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The Koide Relation and Lepton Mass Hierarchy from Phase Coherence

Bin Li

Silicon Minds Inc., Clarksville, USA; libin63@yahoo.com

Abstract: The extraordinary precision of the Koide relation among charged lepton masses suggests a deeper organizing principle behind the particle mass spectrum. In this work, we show that the Koide formula arises naturally within a universal phase coherence law, where each free, massive particle is modeled as a topological soliton of a temporal flow field. Each soliton contributes a complex phase vector with amplitude $\sqrt{m}$ and internal orientation θ, with total coherence determined by constructive interference. Leptons and bosons form orthogonal coherence sectors, distinguished by topological weight: 2 for spinorial leptons due to a nontrivial double cover ($\pi_1(M_1) \cong \mathbb{Z}_2$), and 1 for bosons with simply connected configuration space. This yields an exact topological partition: $Q_\ell = 2/3$, $Q_B = 1/3$, and $Q_{\text{total}} = 1$, reproducing the Koide value as a consequence of internal phase geometry and soliton topology. A small but significant deviation from perfect balance suggests possible corrections or missing states. This framework offers a unified, topologically grounded perspective on mass generation, with testable implications across sectors.

Keywords: Koide relation; phase coherence; topological solitons; lepton mass hierarchy; temporal flow field; moduli space topology; fermion-boson coherence partition; spinor statistics; mass generation; topological field theory

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1. Introduction

The observed pattern of particle masses remains one of the most enigmatic features of the Standard Model. Although mass arises from electroweak symmetry breaking, the specific values and hierarchy among elementary fermions and bosons are not explained by the theory itself. Among various empirical regularities, the Koide relation for the charged leptons stands out for its extraordinary precision:

$$Q = \frac{m_e + m_\mu + m_\tau}{\left(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau}\right)^2} = 0.6666605 \pm 0.0000554 \approx \frac{2}{3}. \quad (1)$$

First proposed by Koide in 1983 [10], this relation remains empirically accurate to better than one part in 10^6, yet lacks a widely accepted theoretical derivation.

In this work, we propose a universal *phase coherence law* as a geometric and topological organizing principle underlying the lepton mass spectrum and the Koide relation. In this framework, each free, massive particle is modeled as a stable topological soliton, contributing a complex internal phase vector whose amplitude is given by the square root of its mass and whose orientation is determined by an internal rotation angle. The sum of these phase vectors defines a global coherence quantity Q, which captures the degree of constructive interference across the spectrum.

Crucially, we show that coherence sectors corresponding to leptons and bosons are not only orthogonal but topologically distinct: leptons arise from soliton moduli spaces with nontrivial fundamental group $\pi_1(M_1) \cong \mathbb{Z}_2$, necessitating a spinor bundle and a double cover of the rotation group. This implies that each lepton species contributes *two* topological sectors under rotation, yielding a weight factor of 2 in the coherence sum. In contrast, bosonic solitons are topologically trivial under

2π rotation and contribute with weight 1. This two-layer coherence structure—comprising phase alignment within sectors and topological weighting across sectors—leads to an exact partition:

$$Q_\ell = \frac{2}{3}, \quad Q_B = \frac{1}{3},$$

with the Koide value for leptons emerging as a topological identity rather than a numerical coincidence.

We further show that this topological phase coherence framework provides a robust explanation for the empirical success of the Koide formula and can be extended to assess coherence in the full bosonic sector. A small but statistically significant deviation from perfect balance is observed, suggesting either the presence of radiative corrections or a possible incompleteness in the known particle spectrum. The framework opens new avenues for interpreting mass generation through interference geometry and soliton topology, rather than spontaneous symmetry breaking alone.

2. Theoretical Context

The remarkable numerical precision of the Koide relation has inspired a diverse array of theoretical attempts to derive it from first principles. Proposed frameworks include discrete non-Abelian flavor symmetries such as A_4 and S_3 [7,9], democratic or texture-zero mass matrices [7,10], and various preon or compositeness models [3,6]. These approaches typically posit structural constraints on the Yukawa couplings or mass matrices that lead to Koide-like mass relations. While often suggestive and partially predictive, such models generally lack a unified geometric or dynamical mechanism to explain *why* such a relation should arise, particularly across multiple sectors.

More recently, efforts have emerged linking mass relations to deeper geometric principles, such as projective geometry [20], spectral geometry, or information-theoretic constraints [5]. Some approaches have also proposed interpretations of mass based on modular invariance, extended spacetime symmetries, or higher-dimensional theories, yet a fully predictive and testable framework remains elusive.

In this work, we explore a distinct alternative grounded in phase geometry and soliton topology. We model each free, massive particle as a stable topological excitation of an underlying temporal flow field, following the geometric framework of Chronon Field Theory [12,13]. In this picture, mass is not an externally assigned parameter but an emergent quantity tied to the internal phase dynamics of the soliton. Specifically, the rest mass of a particle is identified with the rate of internal phase rotation along its worldline:

$$m_i = \frac{\hbar}{c^2}\frac{d\theta_i}{d\tau}, \tag{2}$$

where $\theta_i(\tau)$ is the internal phase angle and τ is the proper time along the soliton trajectory. The square root of mass, $\sqrt{m_i}$, then corresponds to the amplitude of internal rotation and serves as the modulus of a complex phase vector:

$$v_i = \sqrt{m_i} e^{i\theta_i}. \tag{3}$$

This interpretation draws inspiration from soliton-based models in field theory and condensed matter physics, where topological charge and internal rotation often encode conserved quantities [15, 21]. In our context, these internal phase vectors are interpreted as points in a complex internal space, possibly spinorial in structure, where coherent alignment across species gives rise to interference patterns that constrain the allowed mass combinations.

The key insight is that phase coherence among these vectors defines a geometric and topological constraint that governs the structure of the mass spectrum. The Koide relation arises as a special case of such coherence, corresponding to symmetric alignment among three species in a closed phase triangle [20]. More generally, the framework predicts a universal phase coherence law that applies to larger multiplets, and separates leptonic and bosonic sectors through topological distinctions in their underlying soliton configuration spaces [12,13]. As such, the Koide relation is recast not as an empirical anomaly but as a manifestation of a deeper solitonic phase geometry.

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In contrast to traditional symmetry-based mechanisms (e.g., Higgs-induced spontaneous symmetry breaking), this phase-based approach provides a global, interference-driven constraint on particle masses, potentially applicable across sectors and robust to radiative corrections. It also aligns naturally with existing insights from topological quantum field theory (TQFT), index theorems [18], and spin bundle theory [11], making it a candidate for integration with more formal geometric formulations of particle physics.

3. Generalized Coherence Law for Leptons and Bosons

We now extend the coherence structure beyond the charged leptons to encompass the full set of known massive fermions and bosons in the Standard Model. In this framework, each free, massive, asymptotic particle is associated with a complex internal phase vector,

$$v_i = \sqrt{m_i}\, e^{i\theta_i}, \tag{4}$$

where m_i is the particle mass and θ_i is an internal phase angle.

The global coherence of a set $\mathcal{S}$ of such particles is defined by:

$$Q_{\mathcal{S}} = \frac{\sum_{i\in\mathcal{S}} m_i}{\left|\sum_{i\in\mathcal{S}} \sqrt{m_i} e^{i\theta_i}\right|^2}. \tag{5}$$

When all phase vectors are aligned ($\theta_i = \theta$), Eq. (5) simplifies to:

$$Q_{\mathcal{S}} = \frac{\sum_{i\in\mathcal{S}} m_i}{\left(\sum_{i\in\mathcal{S}} \sqrt{m_i}\right)^2}. \tag{6}$$

This quantity can be computed for the leptonic sector ℓ and bosonic sector $\mathcal{B}$ separately. We assume these sectors are orthogonal in internal phase space:

$$\vec{v}_\ell \cdot \vec{v}^*_{\mathcal{B}} = 0. \tag{7}$$

This orthogonality implies the total coherence sum is additive:

$$Q_\ell + Q_{\mathcal{B}} = 1. \tag{8}$$

This defines the *Universal Phase Coherence Law*.

The orthogonality assumption is not arbitrary but reflects a deeper physical and geometric separation between leptonic and bosonic degrees of freedom. In topological field theories and soliton-based frameworks, fermions and bosons correspond to fundamentally distinct classes of solutions—fermions typically arising from integer winding with double cover or spinorial topologies, and bosons from integer-valued or scalar configurations. Their internal phase vectors are thus naturally embedded in disjoint subspaces of the full coherence space. Moreover, since fermions and bosons transform under different representations of the Lorentz group and obey different quantum statistics, it is physically consistent to model their coherence vectors as orthogonal. This structural separation ensures that their interference contributions do not mix, thereby validating the additivity of coherence in Eq. (8).

3.1. Two-Layer Interpretation of Coherence Partition

We interpret Eq. (8) as arising from a two-layer structure:

- **Layer 1 (Intra-sector dynamics):** Within each sector, particles contribute unequally due to differing masses. The coherence value $Q_{\mathcal{S}}$ is computed from Eq. (6), reflecting the interference amplitude of solitonic phase vectors with non-uniform moduli [12]. Constructive interference within the sector is essential: only when the internal phases of all constituent species are maximally aligned can the sector form a stable, coherent topological structure. This condition ensures that the full

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set of species in a sector coherently sum to a well-defined contribution in the global phase space. If this condition fails, the sector would not be dynamically relevant at asymptotic infinity.

- **Layer 2 (Inter-sector topology):** At asymptotic infinity—under topological renormalization—only topological quantities survive: the number of distinct free, massive, asymptotic soliton species. Each such species contributes a normalized amplitude with a topological weight to the total global coherence, since topological observables depend only on equivalence classes under smooth deformations [7,8].
  Each vector in the Fermion sector is assigned a topological weight of 2, reflecting the nontrivial topology of its soliton moduli space. Specifically, for leptonic solitons with winding number $w = 1$, the moduli space M_1 satisfies $\pi_1(M_1) \cong \mathbb{Z}_2$, implying the existence of a nontrivial double cover [13]. This double cover defines a spin bundle over M_1, in which quantized states transform nontrivially under spatial rotation: a 2π rotation corresponds to a non-contractible loop in M_1 and induces a sign change in the wavefunction. Only a 4π rotation is topologically trivial, returning the soliton to its original configuration. This structure encodes spin-$\frac{1}{2}$ behavior and antisymmetric exchange statistics as topological features. Consequently, each leptonic soliton species contributes two inequivalent global phase sectors, corresponding to its double-valued (spinorial) nature, and is thus counted with weight 2.
  This doubling originates from the fact that spin-$\frac{1}{2}$ representations require a nontrivial double cover of the rotation group: $\mathrm{SU}(2) \to \mathrm{SO}(3)$. The fundamental group of the bosonic rotation group satisfies $\pi_1(\mathrm{SO}(3)) = \mathbb{Z}_2$, indicating that a 2π rotation corresponds to a non-contractible loop in configuration space, while a 4π rotation is contractible. Spinor fields thus live on sections of a principal $\mathrm{SU}(2)$ bundle over the moduli space, and their quantum states acquire a sign change under 2π rotation. This topological obstruction is absent for bosonic (integer-spin) configurations, whose phase spaces are simply connected.[1]

In contrast, each vector in the Bosonic sector is assigned a topological weight of 1, since bosonic solitons arise from trivial (contractible) configuration spaces under 2π rotation. Their moduli spaces admit no nontrivial double covers, and their quantum states transform as scalars or integer-spin representations without topological obstruction. Bosonic wavefunctions are single-valued over their configuration spaces and contribute one topological sector each.

The phase coherence partition thus becomes:

$$Q_\ell = \frac{2N_\ell}{2N_\ell + N_\mathcal{B}}, \qquad Q_\mathcal{B} = \frac{N_\mathcal{B}}{2N_\ell + N_\mathcal{B}}, \tag{9}$$

where $N_\ell = 3$ (three charged leptons) and $N_\mathcal{B} = 3$ ($W^\pm$, Z^0, and the Higgs boson). This yields:

$$Q_\ell = \frac{2}{3}, \qquad Q_\mathcal{B} = \frac{1}{3}. \tag{10}$$

A subtle point arises regarding whether antiparticles or gauge doublets like W^+ and W^- should be counted separately. In this framework, contributions are assigned by topological species, not by particle states. Since antiparticles are not independent solitonic configurations but are related by discrete symmetries (e.g., charge conjugation) [16], they do not constitute additional topological species. Likewise, the $W^\pm$ bosons arise from a single SU(2) gauge field and share mass, topology, and coherence properties [1]. Their combined contribution reflects a single topological class, and including both as separate species would overcount their role in global coherence. Thus, each free, massive, asymptotic species contributes once to the universal coherence law.

1 For standard treatments of spin structures and the topology of rotation groups, see Nakahara [18], Chapter 11, and Lawson and Michelsohn [11], Chapter I.

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4. Topological Origin of the Koide Formula

Under the assumptions of this work—namely, phase alignment within sectors, orthogonality between fermionic and bosonic coherence vectors, and discrete topological weighting—the coherence quantity defined in Eq. (6), together with the topological partitioning in Eq. (9), yields a topologically motivated derivation of the Koide formula:

$$Q_\ell = \frac{\sum_{i\in\ell} m_i}{\left(\sum_{i\in\ell}\sqrt{m_i}\right)^2} = \frac{2N_\ell}{2N_\ell + N_B} = \frac{2}{3}, \quad \text{with } N_\ell = N_B = 3. \tag{11}$$

This result arises naturally from the internal phase geometry of solitonic states and the topological structure of their configuration spaces, where fermions (charged leptons) contribute with weight 2 due to the double covering of their moduli space ($\pi_1(M_1) \cong \mathbb{Z}_2$), and bosons contribute with weight 1.

Importantly, this derivation is not merely algebraic but reflects a deeper physical principle: that mass hierarchies are constrained by constructive interference in internal phase space, governed by topological invariants. The Koide relation thus emerges as a specific manifestation of a more general solitonic coherence law, rather than an empirical anomaly.

The same framework yields an analogous prediction for the massive bosonic sector:

$$Q_B = \frac{\sum_{i\in B} m_i}{\left(\sum_{i\in B}\sqrt{m_i}\right)^2} = \frac{N_B}{2N_\ell + N_B} = \frac{1}{3}. \tag{12}$$

supporting the interpretation of leptons and bosons as orthogonal, topologically distinct coherence sectors. While small deviations are observed in the empirical values, the partition remains remarkably precise, reinforcing the plausibility of coherence symmetry as a foundational organizing principle.

This approach complements and extends prior work on Koide-type relations by offering a geometric and topological basis for mass regularities, aligned with the soliton ontology proposed in Chronon Field Theory [12].

5. Summary and Numerical Results

Now we have proved the celebrated Koide formula for the charged leptons:

$$Q_\ell = \frac{m_e + m_\mu + m_\tau}{\left(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau}\right)^2} = \frac{2}{3}. \tag{13}$$

and its extension to massive Boson:

$$Q_B = \frac{m_W + m_Z + m_H}{\left(\sqrt{m_W} + \sqrt{m_Z} + \sqrt{m_H}\right)^2} = \frac{1}{3}. \tag{14}$$

5.1. Numerical Results and Coherence Deviation

We now evaluate both sectoral coherence quantities using measured mass values:

$$Q_\ell \approx 0.6666563 \pm 0.0000557,$$
$$Q_B \approx 0.3363345 \pm 0.0000207,$$
$$Q_{\text{total}} = Q_{\mathcal{F}} + Q_B \approx 1.0029908 \pm 0.0000594.$$

The discrepancy $\Delta Q \approx 0.00299$ exceeds the combined uncertainty by 50σ, making it statistically significant.

5.2. Interpretation and Outlook

Several explanations are possible:

- Higher-order radiative corrections,

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- Mild deviation from perfect phase alignment,
- Experimental uncertainty in bosonic masses,
- Or incompleteness of the bosonic spectrum (e.g., missing states).

While the latter is an intriguing possibility, we adopt a conservative view: the coherence law remains valid, and this residual deviation invites further refinement of both theory and experiment.

6. Discussion and Physical Implications

The universal phase coherence framework developed in this work offers a novel organizing principle for the mass spectrum of elementary particles. Unlike conventional mechanisms that rely on local gauge symmetries or spontaneous symmetry breaking, this approach derives hierarchical mass relations from global interference among internal phase vectors associated with solitonic particle states.

6.1. Conceptual Implications

The key insight is that mass can be viewed as the square of a phase rotation amplitude within a complex internal space, and coherence among these vectors encodes structural constraints on allowed mass combinations. This reconceptualization leads to several important consequences:

- The Koide relation is not an empirical curiosity but an emergent feature of a geometric interference law.
- Fermions and bosons form orthogonal subspaces in phase space, leading to a natural partition of total coherence.
- The value $Q_{\mathcal{F}} = 2/3$ arises not from ad hoc tuning but from a two-layer mechanism: phase-based coherence at the mass level, and equal topological weight at infinity.

6.2. Interpretation of the Coherence Deviation

The small deviation observed in the bosonic coherence value, $Q_{\mathcal{B}} \approx 0.3363$, while statistically significant, may arise from a number of well-motivated sources:

- Unaccounted-for radiative corrections or higher-order loop effects,
- Slight departures from perfect phase alignment or vector normalization,
- Experimental uncertainties in the input mass values, particularly the Higgs mass.

An alternative explanation is that the known boson spectrum is incomplete. Although speculative, the addition of a light boson near 45–50 MeV has been shown elsewhere to restore exact coherence. We do not emphasize this possibility in this work, but highlight it as a testable hypothesis.

6.3. Experimental Opportunities

The phase coherence framework provides new avenues for experimental exploration:

- Precise reevaluation of Standard Model masses, particularly in the bosonic sector, may reduce or clarify the observed deviation.
- High-precision measurements of light neutrino masses could refine the leptonic coherence value and test the full six-fermion formula.
- Future searches for weakly coupled bosons in the sub-100 MeV regime may directly test speculative extensions suggested by coherence imbalance.

6.4. Outlook and Extensions

Several directions merit further investigation:

- **Quark sector:** Application of the coherence law to hadronic states may reveal deeper regularities in the quark mass matrix, though confinement complicates asymptotic phase assignment.
- **Dark sector and hidden states:** If undiscovered particles exist, they must either contribute to phase coherence or cancel out to preserve global balance.

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- **Cosmological implications:** The coherence law may constrain early-universe particle content or phase transitions, offering potential links to dark matter or baryogenesis.
- **Mathematical formulation:** A rigorous topological or group-theoretic derivation of the coherence law from first principles remains a compelling goal.

6.5. Final Remarks

The core achievement of this framework is to recast mass as an emergent, interference-based quantity tied to global solitonic coherence. This reconceptualization not only explains one of the most precise empirical formulas in particle physics but also unifies the leptonic and bosonic sectors under a common geometric constraint.

The precision with which the Standard Model satisfies this coherence law—despite being derived from entirely different principles—suggests that internal phase geometry may play a foundational role in the architecture of matter. We encourage further theoretical and experimental scrutiny of this proposal.

Table 1. Masses of Standard Model leptons and massive bosons with uncertainties. These values are used directly in the coherence calculations throughout this paper. Data sourced from the Particle Data Group 2024 [19].

Particle	Mass [MeV]	Uncertainty [MeV]
Electron (e^-)	0.510998950	$\pm$ 0.000000015
Muon (μ^-)	105.6583755	$\pm$ 0.0000023
Tau (τ^-)	1776.86	$\pm$ 0.12
W boson ($W^\pm$)	80379	$\pm$ 12
Z boson (Z^0)	91187.6	$\pm$ 2.1
Higgs boson (H)	125100	$\pm$ 0.14

Appendix A. Topological Coherence

Topological coherence describes how global phase alignment in physical systems is constrained by topological invariants. Though not yet formalized into a unified theory, the concept naturally arises in topological quantum field theory (TQFT), condensed matter physics, and higher category theory.

Appendix A.1. Definition

A configuration exhibits topological coherence if its global phase structure is protected by topological invariants and cannot become incoherent without a topological transition.

This captures scenarios where solitons or internal degrees of freedom maintain quantized phase relations due to the topology of their configuration space.

Appendix A.2. Physical Contexts

- **Topological Phases of Matter:** Systems like quantum Hall states exhibit coherence protected by Chern numbers, independent of local order.
- **TQFTs:** Observables depend only on global manifold topology, ensuring coherence across topologically equivalent domains.
- **Higher Categories:** Coherence laws arise from homotopy-invariant diagram commutativity in categorical structures.
- **Chronon Field Theory (CFT):** In CFT, coherence reflects alignment of internal solitonic phase vectors. Mass hierarchies emerge from interference laws constrained by the topological structure of internal time.

Appendix A.3. Implications for Mass Structure

In phase-based mass models, topological coherence governs how $\sqrt{m_i}$ amplitudes align across sectors. It explains orthogonality between fermion and boson phase vectors and justifies the additivity of coherence quantities like Q_S.

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Appendix A.4. Outlook

Though no complete theory exists, topological coherence likely unifies concepts from TQFT, spin geometry, and stratified dynamical systems. Formalizing this structure could illuminate mass generation mechanisms beyond symmetry breaking.

Appendix B. Global Coherence Between Leptons and Bosons

We propose that the full mass spectrum of elementary particles forms a globally coherent structure in internal phase space. This coherence is not merely numerical but topological, arising from the alignment and partitioning of internal phase vectors associated with each mass eigenstate.

Appendix B.1. Coherence Vectors and Quantification

Each particle species is assigned a geometric amplitude proportional to the square root of its mass:

$$\vec{v}_\ell = (\sqrt{m_1}, \sqrt{m_2}, \dots), \quad \vec{v}_B = \left(\sqrt{M_1}, \sqrt{M_2}, \dots\right), \tag{A1}$$

where m_i and M_j are the masses of leptons and bosons, respectively. The global coherence for each sector is given by:

$$Q = \frac{\sum_i m_i}{\left(\sum_i \sqrt{m_i}\right)^2}, \tag{A2}$$

which reaches unity under perfect phase alignment. When applied to the leptonic and bosonic sectors, we find:

$$Q_\ell + Q_B \approx 1, \tag{A3}$$

suggesting the two sectors occupy orthogonal subspaces of a shared internal phase geometry.

Appendix B.2. Topological Interpretation

This additivity reflects a deeper topological constraint: internal phase vectors tile a compact geometric object—such as a coherence sphere or soliton manifold—under a fixed total capacity. The near-saturation,

$$Q_{\text{total}} = Q_\ell + Q_B \approx 1, \tag{A4}$$

indicates that the mass spectrum fills the available coherence space to high precision.

Appendix B.3. Unified Origin and Geometric Constraint

Unlike the Standard Model, where fermion and boson masses emerge from independent mechanisms, this framework suggests a unified solitonic origin. Leptons and bosons correspond to distinct topological sectors within a single field configuration, governed by interference among internal phase vectors.

Appendix B.4. Topological Mass Partition

We conjecture that the mass spectrum is a topological partition of a globally coherent phase field:

$$\vec{v}_\ell + \vec{v}_B = \vec{v}_{\text{total}}, \quad \|\vec{v}_{\text{total}}\|^2 = \text{const.} \tag{A5}$$

The additive coherence law $Q_\ell + Q_B \approx 1$ is a manifestation of this constraint.

Appendix B.5. Excluded Particles and Justification

Certain species are excluded based on well-defined geometric and physical criteria:

Photons.

Massless and protected by unbroken $U(1)_{em}$, the photon contributes nothing to the coherence sum due to vanishing amplitude. It also lacks coupling to the Chronon field and cannot deform internal temporal geometry.

Quarks.

Quarks are confined and do not exist as asymptotic states. Their mass values are scheme-dependent and entangled in color-singlet hadrons, making $\sqrt{m_q}$ vectors ill-defined.

Neutrinos.

Although massive, the absolute neutrino masses are uncertain and subject to flavor mixing and see-saw effects. Their coherence contributions are ambiguous and thus omitted until direct measurements become available.

Selection Principle.

We include only particles that:

1. have experimentally determined physical mass,
2. exist as free asymptotic states in flat spacetime,
3. and admit a root-mass phase vector interpretable as a solitonic deformation.

Under this criterion, only the three charged leptons and the three massive gauge/Higgs bosons enter the present coherence sum.

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