Article

Not peer-reviewed version

Quantum Emergence of Black Holes and Dark Matter

Mauricio Rodriguez *

Posted Date: 7 July 2025

doi: 10.20944/preprints202507.0472.v1

Keywords: Dark matter; Black holes; Wave function; Quantum mechanics; Gravitational singularities; Primordial universe; Quantum gravity; Early universe dynamics; Planck scale physics

Article

Quantum Emergence of Black Holes and Dark Matter

Mauricio Rodríguez López

Independent Researcher; Maouskm9@gmail.com

Abstract

Within the theoretical framework of quantum and relativistic physics, the absence of a more comprehensive explanation regarding the emergence of dark matter and black holes still persists [1]. Such a gap could potentially be bridged by adopting an alternative approach and modifying the relationship between phase and group velocity[2]. The approach developed expands the quantum-relativistic framework without contradicting its fundamental principles, offering a coherent and rigorous reinterpretation of anomalous phenomena that challenge conventional interpretations[3]. Which allows us to examine the effects, properties, and dynimic of **dark matter and black holes**. This proposition enables a deeper understanding of the processes involved in gravitationally collapsing stars and the Big Bang [4], as well as the link between the phase-group velocity relationship and mass or curvature[5], thereby providing new conceptual tools to explore what has, until now, remained misunderstood within the current paradigm.

Keywords: Dark matter; Black holes; Wave function; Quantum mechanics; Gravitational singularities; Primordial universe; Quantum gravity; Early universe dynamics; Planck scale physics

---

Introduction

In wave propagation theory and quantum field theory, two essential quantities describe the behavior of waves: the phase velocity $v_p = \frac{w}{k}$ and the group velocity $v_g = \frac{dw}{dk}$[5]. In general, these two velocities differ, especially indispersive media, where the relationship between angular frequency (w) and wave number (k) is nonlinear. When spacetime curvature effects at microscopic scales are considered, the traditional model becomes insufficient[6].

This Paper Analyzes 2 Different Cases.

The first proposal involves a modification to the free massive scalar Lagrangian by incorporating a forurth-order dispersive term derived from the d'Alembertian operator $\Box$, which enables a nontrivial relationship between the phase velocity (v_p) and the group velocity (v_g) of the field modes[7]. In particular, it is shown that under specific conditions—most likely related to extreme curvature of gravitational collapse—both velocities converge or even become equal for particles with mass[8]. This confers unique properties on particles undergoing this transition, which will be analyzed in further detail in this article, as it may represent the most plausible quantum explanation for the emergence of black holes.

The second proposal considers a modified Lagrangian with non-local operators that generate an altered dispersion relation in which the group velocity can exceed the phase velocity, but diverges in the infrared limit, while the phase velocity tends toward zero[9]. This decoupling prevents phase-based information propagation yet permits energy transport via the group component. This establishes a "frozen" field: non-excitable, non-thermodynamic, but with effective energy. This property makes it "invisible" to all interactions except gravity, making it the most plausible explanation for dark matter[10].

1. First Case:

 doi:10.20944/preprints202507.0472.v1

The condition $V_p = V_g$ emerges as a fundamental criterion for the formation of black holes within a modified massive scalar field Lagrangian[11]. Typically, phase and group velocities only coincide when the dispersion relation w(k) is linear, as in non-dispersive media. For massive particles, the classical relativistic dispersion relation is given by: $w^2 = c^2k^2 + \frac{m^2c^4}{\hbar^2}$ [12]

By modifying the free massive scalar field Lagrangian to include a correction to the dispersion relation, a fourth-order term involving the d'Alembertian squared, $\Box^2$, is introduced[13].

$$L = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}m^2\, \phi^2$$

$$\downarrow$$

$$L = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}\, m^2\, \phi^2 + \left(\frac{a}{2}\right)(\Box\phi)^2$$

The correction term includes a parameter a with dimensions of area, which functions as a squared dispersion length and introduces effects relevant to regions of intense curvature[14].

By tuning this coefficient $a = \frac{m^2c^4}{\hbar^2k^4}$ for a given frequency scale, one can effectively cancel the dispersive behavior of the wave, enforcing the condition $v_p = v_g$, thus emulating a non-dispersive medium within curved spacetime, such as during gravitational collapse[15].

Modification of the Equation of Motion

Applying the principle of minimum action to the modified Lagrangian:

$$S = \int d^4x\, L$$

where the variation S=0 for arbitrary variations $\delta\phi$ gives us the equation of motion

New term:

$$\left(\frac{\partial L}{\partial \phi}\right) - \partial_\mu\left(\frac{\partial L}{\partial(\,\partial_\mu \phi\,)}\right) + \partial_\mu\partial_\nu\left(\frac{\partial L}{\partial(\,\partial_\mu\partial_\nu \phi\,)}\right) = 0$$

Then:

$$\frac{\partial L}{\partial \phi} = -m^2\phi$$

$$\frac{\partial L}{\partial(\,\partial_\mu \phi\,)} = \partial^\mu \phi$$

$$\frac{\partial L}{\partial(\,\partial_\mu\partial_\nu \phi\,)} = a\eta^{\mu\nu}\Box\phi, because\ \Box\phi = \eta^{\rho\sigma}\partial_\rho\partial_\sigma\phi$$

Deriving:

$$-m^2\phi - \partial_\mu\partial^\mu\phi + a\partial_\mu\partial^\mu\Box\phi = 0$$

Or

$$(\Box + m^2)\phi - a\Box^2\phi = 0$$

Fourier Space Analysis

Now we use the fourier transform for plane wave type solutions:

$$\phi(x,t) = e^{-i(\omega t - \vec{k}\cdot\vec{x})}$$

The $\Box$ operator is replaced by its action in wave mode:

$$\Box \rightarrow -\omega^2 + c^2k^2, defining: X := -\omega^2 + c^2k^2$$

The fourier equation of motion would look like a quadratic equation in X :

$$X\phi + m^2\phi - aX^2\phi = 0 \implies \quad aX^2 - X - m^2 = 0$$

The Modified Dispersion Equation Through the Solution of the Quadratic Equation in X

We solve:

$$X = \frac{1 \pm \sqrt{1 + 4a \cdot m^2}}{2a}$$

We clear:

$$\omega^2 = c^2k^2 - X = c^2k^2 - \frac{1 \pm \sqrt{1 + 4a \cdot m^2}}{2a}$$

We obtain

Condition $v_p = v_g$ and Solution for k

Modified dispersion relation: $\omega^2 = c^2k^2 + a \cdot k^4 + \frac{m^2c^4}{\hbar^2}$

Phase velocity: $v_p = \frac{\omega}{k} = \left(\frac{1}{k}\right) \cdot \sqrt{c^2k^2 + a \cdot k^4 + \frac{m^2c^4}{\hbar^2}}$

Group velocity: $v_g = \frac{d\omega}{dk} = \left(\frac{1}{2\omega}\right) \cdot \frac{d}{dk}\left[c^2k^2 + a \cdot k^4 + \frac{m^2c^4}{\hbar^2}\right] = \left(\frac{1}{\omega}\right) \cdot (c^2k + 2a \cdot k^3)$

Deriving:

$$\frac{\omega}{k} = \frac{(c^2k + 2a \cdot k^3)}{\omega} \implies \omega^2 = c^2k^2 + 2a \cdot k^4$$

Equating both terms:

$$c^2k^2 + 2a \cdot k^4 = c^2k^2 + a \cdot k^4 + \frac{m^2c^4}{\hbar^2} \implies a \cdot k^4 = \frac{m^2c^4}{\hbar^2} \implies k^4 = \frac{m^2c^4}{a \cdot \hbar^2}$$

Critical Condition

$$k^2 = \sqrt{\frac{m^2c^4}{a \cdot \hbar^2}}$$

As a result, through this modified Lagrangian, the phase and group velocities become equivalent within a specific regime of k, enabling non-dispersive wave propagation under extreme spacetime conditions[16].

The Quantum Reality of Black Holes

In the following, we explore a possible interpretation of the results obtained in the previous section. This interpretation is not intended to be definitive, but rather to offer a coherent reading fron the quantum point of view

The quantum phenomena that occur within gravitationally collapsing stars—particulary in the case of those transforming into black holes—remain largely unknown, yet they are key to understanding the transition from stellar matter to a black hole[17]. Broadly speaking, the most plausible explanation for the phenomenon we know as a “black hole” arises from a modification in the relationship between group velocity and phase velocity under extreme conditions, with a

 doi:10.20944/preprints202507.0472.v1

collapsing star providing the ideal environment for such a transformation[14], more specifically, when a massive particle reaches a regime in which its phase velocity (v_p) equals its group velocity (v_g), the associated wave packet becomes stationary and minimally dispersive, enabling exceptionally high quantum coherence between its spectral components[5]. This coincidence means that all modes of the wave packet propagate in phase, generating highly stable constructive interference across spacetime[7]. As this transformation occurs under extreme pressure gradients, where each collapsing layer transfers its energy into the next, the process proceeds explosively fast due to immense gravitational pressure. The transitions halts the collapse just before the formation of a singularity, entering a fermionic state that behaves bosonically, giving rise to stable, frozen core at the mass-light boundary[18].

Time ceases to be experienced at this point, and no futher compression is possible. This explains why, upon the merger of two black holes, the resulting object grows in size rather than compressing, as no external matter or energy can enter the core without losing causal structure so it can neither emit nor reflect light, that implies that, nothing can enter without undergoing to the same state of gravitational condensation[19].

In such a regime, particles no longer behave as isolated excitations but as part of a coherent condensate—a macroscopic excitation of the field with perfect phase correlations between its components[20]. Classical notions of "density" and "volume" lose all meaning. While individual particle identities may be preserved through quantum numbers (as in a Bose–Einstein condensate), the entire system behaves as a single collective wavefuction, reflecting the underlying symmetry of the field[21]. Locality is preserved, but the dynamics are governed by total synchronization and correlation among the system's components. This is what we refer to as a **black hole**

2. Second Case

$V_g > V_p$ as an essential condition for the quantum emergence of dark matter through the modification of phase and group velocity propagation

In conventional scalar fields, the dispersion relation governs both phase and group propagation, linking them to energy and information. In this particular case, however, we aim to construct a field that precisely describes the behavior of dark matter[1].

In this way, the effective scalar field X(x) is introduced in order to determine the dynamics and fundamental properties of a non-excitable physical sistem:

$$L = -\frac{1}{2}X(x)\boldsymbol{\mathcal{O}}X(x)$$

where $\mathcal{O} = \sqrt{-\square} + \frac{m^2c^3}{\sqrt{-\square}}$ is a non-standard operator that depends on $\square$ and the mode k

In the theory under consideration, the effective scalar field X(x) obeys a modified dispersion relation:

$$w(k) = c|k| + \frac{m^2c^3}{|k|}$$

By deriving a modified dispersion relation, we find that the group velocity $v_g = \frac{dw}{dk} = c - \frac{m^2c^3}{k^2}$ can exceed the phase velocity $\frac{w}{k}$ and diverges in the infrared limit (k→0), while the phase velocity tends to zero[22]. This dissociation prevents the propagation of information (no phase), while still allowing energy transport via the group component.

This leads to a "frozen" or "static" field—non-interactive, non-excitable, non-thermodynamic, but possessing effective energy. Such a structure becomes invisible to all interactions except the gravitational one[10].

Non-Local Lagrangian and Its Geometric Extension

 doi:10.20944/preprints202507.0472.v1

The effective non-local Lagrangian in flat space is:

$$L = -\frac{1}{2}X(x)\left(\sqrt{-\square}+\frac{m^2c^3}{\sqrt{-\square}}\right)X(x)$$

The operador $\sqrt{-\square}$ establishes a linear relation with |k|: representing a form of wave that lacks phase–i.e.,without interference or information, but capable of pure energy transmission[23].

The expression $\sqrt{-\square}+\frac{m^2c^3}{\sqrt{-\square}}$ represents an inverse effective mass that diverges at low energies, effectively preventing any excitation at low frequencies or large wavelengths. This results in a field gravitationally confined, undetectable by any non-gravitational interaction[24].

This field transmits energy solely through its group mode, not through phase. That means it cannot be excited locally as localized particles, cannot be detected via photons, and cannot be heated. It is fundamentally dark and unmeasurable by non-gravitational means, yet possesses effective mass[18]

We further introduce a coupling term (ξ) to the Ricci scalar R, allowing this non-local Lagrangian to be extended to curved spacetime[6]:

$$L = -\frac{1}{2}X(x)\left(\sqrt{-\square+\ \xi R}\ +\frac{m^2c^3}{\sqrt{-\square+\ \xi R}}\right)X(x)$$

The extended non-local Lagrangian depends functionally on □ + ξR, and the coupling allows the field to respond to local geometry

Behavior Near Large Masses (Curvatures)

In regions of high curvature (*R*>0) the operator$\sqrt{-\square+\ \xi R}$ smooths out the infrared divergence, regulating the behavior of the field under extreme conditions[25].

$$\frac{1}{|k|}\to\frac{1}{\sqrt{k^2+\ \xi R}}$$

This implies that the effective energy of the field decreases in regions of high curvature *R* (such as in the presence of strong gravitational mass, e.g., a galaxy) thereby energetically favoring the concentration of X(x) specifically in such zones[10].

The field does not move toward these regions due to inertia, but rather stabilizes there energetically; its minimum energy configuration lies in areas of high curvature. Thus, it condenses or accumulates into a *static halo,* not as a freely falling particle but as a gravitationally-bound structure[26].

At low energy (k⟶0)

The $\frac{m^2c^3}{\sqrt{-\square+\ \xi R}}$ *terminus dominate*

At high energy (k⟶∞)

The $\sqrt{-\square+\ \xi R}$ *terminus dominate*

Calculation of the Energy-Momentum Tensor

In the case of a flat background, the energy-momentum tensor is obtained by varying the Lagrangian with respect to the metric:

$$T_{\mu\nu} = \left(\frac{-2}{\sqrt{-g}}\right)\left(\frac{\delta S}{\delta g^{\mu\nu}}\right)\,|_{\,g=\eta}$$

 doi:10.20944/preprints202507.0472.v1

The differential operator $\square = \eta^{\mu\nu}\partial_\mu\partial_\nu$ and the action S is obtained by intergrating the corresponding Lagrangian density. In this case, since the operator $\sqrt{-\square}$ is non-local, its variation leads to non-trivial terms in the functional derivation of the energy-momentum tensor[27].

By applying the rules of non-local functional calculus—such as generalized commutators, Leibniz-like rules, and regularized functional variations— the tensor is effectively reduced[28]:

$$T_{\mu\nu} = \partial_\mu X(x)\left(\frac{\delta \mathcal{O}}{\delta\partial^\nu X(x)}\right) - \eta_{\mu\nu}L$$

Remember: $\mathcal{O} = \sqrt{-\square} + \frac{m^2c^3}{\sqrt{-\square}}$

However, this result is formally obtained while neglecting the full complexity of the operator in curved spacetime, particularly the variation of $\square$. This limitation motivates a natural extension to its covariant form[25].

Transition to Curved Space and Covariant Operators

The covariant extension is archieved by replacing the flat-space d'Alembertian $\square$ with its geometrical counterpart:

$$\square \rightarrow -\nabla^\mu\nabla_\mu$$

And by adding a scalar coupling term to the Ricci curvature (ξR), we obtain the effective operator[6]:

$$\mathcal{O}_g = \sqrt{-\nabla^2 + \xi R} + \frac{m^2c^3}{\sqrt{-\nabla^2 + \xi R}}$$

Then the action in curved space becomes:

$$S = \int d^4x\sqrt{-g}\,X(x)\left(\sqrt{-\nabla^2 + \xi R} + \frac{m^2c^3}{\sqrt{-\nabla^2 + \xi R}}\right)X(x)$$

Now the calculation of the energy-momentum tensor requires the evaluation of the functional variation:

$$T_{\mu\nu} = \left(\frac{-2}{\sqrt{-g}}\right)\left(\frac{\delta S}{\delta g^{\mu\nu}}\right)$$

The key difficulty lies in the variation of the non-local operator with respect to the metric, which is non-trivial an involves functional derivates of ∇^2 with respect to $g^{\mu\nu}$[29].

These derivations also involve connection terms ($\Gamma^\lambda_{\mu\nu}$) and potential regularization schemes to define $\delta\sqrt{\mathcal{O}}$ through spectral techniques such as the Seeley-DeWitt method or the heat kernel expansion[30].

Functional Derivation of the Action

We need to calculate:

$$\delta S = \int d^4x\,\delta(\sqrt{-g})X\mathcal{O}_gX + \sqrt{-g^\delta}\,(X\mathcal{O}_gX)$$

where $\delta(\sqrt{-g})$ is:

$$\delta(\sqrt{-g}\,) = -\frac{1}{2}\sqrt{-g}\,g_{\mu\nu}\delta g^{\mu\nu}$$

Now the second term requires:

$$\delta(X\mathcal{O}_g X) = X\delta(\mathcal{O}_g)X$$

We expand the variation of the operator:

$$\delta(\mathcal{O}_g) = \delta\left(\sqrt{P} + \frac{m^2c^3}{\sqrt{P}}\right),\quad P = -\nabla^2 + \xi R$$

We make use of the functional formula:

$$\delta\sqrt{P} = \frac{1}{2}\int_0^\infty \frac{ds}{\sqrt{\pi s}} e^{-sP}\delta P$$

Similarly:

$$\delta\left(\frac{1}{\sqrt{P}}\right) = -\frac{1}{2}\int_0^\infty \frac{ds}{\sqrt{\pi s^3}} e^{-sP}\delta P$$

Thus, We Obtain $\delta(\mathcal{O}_g)$:

$$\delta(\mathcal{O}_g) = \int_0^\infty \frac{ds}{\sqrt{\pi s}} e^{-sP}\delta P - \frac{m^2c^3}{2}\int_0^\infty \frac{ds}{\sqrt{\pi s^3}} e^{-sP}\delta P$$

Variation of P with respect to $g^{\mu\nu}$:

$$P = -g^{p\sigma}\nabla_p\nabla_\sigma + \xi R$$

Now:

$$\delta P = -\delta g^{p\sigma}\nabla_p\nabla_\sigma - g^{p\sigma}\delta(\Gamma^\lambda_{p\sigma})\nabla_\lambda + \xi\delta R$$

where:

$$\delta\Gamma^\lambda_{\mu\nu} = \frac{1}{2}g^{\lambda a}(\nabla_\mu\delta\, g_{\nu a} + \nabla_\nu\delta g_{\mu a} - \nabla_a\delta g_{\mu\nu})$$

Finally:

$$\delta R = \nabla_\mu\nabla_\nu\delta g^{\mu\nu} - \square(g_{\mu\nu}\delta g^{\mu\nu})$$

Resultant Energy-Momentum Tensor

$$T_{\mu\nu} = \delta_{g^{\mu\nu}}\left[\sqrt{-g}\,X(x)\mathcal{O}_g X(x)\right] = T^{geom}_{\mu\nu} + T^{op}_{\mu\nu}$$

$T^{geom}_{\mu\nu}$: comes from the variation $\sqrt{-g}$
$T^{op}_{\mu\nu}$: comes from $\delta\mathcal{O}_g$
The final result involves functional integrals of heat kernel type:

$$T_{\mu\nu} \supset \int_0^\infty ds\, Tr\left[K_s(x,x)\left(\frac{\delta P}{\delta g^{\mu\nu}}\right)\right]$$

This framework allows one to express the functional variation of the energy-momentum tensor in non-local field theories on curved backgrounds[31].

By using integral representations (heat kernel methods), one can regularize $\delta(\mathcal{O}_g)$ and obtain closed-form expressions for the contributions arising from curvature, connection, and metric variations. This method provides a mathematically precise framework to define effective energies and energy-momentum tensors, especially in context where fields do not propagate or excite classically, yet still contribute gravitationally[32].

The Quantum Reality of Dark Matter

In the following, we explore a possible interpretation of the results obtained in the previous section. This interpretation is not intended to be definitive, but rather to offer a coherent reading fron the quantum point of view

During the Planck era, when the universe itself still lacked a well-defined metric and distinct classical fields, reality was governed by extreme quantum fluctuations, exotic phenomena, and a total absence of conventional causality[33]. In this *pre-geometric* scenario, the notions of group velocity and phase velocity were fundamentally meaningless, as spacetime itself had not yet emerged[34].

This transition toward a geometrical background likely occurred between the Planck time and the Grand Unified Theory (GUT) epoch, a period during which a profound modification of the group-phase velocity relation is hypothesized to have taken place. This modification would stem from intense temporal anisotropy and symmetry-breaking processes associated with the formation of the geometric fabric of spacetime[35].

Unlike a classical medium, the vacuum during this primordial phase of the Big Bang was inhomogeneous, featuring regions of differential coupling among modes, which could have supported more exotic states of matter but also stable. These states may have acquired an inverted dispersion relation, such as that of a non-local field with mass, wherein the group velocity exceeds the phase velocity[36]. This inversion is not a violation of causality, but rather reflects a deeper form of connectivity—a quantum-coherent network extending beyond classical spacetime[37]. In such a regime, energetic excitations no longer behave as localized particles in the quantum-relativistic sense, but rather manifest as extended field excitations, akin to condensates. Their identity as localized, well-defined entities dissolves, thereby breaking the correspondence with the postulates of causality and localization inherent in standard particle-based quantum field theory[21].

The propagation of energy in this scenario does not occur via localized, point-like excitations, but through spatially non-localized perturbations. These are characterized by coherent phase distributions across the entire Fourier spectrum of the field, making it impossible to assign a precise location to the excitation[12]. Consequently, energy transport is better described as a collective field dynamic, wherein both information and energy are distributed over an extended spatial region. This distribution does not necessarily adhere to localization and causality constraints typically required for massive energetic excitations in standard quantum field theory[38].

This phenomenon corresponds to what we identify as **dark matter:** a non local, non-thermal, and non-interacting condensate-like field excitation that remains invisible to all forces except gravity[26].

References

1. Bertone, G., Hooper, D., & Silk, J. (2005). Particle dark matter: evidence, candidates and constraints. Physics Reports, 405(5-6), 279–390.
2. Magueijo, J., & Smolin, L. (2002). Lorentz invariance with an invariant energy scale. Physical Review Letters, 88(19), 190403.
3. Amelino-Camelia, G. (2002). Doubly-special relativity: First results and key open problems. International Journal of Modern Physics D, 11(10), 1643–1669.
4. Misner, C. W., Thorne, K. S., & Wheeler, J. A. (1973). Gravitation. W. H. Freeman.

5. Brillouin, L. (1960). Wave Propagation and Group Velocity. Academic Press.
6. Parker, L., & Toms, D. J. (2009). Quantum Field Theory in Curved Spacetime: Quantized Fields and Gravity. Cambridge University Press.
7. Barcelo, C., Liberati, S., & Visser, M. (2005). Analogue gravity. Living Reviews in Relativity, 8(1), 12.
8. Calmet, X., & Casadio, R. (2017). Quantum Black Holes from General Relativity. European Physical Journal C, 77, 318.
9. Nicolini, P., Smailagic, A., & Spallucci, E. (2006). Noncommutative geometry inspired Schwarzschild black hole. Physics Letters B, 632(4), 547–551.
10. Hui, L., Ostriker, J. P., Tremaine, S., & Witten, E. (2017). Ultralight scalars as cosmological dark matter. Physical Review D, 95(4), 043541.
11. Visser, M. (1998). Acoustic black holes: Horizons, ergospheres and Hawking radiation. Classical and Quantum Gravity, 15(6), 1767–1791.
12. Peskin, M. E., & Schroeder, D. V. (1995). An Introduction to Quantum Field Theory. Addison-Wesley.
13. Bekenstein, J. D. (2004). Black holes and information theory. Contemporary Physics, 45(1), 31–43.
14. Magueijo, J., & Smolin, L. (2002). Generalized Lorentz invariance with an invariant energy scale. Physical Review D, 67(4), 044017.
15. Barvinsky, A. O., & Vilkovisky, G. A. (1985). The Generalized Schwinger-DeWitt Technique in Gauge Theories and Quantum Gravity. Physics Reports, 119(1), 1–74.
16. Calmet, X., & Casadio, R. (2014). The horizon of quantum black holes. The European Physical Journal C, 74, 2685.
17. Penrose, R. (1965). Gravitational Collapse and Space-Time Singularities. Physical Review Letters, 14(3), 57–59.
18. Dvali, G., & Gomez, C. (2013). Black hole's quantum N-portrait. Fortschritte der Physik, 61(7-8), 742–767.
19. Bekenstein, J. D. (1973). Black holes and entropy. Physical Review D, 7(8), 2333–2346.
20. Mazur, P. O., & Mottola, E. (2004). Gravitational vacuum condensate stars. Proceedings of the National Academy of Sciences, 101(26), 9545–9550.
21. Hu, B. L., & Verdaguer, E. (2008). Stochastic Gravity: Theory and Applications. Living Reviews in Relativity, 11(1), 3.
22. Mattingly, D. (2005). Modern tests of Lorentz invariance. Living Reviews in Relativity, 8(1), 5.
23. Bender, C. M., & Orszag, S. A. (1999). Advanced Mathematical Methods for Scientists and Engineers. Springer.
24. Barcelo, C., Liberati, S., & Visser, M. (2001). Analog gravity from Bose–Einstein condensates. Classical and Quantum Gravity, 18(6), 1137–1156.
25. Donoghue, J. F. (1994). General relativity as an effective field theory: The leading quantum corrections. Physical Review D, 50(6), 3874–3888.
26. Ferreira, E. G. M. (2021). Ultralight dark matter. Astronomy and Astrophysics Review, 29(1), 7.
27. Woodard, R. P. (2014). Nonlocal models of cosmic acceleration. Foundations of Physics, 44, 213–233.
28. Barvinsky, A. O., & Vilkovisky, G. A. (1990). Covariant perturbation theory (II). Nuclear Physics B, 333(2), 471–511.
29. Birrell, N. D., & Davies, P. C. W. (1982). Quantum Fields in Curved Space. Cambridge University Press.
30. Vassilevich, D. V. (2003). Heat kernel expansion: User's manual. Physics Reports, 388(5), 279–360.
31. Barvinsky, A. O., & Nesterov, D. V. (2006). Nonlocal effective action and heat kernel in curved spacetime. Physical Review D, 73(6), 066012.
32. Elizalde, E., Odintsov, S. D., Romeo, A., Bytsenko, A. A., & Zerbini, S. (1994). Zeta regularization techniques with applications. World Scientific Publishing.
33. Rovelli, C. (2020). Helgoland: Making Sense of the Quantum Revolution. Penguin Random House.
34. Isham, C. J. (1995). Structural issues in quantum gravity. In General Relativity and Gravitation (pp. 167-209). World Scientific.
35. Kiefer, C. (2012). Quantum Gravity. Oxford University Press.
36. Barvinsky, A. O., & Nesterov, D. V. (2012). Nonlocal gravity and the diffusion equation. Physical Review D, 86(12), 124015.

 doi:10.20944/preprints202507.0472.v1

37. Hossenfelder, S. (2010). Experimental Search for Quantum Gravity. Springer.
38. Padmanabhan, T. (2010). Gravitation: Foundations and Frontiers. Cambridge University Press.