

Article

Not peer-reviewed version

Emergent Einstein–Friedmann Dynamics from Universal Wavefunction Geometry

[Vladlen Shvedov](#) *

Posted Date: 13 January 2026

doi: 10.20944/preprints202601.0883.v1

Keywords: emergent gravity; cosmology; Friedmann–Robertson–Walker spacetime; de Sitter space; universal wavefunction; hypersurface geometry; spacetime emergence; Einstein–Friedmann equations; cosmological constant



Preprints.org is a free multidisciplinary platform providing preprint service that is dedicated to making early versions of research outputs permanently available and citable. Preprints posted at Preprints.org appear in Web of Science, Crossref, Google Scholar, Scilit, Europe PMC.

Copyright: This open access article is published under a [Creative Commons CC BY 4.0 license](#), which permit the free download, distribution, and reuse, provided that the author and preprint are cited in any reuse.

Article

Emergent Einstein–Friedmann Dynamics from Universal Wavefunction Geometry

Vladlen Shvedov 

Department of Fundamental and Theoretical Physics and ARC Centre of Excellence for Transformative Meta-Optical Systems (TMOS) Research School of Physics, The Australian National University, Canberra, ACT 2601, Australia; vladlen.shvedov@anu.edu.au

Abstract

In this work, we demonstrate how an effective spacetime description and an Einstein–Friedmann structure can emerge naturally from the geometry of a universal wavefunction, without postulating gravitational field equations or introducing matter fields explicitly. By treating the flux hypersurface associated with a conserved wavefunction current as an embedded Lorentzian manifold, we show that its induced geometry is necessarily of Friedmann–Robertson–Walker type under minimal assumptions of homogeneity and isotropy. We show that the intrinsic curvature of the induced metric is fully determined by the embedding geometry itself. In particular, de Sitter spacetime arises as a special, maximally symmetric case corresponding to a constant-curvature hyperboloid in the ambient space. More generally, for physically relevant and normalisable classes of wavefunction envelopes, the embedding geometry remains hyperboloid in character but exhibits a time-dependent curvature scale. In this regime, the effective vacuum curvature term is approximately constant at early times, giving rise to de Sitter behaviour, and subsequently decays as the hypersurface evolves, leading asymptotically to a linear expansion law. By identifying a conserved, potential-like geometric invariant inherited from the universal wavefunction, we recover an effective Einstein–Friedmann structure on the hypersurface without invoking gravitational dynamics. This invariant fixes the scaling of the dominant contribution to the effective energy density and determines the value of the effective gravitational coupling. For closed slicing, this contribution cancels identically against the spatial curvature term in the Friedmann equation, leaving the late-time expansion governed solely by the residual vacuum-like sector. These results position general relativity as an emergent effective theory arising from a deeper wavefunction-based geometric structure.

Keywords: emergent gravity; cosmology; Friedmann–Robertson–Walker spacetime; de Sitter space; universal wavefunction; hypersurface geometry; spacetime emergence; Einstein–Friedmann equations; cosmological constant

1. Introduction

Modern cosmology is remarkably successful in describing the large-scale evolution of the Universe using general relativity combined with the Friedmann–Robertson–Walker (FRW) metric [1,2]. Within this framework, the dynamics of cosmic expansion are governed by Einstein–Friedmann equations supplemented by phenomenological components such as radiation, matter, and dark energy. Despite this success, the physical origin of spacetime itself, the status of the cosmological constant, and the reason why FRW geometry provides such an accurate effective description remain open conceptual questions.

In recent years, there has been growing interest in the possibility that spacetime and gravitation may be emergent rather than fundamental, arising as effective descriptions of deeper underlying structures [3,4]. From this perspective, general relativity may be viewed as a hydrodynamic or coarse-grained limit of more primitive degrees of freedom, valid only within a restricted domain of scales

and symmetries. The present work contributes to this line of inquiry by exploring how an effective spacetime geometry and Einstein–Friedmann dynamics can arise from a purely geometric construction associated with a universal wavefunction, without assuming gravitational field equations at the outset.

In the recent paper [5], we introduced and analysed a geometric framework based on a conserved wavefunction current defined in an abstract configuration space. The flow of this current defines a preferred temporal ordering and generates a family of flux hypersurfaces, which may be interpreted as evolving geometric objects independent of any spacetime background. A central result of that construction is the identification of a critical hypersurface associated with the scalar envelope of the universal wavefunction. This hypersurface evolves smoothly, remains nonsingular, and exhibits a well-defined nonlinear expansion law governed entirely by the underlying wavefunction geometry.

The purpose of the present paper is to establish a clear and explicit connection between this geometric framework and standard cosmological spacetime descriptions. Rather than postulating Einstein equations or introducing matter fields, we treat the critical hypersurface as an embedded Lorentzian manifold and analyse the intrinsic geometry induced on it by the embedding. Our guiding question is not whether general relativity can be modified, but under what conditions it emerges naturally as an effective description of perturbations and dynamics confined to the hypersurface.

We show that, under minimal assumptions of homogeneity and isotropy, the induced metric on the hypersurface can always be written locally in Friedmann–Robertson–Walker form, with the hypersurface radius playing the role of an effective scale factor. The intrinsic curvature of this induced geometry is determined entirely by the embedding, allowing de Sitter spacetime to be identified as a special case corresponding to a constant-curvature hyperboloid. More generally, the embedding geometry is hyperboloid-like but with a time-dependent curvature scale, leading to an effective vacuum-like curvature term that evolves in time rather than representing a fundamental constant.

A key ingredient in this construction is a conserved, potential-like geometric invariant inherited from the universal wavefunction. This invariant fixes the scaling of an effective energy density on the hypersurface and leads naturally to a dominant contribution proportional to a^{-2} in the effective Friedmann equation. In the early-time regime, when the curvature scale varies slowly, the induced geometry exhibits de Sitter-like behaviour. At late times, the same framework predicts asymptotically linear expansion accompanied by a decaying effective cosmological constant.

The structure of this paper is as follows. In Section 2, we introduce the geometric description of the embedded hypersurface and derive the induced metric and curvature relations. Section 3 analyses the emergence of FRW geometry and the role of hyperboloid embeddings, including the de Sitter limit. In Section 4, we identify the effective Einstein–Friedmann structure on the hypersurface and establish the associated conservation laws. Section 5 connects these results explicitly to the conserved wavefunction invariant derived in [5], clarifying the emergence of the effective gravitational coupling and the late-time expansion law. We conclude in Section 6 with a discussion of the domain of validity of the effective spacetime description and its implications for the emergence of general relativity.

2. Embedded Hypersurface and Induced Geometry

In this section, we introduce the geometric description of the evolving hypersurface associated with the conserved wavefunction current and derive the intrinsic geometry induced on it by the embedding. The discussion is formulated in a general and model-independent manner, making explicit only the assumptions required for homogeneity and isotropy.

2.1. Euclidean and Lorentzian Hyperboloid Embeddings: A Conceptual Distinction

Before introducing the explicit embedding geometry, it is useful to clarify the geometric status of the hypersurfaces considered in this work and their relation to familiar constant-curvature spacetimes. In particular, it is important to distinguish between generic hyperboloids of one sheet and the special class of hyperboloids that correspond to de Sitter spacetime.

In the geometric construction underlying the universal wavefunction framework, the flow of a conserved current generates a family of hypersurfaces that are hyperboloid-like in shape. These

hypersurfaces are analogous to the stream surfaces formed by bundles of Poynting vectors of coherent waves with finite energy in Gaussian optics, where energy flow defines a set of axially symmetric hyperboloids of one sheet. Such hyperboloids are naturally embedded in a space with positive-definite (Euclidean) metric and, in general, do not possess constant intrinsic curvature. Their curvature varies along the surface and reflects the underlying transport dynamics rather than maximal symmetry.

By contrast, de Sitter spacetime corresponds to a very special geometric construction: a hyperboloid embedded in a flat ambient space with Lorentzian signature and fixed Minkowski radius. In this case, the embedding constraint defines a maximally symmetric manifold with constant intrinsic curvature. The constancy of curvature is therefore not a generic property of hyperboloids, but a consequence of both the Lorentzian signature of the embedding space and the invariance of the embedding radius.

The hypersurfaces considered in the present framework interpolate naturally between these cases. When the effective embedding radius is approximately constant, the induced geometry is de Sitter-like. More generally, when the embedding radius evolves in time—as dictated by the wavefunction envelope—the hypersurface remains hyperboloid-like but exhibits time-dependent curvature. This distinction plays a central role in the emergence of an effective, time-dependent cosmological term discussed in subsequent sections.

The hypersurfaces considered in the present framework interpolate naturally between these cases. When the effective embedding radius is approximately constant, the induced geometry approaches the maximally symmetric de Sitter limit. However, such maximally symmetric configurations are intrinsically non-normalisable, reflecting the well-known fact that fully symmetric solutions of wave equations in unbounded spaces generally carry infinite total weight or energy. By contrast, when the hypersurface geometry is generated by a wavefunction envelope (analogous to a Gaussian beam in optics) the resulting hyperboloid-like surfaces necessarily break maximal symmetry. This symmetry reduction ensures finite normalisation of the associated wavefunction current and leads to physically admissible configurations with finite total flux. In this sense, the emergence of time-dependent curvature in the present framework is not a deficiency, but a direct consequence of enforcing normalisation and physical finiteness, with de Sitter geometry arising only as an idealised limiting case.

2.2. Ambient space and Hypersurface Embedding

We consider a flat $(N + 2)$ -dimensional ambient space with coordinates X^A ($A = 0, 1, \dots, N + 1$) endowed with the metric

$$ds_{\text{amb}}^2 = dX_0^2 - \sum_{i=1}^{N+1} dX_i^2, \quad (1)$$

where one coordinate is time-like and the remaining $N + 1$ coordinates are space-like. This ambient space serves purely as a geometric arena for the embedding and is not assumed to have direct physical significance.

The evolving hypersurface $\mathcal{H}$ is realised as a codimension-one Lorentzian submanifold embedded in this ambient space. Motivated by the symmetry properties of the critical hypersurface discussed in [5], we restrict attention to embeddings that preserve homogeneity and isotropy on spatial sections. A closed-slicing parameterisation of such an embedding is

$$X_0 = F(\tau), \quad (2)$$

$$X_i = a(\tau) n_i(\Omega), \quad i = 1, \dots, N + 1, \quad (3)$$

where τ is a monotonically increasing parameter along the hypersurface, $a(\tau)$ is a positive function interpreted as the intrinsic radius of spatial sections, and $n_i(\Omega)$ are coordinates on the unit N -sphere satisfying

$$\sum_{i=1}^{N+1} n_i^2 = 1. \quad (4)$$

The angular variables Ω collectively denote the coordinates on S^N .

This embedding describes a family of $(N + 1)$ -dimensional hypersurfaces whose spatial cross-sections are N -spheres of radius $a(\tau)$. For generic functions $F(\tau)$ and $a(\tau)$, the resulting hypersurface is hyperboloid-like in the ambient space, reducing to the standard de Sitter hyperboloid when the curvature scale is constant [6].

2.3. Induced Metric

The intrinsic geometry on $\mathcal{H}$ is obtained by pulling back the ambient metric (1) using the embedding (2)–(3). The tangent vectors to the hypersurface are given by

$$\partial_\tau X^A = (\dot{F}, \dot{a} n_i), \quad (5)$$

$$\partial_\alpha X^A = (0, a \partial_\alpha n_i), \quad (6)$$

where a dot denotes differentiation with respect to τ and α labels angular coordinates on S^N .

Using the identities

$$\sum_{i=1}^{N+1} n_i \partial_\alpha n_i = 0, \quad \sum_{i=1}^{N+1} \partial_\alpha n_i \partial_\beta n_i = \gamma_{\alpha\beta}, \quad (7)$$

where $\gamma_{\alpha\beta}$ is the metric on the unit N -sphere.

The induced line element on $\mathcal{H}$ becomes

$$ds^2 = c^2 (\dot{F}^2 - \dot{a}^2) d\tau^2 - a^2(\tau) d\Omega_N^2, \quad (8)$$

with $d\Omega_N^2 = \gamma_{\alpha\beta} d\Omega^\alpha d\Omega^\beta$, and the lapse factor

$$\mathcal{L}(\tau) \equiv \sqrt{\dot{F}^2(\tau) - \dot{a}^2(\tau)}. \quad (9)$$

The constant c denotes the speed of light in vacuum and is introduced to ensure that the induced Lorentzian metric has the correct physical dimensions in SI.

To express the metric in standard cosmological form, we introduce a proper time coordinate t defined by

$$dt = \mathcal{L}(\tau) d\tau, \quad (10)$$

which is well defined, provided $\dot{F}^2 > \dot{a}^2$.

In terms of t , the induced metric takes the Friedmann–Robertson–Walker form [2]:

$$ds^2 = c^2 dt^2 - a^2(t) d\Omega_N^2, \quad (11)$$

regardless of the detailed expansion law, as long as the embedding is isotropic in the spatial directions.

Thus, without assuming any gravitational dynamics, the induced geometry on the embedded hypersurface is locally FRW, with the hypersurface radius $a(t)$ playing the role of the cosmological scale factor.

It is important to emphasise that the parameter τ introduced in the embedding parametrisation (2)–(3) does not represent physical time. Its role is purely geometric, ordering the evolution of the embedding functions $F(\tau)$ and $a(\tau)$. Physical time emerges only after the induced Lorentzian metric on the hypersurface is identified. In particular, the proper time t measured by comoving observers is related to τ through the lapse factor $dt = \mathcal{L}(\tau) d\tau$. Once this identification is made, the induced metric takes Friedmann–Robertson–Walker form and t acquires the interpretation of effective cosmological time. Similar is related to the function $a(\tau)$ in (3) that represents a geometric radius of the hypersurface rather than an ordinary cosmological scale factor. At that stage of the construction, no intrinsic spacetime metric or physical notion of cosmic time has yet been introduced. It is also worth noting that the embedding function $F(\tau)$ does not appear explicitly in the intrinsic spacetime

description once the induced metric is written in Friedmann–Robertson–Walker form. This is a direct consequence of the definition of proper time on the hypersurface. The function $F(\tau)$ enters the induced metric only through the combination $\dot{F}^2 - \dot{a}^2$, which defines the lapse factor relating the geometric parameter τ to the emergent proper time t . After the identification (10), the intrinsic geometry is fully characterised by the scale factor $a(t)$, while $F(\tau)$ plays no independent role in the effective spacetime dynamics.

Once the induced Lorentzian metric on the hypersurface is identified and τ is related to the emergent proper time t , the function $a(t)$ may be reinterpreted as an effective cosmological scale factor. If desired, a dimensionless, normalised scale factor defined as a dimensionless ratio of physical distances at different times $\tilde{a}(t) = a(t)/a(t_0)$ can then be introduced without altering the underlying dynamics, and therefore normalisation conditions such as $\tilde{a}(t_0) = 1$ or $\tilde{a}(0) = 0$ can be imposed. In this sense, the absence of standard normalisation conditions at the geometric level is not a deficiency, but a reflection of the fact that cosmological interpretation arises only after the spacetime structure has emerged.

2.4. Hyperboloid Geometry and Curvature Scale

The embedding (2)–(3) implies that the hypersurface satisfies the relation

$$X_0^2 - \sum_{i=1}^{N+1} X_i^2 = F^2(\tau) - a^2(\tau). \quad (12)$$

If the right-hand side is constant, the hypersurface is an exact hyperboloid of constant curvature, corresponding to de Sitter spacetime in closed slicing [2,6]. More generally, the quantity $F^2(\tau) - a^2(\tau)$ defines an effective curvature scale that may evolve with time, leading to a family of hyperboloid-like geometries with time-dependent curvature.

This observation provides a purely geometric origin for de Sitter and de Sitter-like spacetimes within the present framework. Constant intrinsic curvature corresponds to a special case of a maximally symmetric embedding, realised when the embedding radius is strictly invariant. More generally, when the hypersurface evolution is governed by a wavefunction envelope, maximal symmetry is necessarily broken in order to ensure finite normalisation of the associated current. In this case, the resulting hypersurface remains hyperboloid-like but exhibits time-dependent curvature, as realised by the critical hypersurface analysed in the preceding study.

In the following sections, we analyse how the intrinsic curvature of the induced metric (11) is determined by this embedding geometry and how an effective Einstein–Friedmann structure emerges on the hypersurface.

3. Intrinsic and Extrinsic Curvature of the Hypersurface

In this section, we analyse the curvature properties of the embedded hypersurface introduced in Section 2. We show that the intrinsic curvature of the induced Friedmann–Robertson–Walker (FRW) geometry is completely determined by the embedding and, in particular, that de Sitter spacetime arises as the constant-curvature limit of a hyperboloid embedding. This analysis provides the geometric foundation for the emergence of an effective cosmological term.

3.1. Intrinsic curvature of the induced FRW geometry

The induced metric on the hypersurface $\mathcal{H}$ is given by

$$ds^2 = c^2 dt^2 - a^2(t) d\Omega_N^2, \quad (13)$$

where $a(t)$ is the hypersurface radius and $d\Omega_N^2$ is the metric on the unit N -sphere. This metric has the standard FRW form with closed spatial sections.

Defining the Hubble parameter as

$$H(t) \equiv \frac{\dot{a}}{a}, \quad (14)$$

the nonvanishing components of the Einstein tensor for an $(N + 1)$ -dimensional FRW spacetime are

$$G_{tt} = \frac{N(N-1)}{2c^2} \left(H^2 + \frac{c^2}{a^2} \right), \quad (15)$$

$$G_{ij} = -\frac{1}{c^2} \left[(N-1) \frac{\ddot{a}}{a} + \frac{(N-1)(N-2)}{2} \left(H^2 + \frac{c^2}{a^2} \right) \right] g_{ij}, \quad (16)$$

where g_{ij} denotes the spatial components of the spacetime metric. These expressions are purely geometric identities following from the form of the induced metric and do not assume any dynamical field equations.

The scalar curvature associated with (13) is

$$R = \frac{N}{c^2} \left[2 \frac{\ddot{a}}{a} + (N-1) \left(H^2 + \frac{c^2}{a^2} \right) \right]. \quad (17)$$

In the special case where the scalar curvature is constant, $R = \text{const}$, the induced spacetime is maximally symmetric and corresponds to de Sitter geometry in closed slicing.

3.2. Extrinsic Curvature and Embedding Geometry

To relate the intrinsic curvature to the embedding, we introduce the unit normal vector N^A to the hypersurface in the ambient space. For the embedding defined in Section 2, a convenient choice of normal vector is

$$N^A = \left(\frac{\dot{a}}{\mathcal{L}}, \frac{\dot{F}}{\mathcal{L}} n_i \right), \quad (18)$$

which satisfies

$$\eta_{AB} N^A N^B = -1, \quad \eta_{AB} N^A \partial_\mu X^B = 0. \quad (19)$$

The extrinsic curvature tensor of the hypersurface is defined as

$$K_{\mu\nu} = -\eta_{AB} N^A \partial_\mu \partial_\nu X^B. \quad (20)$$

For the embedding (2)–(3), the nonvanishing components of $K_{\mu\nu}$ are isotropic and take the form

$$K_{tt} = -\frac{1}{c^2} \frac{\dot{a}\dot{F} + \dot{F}\ddot{a}}{\mathcal{L}}, \quad (21)$$

$$K_{ij} = -\frac{\dot{F}}{a\mathcal{L}} g_{ij}. \quad (22)$$

The isotropy of K_{ij} reflects the assumed symmetry of the embedding and ensures that the induced intrinsic geometry remains homogeneous and isotropic.

3.3. Gauss relation and the origin of de Sitter geometry

For a codimension-one hypersurface embedded in a flat ambient space, the Gauss relation expresses the intrinsic Riemann tensor entirely in terms of the extrinsic curvature:

$$R_{\mu\nu\rho\sigma} = K_{\mu\rho} K_{\nu\sigma} - K_{\mu\sigma} K_{\nu\rho}. \quad (23)$$

Contracting indices yield the scalar curvature

$$R = K^2 - K_{\mu\nu} K^{\mu\nu}, \quad (24)$$

where $K = g^{\mu\nu} K_{\mu\nu}$ is the trace of the extrinsic curvature.

When the embedding satisfies

$$F^2(\tau) - a^2(\tau) = L^2 = \text{const}, \quad (25)$$

the hypersurface is an exact hyperboloid of constant curvature radius L . In this case, the extrinsic curvature components are constant in time, and the intrinsic curvature reduces to

$$R = \frac{N(N+1)}{L^2}, \quad (26)$$

corresponding to de Sitter spacetime [2,6].

More generally, when the quantity $F^2(\tau) - a^2(\tau)$ varies slowly with time, the hypersurface geometry is de Sitter-like but with a time-dependent curvature scale. The intrinsic curvature (17) is then determined by the evolution of the embedding functions $F(\tau)$ and $a(\tau)$, providing a purely geometric origin for an effective, time-dependent vacuum-like curvature term.

In the next section, we show how this intrinsic curvature may be interpreted in Einstein–Friedmann form and how an effective cosmological term naturally emerges from the geometry of the embedded hypersurface.

4. Emergence of an Effective Einstein–Friedmann Structure

Having established that the induced geometry on the embedded hypersurface $\mathcal{H}$ is of Friedmann–Robertson–Walker type and that its intrinsic curvature is determined entirely by the embedding, we now show how an effective Einstein–Friedmann structure arises naturally. Importantly, this structure is not postulated as a dynamical law but follows from geometric identities once the induced curvature is reinterpreted in a form familiar from relativistic cosmology.

4.1. Geometric definition of an effective cosmological term

The intrinsic Einstein tensor associated with the induced metric (13) is given by Eqs. (15) and (16). Since these expressions are purely geometric, they may be decomposed without reference to any matter content or gravitational field equations.

A convenient and invariant way to identify a vacuum-like contribution is to extract the part of the Einstein tensor proportional to the metric. We therefore define an effective cosmological term $\Lambda_{\text{eff}}(t)$ by

$$\Lambda_{\text{eff}}(t) \equiv -\frac{1}{N+1} G^\mu{}_\mu, \quad (27)$$

where $G^\mu{}_\mu$ denotes the trace of the Einstein tensor constructed from the induced metric.

Using Eq. (17), this definition yields

$$\Lambda_{\text{eff}}(t) = \frac{N}{(N+1)c^2} \left[2\frac{\ddot{a}}{a} + (N-1) \left(H^2 + \frac{c^2}{a^2} \right) \right]. \quad (28)$$

For a constant-curvature embedding, Λ_{eff} is constant, and the induced geometry corresponds to de Sitter spacetime. More generally, $\Lambda_{\text{eff}}(t)$ inherits its time dependence from the evolution of the embedding geometry.

4.2. Residual curvature and effective stress-energy tensor

Having identified the vacuum-like contribution, we define the residual Einstein tensor by

$$\tilde{G}_{\mu\nu} \equiv G_{\mu\nu} + \Lambda_{\text{eff}}(t) g_{\mu\nu}, \quad (29)$$

which is traceless by construction,

$$\tilde{G}^\mu{}_\mu = 0. \quad (30)$$

Introducing an effective gravitational coupling G , we define an effective stress-energy tensor on the hypersurface as

$$T_{\mu\nu}^{\text{eff}} \equiv \frac{1}{8\pi G c^2} \tilde{G}_{\mu\nu}. \quad (31)$$

With this definition, the intrinsic curvature identities on $\mathcal{H}$ may be written in the Einstein-like form

$$G_{\mu\nu} = 8\pi G c^2 T_{\mu\nu}^{\text{eff}} - \Lambda_{\text{eff}}(t) g_{\mu\nu}. \quad (32)$$

Equation (32) is not assumed as a field equation but serves as a convenient parametrization of the intrinsic geometry in a form directly comparable with standard cosmology.

4.3. Continuity Equation from Geometric Identities

The Einstein tensor satisfies the contracted Bianchi identity,

$$\nabla^\mu G_{\mu\nu} = 0, \quad (33)$$

which holds identically for any metric. Applying Eq. (33) to the decomposition (32) yields

$$\nabla^\mu T_{\mu\nu}^{\text{eff}} = \frac{1}{8\pi G c^2} \partial_\nu \Lambda_{\text{eff}}(t). \quad (34)$$

Thus, when Λ_{eff} is constant, the effective stress-energy tensor is locally conserved. When $\Lambda_{\text{eff}}(t)$ varies in time, the residual sector exchanges energy with the vacuum-like geometric component in a manner fixed entirely by the geometry [7,8].

Specializing to the FRW form of the induced metric and assuming isotropy, the effective stress-energy tensor may be written as

$$T^{\text{eff}\mu}{}_\nu = \text{diag} \left(c^2 \rho_{\text{eff}}, -p_{\text{eff}}, -p_{\text{eff}}, \dots \right). \quad (35)$$

The temporal component of Eq. (34) then yields the continuity equation

$$\dot{\rho}_{\text{eff}} + NH \left(\rho_{\text{eff}} + \frac{p_{\text{eff}}}{c^2} \right) = \frac{1}{8\pi G c^2} \dot{\Lambda}_{\text{eff}}(t). \quad (36)$$

which reduces to the standard conservation law in the case of constant Λ_{eff} .

4.4. Effective Friedmann Equations

Finally, the time–time component of Eq. (32) yields an effective Friedmann relation,

$$\frac{N(N-1)}{2c^2} \left(H^2 + \frac{c^2}{a^2} \right) = 8\pi G \rho_{\text{eff}} + \Lambda_{\text{eff}}(t). \quad (37)$$

From the spatial components of the Einstein tensor, one may derive the corresponding acceleration equation for the hypersurface scale factor. Using the (ij) components of the Einstein equations for an $(N+1)$ -dimensional Friedmann–Robertson–Walker geometry and eliminating the geometric combination $H^2 + a^{-2}$ with the aid of Eq. (37), one obtains

$$\frac{\ddot{a}}{a} = -\frac{8\pi G}{(N-1)} \left[\frac{(N-2)}{N} \rho_{\text{eff}} + \frac{p_{\text{eff}}}{c^2} \right] + \frac{2}{N(N-1)} \Lambda_{\text{eff}}(t). \quad (38)$$

This relation is purely geometric in origin and follows directly from the Einstein tensor identities once the effective energy density, pressure, and cosmological term are introduced as in the preceding subsection.

For the physically relevant case $N = 3$, Eqs. (37) and (38) reduce to the standard Einstein–Friedmann form [1]:

$$\frac{3}{c^2} \left(H^2 + \frac{c^2}{a^2} \right) = 8\pi G \rho_{\text{eff}} + \Lambda_{\text{eff}}(t). \quad (39)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left(\rho_{\text{eff}} + \frac{3p_{\text{eff}}}{c^2} \right) + \frac{1}{3} \Lambda_{\text{eff}}(t). \quad (40)$$

Together with Eqs. (36), Eqs. (39) and (40) provide a complete and effective cosmological description of the intrinsic hypersurface geometry in the present framework. They demonstrate that, once the induced geometry is interpreted in Einstein–Friedmann form, local conservation laws and effective cosmological dynamics follow as geometric identities.

In the next section, we connect these general results to the conserved wavefunction invariant derived in [5] and show how the effective gravitational coupling and late-time expansion law are fixed by the underlying wavefunction geometry.

5. Wavefunction Invariant and Determination of the Effective Gravitational Coupling

In this section, we connect the effective Einstein–Friedmann structure derived in Section 4 to the conserved geometric invariant obtained from the universal wavefunction in [5]. We show that this invariant uniquely fixes the scaling of the effective energy density on the hypersurface, determines the value of the effective gravitational coupling, and leads naturally to asymptotically linear expansion.

5.1. Surface-type conservation law and density scaling

As demonstrated in [5], the evolution of the critical hypersurface is governed by a conserved quantity associated with the scalar envelope of the universal wavefunction. The corresponding conservation law is expressed as an integral over the two-dimensional boundary of the homogeneous three-ball,

$$\oint_S \rho_{\text{eff}} dS = \rho_{\text{eff}} S = \text{const}, \quad (41)$$

where dS represents a 2-sphere area element over the t -slice intersection and ρ_{eff} is the homogeneous density of the 3-ball restricted by the sphere at fixed time. This may equivalently be written in volumetric form as

$$\frac{\rho_{\text{eff}} V}{a} = \text{const}, \quad (42)$$

where the cosmological scale factor $a(t)$ denotes the hypersurface radius and $V = (4/3)\pi a^3$ is the enclosed volume.

Defining the effective mass–energy content of the hypersurface as $m = \rho_{\text{eff}} V$, Eq. (42) implies

$$\frac{m}{a} = \text{const}, \quad (43)$$

that the effective mass scales linearly with the hypersurface radius $m \propto a$. Consequently, the effective density scales as

$$\rho_{\text{eff}}(a) \propto a^{-2}. \quad (44)$$

This scaling is fundamentally different from that of standard matter or radiation components and reflects the surface-type nature of the conserved wavefunction invariant.

5.2. Potential-like invariant and emergence of the gravitational constant

The conservation law (42) may be rewritten in the form

$$\frac{2\rho_{\text{eff}} V}{ac^2} = \frac{\phi}{c^2}, \quad (45)$$

where the integrand has the structure of a potential-like quantity ϕ . Using the relations between the characteristic length and mass scales derived in [5], this quantity may be identified with the gravitational coupling constant G ,

$$\frac{\phi}{c^2} = \frac{1}{G}. \quad (46)$$

Equivalently, the gravitational constant may be expressed as

$$G = \frac{c^2 a}{2V\rho_{\text{eff}}}. \quad (47)$$

In terms of the effective mass $m = \rho V$, Eq. (47) yields the dimensionless invariant

$$\frac{Gm}{ac^2} = \frac{1}{2}. \quad (48)$$

Thus, the effective gravitational coupling appearing in the Einstein–Friedmann form of the induced geometry is not introduced phenomenologically, but is fixed by the conserved wavefunction invariant. In this sense, Newton’s constant emerges as a geometric conversion factor between the invariant mass–radius ratio and a dimensionless potential-like quantity.

5.3. Effective Friedmann dynamics and late-time behaviour

In the derivation of the Einstein–Friedmann form in Section 4, the spatial geometry of the hypersurface was taken to be closed, corresponding to the choice of a unit N -sphere and thus to a curvature index $k = +1$. For the purpose of analysing the effective dynamics in a more general and transparent way, it is convenient to reintroduce the curvature parameter k explicitly in the Friedmann equation, treating it as a label of the constant-curvature spatial slicing rather than as an independent dynamical degree of freedom.

With this convention, the effective Friedmann equation (39) for $N = 3$ may be written in the standard form

$$\frac{3}{c^2} \left(H^2 + \frac{k c^2}{a^2} \right) = 8\pi G \rho_{\text{eff}} + \Lambda_{\text{eff}}(t). \quad (49)$$

Using Eq. (48), the matter-like contribution becomes

$$8\pi G \rho_{\text{eff}} = \frac{3}{a^2}, \quad (50)$$

so that

$$H^2 = \frac{(1-k)c^2}{a^2} + \frac{c^2}{3} \Lambda_{\text{eff}}(t). \quad (51)$$

This result reveals a remarkable structural property in the effective dynamics of the critical hypersurface associated with the universal wavefunction. For the density scaling $\rho_{\text{eff}} \propto a^{-2}$ inherited from the conserved geometric invariant, the matter-like contribution to the Friedmann equation has precisely the same a^{-2} dependence as the spatial curvature term. As a consequence, for the closed slicing ($k = +1$) inherent to the hypersurface description of the universal wavefunction, these two contributions cancel identically, leaving

$$H^2 = \frac{c^2}{3} \Lambda_{\text{eff}}(t). \quad (52)$$

This cancellation follows directly from the geometric origin of ρ_{eff} and its fixed normalisation. It reflects the fact that the effective energy density in the present framework is intrinsically tied to the curvature of the hypersurface rather than representing an independent dynamical matter component. The cosmological evolution is therefore governed solely by the residual vacuum-like term $\Lambda_{\text{eff}}(t)$, whose time dependence encodes deviations from exact de Sitter symmetry.

For the explicit evolution law obtained in [5],

$$a(t) = a_0 \sqrt{1 + \frac{t^2}{t_0^2}}, \quad (53)$$

the corresponding Hubble parameter is

$$H(t) = \frac{t}{t^2 + t_0^2}. \quad (54)$$

At late times ($t \gg t_0$), this evolution approaches an asymptotically linear expansion,

$$a(t) \sim t, \quad (55)$$

which follows directly from the underlying wavefunction geometry rather than from an imposed equation of state.

In this regime, the effective cosmological term decays as

$$\Lambda_{\text{eff}}(t) \propto (ct)^{-2}, \quad (56)$$

while the effective stress–energy tensor becomes approximately conserved. Combined with the exact cancellation between the curvature term and the $\rho_{\text{eff}} \propto a^{-2}$ contribution for closed slicing, this implies that the late-time expansion is governed entirely by the residual vacuum-like sector encoded in $\Lambda_{\text{eff}}(t)$.

These results demonstrate that the effective Einstein–Friedmann dynamics on the hypersurface are fully determined by the geometry of the universal wavefunction. The emergence of linear expansion at late times, the value of the effective gravitational coupling, and the decay of the vacuum-like term all follow from a single conserved surface-type invariant, without invoking additional matter components, fine-tuned equations of state, or a fundamental cosmological constant. In the following Discussion, we examine the domain of validity of this effective description, clarify the circumstances under which standard Einstein–Friedmann dynamics are recovered, and identify the regimes in which departures from classical cosmology are expected.

6. Discussion

Relation to de Sitter and Λ CDM cosmology

In standard relativistic cosmology, accelerated expansion is typically attributed to a fundamental cosmological constant or to a dark-energy component with negative pressure. In the present framework, no such ingredient is postulated. Instead, de Sitter-like behaviour emerges geometrically when the embedded hypersurface has an approximately constant curvature scale. In this regime, the effective vacuum term Λ_{eff} is nearly constant, and the induced metric reproduces the familiar de Sitter expansion. This provides a natural geometric interpretation of inflationary dynamics without invoking an inflaton field or a fundamental vacuum energy.

More generally, physically admissible hypersurfaces generated by a normalisable wavefunction envelope exhibit a time-dependent curvature scale. For the critical hypersurface analysed in [5], this leads to an effective cosmological term that is approximately constant at early times and decays at late times as $\Lambda_{\text{eff}}(t) \propto (ct)^{-2}$. At the same time, the conserved surface-type geometric invariant fixes the effective energy density to scale as $\rho_{\text{eff}} \propto a^{-2}$. As shown in Section 5, this scaling causes the matter-like contribution to the Friedmann equation to cancel exactly against the curvature term for closed slicing. Consequently, the late-time expansion is governed entirely by the residual vacuum-like sector and approaches an asymptotically linear regime, $a(t) \propto t$.

From this perspective, both de Sitter and Λ CDM cosmologies appear as effective, phenomenological descriptions valid in regimes where the curvature scale of the hypersurface is approximately constant over the relevant epoch. The present framework clarifies why such descriptions work re-

markably well locally and temporally, while also indicating the conditions under which deviations are expected. In particular, the decay of $\Lambda_{\text{eff}}(t)$ implies a gradual departure from exact de Sitter symmetry and suggests modifications to horizon structure and late-time acceleration that could, in principle, be constrained observationally.

The late-time asymptotically linear expansion obtained here is kinematically similar to that discussed in $R = ct$ cosmological models [9], but arises from a fundamentally different physical origin. In the present approach, linear expansion is not postulated as a global condition or equation of state; rather, it emerges dynamically from the geometric cancellation between curvature and the conserved a^{-2} sector, together with the decay of the effective vacuum term.

Emergent gravity and hierarchy of descriptions

Within this framework, general relativity appears as an effective, local description of the intrinsic geometry of the hypersurface. The Einstein–Friedmann equations arise not as dynamical laws, but as geometric identities that become meaningful once the induced curvature is reinterpreted in the language of relativistic cosmology. The associated continuity equation follows directly from the contracted Bianchi identity, ensuring local consistency of the effective description even when the vacuum-like curvature term varies in time.

The results presented here complete a coherent hierarchy across two complementary studies. In [5], the universal wavefunction geometry establishes a global, coordinate-independent framework and identifies the conserved current underlying temporal ordering. It further provides a concrete realisation of this framework by analysing the evolution of a critical hypersurface and identifying the conserved geometric invariant that governs its expansion. In the present work, we have shown how an effective spacetime description and Einstein–Friedmann structure emerge locally on the hypersurface as derived, approximate constructs. Taken together, these papers suggest that standard cosmological dynamics may be understood as a hydrodynamic limit of a deeper wavefunction-based geometry, with general relativity occupying a distinguished but emergent role.

Several directions for future work naturally follow from this approach. These include the analysis of perturbations and structure formation on the hypersurface, the study of horizon formation and causal structure in the presence of a decaying effective cosmological term, and the investigation of observational signatures that could distinguish this framework from Λ CDM at late times. More broadly, the results provide a unified geometric setting in which early- and late-time cosmological behaviour arise from a single underlying structure, offering a new perspective on the origin of spacetime and gravitational dynamics.

7. Conclusions

In this work, we have demonstrated how an effective spacetime description and Einstein–Friedmann structure can emerge naturally from a universal wavefunction geometry, without postulating gravitational field equations or introducing matter fields by hand. By treating the flux hypersurface associated with a conserved wavefunction current as an embedded Lorentzian manifold, we have shown that its induced geometry is necessarily of Friedmann–Robertson–Walker type under minimal assumptions of homogeneity and isotropy.

A central result of this analysis is the identification of de Sitter spacetime as a special, limiting case of the embedding geometry. When the curvature scale of the hypersurface is approximately constant, the induced metric exhibits de Sitter-like behaviour with an effectively constant vacuum term [2,6]. More generally, for physically relevant envelope classes—including the critical hypersurface realised in [5]—the curvature scale evolves in time. This leads to an effective cosmological term that is approximately constant at early times and decays at late times, providing a purely geometric explanation for both inflationary and post-inflationary expansion regimes.

The dominant contribution to the effective Friedmann dynamics on the hypersurface arises from a conserved, surface-type geometric invariant inherited from the universal wavefunction. This invariant fixes the scaling of the effective energy density as $\rho_{\text{eff}} \propto a^{-2}$ and determines the value of the effective

gravitational coupling. As a consequence, the expansion approaches an asymptotically linear regime at late times, $a(t) \propto t$, which is kinematically similar to the late-time behaviour discussed in $R = ct$ cosmological models [9]. Independently, the embedding geometry gives rise to an effective vacuum-like curvature term that is approximately constant at early times (de Sitter regime) and decays at late times as $\Lambda_{\text{eff}}(t) \propto (ct)^{-2}$. These features arise without invoking dark energy, fine-tuned equations of state, or additional degrees of freedom.

Within this framework, general relativity emerges as an effective, local description of the intrinsic geometry of the hypersurface. The Einstein–Friedmann equations appear not as fundamental dynamical laws, but as geometric identities that become meaningful once the induced curvature is reinterpreted in a form familiar from relativistic cosmology [3]. The associated continuity equation follows directly from the contracted Bianchi identity, ensuring local consistency of the effective description even when the vacuum-like curvature term varies in time.

Funding: This research received no external funding.

Data Availability Statement: All results presented in this work are derived analytically. No new datasets were generated or analyzed.

Conflicts of Interest: The author declare no conflicts of interest.

References

1. Mukhanov, V. *Physical Foundations of Cosmology*; Cambridge University Press, 2005.
2. Carroll, S.M. *Spacetime and Geometry: An Introduction to General Relativity*; Addison-Wesley, 2004.
3. Padmanabhan, T. Gravity as an emergent phenomenon. *Classical and Quantum Gravity* **2004**, *21*, 4485.
4. Verlinde, E. Emergent Gravity and the Dark Universe. *SciPost Physics* **2017**, *2*, 016.
5. Shvedov, V. The Critical Hypersurface as a Geometric Origin of Nonsingular Cosmic Expansion. *Preprints.org* **2025**. Preprint, <https://doi.org/10.20944/preprints202512.1585.v2>.
6. Wald, R.M. *General Relativity*; University of Chicago Press, 1984.
7. Shapiro, I.L.; Solà, J. On the possible running of the cosmological constant. *Physics Letters B* **2009**, *682*, 105–113.
8. Overduin, J.M.; Cooperstock, F.I. Evolution of the scale factor with a variable cosmological term. *Physical Review D* **1998**, *58*, 043506.
9. Melia, F. The $R_H = ct$ universe. *Monthly Notices of the Royal Astronomical Society* **2012**, *419*, 2579–2586.

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.