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Article

On the Hypothesis of Exact Conservation of Charged Weak Hadronic Vector Current in the Standard Model

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Abstract: We investigate a reliability of conservation of the vector current (CV) hypothesis in the neutron β^--decay. We calculate the contribution of the phenomenological term, responsible for the CVC in the hadronic current of the neutron β^--decay (or the CVC effect), to the neutron lifetime. We show that the CVC effect increases the neutron lifetime with a relative contribution 8.684×10^{-2}. This leads to the increase of the neutron lifetime by 76.4 s with respect to the world averaged value $\tau_n = 880.2(1.0)$s (Particle Data Group, Chin. Phys. 40, 100001 (2016)). We show that since in the Standard Model (SM) there are no interactions, which are able to cancel such a huge increase of the neutron lifetime, we have to turn to the interactions beyond the SM the contribution of which to the neutron lifetime reduces to the Fierz interference term b_F only. Cancelling the CVC effect at the level of the experimental accuracy we get $b_F = 0.1219(12)$. If this value cannot be accepted for the Fierz interference term, the CVC effect induces irresistible problems for description and understanding of the neutron β^--decay.

Keywords: Neutron beta decay; ECVC

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1. Introduction

The neutron lifetime with the account for the complete set of corrections of order 10^{-3}, caused by the weak magnetism, proton recoil and electromagnetic interaction, has been calculated in [1]. The theoretical value $\tau_n = 879.6(1.1)$s agrees well with the world averaged one $\tau_n = 880.2(1.0)$s [2] and recent experimental value $\tau_n = 880.2(1.2)$s [3]. The theoretical uncertainty ± 1.1 is fully defined by the experimental uncertainties of the axial coupling constant $\lambda = -1.2750(9)$ [4] and the Cabibbo-Kobayashi-Maskawa (CKM) matrix element $|V_{ud}| = 0.97425(22)$ [5], which agrees well with a new value $|V_{ud}| = 0.97417(21)$ [2] reported by Hardy and Tower [6]. Both values of the CKM matrix elements have been extracted from the $0^+ \to 0^+$transitions with the errors dominated by the theoretical uncertainties caused by nuclear Coulomb distortion and radiative corrections [2,6].

According to recent analysis by Hardy and Tower [6] (see also [7]), the effect of conservation of the vector current (CVC) in the $0^+ \to 0^+$transitions (or in the pure Fermi transitions) is being observed at the level of 1.2×10^{-4}. In turn, as has been found by Naviliat-Cuncic and Severijns [8], in the mirror decays of ^{19}Ne, ^{21}Na, ^{29}P, ^{35}Ar, and ^{17} K, caused by the Gamow-Teller mirror transitions, a new independent test of the CVC effect may be performed at the level of 4×10^{-3} [7]. Recently, the CVC effect has been investigated by Ankowski [9] and Giunti [10] in the inverse β-decay $\bar{\nu}_e + p \to n + e^+$.

2. Precision Analysis of Neutron Lifetime to order 10^{-4}

This work is addressed to the analysis of the reliability of the CVC hypothesis or the CVC effect in the neutron lifetime. The effective low-energy Lagrangian of $V - A$ weak interactions takes the standard form [1,11]

$$\mathcal{L}_W(x) = -\frac{G_F}{\sqrt{2}} V_{ud} J_\mu(x) \left[\bar{\psi}_e(x)\gamma^\mu \left(1 - \gamma^5\right) \psi_{\nu_e}(x)\right] \quad (1)$$

where $G_F = 1.1664 \times 10^{-11}\mathrm{MeV}^{-2}$ and $V_{ud} = 0.97417(21)$ are the Fermi weak constant and the Cabibbo-KobayashiMaskawa (CKM) matrix element [2], respectively, $J_\mu(x)$ is the hadronic $V-A$ current, $\psi_e(x)$ and $\psi_{\nu_e}(x)$ are the operators of the electron and electron neutrino (antineutrino) fields. The amplitude of the neutron β^--decay $n \to p + e^- + \bar{\nu}_e$ is equal to (for more details see Appendix A)

$$M(n \to p e^- \bar{\nu}_e) = -\frac{G_F}{\sqrt{2}} V_{ud} \left\langle p\left(\vec{k}_p, \sigma_p\right) \middle| J_\mu(0) \middle| n\left(\vec{k}_n, \sigma_n\right) \right\rangle \left[\bar{u}_e\left(\vec{k}_e, \sigma_e\right) \gamma^\mu \left(1 - \gamma^5\right) v_\nu\left(\vec{k}, +\frac{1}{2}\right)\right] \quad (2)$$

where $\left|p\left(\vec{k}_p, \sigma_p\right)\right\rangle, \left|n\left(\vec{k}_n, \sigma_n\right)\right\rangle$ are the wave functions of the free proton and neutron with 3-momenta $\vec{k}_p$ and $\vec{k}_n = \vec{0}$ and polarizations $\sigma_p = \pm 1$ and $\sigma_n = \pm 1$, respectively. Then, $\bar{u}_e\left(\vec{k}_e, \sigma_e\right)$ and $v_\nu\left(\vec{k}_\nu, +\frac{1}{2}\right)$ are the Dirac wave functions of the free electron and electron antineutrino with 3− momenta $\vec{k}_e$ and $\vec{k}_\nu$ and polarizations $\sigma_e = \pm 1$ and $+\frac{1}{2}$ [1,11], respectively. In the Standard Model (SM) the matrix element of the hadronic current we take in the following form

$$\begin{aligned} &\left\langle p\left(\vec{k}_p, \sigma_p\right) \middle| J_\mu(0) \middle| n\left(\vec{k}_n, \sigma_n\right) \right\rangle = \\ &\bar{u}_p\left(\vec{k}_p, \sigma_p\right) \left[\left(\gamma_\mu - \frac{q_\mu \hat{q}}{q^2}\right) + \frac{\kappa}{2M} i\sigma_{\mu\nu} q^\nu + \lambda\left(-\frac{2M q_\mu}{q^2 - m_\pi^2} + \gamma_\mu\right)\gamma^5\right] u_n\left(\vec{k}_n, \sigma_n\right) \end{aligned} \quad (3)$$

which is similar to that used by Nowakowski et al. [12] and Leitner et al. [13], where $\bar{u}_p\left(\vec{k}_p, \sigma_p\right)$ and $u_n\left(\vec{k}_n, \sigma_n\right)$ are the Dirac wave functions of the free proton and neutron. Then, $-q_\mu \hat{q}/q^2$, where $q = k_p - k_n$ is a 4− momentum transferred, is the phenomenological term responsible for the CVC in the neutron β^--decay. The term $\kappa/2M$, where $2M = m_n + m_p$ and $m_n = 939.5654\mathrm{MeV}$ and $m_p = 938.2720\mathrm{MeV}$ are the neutron and proton masses, defines the contribution of the weak magnetism, where $\kappa = \kappa_p - \kappa_n = 3.7058$ is the isovector anomalous magnetic moment of the nucleon, defined by the anomalous magnetic moments of the proton $\kappa_p = 1.7928$ and the neutron $\kappa_n = -1.9130$ and measured in nuclear magneton [2]. The contribution of the axial current is given by the last term in Eq.(3), where $\lambda = -1.2750(9)$ is the axial coupling constant [4] (see also [1,11]) and m_π is the charged pion mass [2]. In the limit $m_\pi \to 0$ (or in the chiral limit) the axial current is also conserved [14,15]. Skipping standard calculations [1] we arrive at the rate of the neutron β^--decay given by

$$\frac{1}{\tau_n} = \frac{1}{\tau_n^{(\mathrm{SM})}}\left(1 + \frac{f_n^{(\mathrm{CVC})}}{f_n}\right) \quad (4)$$

where $\tau_n^{(\mathrm{SM})} = 879.6(1.1)\mathrm{s}$ is the theoretical value of the neutron lifetime, calculated in [1] for $\lambda = -1.2750(9)$. It agrees perfectly well with the world averaged value $\tau_n = 880.2(1.0)\mathrm{s}$ [2] and recent experimental one $\tau_n = 880.2(1.2)\mathrm{s}$ [3]. Then, the phase space factor f_n of the neutron, calculated order $O(1/M)$ and $O(\alpha/\pi)$ caused by the contributions of the weak magnetism and proton recoil and the radiative corrections, respectively, is equal to $f_n = 6.116 \times 10^{-2}\mathrm{MeV}^5$. The phase space factor of the neutron $f_n^{(\mathrm{CVC})}$, caused by the CVC effect, is given by the expression

$$\begin{aligned} f_n^{(\mathrm{CVC})} = &\frac{1}{1+3\lambda^2}\int_{m_e}^{E_0} dE_e k_e (E_0 - E_e)^2 F(E_e, Z=1) \int \frac{d\Omega_{e\nu}}{4\pi}\left\{-\frac{2m_e^2\Delta}{m_e^2 + 2E_e(E_0 - E_e) - 2\vec{k}_e\cdot\vec{k}_\nu}\right. \\ &\left. + \frac{m_e^2\Delta^2}{\left(m_e^2 + 2E_e(E_0 - E_e) - 2\vec{k}_e\cdot\vec{k}_\nu\right)^2}\left(E_e - \frac{\vec{k}_e\cdot\vec{k}_\nu}{E_\nu}\right)\right\} \end{aligned} \quad (5)$$

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where $\Delta = m_n - m_p$, $E_0 = \left(m_n^2 - m_p^2 + m_e^2\right)/2m_n = 1.2927\mathrm{MeV}$ is the end-point energy of the electron energy spectrum of the neutron β^--decay [1], $k_e = \sqrt{E_e^2 - m_e^2}$ is an absolute value of the electron 3 -momentum, $F(E_e, Z = 1)$ is the relativistic Fermi function describing the proton-electron Coulomb final-state interaction [1]. Then, $d\Omega_{e\nu}$ is an element of the solid angle of the electron-antineutrino momentum correlations. Since we analyse the main contribution of the CVC effect, the integrand of the phase space factor $f^{(CVC)}$ is calculated to leading order in the large nucleon mass expansion. Because of the numerical value $f_n^{(\mathrm{CVC})} = -4.887 \times 10^{-3}\mathrm{MeV}^5$ the contribution of the CVC effect to the rate of the neutron β^--decay is equal to $f_n^{(\mathrm{CVC})}/f_n = -7.991 \times 10^{-2}$. This corresponds to the relative correction to the neutron lifetime $\Delta\tau_n^{(\mathrm{CVC})}/\tau_n = 8.684 \times 10^{-2}$ that gives $\Delta\tau_n^{(\mathrm{CVC})} = 76.4$ s. Unfortunately, such a huge increase of the lifetime by the CVC effect cannot be accepted for the neutron and should be substantially suppressed for the correct agreement with recent experimental data $\tau_n = 880.2(1.2)$s [3] and world averaged value $\tau_n = 880.2(1.0)$s [2]. In this connection it is important to emphasize that in the SM there are no contributions, which are able to diminish such a huge increase of the neutron lifetime, induced by the phenomenological term $-q_\mu\hat{q}/q^2$ responsible for the CVC in the neutron β^--decay [16–18]. Indeed, the contribution of the pseudoscalar term for a physical mass of the charged pion $m_\pi = 139.570\mathrm{MeV}$ decreases the neutron lifetime at the level of $\Delta\tau_n^{(\pi)}/\tau_n = 4.691 \times 10^{-6}$. However, in the chiral limit $m_\pi \to 0$ the contribution of the charged pion may only aggravate the problem. Hence, in order to reduce a huge contribution of the CVC effect at the level of $f_n^{(\mathrm{CVC})}/f_n = -7.991 \times 10^{-2}$ to the level of 1.2×10^{-4} one has to turn to interactions beyond the SM.

3. Fierz Interference Term

It is well-know [20-23][20–23] (see also the review papers [4,19]) that the simplest contribution of interactions beyond the SM to the neutron β^--decay is the Fierz interference term $b_F m_e/E_e$, caused by scalar and tensor interactions beyond the SM. Below for the analysis of the contribution of the Fierz interference term we use the results, obtained in [1]. As has been shown in [1] all possible interactions beyond the SM [22] give the contribution to the neutron lifetime only in the form of the Fierz interference term. As a result, the Fierz interference term changes the rate of the neutron β^--decay as follows

$$\frac{1}{\tau_n} = \frac{1}{\tau_n^{(\mathrm{SM})}}\left(1 + b_F\left\langle\frac{m_e}{E_e}\right\rangle_{\mathrm{SM}}\right) \tag{6}$$

where $\langle m_e/E_e\rangle_{\mathrm{SM}} = 0.6556$ is calculated with the electron energy spectrum density Eq.(D-59) of Ref. [1]. Taking into account the contribution of the CVC effect we get

$$\frac{1}{\tau_n} = \frac{1}{\tau_n^{(\mathrm{SM})}}\left(1 + \frac{f_n^{(\mathrm{CVC})}}{f_n} + b_F\left\langle\frac{m_e}{E_e}\right\rangle_{\mathrm{SM}}\right) \tag{7}$$

where the right-hand-side (r.h.s.) of Eq.(7) contains a complete set of phenomenological contributions within the SM and contributions beyond the SM. In the obtained expression for the neutron lifetime, given by Eq.(7), effectively the contribution of the CVC effect and the Fierz interference term can be kept at the required level of 1.2×10^{-4} if the Fierz interference term is equal to $b_F = 0.12189(12)$.

4. Conclusions

We have analyzed the CVC hypothesis in the neutron β^--decay and calculated the contribution of the CVC effect, i.e. the contribution of the phenomenological term $-q_\mu\hat{q}/q^2$ responsible for the CVC of the hadronic weak current of the neutron β^--decay. We have shown that the CVC effect gives a relative contribution to the rate of the neutron β^--decay at the level of $f_n^{(\mathrm{CVC})}/f_n = -7.991 \times 10^{-2}$, which is one order of magnitude large compared to the level of 4×10^{-3} of the CVC test in the Gamow-Teller mirror transitions reported by Naviliat-Cuncic and Severijns [8]. As a result, the phenomenological term $-q_\mu\hat{q}/q^2$, providing the CVC of the hadronic current in the neutron β^--decay, changes the lifetime of the neutron by $\Delta\tau_n = 76.4$ s. Since in the SM there are no interactions, which are able to cancel such a huge increase of the neutron lifetime, we have turned to interactions beyond the SM. As has been shown in [1], the contributions of all possible interactions beyond the SM [22], which may affect the energy spectra and angular distributions of the neutron β^--decay, reduce themselves to the contribution to the neutron lifetimes in the form of the Fierz interference term only. Keeping the effective contribution, caused by the CVC effect and the Fierz interference term, at the level of the experimental uncertainty of the neutron lifetime 1.2×10^{-3} we have obtained the Fierz interference term $b_F = 0.1219(12)$, which seems to be also huge in comparison with the results $b_F < 0.01$ Herczeg [24], $b_F = 0.0032$ (23) Faber et al. [25], $b_F = -0.0028(26)$ Hardy and Tower [6] (see also discussion below Eq.(7) of Ref.[6]), and and $|b_F| < 0.03$, reported by H. Saul on behalf of the PERKEO III Collaboration [26]. Thus, if the value of the Fierz interference term $b_F = 0.1219(12)$ is not acceptable one may conclude that the phenomenological realization of the CVC hypothesis in the neutron β^--decay in terms of the contribution $-q_\mu\hat{q}/q^2$ induces irresistible problems for description and understanding of the neutron β^--decay.

Acknowledgments: We want to thank our dear colleague Andrey Nikolaevich Ivanov, who was the main investigator of this work until he sadly passed away on December 18, 2021. We see it as our professional and personal duty to honor his legacy by continuing to publish our collaborative work. Andrey was born on June 3, 1945 in what was then Leningrad. Since 1993 he was a university professor at the Faculty of Physics, named "Peter The Great St. Petersburg Polytechnic University" after Peter the Great. Since 1995 he has been a guest professor at the Institute for Nuclear Physics at the Vienna University of Technology for several years and has been closely associated with the institute ever since. This is also were we met Andrey and have been collaborating with him closely over more than 20 years resulting in 40 scientific publications, see also [27–37]. We will miss Andrey as a personal friend and his immense wealth of ideas, scientific skills and his creativity. See also the official obituary for Andrey Nikolaevich Ivanov. The work of M. Wellenzohn was supported by MA 23 (p.n. 27-07 and p.n. 30-22). The sole responsibility for the content of this publication lies with the authors.

Appendix A. The Amplitude of the Rate of the Neutron Radiative β^--Decay.

In the tree-approximation the Feynman diagrams of the amplitude of the neutron radiative β^--decay are shown in Figure A1. The amplitude of the neutron radiative β^--decay, defined by the diagrams in Figure A1, we describe by the expression [1]

$$M(n\to pe^-\bar{\nu}_e\gamma)_{\lambda'} = e\frac{G_F}{\sqrt{2}}V_{ud}\mathcal{M}(n\to pe^-\bar{\nu}_e\gamma)_{\lambda'} \tag{A1}$$

where e is the proton electric charge, G_F and V_{ud} are the Fermi weak constant and the Cabibbo-Kobayashi-Maskawa (CKM) matrix element, respectively, and the amplitude $\mathcal{M}(n\to pe^-\bar{\nu}_e\gamma)_{\lambda'}$ is equal to

$$\mathcal{M}(n\to pe^-\bar{\nu}_e\gamma)_{\lambda'} = \Big[\bar{u}_p\Big(\vec{k}_p,\sigma_p\Big)\hat{\varepsilon}^*_{\lambda'}(k)\frac{1}{m_p-\hat{k}_p-\hat{k}-i0}O_\mu u_n\Big(\vec{k}_n,\sigma_n\Big)\Big]\Big[\bar{u}_e\Big(\vec{k}_e,\sigma_e\Big)\gamma^\mu\Big(1-\gamma^5\Big)v_\nu\Big(\vec{k},+\frac{1}{2}\Big)\Big]$$
$$-\Big[\bar{u}_p\Big(\vec{k}_p,\sigma_p\Big)O_\mu u_n\Big(\vec{k}_n,\sigma_n\Big)\Big]\Big[\bar{u}_e\Big(\vec{k}_e,\sigma_e\Big)\hat{\varepsilon}^*_{\lambda'}(k)\frac{1}{m_e-\hat{k}_e-\hat{k}-i0}\gamma^\mu\Big(1-\gamma^5\Big)v_\nu\Big(\vec{k},+\frac{1}{2}\Big)\Big] \tag{A2}$$

where $\bar{u}_p\Big(\vec{k}_p,\sigma_p\Big), u_n\Big(\vec{k}_n,\sigma_n\Big), \bar{u}_e\Big(\vec{k}_e,\sigma_e\Big)$ and $v_\nu\Big(\vec{k}_\nu,+\frac{1}{2}\Big)$ are the Dirac wave functions of the free proton, neutron, electron and electron antineutrino with 3-momenta $\vec{k}_p,\vec{k}_n = \vec{0},\vec{k}_e$ and $\vec{k}_\nu$ and polarizations $\sigma_p = \pm1, \sigma_n = \pm1$, $\sigma_e = \pm1$ and $+\frac{1}{2}$ [1,11], respectively, $\varepsilon^{*\alpha}_{\lambda'}(k)$ is the polarization vector of the photon in the polarization state $\lambda' = 1,2$ with a 4 -momentum k, obeying the constraint $\varepsilon^*_{\lambda'}(k)\cdot k = 0$. The amplitude Eq. A2 is gauge invariant. Indeed, one may show that, replacing $\varepsilon^{\alpha*}_{\lambda'}(k) \to k^\alpha$ and using the Dirac equations for the free proton and electron, the amplitude Eq. A2 vanishes. Then, the matrix O_μ is given by

$$O_\mu = \Big(\gamma_\mu - \frac{q_\mu \hat{q}}{q^2}\Big) + \lambda\gamma_\mu\gamma^5 + i\frac{\kappa}{2M}\sigma_{\mu\nu}(k_p - k_n)^\nu \tag{A3}$$

where the matrix O_μ, used in [5,8], is modified by the term $-q_\mu\hat{q}/q^2$, which is introduced according to the CVC hypothesis (see [9,12,13]) with $q = k_n - k_p, \lambda$ is the axial coupling, which we set $\lambda = -1.2750(9)$ (see [1,4,38]), $\kappa = \kappa_p - \kappa_n = 3.7058$ is the isovector anomalous magnetic moment of the nucleon, defined by the anomalous magnetic moments of the proton $\kappa_p = 1.7928$ and the neutron $\kappa_n = -1.9130$ and measured in nuclear magneton [2]. In the tree-approximation the contribution of the term $-q_\mu\hat{q}/q^2$ to the amplitude of the neutron radiative β^--decay, calculated in the baryon non-relativistic approximation to leading order in the large baryon mass expansion, is equal to

$$\delta\mathcal{M}(n \to p e^- \bar{\nu}_e \gamma)_{\lambda'} = \frac{2m_n\Delta}{(k_n - k_p)^2}\Big[\varphi^\dagger_p\varphi_n\Big]\Big[\bar{u}_e\Big(\vec{k}_e, \sigma_e\Big)\frac{1}{2k_e\cdot k}Q_{e\lambda'}\Big(1-\gamma^5\Big)v_\nu\Big(\vec{k}, +\frac{1}{2}\Big)\Big] \tag{A4}$$

where $\Delta = m_n - m_p, Q_{e\lambda'} = 2m_e k_e\cdot\varepsilon^*_{\lambda'} + 2k_e\cdot k\varepsilon^*_{\lambda'} + m_e\hat{\varepsilon}^*_{\lambda'}\hat{k}$ and we have used the Dirac equations for the free proton, neutron and electron antineutrino (see the Appendix of Ref. [38]). We have to emphasize that following [1,38,39] we have kept only the contribution of the photon emitted by the electron, which survives for the physical degrees of freedom of the photon. The contribution of the CVC effect to the branching ratio of the neutron radiative β^--decay is given

$$\begin{aligned}\mathrm{BR}^{(\mathrm{CVC})}_{\beta\gamma} &= \tau_n\frac{\alpha}{\pi}\frac{G^2|V_{ud}|^2}{\pi^3}\int^{\omega_{\max}}_{\omega_{\min}}d\omega\int^{E_0-\omega}_{m_e}dE_e\sqrt{E^2_e - m^2_e}F(E_e, Z=1)(E_0 - E_e - \omega)^2\int\frac{d\Omega_{\nu e}}{4\pi}\int\frac{d\Omega_{\nu\gamma}}{4\pi}\\ &\quad\frac{E_0}{E^2_0 - \Big(\vec{k}_e + \omega\vec{n}_{\vec{k}} + \vec{k}_\nu\Big)^2}\left\{\frac{m^2_e}{\omega}\frac{k^2_e - \Big(\vec{k}_e\cdot\vec{n}_{\vec{k}}\Big)^2}{\Big(E_e - \vec{k}_e\cdot\vec{n}_{\vec{k}}\Big)^2} + \frac{1}{E_e - \vec{k}_e\cdot\vec{n}_{\vec{k}}}\Big[\Big(k^2_e - \Big(\vec{k}_e\cdot\vec{n}_{\vec{k}}\Big)^2\Big)\right.\\ &\quad\left. + 2E_e\frac{\vec{k}_e\cdot\vec{k}_\nu}{E_\nu} - E_e(E_e+\omega)\frac{\vec{n}_{\vec{k}}\cdot\vec{k}_\nu}{E_\nu} - E_e(E_e - \omega)\Big] + (E_e+\omega)\frac{\vec{n}_{\vec{k}}\cdot\vec{k}_\nu}{E_\nu} + \omega\right\},\end{aligned} \tag{A5}$$

where the branching ratio is defined for the photon emitted with an energy from the interval $\omega_{\min} \le \omega \le \omega_{\max}$. Then, $E_0 = 1.2927\mathrm{MeV}$ is the end-point energy of the electron energy spectrum of the neutron β^--decay [1], $F(E_e, Z=1)$ is the relativistic Fermi function taking into account the Coulomb proton-electron final-state interaction [1,38], $d\Omega_{\nu e} = \sin\vartheta_{\nu e}d\vartheta_{\nu e}d\varphi_{\nu e}$ and $d\Omega_{\nu gamma} = \sin\vartheta_{\nu\gamma}d\vartheta_{\nu\gamma}d\varphi_{\nu\gamma}$ are elements of solid angles of antineutrino-electron and antineutrino-photon momentum correlations, respectively, such as $\vec{k}_e\cdot\vec{k}_\nu = k_e(E_0 - E_e - \omega)\cos\vartheta_{\nu e}, \vec{n}_{\vec{k}}\cdot\vec{k}_\nu = (E_0 - E_e - \omega)\cos\vartheta_{\nu\gamma}$ and $\vec{k}_e\cdot\vec{n}_{\vec{k}} = k_e(\cos\vartheta_{\nu e}\cos\vartheta_{\nu\gamma} + \sin\vartheta_{\nu e}\sin\vartheta_{\nu\gamma}\cos(\varphi_{\nu e} - \varphi_{\nu\gamma}))$. For the numerical analysis of the branching ratio $\mathrm{B}^{(\mathrm{CVC})}_{\beta\gamma}$ we use the theoretical value of the neutron lifetime $\tau_n = 879.6(1.1)\mathrm{s}$, which agrees perfectly well with the world averaged value $\tau_n = 880.2(1.0)\mathrm{s}$ [2]. The numerical values of the branching ratio we adduce in Table A1, more details on the numerical solution of Eq. (A5) find in Appendix B.

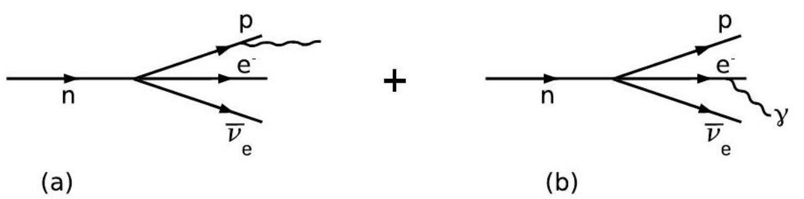

Figure A1. Feynman diagrams, defining the amplitude of the neutron radiative β^--decay in the tree-approximation.

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Table A1. The contribution of the CVC effect to the branching ratio of the neutron radiative β^--decay for three photon energy regions. In the last column we give a total theoretical value of the branching ratio of the neutron radiative β^--decay, calculated for three photon energy regions.

ω [keV]	$\mathrm{BR}_{\beta\gamma}$(Experiment)	$\mathrm{BR}^{(\mathrm{CVC})}_{\beta\gamma}$(Theory)	$\mathrm{BR}_{\beta\gamma}$ (Theory)
$15 \le \omega \le 340$	$(3.09 \pm 0.32) \times 10^{-3}$ [4]	3.57×10^{-4}	3.25×10^{-3}
$14 \le \omega \le 782$	$(3.35 \pm 0.05\,[\mathrm{stat}] \pm 0.15\,[\mathrm{syst}]) \times 10^{-3}$ [1]	3.78×10^{-4}	3.42×10^{-3}
$0.4 \le \omega \le 14$	$(5.82 \pm 0.23\,[\mathrm{stat}] \pm 0.62\,[\mathrm{syst}]) \times 10^{-3}$ [1]	8.07×10^{-4}	5.89×10^{-3}

The contribution of the CVC effect to the amplitude of the neutron β^--decay is equal to

$$\mathcal{M}(n \to p e^- \bar{\nu}_e) = \Big[\bar{u}_p\big(\vec{k}_p, \sigma_p\big)\Big(\gamma_\mu - \frac{q_\mu \hat{q}}{q^2} + \lambda\gamma_\mu\gamma^5\Big)u_n\big(\vec{k}_n, \sigma_n\big)\Big]\Big[\bar{u}_e\big(\vec{k}_e, \sigma_e\big)\gamma^\mu\big(1 - \gamma^5\big)v_\nu\Big(\vec{k}, +\frac{1}{2}\Big)\Big]. \quad \text{(A6)}$$

This changes the neutron lifetime as follows (see also Appendix C

$$\Delta\tau_n^{(\mathrm{CVC})} = -\tau_n^2 \frac{G_F^2|V_{ud}|^2}{2\pi^3}\int_{m_e}^{E_0} dE_e k_e (E_0 - E_e) F(E_e, Z = 1)\int \frac{d\Omega_{e\nu}}{4\pi}$$
$$\frac{E_0 m_e^2}{\big(m_e^2 + 2E_e(E_0 - E_e) - 2\vec{k}_e\cdot\vec{k}_\nu\big)^2}\Big\{(4E_e - 3E_0)\Big[E_e(E_0 - E_e) - \vec{k}_e\cdot\vec{k}_\nu\Big] - 2m_e^2(E_0 - E_e)\Big\}. \quad \text{(A7)}$$

The contribution of the CVC effect to the rate of the neutron β^--decay is defined by

$$\lambda_n^{(\mathrm{CVC})} = \big(1 + 3\lambda^2\big)\frac{G_F^2|V_{ud}|^2}{2\pi^3} f_n^{(\mathrm{CVC})}(E_0) \quad \text{(A8)}$$

where $f_n^{(\mathrm{CVC})}(E_0)$ is given by

$$f_n^{(\mathrm{CVC})}(E_0) = \frac{1}{1 + 3\lambda^2}\int_{m_e}^{E_0} dE_e k_e (E_0 - E_e)^2 F(E_e, Z = 1)\int\frac{d\Omega_{e\nu}}{4\pi}\Big\{\Big[-\frac{2E_0 m_e^2}{m_e^2 + 2E_e(E_0 - E_e) - 2\vec{k}_e\cdot\vec{k}_\nu}$$
$$+ \frac{E_0^2 m_e^2}{\big(m_e^2 + 2E_e(E_0 - E_e) - 2\vec{k}_e\cdot\vec{k}_\nu\big)^2}\Big(E_e - \frac{\vec{k}_e\cdot\vec{k}_\nu}{E_\nu}\Big)\Big]$$
$$+ \lambda^2\Big[\frac{m_e^2}{m_e^2 + 2E_e(E_0 - E_e) - 2\vec{k}_e\cdot\vec{k}_\nu}\Big(E_0 - E_e + \frac{\vec{k}_e\cdot\vec{k}_\nu}{E_\nu}\Big)$$
$$+ \frac{m_e^2}{\big(m_e^2 + 2E_e(E_0 - E_e) - 2\vec{k}_e\cdot\vec{k}_\nu\big)^2}\Big((E_0 - E_e)^2 + k_e^2 + 2\vec{k}_e\cdot\vec{k}_\nu\Big)\Big(E_e - \frac{\vec{k}_e\cdot\vec{k}_\nu}{E_\nu}\Big)\Big]\Big\} \quad \text{(A9)}$$

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Appendix B. Numerical Analysis of the Branching Ratio in Eq. (A5)

The numerical analysis of the contribution of the CVC effect to the branching ratio of the neutron radiative β^-– decay reduces to the calculation of the following in integral

$$\begin{aligned}I(\omega_{\max},\omega_{\min}) =& \frac{E_0}{8\pi}\int_{\omega_{\min}}^{\omega_{\max}} d\omega \int_{m_e}^{E_0-\omega} dE_e k_e F(E_e, Z=1)(E_0-E_e-\omega)^2 \int_{-1}^{+1} dx \int_{-1}^{+1} dy \int_0^{2\pi} d\varphi \\ &\Big[E_0^2 - k_e^2 - \omega^2 - (E_0-E_e-\omega)^2 - 2k_e(E_0-E_e-\omega)x - 2\omega(E_0-E_e-\omega)y \\ &-2k_e\omega\Big(xy + \sqrt{(1-x^2)(1-y^2)}\cos\varphi\Big)\Big]^{-1}\Big\{\frac{m_e^2}{\omega}\frac{k_e^2\Big[1-\Big(xy+\sqrt{(1-x^2)(1-y^2)}\cos\varphi\Big)^2\Big]}{\Big[E_e - k_e\Big(xy+\sqrt{(1-x^2)(1-y^2)}\cos\varphi\Big)\Big]^2} \\ &+ \frac{1}{E_e - k_e\Big(xy+\sqrt{(1-x^2)(1-y^2)}\cos\varphi\Big)}\Big\{k_e^2\Big[1-\Big(xy+\sqrt{(1-x^2)(1-y^2)}\cos\varphi\Big)^2\Big] \\ &+ 2k_eE_e x - E_e(E_e+\omega)y - E_e(E_e-\omega)\Big\} + (E_e+\omega)y + \omega\Big\},\end{aligned} \tag{A10}$$

where $k_e = \sqrt{E_e^2 - m_e^2}$. The integral should be calculated for three intervals i) $15\times10^{-3}\mathrm{MeV} \le \omega \le 340\times10^{-3}\mathrm{MeV}$, ii) $14\times10^{-3}\mathrm{MeV} \le \omega \le 782\times10^{-3}\mathrm{MeV}$ and iii) $0.4\times10^{-3}\mathrm{MeV} \le \omega \le 14\times10^{-3}\mathrm{MeV}$ with $E_0 = 1.2927\mathrm{MeV}$ and $m_e = 0.511\mathrm{MeV}$. The Fermi function $F(E_e, Z=1)$ is equal to

$$F(E_e, Z=1) = \Big(1+\frac{1}{2}\gamma\Big)\frac{4(2r_p m_e\beta)^{2\gamma}}{\Gamma^2(3+2\gamma)}\frac{e^{\pi\alpha/\beta}}{(1-\beta^2)^\gamma}\Big|\Gamma\Big(1+\gamma+i\frac{\alpha}{\beta}\Big)\Big|^2 \tag{A11}$$

where $\beta = k_e/E_e = \sqrt{E_e^2-m_e^2}/E_e$ is the electron velocity, $\gamma = \sqrt{1-\alpha^2}-1$, $r_p = 4.262\times10^{-3}\mathrm{MeV}^{-1}$ is the electric radius of the proton and $\alpha = 1/137.036$ is the fine-structure constant.

Appendix C. Numerical Analysis of Eq. (A7)

The numerical analysis of Eq. (A7) reduces to the calculation of the integral

$$\begin{aligned}I =& \frac{1}{2}m_e^2 E_0 \int_{m_e}^{E_0} dE_e \sqrt{E_e^2-m_e^2}(E_0-E_e)^2 F(E_e, Z=1) \\ &\int_{-1}^{+1} dx \frac{1}{\Big(m_e^2 + 2(E_0-E_e)\Big(E_e - x\sqrt{E_e^2-m_e^2}\Big)\Big)^2}\Big\{(4E_e - 3E_0)\Big(E_e - x\sqrt{E_e^2-m_e^2}\Big) - 2m_e^2\Big\}\end{aligned} \tag{A12}$$

for $E_0 = 1.2927\mathrm{MeV}$ and $m_e = 0.511\mathrm{MeV}$ with $k_e = \sqrt{E_e^2-m_e^2}$. The Fermi function is given by Eq.(A-2).

One has to calculate the following integral

$$\begin{aligned}f_n^{(\mathrm{CVC})}(E_0) =& \frac{1}{1+3\lambda^2}\int_{m_e}^{E_0} dE_e k_e (E_0-E_e)^2 F(E_e, Z=1)\frac{1}{1}\int_{-1}^{+1} dx \\ &\Big\{k_1\Big[-\frac{2E_0 m_e^2}{m_e^2 + 2(E_0-E_e)(E_e-k_e x)} + \frac{E_0^2 m_e^2}{(m_e^2+2(E_0-E_e)(E_e-k_e x))^2}(E_e - k_e x)\Big] \\ &+ k_2\lambda^2\Big[\frac{m_e^2}{m_e^2+2(E_0-E_e)(E_e-k_e x)}(E_0 - E_e + k_e x) \\ &+ \frac{m_e^2}{(m_e^2+2(E_0-E_e)(E_e-k_e x))^2}\Big((E_0-E_e)^2 + k_e^2 + 2(E_0-E_e)k_e x\Big)(E_e-k_e x)\Big]\Big\}\end{aligned} \tag{A13}$$

where $\lambda = -1.2750$, $E_0 = 1.2927\mathrm{MeV}$ and $m_e = 0.511\mathrm{MeV}$ with $k_e = \sqrt{E_e^2-m_e^2}$. The Fermi function is given by Eq. A11). The calculation to perform for i) $k_1 = k_2 = 1$, ii) $k_1 = 1$ and $k_2 = 0$ and iii) $k_1 = 0$ and $k_2 = 1$.

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Taking into account the non-vanishing mass of charged pions one has to calculate the following integral

$$
\begin{aligned}
f_n^{(\mathrm{CVC})}(E_0) =& \frac{1}{1+3\lambda^2}\int_{m_e}^{E_0} dE_e k_e (E_0 - E_e)^2 F(E_e, Z=1)\frac{1}{1}\int_{-1}^{+1} dx \\
&\Big\{k_1\Big[-\frac{2E_0 m_e^2}{m_e^2 + 2(E_0 - E_e)(E_e - k_e x)} + \frac{E_0^2 m_e^2}{(m_e^2 + 2(E_0 - E_e)(E_e - k_e x))^2}(E_e - k_e x)\Big] \\
&+ k_2\lambda^2\Big[\frac{m_e^2}{m_e^2 + 2(E_0 - E_e)(E_e - k_e x) - m_\pi^2}(E_0 - E_e + k_e x) \\
&+ \frac{m_e^2}{(m_e^2 + 2(E_0 - E_e)(E_e - k_e x) - m_\pi^2)^2}\Big((E_0 - E_e)^2 + k_e^2 + 2(E_0 - E_e)k_e x\Big)(E_e - k_e x)\Big]\Big\},
\end{aligned}
\tag{A14}
$$

where $m_\pi = 139.570\mathrm{MeV}$, $\lambda = -1.2750$, $E_0 = 1.2927\mathrm{MeV}$ and $m_e = 0.511\mathrm{MeV}$ with $k_e = \sqrt{E_e^2 - m_e^2}$. The Fermi function is given by Eq.(A-2). The calculation should be performed for $k_1 = 0$ and $k_2 = 1$.

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