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An Exact Formula for Cosmic Entropy in Rh=ct Cosmological Model

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Abstract

The question of the entropy of the universe is crucial and remains unanswered in cosmology. Assuming a flat universe, we derive, which we then demonstrate, an exact heuristic formula for the entropy of the apparent universe: $S_{Rh} = \frac{16\,\pi^2 Rh^2 E_{Pl}}{Rh\, l_{Pl}\, T_{Pl}} \frac{Tcmb}{T_{Pl}} \frac{t_{Rh}}{t_{Pl}}\ J.K^{-1}$ at the apparent horizon, i.e. at the Hubble radius. This approach forms part of a quantum thermodynamic cosmology framework of the Rh = ct type, which is in the field of classical mechanics, and could help to quantify the Planck era of Big Bang theory. It assumes that the universe would exist before Planck time at Planck temperature. Furthermore, it could shed new light on the standard cosmological model with regard to entropy.

Keywords: thermodynamics; cosmic entropy; cosmology; temperature of CMB; black hole; Planck era

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1. Introduction

Einstein said about thermodynamics: "A theory is the more impressive the greater the simplicity of its premises is, the more different kinds of things it relates, and the more extended is its area of applicability. Therefore, the deep impression which classical thermodynamics made upon me. It is the only physical theory of universal content concerning which I am convinced that within the framework of the applicability of its basic concepts, it will never be overthrown." [1]

Entropy is a measure of the disorder or randomness of a system. According to the second law of thermodynamics, the entropy of an isolated system increases over time, or at best remains constant. This law gives time a fundamental direction, often referred to as the 'arrow of time'.

A major challenge in the standard cosmological model is explaining why the universe began its expansion with abnormally low entropy, which then increased dramatically to reach values much higher than those observed at decoupling (approximately 380,000 years after the Big Bang). This 'initial entropy problem' appears to contradict the observed cosmic microwave background (CMB), which indicates that the early universe was close to thermal and chemical equilibrium, a state typically associated with high entropy.

Assuming our universe is an isolated system at the temperature of the CMB and based on recent thermodynamic cosmology research of the Rh = ct type, we propose a formula for the entropy of our universe that is consistent with its energy at the apparent horizon.

2. Background

In 2015, Tatum et al. [2] proposed an equation for the CMB temperature, noted T_{cmb}, that has since been formally derived from the Stefan-Boltzmann law by Haug and Wojnow [3,4].

$$T_{cmb} = T_{Rh} = \frac{\hbar c}{k_b 4\pi \sqrt{R_h 2 l_{Pl}}} \quad (1)$$

Witch can be derived as follows:

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$$T_{cmb} = T_{Rh} = \frac{\hbar}{k_b 4\pi\sqrt{t_{Rh}\, 2t_{Pl}}} \tag{2}$$

Where $\hbar$ is the reduced Planck constant, c is the speed of light in a vacuum, k_b is Boltzmann's constant, the Hubble radius is defined by $R_h = \frac{c}{H}$ where H is the Hubble parameter, T_{Rh} is the temperature of the Hubble sphere, l_{Pl} is the Planck length, t_{Rh} is the Hubble time defined by $t_{Rh} = \frac{1}{H}$, and t_{Pl} is the Planck time.

From Eq.2 we derive directly:

$$t_{Rh} = \frac{\hbar^2}{T_{cmb}^2 k_b^2 16\pi^2 2t_{Pl}} \tag{3}$$

These values, together with Planck's energy, $E_{Pl} = m_{Pl}c^2$, where m_{Pl} is Planck's mass, are necessary and sufficient to lead us to the formulation of the entropy S_{Rh} of the apparent universe, i.e. at the Hubble radius, compatible with the energy contained in the Hubble sphere.

Note: It should be noted that Eq.1 is an adaptation of the Hawking temperature of black holes[2]. This leads to the idea that our universe is the interior of an expanding black hole and that, in thermodynamic cosmology, an isolated system can also be linked to the interior of a black hole. Thus, our universe is a simple part of an infinite flat universe populated by black holes, which themselves contain their own universes.

3. Heuristic Formulation of the Entropy of Our Apparent Universe

First, we are in the field of classical thermodynamic cosmological models, so the energy contained in the Hubble sphere, $E_{Rh} = \frac{c^4 Rh}{2G}$, where G is the gravitational constant, $S_{Rh}\, T_{Rh}$ must be equal to E_{Rh} in our model.

For example, in Haug and Tatum's approach to the entropy of our apparent universe, the energy E_{Rh} is correct at Planck temperature, which should be noted, but diverges, by a factor of 10^{52} today, in our model when it is applied naively to the temperature of Hubble sphere as follows: $S_{BH}\, Tcmb$. This is incorrect because we don't respect the law of conservation of energy which imposes: $S_{Rh}\, T_{Rh} = E_{Rh}$. We must present our apologies to Haug and Tatum here. Indeed, in previous versions I had not found their version, see [5], how they arrive at the law of the conservation of energy which is correct is equivalent to our (see Annex A). Their formula is as follows, see [8], $S_{BH}\, T_{Haw,p} = \frac{c^4 Rh}{2\,G}$, with $T_{Haw,p} = \frac{\hbar c}{2\pi l_p}\frac{1}{k_b}$, i.e. independent of H, and $S_{BH} = k_b\, \frac{\pi\, Rh^2}{l_p^2}$.

The entropy S_{Rh} proposed by Haug and Tatum [5], although logically incorrect for all Rh in classical mechanics when it is applied naively to the temperature of Hubble sphere, has the advantage of being correct at Planck temperature. They assumed in Rh=ct cosmology the Bekenstein-Hawking formula for the entropy of a black hole as follows:

$$S_{Rh} = k_b\, \frac{4\pi\, R_h^2}{4 l_{Pl}^2} \tag{4}$$

We have noticed that the geometric means are commonly used in our particular approach to Rh=ct thermodynamic cosmological models [2,6], between unit quantum values and Rh=ct model values.

We therefore replaced l_{Pl}^2 with $\sqrt{R_h^2 l_{Pl}^2} = R_h l_{Pl}$ to preserve the exact result at Planck temperature, when $R_h = c\, t_{Pl}$. Despite this modification, $S_{Rh}T_{Rh}$ still diverged from $E_{Rh} = \frac{c^4 Rh}{2G}$ for more contemporary values of Rh. We then applied the principle of the ratio of quantum values to values in the Rh = c t model to count the number of Planck units. For example [7], $\frac{t_{Rh}}{t_{Pl}}$. When $S_{Rh}T_{Rh}$ was sufficiently close to E_{Rh}, we searched for constants, particularly simple powers of π, to arrive at this formula for the entropy of the apparent universe, which is compatible with its energy at the CMB temperature

$$S_{Rh} = \frac{16\,\pi^2\,Rh^2 E_{Pl}}{Rh\,l_{Pl}\,T_{Pl}}\,\frac{Tcmb}{T_{Pl}}\,\frac{t_{Rh}}{t_{Pl}}\;J.K^{-1} \tag{5}$$

With $Rh = c\,t_{Rh}, l_{Pl} = c\,t_{pl}$ and $T_{Pl} = \frac{E_{Pl}}{k_B}$ Eq.5, i.e. the formula of cosmic entropy in this Rh = c t model, can simplify as follows:

$$S_{Rh} = 16\,\pi^2 k_B\,\frac{Tcmb}{T_{Pl}}\,\frac{t_{Rh}^2}{t_{Pl}^2}\;J.K^{-1} \tag{6}$$

It is important to emphasize and remember that, in this approach,

$$T_{cmb} = T_{Rh} = \frac{\hbar}{k_b 4\pi\sqrt{t_{Rh}\,2t_{Pl}}}\;K \tag{7}$$

and

$$t_{Rh} = \frac{\hbar^2}{T_{cmb}^2 k_b^2 16\pi^2 2t_{Pl}}\;s \tag{8}$$

Then we can verify numerically $S_{Rh}T_{cmb} = S_{Rh}T_{Rh} = E_{Rh}$, i.e. the law of energy conservation:

$$S_{Rh}T_{Rh} = 16\,\pi^2\,k_B\,\frac{Tcmb}{T_{Pl}}\,\frac{t_{Rh}^2}{t_{Pl}^2}Tcmb = E_{Rh} = \frac{c^4 Rh}{2G}\;J \tag{9}$$

$$S_{Rh}T_{Rh} = 16\,\pi^2\,k_B\,\frac{T_{cmb}^2}{T_{Pl}}\,\frac{t_{Rh}^2}{t_{Pl}^2} = E_{Rh} = \frac{c^4 Rh}{2G}\;J \tag{10}$$

As $Tcmb$ decreases, the cosmic entropy S_{Rh} of the universe increases. The temperature and the entropy of universe are transformed into Hubble volume and Hubble mass (i.e. energy). This is a global state change in the temperature of the universe, which simultaneously affects its volume and mass (i.e. energy).

4. Demonstration of This Formula for Cosmic Entropy in This Model Rh=ct

$$S_{Rh}T_{Rh} = 16\,\pi^2\,k_B\,\frac{T_{cmb}^2}{T_{Pl}}\,\frac{t_{Rh}^2}{t_{Pl}^2}\;J \tag{11}$$

With $T_{cmb} = \frac{\hbar}{4\pi\,k_b\,\sqrt{t_{Rh}\,2t_{Pl}}}\;K$, we derive Eq.11 as follows:

$$S_{Rh}T_{Rh} = \frac{\hbar^2}{k_b\,T_{Pl}}\,\frac{t_{Rh}}{2t_{Pl}^3}\;J \tag{12}$$

With $T_{Pl} = \sqrt{\frac{\hbar\,c^5}{G\,k_B^2}}$ and $t_{Pl} = \sqrt{\frac{\hbar\,G}{c^5}}$, we derive Eq.12 as follows:

$$S_{Rh}T_{Rh} = \frac{\hbar^2}{\sqrt{\frac{\hbar\,c^5}{G}}}\,\frac{t_{Rh}}{2\sqrt{\frac{\hbar\,G}{c^5}\frac{\hbar\,G}{c^5}}} = \frac{c^5 t_{Rh}}{2\,G} = \frac{c^4\,ct_{Rh}}{2\,G} = \frac{c^4\,Rh}{2\,G}J \tag{13}$$

Since $E_{Rh} = \frac{c^4 \, Rh}{2 \, G}$, we have shown that $S_{Rh}T_{Rh} = \frac{c^4 \, Rh}{2 \, G} = E_{Rh}$ to satisfy the law of the conservation of energy in the Rh=ct model thanks to the formulas of T_{Rh} and t_{Rh} .

5. Contribution of the Entropy $R_h = \ c \ t$ to Duration and Energy in the Planck Era

It is widely accepted that the Planck era is characterized by Planck energy and Planck temperature. However, the concept of time in the Planck era is poorly defined. By setting $T_{cmb} = T_{Rh} = \ T_{Pl}$, we calculate $t_{Rh} = \frac{t_{Pl}}{32\pi^2}$, i.e. a time shorter than the Planck time at Planck era. In an other hand, we also can calculate in this model that for $E_{Pl} = S_{Rh}T_{Rh} = S_{Rh}T_{cmb}$, we need to set $T_{cmb} = \frac{T_{Pl}}{8\pi}$.

6. Conclusion

The contribution of the universe entropy formula Rh=ct to this emerging quantum thermodynamic cosmological model is an important advance. It provides a reliable formula in this field of research, paving the way for new developments and perspectives on the issues faced by the contemporary standard cosmological model. Indeed, there is a potential link between the $Rh = ct = \frac{c}{H}$ models and the cosmological standard model with the formula of observable radius: $R_{Obs} = \frac{c}{H_0}\int_{a=0}^{a=1} \frac{da}{a^2\sqrt{\Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_k a^{-2} + \Omega_\Lambda}}$. This could precision and refine the standard cosmological model.

7. Acknowledgments

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Annex A: Demonstration of Equivalence of These Two Different Approaches

Haug, Tatum formula for the Hubble sphere is as follows [8]:

$$S_p T_{Haw,p} = k_b \frac{\pi Rh}{l_p} \frac{\hbar c}{2\pi l_p} \frac{1}{k_b} J \tag{A.1}$$

Witch can be simplified as follows:

$$S_p T_{Haw,p} = \frac{Rh}{l_p} \frac{\hbar c}{2 l_p} J \tag{A.2}$$

My Eq.12 is:

$$S_{Rh} T_{Rh} = \frac{\hbar^2}{k_b \, T_{Pl}} \frac{t_{Rh}}{2 t_{Pl}^3} J \tag{A.3}$$

Witch can be rearranged as follows:

$$S_{Rh} T_{Rh} = \frac{\hbar^2}{2 \, k_b \, t_{Pl} \, T_{Pl} \, t_{Pl}} \frac{t_{Rh}}{t_{Pl}} J \tag{A.4}$$

With $t_{Pl} = \sqrt{\frac{\hbar \, G}{c^5}}$ and $T_{Pl} = \sqrt{\frac{\hbar \, c^5}{G \, k_B^2}}$, we derive Eq.A.4 as follows:

$$S_{Rh}T_{Rh} = \frac{t_{Rh}}{t_{Pl}} \frac{\hbar^2}{2\, k_b \sqrt{\frac{\hbar\, c^5}{G\, k_B^2}} \sqrt{\frac{\hbar\, G}{c^5}}\, t_{Pl}}\ J \qquad (A.5)$$

Witch can be simplified as follows:

$$S_{Rh}T_{Rh} = \frac{c\, t_{Rh}}{c\, t_{Pl}} \frac{\hbar^2\, c}{\frac{k_b}{k_b}\, 2\, \hbar\, c\, t_{Pl}}\ J \qquad (A.6)$$

With $Rh = c\, t_{Rh} = \frac{c}{H}$ and $l_{Pl} = c\, t_{Pl}$, Eq. $A.6$ can be simplified as follows:

$$S_{Rh}T_{Rh} = \frac{Rh}{l_{Pl}} \frac{\hbar\, c}{2\, l_{Pl}}\ J \qquad (A.7)$$

We have proven that Eq $A.2$ is equal to Eq $A.7$ which is derived from Eq.$A.3$. So that we have the conservation of energy at the apparent horizon in both cases:

$$S_p T_{Haw,p} = S_{Rh}T_{Rh} = \frac{c^4\, Rh}{2\, G} = \frac{c^5}{2\, G\, H}\ J \qquad (A.8)$$

Annex B: The Photon Energy Density Parameter

Haug, in his article [8], develops an interesting argument around the photon energy density parameter. The current CMB photon density is about (See PDG [9]):

$$\Omega_\gamma = \frac{\rho_\gamma}{\rho_{cr}} = 2.473\ 10^{-5}(T_{Rh}/2.7255)^4 h^{-2} \approx 5.38\ 10^{-5} \pm 0\ .3 \qquad (B.1)$$

The parameter h is define as $h = H/100$, where H is expressed in $km/s/Mpc$. The Megaparsec is:

$$1\ Mpc = 3.085677584\ 10^{22}\ m \qquad (B.2)$$

Now let's use Eq.8 to, in a first time, calculate t_{Rh} based on T_{Rh}. For $T_{Rh} = 2.7255\ K$ we have an exact value for $t_{Rh} = 4{,}6127598948 \ldots\ 10^{17} s$ which will use from $(B.3)$ to $(B.5)$.

In $Rh = ct = \frac{c}{H}$ models so $t_{Rh} = 1/H$, in S.I. units. Then we calculate:

$$H = 1/1000\ \frac{1}{t_{Rh}} 3.085677584\ 10^{22} = 66{,}895371558 \ldots\ km/s/Mpc \qquad (B.3)$$

Which can be simplified as follows:

$$H = \frac{1}{t_{Rh}}\ 3.085677584\ 10^{19} = 66{,}895371558 \ldots\ km/s/Mpc \qquad (B.4).$$

Since $h = H/100$, we can derive h as follows:

$$h = \frac{1}{t_{Rh}}\ 3.085677584\ 10^{17} = 0.6689537 \ldots \qquad (B.5)$$

If T_{Rh} is set, we calculate t_{Rh} with Eq.8 or alternatively if t_{Rh} is set, we calculate T_{Rh} with Eq.7. Then we can calculate a constant value for Ω_γ as follows determinate by Eq.B.1:

$$\Omega_\gamma = \frac{\rho_\gamma}{\rho_{cr}} = 2.473\ 10^{-5}\ (T_{Rh}/2.7255)^4\ h^{-2} = 5.526268\ 10^{-5} \qquad (B.6)$$

Which is close to exact the value $\Omega_\gamma = \frac{1}{5760\,\pi} = 5.526213 \ldots$ determinate by Haug [10] [8]. We are limited by the precision of $2.473\ 10^{-5}$ value. So, it seems possible to precise the $2.473\ 10^{-5}$ value, as for example $2{,}472975333 \ldots\ 10^{-5}$, with $\Omega_\gamma = \frac{1}{5760\,\pi} = 5.526213302 \ldots 10^{-5}$.

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Annex C: Similarity Between Haug Tatum Entropy and Wojnow Entropy

In annex A, we have already demonstrated that these two entropies are equivalent. Now we examine the $\frac{S_{BH}}{k_B} = 2.29\ 10^{122}$ value proposed by Haug [8].

For t_{Rh} interdepend from $T_{Rh} = 2.7255K$, we have exactly:

$$2\pi \frac{t_{Rh}}{t_{Pl}} \frac{T_{Pl}^2}{(8\pi)^2} \frac{1}{T_{cmb}^2} = 2.29\ 10^{122} = \frac{S_{BH}}{k_B} = \frac{\pi t_{Rh}^2}{t_{Pl}^2} = \frac{\pi\ Rh^2}{l_{Pl}^2} \quad (C.1)$$

Where $T_{Pl}/8\pi$ is the value needed to fix $E_{Rh} = E_{cr} = E_{Pl}$ in our model (see section 5). So, Haug Tatum entropy is clearly linked to $T_{cmb} = T_{Rh}$ despite the critics formulated by Haug, see [8]. It's only another point of view on the same subject.

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