

Article

Fuzzy Ensemble Ideal Solution Based Multi-Criteria Decision-Making Support for Heat Energy Transition in Danish Households

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Abstract: More than 110 countries including 500 cities worldwide have set the goal of reaching carbon neutrality. Heating contributes to most of the residential energy consumption and carbon emissions. The green energy transition of fossil-based heating systems is needed to reach the emission goals. However, the heating systems vary in energy source, heating technology, equipment location, and these complexities make it challenging for households to compare heating systems and make decisions. Hence, a decision support tool that provides a generalized ranking of individual heating alternatives is proposed for households as decision-makers to identify the optimal choice. This paper presents an analysis of 13 heating alternatives and 19 quantitative criteria in technological, environmental, and financial aspects, combines ideal solution based Multi-Criteria Decision Making with 6 weighting methods and 4 normalization methods, and introduces ensemble learning with a fuzzy membership function derived from Cauchy distribution to finalize the final ranking. The robustness of the proposed method is verified by 3 sensitive analyses from different aspects. Air to water heat pump, solar heating and direct district heating are the top three rankings in the final result under Danish national average data. A framework is designed to guide the decision-makers apply this ranking guideline with their practical feasible situations.

Keywords: MCDM; Individual Heating; Fuzzy; Energy Transition; Ensemble

1. Introduction

Climate change is currently considered one of the most significant global crises. Limiting global warming to 1.5°C requires rapid and deep transitions in energy, land, urban and infrastructure, and industrial systems [1]. Governments are seriously taking this into the agenda. More than 110 countries including 500 cities worldwide have set the goal of reaching carbon neutrality [2]. In addition, 78% of European cities have greenhouse gas (GHG) mitigation targets [3]. Challenges have been raised in the long-term planning and decision making of the energy transition for municipalities in order to reach the emission goals [4]. Especially in the Danish context of our research, the municipality has a key role in the national transition to a fossil-free society because strategic energy planning in Denmark is clearly defined as a responsibility of the municipalities.

Heat is the key part of the energy transition. Heating for buildings takes nearly 25% of global energy end-use, where fossil fuels heating is responsible for 8% of global CO₂ emissions [5]. Danish Climate Agreement for Energy and Industry 2020 [6] emphasizes oil and gas boilers must be phased out and replaced by green district heating or electric heat pumps to achieve green heating. This agreement [7] allocates DKK 2.3 billion to support the replacements for the next 10 years, which includes subsidies for heat pumps and free disconnection of gas networks. Hence, there is an increasing need for decision supports to identify the optimal heating choice, especially the green transition path for those

using natural gas and oil as heating sources, in order to achieve the climate commitments of local governments.

Heating systems differ in energy source, heat technology, equipment location, heat carrier, transfer mechanism and heat requirement in the heated spaces [8,9]. Energy sources are the major factor impacting the environment [10], which can be divided as fossil fuels including oil, gas and coal, and renewables including biomass, solar, geothermal, air, water and waste [9]. Apart from the traditional single energy source, research for combinations of multiple energy sources is increasing, like hybrid source heat pumps [11] and district heating [12]. Heat technology has been developed in each category, including fireplace, stove, boiler, heater, heat pump, solar thermal collector and cogeneration. Heating systems can also be classified as local, central and district heating systems by equipment location[8]. Therefore, there are many different kinds of combinations that can be made for a heating system, and the complexity of it would increase the difficulty to make an optimal decision, especially for regular households.

Thus, how one can choose an optimal heating system becomes the next challenge. There are mainly three general aspects that need to be considered when one makes a heating choice, which are financial costs, technical considerations, and climate friendliness. Financial costs are costs covering all usage span, such as cost for purchasing equipment, installation, maintenance and periodical consumption. Technical considerations usually include considerations for heating efficiency, lifespan, difficulties comparing with currently installed old technologies, and other technical problems such as noise level. Climate change calls more and more attention to the environment not only by governments but also residents. A recent survey conducted by Evida, which is a national natural gas supplier in Denmark, shows that over 44% of individuals in Denmark believe climate-friendly energy technology would be one out of three most important features to be considered if they need to buy a new heating technology, and it ranks on the third position among total twelve choices (the top two are both financial costs related)[13]. However, considering too many indicators, especially sometimes contradictory regarding financial cost and climate friendliness, could be a huge challenge. Hence, a decision support instrument, such as Multi-Criteria Decision-Making (MCDM) which is especially effective facing such a circumstance[14], becomes necessary.

Many have used MCDM in evaluating and selecting optimal renewable and non-renewable energy sources [15]. However, there has been little scientific literature focusing on household-level energy technology (Only 14 research assessed renewable energy technologies in households through MCDM in 30 years), although its significant potential for decarbonization is acknowledged [16]. None of them compares more than 7 technology alternatives, which are far less for individuals to make a full comparison towards all potential alternatives, and only one of them focus on Denmark.

This research aims to propose a decision support tool that provides a generalized ranking of individual heating alternatives for decision-makers to identify the optimal choice. The solution is based on an analysis of 13 heating alternatives and 19 quantitative criteria in technological, environmental, and financial aspects. It combines ideal solution based MCDM with 6 weighting methods and 4 normalization methods, and introduce ensemble learning with a fuzzy membership function derived from Cauchy distribution to finalize the final ranking.

This study contributes both theoretically and practically. The theoretical contribution lies in the proposed general framework to support objective decision making by optimizing the rankings from multiple ideal solutions based MCDMs through a fuzzy ensemble approach, in order to achieve the optimal combination and decrease the variance of ranking results from a single MCDM. The practical contribution lies in providing a full, detailed, yet generalized ranking of heating alternatives. It could be used as a guidebook for both residence and municipalities for the heat energy transition. Moreover, the proposed tool can be easily generalized to countries or regions with similar characteristics, based on the individual energy data in that country or region.

2. Methods

Multi-criteria analysis is needed considering sustainability's multi-dimensional nature, energy systems' complexity [17], and variation of households' situations. Thus, numerous studies are applying MCDM methods in energy sectors regarding sustainability analysis. [18] applied MCDM for the development of renewable energy systems on islands. [19] introduced the framework of dividing MCDM application methodological process into 5 steps. It starts from alternative selection, criteria selection, weighting and evaluation, followed by final treatment as the end, which only include sensitive, reliability analysis and Monte Carlo Simulation. [20] shows that recent surveys are focusing on MDDM for sustainability and renewable energy support.

Although there is much development of MCDM towards application in energy sectors, certain limitations of this approach have been identified as well. [21] argued the complexity of energy planning issues regarding the presence of different approaches. The different results of all approaches with uncertain final decision values showed that there was a need for appropriate quantitative techniques to deal with the imprecise information and to evaluate the real effect of uncertainties on the final results, such as ranking the alternatives. [22] argued that results from MCDM could be easily altered because of its alternatives nature in the underlying assumptions, such as criterion weights.

Ensemble learning is an interpretation for the wisdom of the crowd and ensemble methodology can be explained from the tendency of human nature to collect various opinions and information and weigh and combine them in order to make a more complicated and reasonable decision which is believed that aggregation of a group of ideas is better than choosing only one from all [23]. Ensemble learning is introduced to the approach to eliminate the limitations of MCDM mentioned above.

The decision support methodology in this paper is shown in Figure 1 and tested in the Danish heating ranking. It starts with the selected alternatives and criteria according to the defined problem, follows with data preprocessing, and forms alternatives and criteria matrix M_{ij} as input to the next step. Then, the objective weights w_j and normalized matrix \bar{M}_{ij} are calculated and combined with the evaluation methods to generate m ranking data sets R_{m^*i} . Finally, the final ranking \bar{R}_i is aggregated by an ensemble approach using fuzzy membership function derived from Cauchy and logarithm function to support the decision-making process.

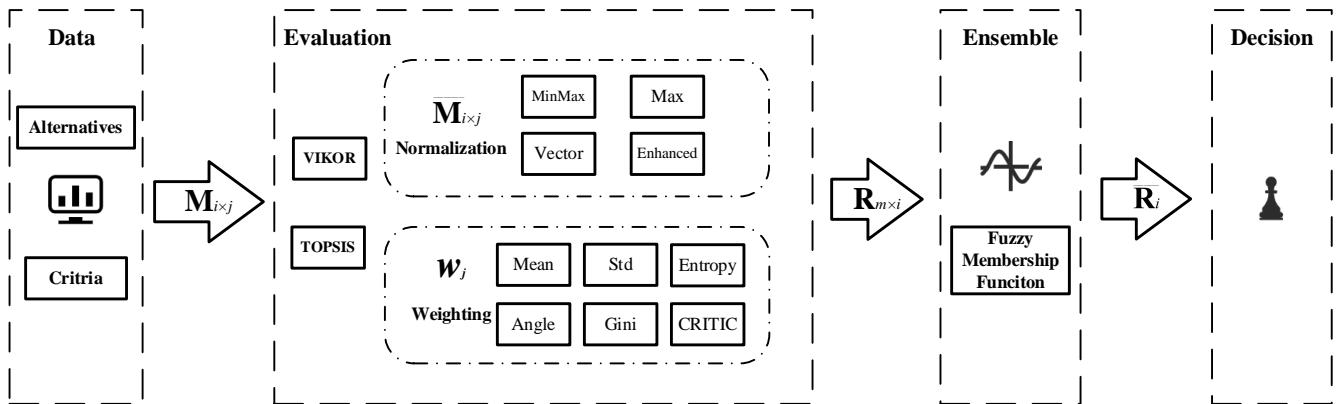


Figure 1. Decision support methodology.

2.1. Multi-Criteria Decision-Making

MCDM has rapidly developed a large number of objective and subjective methods in recent years. Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) becomes one of the most popular approaches in the field of solving energy sector issues due to its relatively rational logic [16]. It selects the alternative with the shortest distance from the positive ideal solution and the longest distance from the negative ideal solution [19]. Vise Kriterijumska Optimizacija Kompromisno Resenje (VIKOR) has the advantage of maximizing “group utility” for the “majority” and minimizing individual regret for the “opponent” [24]. Through a proximity analysis of the ideal solution, VIKOR could develop a ranking with several, usually conflicting alternative criteria [19]. This paper focuses on these two ideal solution based methods with the combination of objective weighting and normalization.

Table 1 shows the equations used for weighting and normalization methods with the corresponding notations defined in Table 2. The normalization equation has separated for a different target. For criteria that target maximization (max), it means the larger the value of that criterion is, the better it is. For criteria that target minimization (min), it means the smaller value of that criterion is, the better it is.

Table 1. Equations for Weighting and Normalization.

Weighting Method	Equation	
Mean	$w_j = \frac{1}{n}$	
Std	$w_j = \frac{\sigma(M_j)}{\sum_{j=1}^J \sigma(M_j)}$	
Entropy	$d_j = 1 + \frac{1}{\ln(\frac{x_{ij}}{\sum_{i=1}^J x_{ij}})} \sum_{j=1}^J \left(\frac{x_{ij}}{\sum_{i=1}^J x_{ij}} \ln(\frac{x_{ij}}{\sum_{i=1}^J x_{ij}}) \right), w_j = \frac{d_j}{\sum_{j=1}^J d_j}$	
CRITIC	$C_j = \sum_{i=1}^I (1 - r_{ij}) \sqrt{\frac{\sum_{i=1}^I (x_{ij} - \frac{1}{n} \sum_{i=1}^n x_{ij})^2}{n-1}}, w_j = \frac{C_j}{\sum_{j=1}^J C_j}$	
Angle	$u_j = \arccos \left(\frac{\sum_{i=1}^I \overline{x_{ij}}}{\ \overline{M_i} \ _2 \ \overline{M_j} \ _2} \right), w_j = \frac{u_j}{\sum_{j=1}^J u_j}$	
Gini	$v_i = \sum_{j=1}^J \frac{ x_{ij} - M_j }{\sum_{i=1}^I x_{ij}}, w_j = \frac{v_j}{\sum_{j=1}^J v_j}$	
Normalization Method	Target: max	Target: min
MinMax	$\overline{x_{ij}} = \frac{x_{ij} - \min(M_i)}{\max(M_i) - \min(M_i)}$	$\overline{x_{ij}} = \frac{\max(M_i) - x_{ij}}{\max(M_i) - \min(M_i)}$
Max	$\overline{x_{ij}} = \frac{x_{ij}}{\max(M_i)}$	$\overline{x_{ij}} = 1 - \frac{x_{ij}}{\max(M_i)}$
Vector	$\overline{x_{ij}} = \frac{x_{ij}}{\ \overline{M_i} \ _2}$	$\overline{x_{ij}} = 1 - \frac{x_{ij}}{\ \overline{M_i} \ _2}$
Enhanced	$\overline{x_{ij}} = 1 - \frac{\max(M_i) - x_{ij}}{\sum_{j=1}^J \max(M_i) - x_{ij}}$	$\overline{x_{ij}} = 1 - \frac{x_{ij} - \min(M_i)}{\sum_{j=1}^J x_{ij} - \min(M_i)}$

Table 2. Nomenclature.

Notation	Definition
I	Alternative set
J	Criteria set
w_j	j_{th} criteria weight
M_i	i_{th} alternative's score vector
M_j	j_{th} criteria's score vector
x_{ij}	i_{th} alternative j_{th} criteria score
$\sigma(\cdot)$	Standard deviation of a dataset
\bar{x}_{ij}	x_{ij} after normalization
n	Criteria number
r_{ij}	Correlation coefficient
\bar{M}_i	M_i after normalization
\bar{M}_j	M_j after normalization

2.2. Ensemble

An ensemble learning methodology originally from machine learning is adopted to generate a stronger ranking by utilising relatively weaker ranking sets from each MCDM. Fuzzy membership distribution with Cauchy distribution is introduced[25].

By assuming a) when ranking = 1, membership = 0.9, b) when ranking = 6, membership = 0.1, and c) when ranking = 13, membership = 0, $f(x)$ can be calculated through Cauchy distribution as shown in Equation (1) and (2).

$$f(x) = \begin{cases} 1 - [1 + a(x - b)^{-2}]^{-1}, & 1 \leq x \leq 6 \\ 1 - c \ln x + d, & 6 \leq x \leq 13 \end{cases} \quad (1)$$

$$a = \frac{225}{64}, b = \frac{3}{8}, c = \frac{1}{10 \ln \frac{13}{6}}, d = \frac{\ln 13}{10 \ln \frac{13}{6}} - 1 \quad (2)$$

All the MCDM rankings will be summarized to the final ranking through a fuzzy transition. The final ranking calculation is shown in Equation (3), where $\text{rank}(A, S)$ means A's ranking in set S, S_{ij} means i_{th} technology and j_{th} ranking.

$$r(i) = \text{rank} \left(\sum_j^{j=J} f(s_{ij}), \sum_i^{i=I} \sum_j^{j=J} f(s_{ij}) \right) \quad (3)$$

3. Results

The represented data for individual heating systems is considered and calculated based on a typical single-family house with average characteristics including 150 m² area, 18MWh annual heat demand, 8kW peak demand, 4kW hot water capacity and average improvements for building before 1979 [26]. This assumption also corresponds to the Evida report [13], where over 75% of participants in Denmark reside in a residential area between 100 and 200 m².

3.1. Alternatives and Criteria Matrix

The heating alternatives are selected based on the principle of representative technologies and available mature products in the Danish market but also international individuals to support a generic comparison [26]. Both possible currently installed heating

technology and available green heating technology are listed, which could provide a clear comparison to the decision-makers. A total of 13 heating technology alternatives has been investigated as shown in Table 3. These comprehensive alternatives could provide a complete ranking to decision-makers.

Table 3. Lists of 13 heating alternatives.

Heating alternatives	ID
Oil boiler	H1
Gas boiler	H2
Biomass boiler (auto)	H3
Biomass boiler (manual)	H4
District heating (indirect)	H5
District heating (direct)	H6
Heat pump (air to water)	H7
Heat pump (air to water, low-price product)	H8
Heat pump (ground source)	H9
Heat pump (gas-hybrid)	H10
Heat pump (air-to-air)	H11
Woodstove	H12
Solar heating	H13

Since the decision-makers are households all over Denmark with different locations, currently installed heating technologies, financial situations, application of regulations and personal preferences, the feasible heating sets considered by each decision-maker vary a lot. Specifically, whether the house is located in a current district heating or future district heating area decides its feasibility of choosing district heating. The functioning condition, remaining lifespan, and possibilities of currently installed heating combining with additional heating also influence the decision-maker's choices. The financial situation will influence much while considering technologies with high investment. Some decision-makers could face regulations that constrain solar panels or other external units (e.g. heat pumps) installation due to architectural concerns. Besides, personal preferences, such as enjoying the warmth of woodstove, would also largely influence the choice of decision-makers. Therefore, the designed framework operate the above individual indicators to the decision-maker's own subjective consideration and keep the ideal solution based ensemble heating ranking objective. Hence, all criteria selected are quantitative indicators regarding technical, environmental, and financial data. Since there are both full-year heating and supplement-only heating alternatives, the expected share of space heating demand and hot tap water demand can be covered by each alternative are listed. Table 4 summarises all selected criteria with their measurement, id, data source and max or min ideal target.

Table 4. Criteria sets for heating alternatives assessment.

Dimension	Criteria	Measurement	ID	Target	Source
Technical	Expected covered space heating demand	share	T1	max	[26]
	Expected covered hot tap water demand	share	T2	max	[26]
	Annual average heat efficiency	net heat/fuel consumption	T3	max	[26]
	Technical economic lifespan	years	T4	max	[26]
	Time spends on manual maintenance	hours/y	T5	min	[26]
	Noise	dB	T6	min	[26,27]
Environmental	SO ₂ emission	g/GJ	E1	min	[26,28,29]
	PM _{2.5} emission	g/GJ	E2	min	[26,28,29]
	NO _x emission	g/GJ	E3	min	[26,28,29]

	CH ₄ emission	g/GJ	E4	min	[26,28,29]
	N ₂ O emission	g/GJ	E5	min	[26,28,29]
	CO ₂ emission	kg/GJ	E6	min	[28–30]
Financial	Nominal equipment investment	k€	F1	min	[26]
	Nominal install investment	k€	F2	min	[26]
	Nominal additional investment	k€	F3	min	[26]
	Fixed electricity cost	€/y	F4	min	[26]
	Fixed operating and maintenance cost	€/y	F5	min	[26]
	Fuel cost	€/GJ	F6	min	[28,31]
	Subsidy	k€	F7	max	[32]

The data is mainly collected from Danish Energy Agency and its subsidiary. The measurement unit is unified through a calculation based on 1 MWh = 1000 kWh = 3.6 GJ and 1 EUR = 7.45 DKK. The emission of district heating and heat pumps are calculated based on the used fuel distribution in district heating production and electricity production and losses on the transition. The noise is set by the influence level and average dB of the available product. The fuel cost is a national average, and the subsidy is for specific heat pumps according to Danish Executive Order. Alternatives and criteria matrix with all quantitative results is provided in Table 5.

Table 5. Alternatives and criteria matrix.

Criteria	H1	H2	H3	H4	H5	H6	H7	H8	H9	H10	H11	H12	H13
T1	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.3	0.3	0.1
T2	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.0	0.0	0.7
T3	0.9	1.0	0.8	0.8	1.0	1.0	3.2	3.0	3.5	2.2	4.9	0.7	0.2
T4	20.0	20.0	20.0	20.0	25.0	25.0	16.0	12.0	20.0	18.0	12.0	20.0	25.0
T5	0.0	0.0	20.0	60.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	50.0	0.0
T6	42.0	42.0	42.0	42.0	25.0	25.0	52.0	67.0	42.0	50.0	64.0	49.0	25.0
E1	6.7	0.4	25.0	25.0	0.7	0.7	9.3	9.8	8.5	11.0	6.1	25.0	0.0
E2	5.0	0.1	14.0	14.0	0.1	0.1	0.3	0.3	0.2	0.3	0.2	28.8	0.0
E3	52.0	20.4	70.0	70.0	4.7	4.7	18.9	19.8	17.2	24.0	12.3	90.0	0.0
E4	0.0	1.0	2.0	2.0	0.2	0.2	0.1	0.1	0.1	0.2	0.1	125.0	0.0
E5	0.0	1.0	4.0	4.0	0.5	0.5	6.3	6.7	5.8	7.5	4.1	4.0	0.0
E6	74.1	49.5	0.0	0.0	2.6	2.6	11.5	12.0	10.5	13.4	7.4	0.0	0.0
F1	4.3	2.7	3.8	4.5	1.3	1.0	6.9	4.6	7.1	6.7	1.2	2.1	2.9
F2	1.3	1.2	1.1	1.9	1.1	1.1	4.0	4.0	7.5	4.6	0.5	0.4	1.2
F3	0.0	2.0	6.0	0.0	3.0	3.0	0.0	0.0	0.0	2.0	0.0	1.6	0.0
F4	9.7	9.7	16.6	13.8	8.3	2.8	0.0	0.0	0.0	0.0	0.0	0.0	3.5
F5	174.9	181.9	357.6	420.3	37.8	46.0	311.4	359.7	287.3	375.8	150.3	145.0	49.0
F6	14.0	10.4	14.2	14.2	26.1	26.1	26.0	26.0	26.0	26.0	26.0	7.5	0.0
F7	0.0	0.0	0.0	0.0	0.0	0.0	3.5	2.8	4.3	0.0	0.0	0.0	0.0

3.2. Weighting Matrix

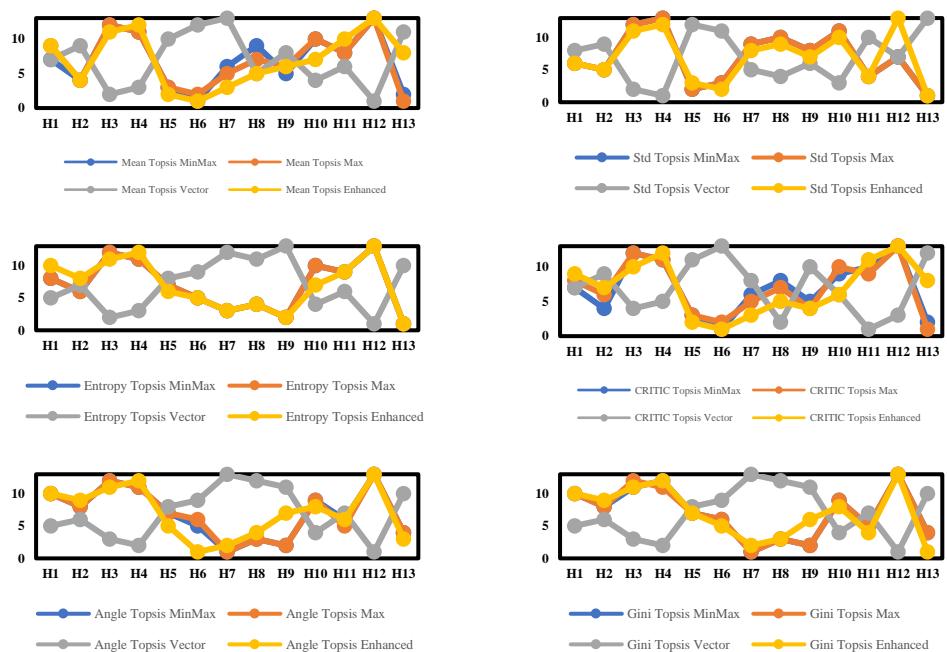
Table 6 lists 5 weighting results from different weighting methods. Most weights have a good balance between different criteria with low deviation. However, a large variance can be noticed in the Std weighting method, which highlight criteria F5 with 44.4% weight. It will be evaluated by sensitive analysis.

Table 6. Weights results of 19 criteria from 5 different weighting methods.

Criteria	Mean	Std	Entropy	CRITIC	Angle	Gini
T1	0.0526	0.0011	0.0033	0.0542	0.0279	0.0188
T2	0.0526	0.0012	0.0804	0.0601	0.0307	0.0200
T3	0.0526	0.0046	0.0086	0.0520	0.0483	0.0447
T4	0.0526	0.0139	0.0007	0.0669	0.0154	0.0126
T5	0.0526	0.0679	0.0804	0.0538	0.0818	0.0911
T6	0.0526	0.0437	0.0014	0.0491	0.0212	0.0180
E1	0.0526	0.0308	0.0804	0.0511	0.0550	0.0551
E2	0.0526	0.0289	0.0804	0.0481	0.0776	0.0849
E3	0.0526	0.0955	0.0804	0.0467	0.0544	0.0536
E4	0.0526	0.1127	0.0804	0.0480	0.0943	0.1012
E5	0.0526	0.0089	0.0804	0.0496	0.0483	0.0478
E6	0.0526	0.0728	0.0804	0.0546	0.0730	0.0755
F1	0.0526	0.0070	0.0049	0.0493	0.0370	0.0345
F2	0.0526	0.0068	0.0103	0.0438	0.0530	0.0488
F3	0.0526	0.0060	0.0804	0.0533	0.0678	0.0728
F4	0.0526	0.0194	0.0804	0.0610	0.0633	0.0685
F5	0.0526	0.4441	0.0061	0.0451	0.0392	0.0371
F6	0.0526	0.0295	0.0804	0.0568	0.0322	0.0271
F7	0.0526	0.0052	0.0804	0.0566	0.0796	0.0882

3.3. MCDM Ranking Results and Analysis

The 48 MCDM rankings resulted from the combination of 6 weighting methods * 2 evaluation methods * 4 normalization methods are listed in Table A1. The ranking results of TOPSIS have shown a larger variance compared to VIKOR. As shown in Figure 2, normalization methods heavily influence the ranking results in the same weighting method, but a similar trend can be noticed in the same normalization methods using different weighting. Besides, the ranking generated by VIKOR apparently will not change through different normalization methods and share similarities when using different weighting methods as shown in Figure 3.

**Figure 2.** Ranking changes comparison.

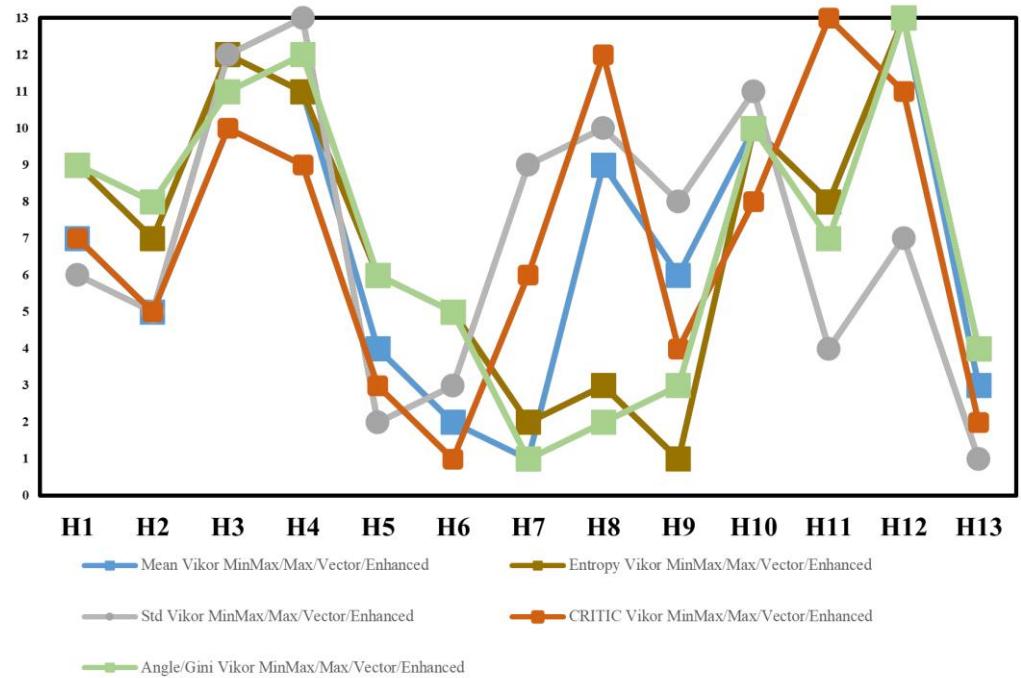


Figure 3. Ranking comparison in VIKOR.

3.3.1. Sensitive Analysis to v in VIKOR

As the final score calculation formula of VIKOR is shown in Equation (4), there is a parameter v to weight the strategy of maximum group utility, which represents the majority of criteria [19]. Normally, v is set to 0.5 to represent the risk-neutral group who weigh the group utility and the individual regret equally. Hence, to generalize this decision making, the value of v has been changed to discover the influence of different preferences existing in a large number of national and even international decision-makers.

$$Q_i = v \frac{S_i - \min(S)}{\max(S) - \min(S)} + (1 - v) \frac{R_i - \min(R)}{\max(R) - \min(R)} \quad (4)$$

where i_{th} negative ideal solution $S_i = \sum w_{ij} \left(\frac{\max(\bar{M}_j) - \bar{x}_{ij}}{\max(\bar{M}_j) - \min(\bar{M}_j)} \right)$

i_{th} positive ideal solution $R_i = \max \left\{ w_{ij} \left(\frac{\max(\bar{M}_j) - \bar{x}_{ij}}{\max(\bar{M}_j) - \min(\bar{M}_j)} \right) \right\}$

The sensitive analysis results are listed in Table 7. The rankings of H3 to H9 are more easily influenced by the change of v , where the ranking will fluctuate in a certain range. However, the range of absolute value change is still logically reasonable on a large scale.

Table 7. Sensitive analysis results of changing parameter v in VIKOR.

V	H1	H2	H3	H4	H5	H6	H7	H8	H9	H10	H11	H12	H13
0.05	9	7	12	13	5	6	1	4	3	10	8	11	2
0.10	9	7	13	12	4	3	1	5	6	10	8	11	2
0.15	9	7	13	12	6	3	1	5	4	10	8	11	2
0.20	9	7	13	12	6	3	1	5	4	10	8	11	2
0.25	9	7	13	12	6	3	1	5	4	10	8	11	2
0.30	9	7	13	12	6	3	1	5	4	10	8	11	2

0.35	9	7	11	12	6	3	1	5	4	10	8	13	2
0.40	9	7	12	11	6	3	1	5	4	10	8	12	2
0.45	9	7	12	11	6	3	1	5	4	10	8	12	2
0.50	9	7	12	11	6	3	1	5	4	10	8	12	2
0.55	9	7	12	11	6	3	1	5	4	10	8	12	2
0.60	9	7	12	11	5	3	1	6	4	10	8	13	2
0.65	9	7	12	11	5	3	1	6	4	10	8	12	2
0.70	9	7	12	11	5	3	1	6	4	10	8	12	2
0.75	8	6	12	10	5	4	1	11	3	9	7	13	2
0.80	8	6	12	11	5	3	2	9	4	10	7	13	1
0.85	9	6	12	11	5	3	4	7	2	10	8	13	1
0.90	9	5	12	11	4	2	8	6	3	10	7	13	1
0.95	9	5	12	11	4	2	6	8	3	10	7	13	1
1.00	9	5	12	11	4	2	6	8	3	10	7	13	1

3.4. Ensemble Results

With the proposed fuzzy Cauchy distribution membership function, the ensemble ranking, shown in the first row of Table 8, is 1. Heat pump (air to water), 2. Solar heating, 3. District heating (direct), 4. Heat pump (ground source), 5. Heat pump (air to water, low-price product), 6. District heating (indirect), 7. Heat pump (air-to-air), 8. Gas boiler, 9. Oil boiler, 10. Heat pump (gas-hybrid), 11. Biomass boiler (manual), 12. Biomass boiler (auto), 13. Woodstove. In order to validate the robustness of the ranking result, sensitive analysis of the possible influencing factors, which are the high variance weight and fuzzy parameter setting, is needed.

3.4.1. Sensitive analysis to with or without high variance weight

Since the Std weighting method generated a high variance weight range from 0.001 to 0.444, a comparison experiment of with and without Std weighting is conducted to evaluate how high variance weight can influence the final ranking result. As shown in Table 8, 6 alternatives change 1 place, H11 and H12 exchange their rankings by changing 2 places, and the rest stays the same ranking. Therefore, the proposed method can handle high weight variance.

Table 8. Final ensemble ranking with or without the ranking results calculated by Std weighting.

	H1	H2	H3	H4	H5	H6	H7	H8	H9	H10	H11	H12	H13
With std	9	8	12	11	6	3	1	5	4	10	7	13	2
Without std	9	7	12	11	6	4	1	5	3	10	8	13	2

3.4.2. Sensitive analysis to the fuzzy ensemble method

The parameter of the original fuzzy Cauchy distribution membership function is set based on the theory that decision-makers prefer top-ranking alternatives more. However, the preference degree could change in a certain range, so the influence of different membership values is tested below.

$$\text{When ranking} = 1, \text{membership} = y_1, a = \frac{25y_1}{(1-y_1)\left(1-\sqrt{\frac{9y_1}{1-y_1}}\right)^2}, b = \frac{6-\sqrt{\frac{9y_1}{1-y_1}}}{1-\sqrt{\frac{9y_1}{1-y_1}}}$$

$$, c = \frac{1}{10\ln\frac{13}{6}}, d = \frac{\ln 13}{10\ln\frac{13}{6}} - 1, \text{then the following } f(x) \text{ can be calculated by}$$

Equation (1) when y_1 vary from 0.6 to 0.999 as shown in Figure 4. The final ensemble ranking results in Table 9 show a rather consistent ranking while facing different membership.

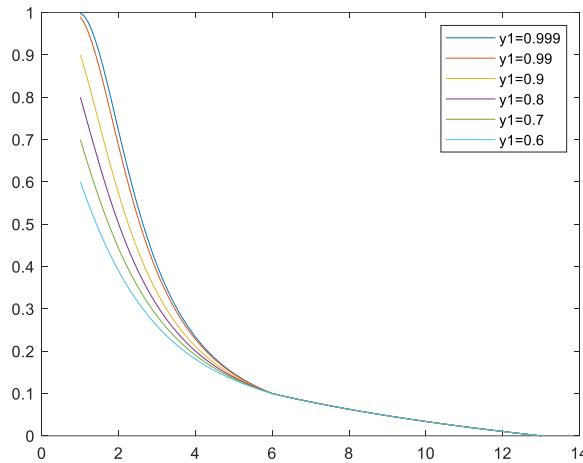


Figure 4. $f(x)$ changes trend in different membership.

Table 9. Final ensemble ranking influenced by different membership.

fuzzy_y1	H1	H2	H3	H4	H5	H6	H7	H8	H9	H10	H11	H12	H13
0.6	10	8	13	11	6	3	1	5	4	12	7	9	2
0.7	10	8	13	11	6	3	1	5	4	12	7	9	2
0.8	11	8	12	10	6	3	1	5	4	13	7	9	2
0.9	11	8	12	10	6	3	1	5	4	13	7	9	2
0.99	12	9	11	10	6	3	1	5	4	13	7	8	2
0.999	12	9	11	10	6	3	1	5	4	13	7	8	2

4. Discussion

Based on previous analysis and comparison between VIKOR and TOPSIS methods [33,34], this paper introduced a framework that combines both methods with multiple weighting and normalization to generate comprehensive rankings for heat alternatives decision support, following by an ensemble learning with fuzzy membership function to generate a final ranking. Although the fuzzy theory has been widely used with MCDM[35], it is mainly used during the weighting and evaluating methods inside MCDM to handle the fuzzy environment and subjective information. In this paper, the fuzzy membership function has been applied after MCDM ranking. Additionally, the ensemble learning methodology is widely used in machine learning, but it has rarely been found in MCDM. The proposed framework achieves a perfect combination under the heating alternatives decision support circumstance. The ensemble learning with fuzzy membership function has shown great robustness by effectively reducing the outcome variance while altering the value of each criterion. Thus, with customized changes in each criterion, it can provide a relatively stable and accurate ranking for heat alternatives decision support.

This framework has provided a full, detailed, yet generalized ranking of heating alternatives. It contains 13 heating alternatives that are available in product markets. Compared to the previous study [36], the comparison is based on more detailed alternatives within the same kind of technology. For example, the biomass boiler is divided into auto and manual categories. The variety ranking of different heat pumps in the final result also proves the necessity to separate heating alternatives for individual households while facing these detailed comparisons in real life. In addition, including all alternatives allows

individuals to compare the currently installed heat technology with possible heat alternatives, so it can provide a clear vision of improvements or gaps. In the meantime, while expanding the comparison list, the ranking result is relatively consistent with previous research on Denmark using single TOPSIS, which concludes solar heating is better than heat pumps and better than wood pellet boilers [36].

Figure 5 shows an example of how this generalized framework derived from the proposed method can apply in real life. In the context of heating transition decisions, the generated ranking could act as a theoretical guideline for both municipalities and households. According to the climate coordinator and energy planner in the partner municipality in Denmark, the households often need advice in changing the heating system. Usually, households turn to local installers for opinions, but these opinions might be subjective and influenced by what they sell and used to install. Therefore, municipalities need a tool to educate households to ask for certain solutions. Hence, the ranking contains all available heating alternatives in the market and can act as a comprehensive guideline. With this information, the decision-makers can easily choose optimal heating from the practical feasible heating sets influence by both personal and realistic factors. Apart from the final rankings, decision-makers can have an overview of the intuitive quantified indicators in summarised alternatives and criteria matrix to match their specific preference.

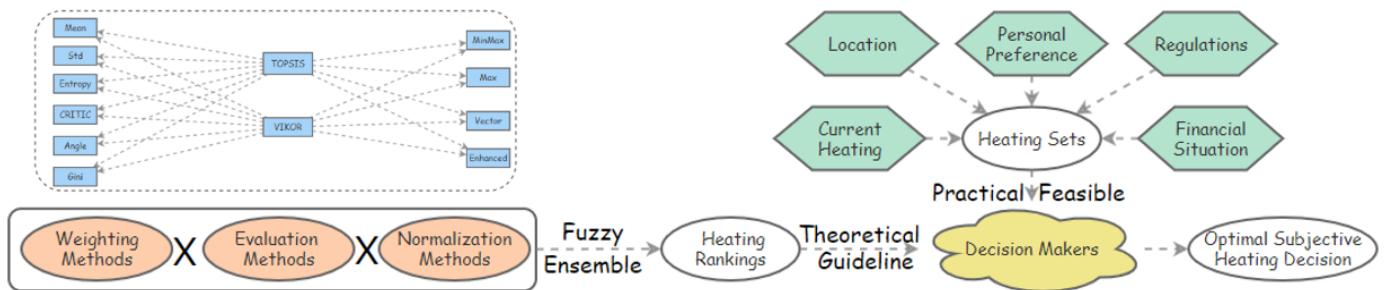


Figure 5. Generalized application framework.

However, the influence of the result on decision-makers has not yet been researched. Hence, further work will focus on the interaction with decision-makers and consider the whole framework in Figure 5 into the modelling to generate tailored optimum alternatives for one's specific needs. Based on this research, an intelligent decision support tool could be developed and open to the public with a user-friendly interface. Rather than using it as a guidebook to compare currently installed technologies with possible heat alternatives, individuals could alter the value of each criterion based on their own situation, so the result would be further customized.

5. Conclusions

While humanity facing the challenge of self-salvation through carbon neutrality, this research contributes to helping the heat energy transition in Denmark and globally. Denmark is one of the pioneer cases within the EU, many municipalities need this kind of decision support instrument to better communicate with residence, in order to achieve a smooth energy transition. The goals of residence, municipalities and researchers are the same with minimizing costs and carbon emissions. Thus, the framework provided in this research and also possible research could greatly help the residence and local municipalities of Denmark to achieve their goals. Furthermore, the generalization of this framework could help more countries in the world.

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funding acquisition, Y.L. All authors have read and agreed to the published version of the manuscript.

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Appendix A

Table A1. 48 MCDM ranking results.

Weighting	Evaluation	Normalization	H1	H2	H3	H4	H5	H6	H7	H8	H9	H10	H11	H12	H13
Mean	Topsis	MinMax	7	4	12	11	3	1	6	9	5	10	8	13	2
Mean	Topsis	Max	9	4	12	11	3	2	5	7	6	10	8	13	1
Mean	Topsis	Vector	10	8	12	11	7	5	1	4	6	9	3	13	2
Mean	Topsis	Enhanced	9	4	11	12	2	1	3	5	6	7	10	13	8
Mean	Vikor	MinMax	7	5	12	11	4	2	1	9	6	10	8	13	3
Mean	Vikor	Max	7	5	12	11	4	2	1	9	6	10	8	13	3
Mean	Vikor	Vector	7	5	12	11	4	2	1	9	6	10	8	13	3
Mean	Vikor	Enhanced	7	5	12	11	4	2	1	9	6	10	8	13	3
Entropy	Topsis	MinMax	8	6	12	11	7	5	3	4	2	10	9	13	1
Entropy	Topsis	Max	8	6	12	11	7	5	3	4	2	10	9	13	1
Entropy	Topsis	Vector	10	9	12	11	7	5	2	3	1	8	6	13	4
Entropy	Topsis	Enhanced	10	8	11	12	6	5	3	4	2	7	9	13	1
Entropy	Vikor	MinMax	9	7	12	11	6	5	2	3	1	10	8	13	4
Entropy	Vikor	Max	9	7	12	11	6	5	2	3	1	10	8	13	4
Entropy	Vikor	Vector	9	7	12	11	6	5	2	3	1	10	8	13	4
Entropy	Vikor	Enhanced	9	7	12	11	6	5	2	3	1	10	8	13	4
Std	Topsis	MinMax	6	5	12	13	2	3	9	10	8	11	4	7	1
Std	Topsis	Max	6	5	12	13	2	3	9	10	8	11	4	7	1
Std	Topsis	Vector	6	5	12	13	3	2	8	10	7	11	4	9	1
Std	Topsis	Enhanced	6	5	11	12	3	2	8	9	7	10	4	13	1
Std	Vikor	MinMax	6	5	12	13	2	3	9	10	8	11	4	7	1
Std	Vikor	Max	6	5	12	13	2	3	9	10	8	11	4	7	1
Std	Vikor	Vector	6	5	12	13	2	3	9	10	8	11	4	7	1
Std	Vikor	Enhanced	6	5	12	13	2	3	9	10	8	11	4	7	1
CRITIC	Topsis	MinMax	7	4	12	11	3	1	6	8	5	9	10	13	2
CRITIC	Topsis	Max	8	6	12	11	3	2	5	7	4	10	9	13	1
CRITIC	Topsis	Vector	10	8	12	11	7	6	1	3	2	9	5	13	4
CRITIC	Topsis	Enhanced	9	7	10	12	2	1	3	5	4	6	11	13	8
CRITIC	Vikor	MinMax	7	5	10	9	3	1	6	12	4	8	13	11	2
CRITIC	Vikor	Max	7	5	10	9	3	1	6	12	4	8	13	11	2
CRITIC	Vikor	Vector	7	5	10	9	3	1	6	12	4	8	13	11	2
CRITIC	Vikor	Enhanced	7	5	10	9	3	1	6	12	4	8	13	11	2
Angle	Topsis	MinMax	10	8	12	11	7	5	1	3	2	9	6	13	4
Angle	Topsis	Max	10	8	12	11	7	6	1	3	2	9	5	13	4
Angle	Topsis	Vector	10	9	12	11	7	6	1	2	3	8	4	13	5
Angle	Topsis	Enhanced	10	9	11	12	5	1	2	4	7	8	6	13	3

Angle	Vikor	MinMax	9	8	11	12	6	5	1	2	3	10	7	13	4
Angle	Vikor	Max	9	8	11	12	6	5	1	2	3	10	7	13	4
Angle	Vikor	Vector	9	8	11	12	6	5	1	2	3	10	7	13	4
Angle	Vikor	Enhanced	9	8	11	12	6	5	1	2	3	10	7	13	4
Gini	Topsis	MinMax	10	8	11	12	7	6	1	3	2	9	5	13	4
Gini	Topsis	Max	10	8	12	11	7	6	1	3	2	9	5	13	4
Gini	Topsis	Vector	10	9	11	12	7	6	1	3	2	8	4	13	5
Gini	Topsis	Enhanced	10	9	11	12	7	5	2	3	6	8	4	13	1
Gini	Vikor	MinMax	9	8	11	12	6	5	1	2	3	10	7	13	4
Gini	Vikor	Max	9	8	11	12	6	5	1	2	3	10	7	13	4
Gini	Vikor	Vector	9	8	11	12	6	5	1	2	3	10	7	13	4
Gini	Vikor	Enhanced	9	8	11	12	6	5	1	2	3	10	7	13	4

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