New Formula of Inertial Force and Tests in Condensed Matter Physics

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Abstract: The energy E takes the place of the gravitational mass m_G to produce gravity [1]. We wonder whether the inertial mass m_i in the inertial force is also replaced by E. The experiments to measure the inertial force $\mathbf{F}_i = -E\frac{\mathbf{a}}{c^2}$ and weight of a carrier are presented. Now we can test the principle of equivalence and study foundations of physics in condensed matter.

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1: Introduction

First, we use the Tolman effect in the accelerating heavy-fermion material to study the formula of the inertial force $\mathbf{F}_i$ on a carrier. Secondly, we compare the result versus the phase shift in a freely falling interferometer caused by the resultant $\mathbf{F}_i + \mathbf{W}$ of the inertial force and gravitational force $\mathbf{W}$ to determine the weight of the carrier.

2. Inertial mass of a carrier

In a magnetic field, the centripetal force $m_i\,\omega_c^2 r$ arises from the Lorentz force qvB,

$$m_i\,\omega_c^2 r = qvB \tag{1}$$

The definition of the the cyclotron frequency ω_c is

$$\omega_c = \frac{2\pi}{T} = \frac{2\pi}{\frac{2\pi r}{v}} = \frac{v}{r} \tag{2}$$

Consequently,

$$\omega_c = \frac{qB}{m_i} \tag{3}$$

In quantum mechanics, the Hamiltonian in a magnetic field is

$$\mathbf{H} = \frac{(\mathbf{p} - q\mathbf{A})^2}{2m_i} \tag{4}$$

The eigenvalue is the Landau energy

$$\left(n + \frac{1}{2}\right)\hbar\omega_c \qquad (\,n = 0,1,2,3......\,) \tag{5}$$

In vacuum, the rest mass of an electron is $m_0 = 9.1\times10^{-31}kg$.

$$\omega_c \approx \frac{qB}{m_0} \quad (v << c) \tag{6}$$

$$m_i = m_0 \tag{7}$$

In condensed matter, the experimental result is

$$\omega_c = \frac{qB}{m^*} \tag{8}$$

It indicates that the inertial mass is

$$m_i = m^* \tag{9}$$

in this case. It is nonsense to say that the real mass of an electron is still $m_0 = 9.1\times10^{-31}kg$ but behaves as if it is m^*. m^* in condensed matter is as real as $9.1\times10^{-31}kg$ in vacuum determined by the same physical laws. Actually, the vacuum is not empty and $m_0 = 9.1\times10^{-31}kg$ is not a bare mass either in the quantum field theory. It is a effective mass in vacuum, just like m^* in condensed matter.

3. Tolman effect and inertial mass

When a conductor or superconductor ceases to rotate, the electric field $\mathbf{E}$ caused by the inertial force is thought to be [2][3]

$$q\mathbf{E} = -m_i\mathbf{a} \tag{10}$$

Thus, the electric current and total electric charge are proportional to the inertial mass-to-electric charge ratio

$$\frac{m_i}{q} \tag{11}$$

Substituting Eq.(9) into Eq.(11),

$$\frac{m_i}{q} = \frac{m^*}{q} \tag{12}$$

In a conventional material such as the normal metal[2],

$$\frac{m^*}{q} \approx \frac{m_0}{q} \tag{13}$$

The effective mass m^* of a heavy fermion can be 1000 times larger than $m_0 = 9.1 \times 10^{-31} kg$ [4][5]. This feature helps us measure the quantity,

$$\frac{m^*}{q} >> \frac{m_0}{q} \tag{14}$$

4. Total energy of a carrier

The energy of a slow electron in vacuum is

$$mc^2 = m_0 c^2 + E_k \approx m_0 c^2 = 9.1 \times 10^{-31} kg\, c^2 = 0.51 \times 10^6 e\mathrm{V} \tag{15}$$

$E_k << m_0 c^2$ is the kinetic energy. In a material, the total energy of the electron is

$$mc^2 + U = m_0 c^2 + E_k + U \tag{16}$$

U is the sum of potential energies. On the Earth, the strongest interaction exists in the nucleus. The average binding energy of most of nuclei does not exceed $10\ million\ e\mathrm{V}$

and the ratio to the rest energy of a nucleon is about $\frac{10\ million\ e\mathrm{V}}{1000\ million\ e\mathrm{V}} \approx 1\%$. The interactions in condensed matter are weaker and the ratio should be lower. That is to say,

$$U << m_0 c^2 \tag{17}$$

$$E_k + U << m_0 c^2 \tag{18}$$

$$mc^2 + U = m_0 c^2 + E_k + U \approx m_0 c^2 \tag{19}$$

One can use the positron annihilation technique (PAT) to verify Eq.(19). The total energy of the incident positron and an electron in the material is around

$$m_0 c^2 + m_0 c^2 \approx 10^6 e\mathrm{V} \tag{20}$$

instead of

$$m^* c^2 + m_0 c^2 \approx m^* c^2 \approx 1000 m_0 c^2 = 500 \times 10^6 e\mathrm{V} \tag{21}$$

Eq.(19) and Eq.(8) imply that the inertial mass $m_i = m^*$ and $\frac{E}{c^2}$ of a carrier in condensed matter are unequal.

5: Tolman effect and energy

In contrast to Eq.(1), the inertial force is the cause of the Tolman effect where the acceleration is known and the electric field $\mathbf{E}$ is the outcome. According to the new equation

$$q\mathbf{E} = -E \frac{\mathbf{a}}{c^2} \tag{22}$$

the experimental results should be in proportion to the energy-to-electric charge ratio

$$\frac{E}{qc^2} \tag{23}$$

In condensed matter (Section.4),

$$\frac{E}{qc^2} = \frac{mc^2 + U}{qc^2} \approx \frac{mc^2}{qc^2} \approx \frac{m_0 c^2}{qc^2} = \frac{m_0}{q} \tag{24}$$

Namely, the coefficients are the same in both conventional materials and heavy-fermion materials. To exclude the possibility that the inertial mass of a carrier in condensed matter is always $m_i \approx m_0$ rather than $m_i = m^*$, an electric scalar potential ϕ ($q\phi >> U$) can be applied (Section.4 in [1]). The following result

$$\frac{E}{qc^2} = \frac{mc^2 + U + q\phi}{qc^2} \approx \frac{mc^2 + q\phi}{qc^2} \approx \frac{m_0 c^2 + q\phi}{qc^2} = \frac{m_0}{q} + \frac{\phi}{c^2} \tag{25}$$

is to distinguish $-E\dfrac{\mathbf{a}}{c^2}$ from $-m_i\mathbf{a}$.

6: Weight of a carrier

An interesting question arises, what the role the effective mass m^* can play in a gravitational interaction. Or, whether a heavy fermion is really heavy ? Taking into account Eq.(9) and the universal recognition $m_G = m_i$, the gravitational mass of a carrier is

$$m_G = m_i = m^* \tag{26}$$

In Newton's law of gravitation, the weight of a heavy fermion on the surface of the Earth whose mass is M should be

$$G\frac{Mm^*}{r^2} = m^* g \qquad (G = 6.674\times10^{-11}\,Nm^2kg^{-2},\ g = \frac{GM}{r^2} = 9.8ms^{-2}) \tag{27}$$

It is much heavier than a free electron. However, we retell the law in terms of the energy [1]. Substituting Eq.(15), the force between the Earth and a slow electron in vacuum whose energies are Mc^2 and mc^2 respectively can be written as

$$G'\frac{Mc^2\,mc^2}{r^2}=G\frac{Mm}{r^2}\approx G\frac{Mm_0}{r^2}=m_0 g \qquad (G'=\frac{G}{c^4}=8.262\times10^{-45}N^{-1}) \quad (28)$$

Note that the energy in a material is Eq.(19), the weight of a carrier is

$$G'\frac{Mc^2(mc^2+U)}{r^2}\approx G'\frac{Mc^2\,m_0c^2}{r^2}=G\frac{Mm_0}{r^2}=m_0 g \quad (29)$$

A carrier is as light as a free electron, although m^* can be extremely large, negative and even anisotropic.

7: How to weight a carrier

In a stationary interferometer (Section.6 in [1]), the momenta p_0 and p of a particle at different heights lead to a phase shift ϑ between 4' and 1 (Fig.1)

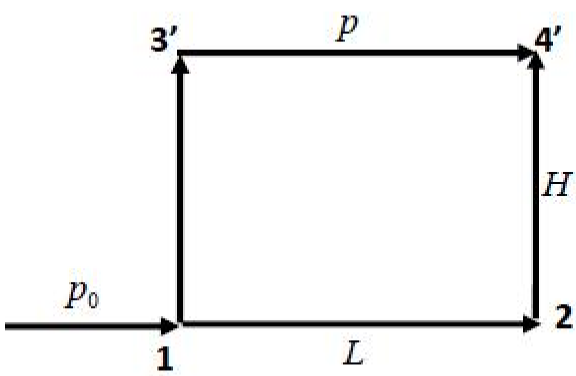

Figure.1 Schematic diagram

$$\vartheta = \frac{p_0}{\hbar}\overline{12} - \frac{p}{\hbar}\overline{3'4'} = \frac{p_0 - p}{\hbar}L \tag{30}$$

In a stationary superconductor, the carrier does not drop because it is dragged by the crystal lattice. When the system is in free fall, a carrier has the same gravitational acceleration

$$\mathbf{a} = \mathbf{g} \tag{31}$$

of the crystal. Here, the carrier behaves as if it is a free particle in the gravitational field whose acceleration is also Eq.(31) which does not affected by the crystal in the vertical direction. In the free fall reference frame, the resultant of the gravitational force $\mathbf{W}$ and inertial force $\mathbf{F}_i$ on a carrier is (Fig.2)

$$\mathbf{W} + \mathbf{F}_i \tag{32}$$

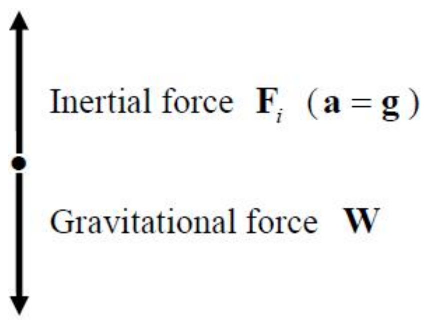

Figure.2 Force analysis

The relation between the momenta is

$$\frac{p^2}{2m_i} = \frac{p_0^2}{2m_i} - (\mathbf{W} + \mathbf{F}_i)\mathbf{H} \tag{33}$$

The form of $\mathbf{F}_i$ can be determined by the Tolman effect (Sections.3-5). Therefore, we combine Eq.(33) and the observed value of ϑ (Eq.30) to study the weight. For instance, suppose the inertial force is $\mathbf{F}_i = -E\frac{\mathbf{a}}{c^2}$ (Section.5) and the observer in the same reference frame cannot discover a shift of the phase to conclude that the system is falling. $\vartheta = 0$ corresponds to $p = p_0$ (Eq.30) and

$$\mathbf{W} + \mathbf{F}_i = 0 \tag{34}$$

in Eq.(33). The inertial force balances the gravitational force (Fig.2),

$$\mathbf{W} = -\mathbf{F}_i = E\frac{\mathbf{a}}{c^2} = \frac{E}{c^2}\mathbf{g} \quad (\mathbf{a} = \mathbf{g}) \tag{35}$$

It is consistent with the idea in [1] that the gravitational mass is replaced by the energy. By contrast, assume $\vartheta = 0$ and $\mathbf{F}_i = -m_i\mathbf{a} = -m^*\mathbf{a}$ (Section.1,3),

$$\mathbf{W} = -\mathbf{F}_i = m_i\mathbf{a} = m^*\mathbf{a} = m^*\mathbf{g} \quad (\mathbf{a} = \mathbf{g}) \tag{36}$$

The gravitational force is that derived from Newton's law (Eq.27).

8. Physical significance

Mass plays dual roles in physics now. One is resistance to acceleration. For example, the Lorentz force in Eq.(1) is the cause and the acceleration $\omega_c^2 r$ depends on m_i. In condensed matter, it equals m^* (Eq.9). Conversely, the other appears as if it is the source of a force like m_G in the gravitational interaction [1] and m_i in the inertial force (Section.3). Since the energy is perceived as enabling gravity [1], it is reasonable for us to think that the mass in the second type should also be replaced by E, e.g. the inertial force is $-E\frac{\mathbf{a}}{c^2}$ (Section.5).

That is to say, mass is always passive and cannot be active. In comparison with Eq.(25), the cyclotron frequency is still described by Eq.(6) in vacuum and Eq.(8) in condensed matter even though an electric scalar potential ϕ is applied where the total energy of the charged particle is $E = mc^2 + q\phi$ (Section.4 in [1]). It is impossible to be

$$\omega_c = \frac{qB}{\frac{E}{c^2}} = \frac{qB}{\frac{mc^2 + q\phi}{c^2}} \approx \frac{qB}{\frac{m_0c^2 + q\phi}{c^2}} = \frac{qB}{m_0 + \frac{q\phi}{c^2}} \tag{37}$$

9. Conclusions

There are too many different masses in physics and the empirical evidences in condensed matter may make a significant breakthrough. The distinction among the gravitational mass, inertial mass and effective mass is unnecessary. They are all redundant concepts .Only one "mass" as resistance ought to be sufficient whose value in condensed matter is m^* (Section.1). In the places where the mass seemed like the source, it may be replaced by the energy E. For this reason, the gravitational charge is E and a heavy fermion is not heavy because the energy is approximated to that of a free electron (Section.6). Additionally, the inertial force is probably $-E\frac{\mathbf{a}}{c^2}$ and the experiments to distinguish it from $-m_i\mathbf{a}$ are proposed (Sections.3-5).

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