continuous bridge**: holographic scaling ($S \propto M^2$) and the discrete Bekenstein–Mukhanov spectrum $\Delta A = 4 \ln 2 l_p^2$ are shown to be mutually compatible once the standard Schwarzschild area law is assumed.
3. A **quantitative measurement–gravity bridge**: the scrambling time $t_* \sim (\hbar/2\pi k_B T_H) \ln(S_{BH}/k_B)$ provides a partial gravitational analogue of Paper A’s reversibility time τ_c . It marks the onset of scrambling and, for an old black hole, the delay after which newly injected information can begin to reappear in Hawking radiation.
4. A **formal erasure proposition**: Proposition 1 frames “black holes erase classical records” via a strict exterior coarse-graining definition whose Holevo corollary ($\chi = 0$ within a fixed-charge sector) is then supported by GSL-compatible entropy bookkeeping and the data-processing inequality.
5. **Robustness**: Landauer saturation is preserved per emission step under the memory burden effect, with \mathcal{R}_L serving as a probe of semiclassical validity.

The supplementary simulation checks the internal consistency of all numerical claims across the Schwarzschild, Kerr, and Reissner–Nordström parameter spaces through 121 checks across 13 groups, every one falsifiable (no hardcoded assertions). These cover Smarr and greybody identities, time-domain evaporation, counterfactual $S \propto M^k$ scaling with negative tests for $k \neq 2$, Kerr and Reissner–Nordström sweeps, the integrated \mathcal{R}_L diagnostic, Page-curve comparison, cross-consistency with Papers A/B, cosmological \mathcal{R}_L derived from the Friedmann equations, the area-quantization sweep, memory-burden rate suppression with \mathcal{R}_L preservation, channel capacity, scrambling-time identities, and Song’s ledger equation (2).

Supplementary Materials: The following supporting information can be downloaded at the website of this paper posted on Preprints.org.

Code Availability: The supplementary simulation code and generated consistency reports are available at <https://github.com/MosesRahnama/paper-c-blackhole-landauer-supplements>.

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