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Article

Interpretation of Gravity by Entropy

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Abstract: In this paper, we introduce generalized entropy, the acceleration of its entropy and its the partial entropy. We assume that generalized entropy can be expressed as a second-order polynomial by applying the idea of logistics function to its entropy. In other words, we assume that the acceleration of generalized entropy is a constant. Besides, we show that the negative inverse of the partial entropy can express Newton's classical gravity, which is an inverse square law. By applying these concepts, we attempt to explain that 1) Gravity is constant within small distances with some conditions. It is possible that gravity have 5-states within small distance. Furthermore, within small distance, we show the possibility that the gravitational potential and the Coulomb potential can be treated in the same way, that 2) The rotation speed of a galaxy does not depend on its radius if the radius is within the size level of the universe. (The galaxy rotation curve problem), and that 3) The gravitational acceleration toward the center changes at long distances compared to the classical theory of gravity. We show that it is an extension of Newton's classical theory of gravity. Furthermore, we show that the possibility of the existence of some constants which controls gravity and the speed of galaxies, and that gravity may relate on entropy.

Keywords: entropy; gravity; galaxy rotation curve; MOND; Planck's law; dynamical system; inverse square law; logistic function

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1. Introduction

In this paper, we will explain in the following order.

1.1.

First, We define generalized entropy $S_D(x,k)$ and partial entropy $S_D(x)$ partitioned by the partition function $D(x)$, and introduce the acceleration of partial entropy $S''_D(x)$, where x is a positive variable, ξ is a positive constant. $Q_D(x)$ is a positive function as satisfied $Q_D(x) = \xi x/D(x)$.

1.2.

Second, by applying the idea of logistics to that entropy, Using the ideas of logistic theory, we derive a function $Q(x)$ that defines the partition function $D(x)$. Moreover, we assume that generalized entropy $S_D(x,k)$ is approximated by second-degree polynomial, that is, the formula $\lambda_2 x^2 + \lambda_1 x$. In other words, we assume that the second derivative of $S_D(x,k)$ is a constant $\lambda_2/2$.

1.3.

Third, the negative inverse of partial entropy $S_D(x)$ is defined as the potential $V_D(x,k)$, and the first derivative of potential $V_D(x,k)$ is defined as the acceleration $V'_D(x,k)$ Namely, it is assumed that the potential and the acceleration are derived from entropy.

1.4.

Finally, according as the theory of gravity, the inverse $1/\lambda_2$ is interpreted as the mass m, the constant k is interpreted as the gravitational constant G, and the variable x is interpreted as the distance R, etc. Thereby, the potential $V_D(x,k)$, and the acceleration $V'_D(x,k)$ are interpreted as the gravitational potential $V(R,G)$ and the gravitational acceleration $V'(R,G)$, Therefore, we show and propose some conclusions :

1. When the distance is small enough, gravity is constant regardless of R and does not become infinite, except some conditions. It is possible that gravity have 5-states within the distance

R is small enough. Furthermore, within small distance, we show that the possibility that the gravitational potential and the Coulomb potential can be treated in the same way.

2. At distances large enough to be within the size of the universe, gravity follows an adjusted inverse law. Within this distances, the rotation speed of a galaxy v follows the gravitational constant G, the mass m and some constants, not depend on its galaxy radius R (the galaxy rotation curve problem).
3. At large distances, gravity follows an adjusted inverse square law. Comparing to conventional gravity, the gravitational acceleration towards the center of rotation is slightly weaker or stronger. This means that the gravitational acceleration towards the center of a rotating substance can be slightly changed at distance. (Pioneer Anomaly)

The gravitational acceleration $V'(R,G)$ is an extension of Newton's classical theory of gravity. Furthermore, it is possible there exists some constants that controls gravity and the speed of galaxies. Besides, some constants depends on the definition of entropy, therefore gravity is thought to depend on entropy.

2. Generalized Entropy and Application to Dynamical Systems

In this section, we introduce generalized entropy, its partial entropy and its acceleration entropy. By using logistic function models, we attempt to discuss adjusted gravity and the rotation speed of galaxy.

2.1. Generalized Entropy $S_D(x,k)$ and Its Partial Entropy $S_D(x)$

We define generalized entropy as follows. In this paper, the function log represents the natural logarithm $\log_e$.

Definition 1. *Generalized Entropy $S_D(x,k)$ and its partial entropy $S_D(x)$.*
Let $x > 1$ be a real variable, and $k \gneqq 0$ and $\xi \gneqq 0$ be real constants. Let $D(x)$ be a positive real valued function that partitioning x. $S_D(x,k)$, $S_D(x)$ and $Q_D(x)$ are defined as follows:

$$S_D(x,k) = kD(x)S_D(x), \tag{1}$$

$$Q_D(x) = \frac{\xi x}{D(x)}, \tag{2}$$

$$\begin{aligned} S_D(x) &= (1+\frac{x}{D(x)})\log(1+\frac{x}{D(x)}) - \frac{x}{D(x)}\log(\frac{x}{D(x)}), \\ &= (1+\frac{Q_D(x)}{\xi})\log(1+\frac{Q_D(x)}{\xi}) - \frac{Q_D(x)}{\xi}\log(\frac{Q_D(x)}{\xi}), \end{aligned} \tag{3}$$

where for any positive variable $x > 0$, the function Q_D is satisfied as follows :

$$Q_D \gneqq 0, \quad Q'_D \gneqq 0. \tag{4}$$

□

On above definition, $S'_D(x)$ and $S''_D(x)$ are expressed as follows :

$$S'_D(x) = \frac{Q'_D(x)}{\xi}\left(\log(1+\frac{Q_D(x)}{\xi}) - \log(\frac{Q_D(x)}{\xi})\right) \tag{5}$$

$$\begin{aligned} S''_D(x) &= \frac{Q'_D(x)}{\xi}\left(\frac{1}{\xi+Q_D(x)} - \frac{1}{Q_D(x)}\right) \\ &+ \frac{Q''_D(x)}{\xi}\left(\log(1+\frac{Q_D(x)}{\xi}) - \log(\frac{Q_D(x)}{\xi})\right). \end{aligned} \tag{6}$$

We will call $S'_D(x)$ entropy generation (velocity) of $S_D(x)$, and $S''_D(x)$ entropy acceleration of $S_D(x)$. The function $Q_D(x)$ can be regard as the position partitioned a real value ξx by $Q_D(x)$. The first order derivative of $Q_D(x)$, that is, $Q'_D(x)$ can be regard as the change of the position by x and ξ. (Refer to Fujino[22] for details on how to derive generalized entropy, entropy acceleration and its partial entropy.)

2.2. The Function $Q_D(x)$ and the Approximated of Generalized Entropy $S_D(x,k)$

Next, we find the function $Q_D(x)$ using the idea behind Planck's radiation formula and logistic function. Put the part of partial entropy $S''_D(x)$ as follows :

$$\frac{Q'_D(x)}{\xi}\left(\frac{1}{\xi + Q_D(x)} - \frac{1}{Q_D(x)}\right) = -\mu(x), \tag{7}$$

where $\mu(x) > 0$ is a positive real function. The left side of above formula (7) looks like spectra partitioned by $\xi x/Q_D(x)$ and the right side of (7) become an approximation by the function $\mu(x)$. We consider $Q'_D(x)$ as follows :

$$Q'_D(x) = \frac{dQ_D}{dx}. \tag{8}$$

Transforming according to formula (8), we can express as follows :

$$dQ_D\left(\frac{1}{\xi + Q_D(x)} - \frac{1}{Q_D(x)}\right) = -\xi\mu(x)dx. \tag{9}$$

Integrating both sides gives as follows :

$$\log(\xi + Q_D(x)) - \log(Q_D(x)) = -\xi \int \mu(x)dx \pm \mu_1, \tag{10}$$

where $\mu_1 \geq 0$. Since the left side of the above equation is a positive number and $\xi > 0$, $\mu_1 > 0$, therefore the right side is also a positive. Therefore, we consider only the case where the sign of μ_1 is positive as follows :

$$-\xi \int \mu(x)dx + \mu_1 > 0. \tag{11}$$

Therefore, the following equation is satisfied :

$$\log(1 + \frac{\xi}{Q_D(x)}) = -\xi \int \mu(x)dx + \mu_1. \tag{12}$$

By transforming the above equation, it is satisfied as follows :

$$1 + \frac{\xi}{Q_D(x)} = \exp(-\xi \int \mu(x)dx + \mu_1). \tag{13}$$

Therefore, the function $Q_D(x)$ is expressed as follows :

$$Q_D(x) = \frac{\xi}{\exp(-\xi \int \mu(x)dx + \mu_1) - 1}. \tag{14}$$

The function $Q_D(x)$ becomes the distribution function of the position which the real value ξx partitioned by $Q_D(x)$. The formula (7) also looks like spectra partitioned by $\xi x/Q_D(x)$. If we actually take $Q_D(x)$ to $\log(x)$, we obtain an equation similar to the expand of Planck's radiation formula, (Refer to Planck[1] and Fujino[22]). If we partition it further into squares of discrete integers, put $Q'_D(x) = 1$, $\mu(x)$ is expressed the wave number, and $1/\xi$ is the Rydberg constant Ry, then it resembles the Rydberg formula that represents a spectral series. Namely, entropy may be related to atomic spectra and its energy levels. We would like to make this a topic of research in the future.

Next, we make assumption about the approximated of generalized entropy $S_D(x,k)$.

Assumption 1. *Assume generalized entropy $S_D(x,k)$ can be approximated by a second-degree polynomial. Hence set as follows :*

$$S_D(x,k) = \lambda_2 x^2 \pm \lambda_1 x, \tag{15}$$

where $\lambda_2 \gneqq 0$ $\lambda_1 \geq 0$ are real numbers, and $S_D(0,k) = 0$. □

Hence, the first derivative $S'_D(x,k)$ is expressed a first-degree polynomial as follows :

$$S'_D(x,k) = 2\lambda_2 x \pm \lambda_1. \tag{16}$$

Besides, the second derivative $S''_D(x,k)$ is constant. Namely, it is satisfied as follows :

$$S''_D(x,k) = 2\lambda_2. \tag{17}$$

In other words, we assume that the second derivative of $S_D(x,k)$ is a constant.

2.3. *The Inverse of Partial Entropy $S_D(x)$ and Potential $V_D(x,k)$*

Next, we focus on the inverse of partial entropy $S_D(x)$ as follows :

$$\frac{1}{S_D(x)} = k\frac{\xi x}{Q_D(x)}\frac{1}{\lambda_2 x^2 \pm \lambda_1 x}. \tag{18}$$

By formula (14), we can express as follows :

$$\frac{1}{S_D(x)} = k\frac{1}{\lambda_2 x \pm \lambda_1}(\exp(-\xi\int \mu(x)dx + \mu_1) - 1). \tag{19}$$

We define the negative inverse of $S_D(x)$ as the potential $V_D(x,k)$:

$$V_D(x,k) = -k\frac{\frac{1}{\lambda_2}}{x \pm \frac{\lambda_1}{\lambda_2}}(\exp(-\xi\int \mu(x)dx + \mu_1) - 1). \tag{20}$$

In other words, the above potential $V_D(x,k)$ can be defined as the product of a constant k, the partition $D(x) = \xi/Q_D(x)$, and the negative inverse of the general entropy $S_D(x,k)$.

Here, let us reorganize the above, that is, we assume as follows :

$$S_D(x,k) = \lambda_2 x^2 \pm \lambda_1 x, \tag{21}$$

$$\frac{Q'_D(x)}{\xi}\left(\frac{1}{\xi + Q_D(x)} - \frac{1}{Q_D(x)}\right) = -\mu(x), \tag{22}$$

where $\lambda_2 \gneqq 0$ is positive real number and $\mu(x) \gneqq 0$ is a positive real function. Therefore, we define the negative inverse of expression $S_D(x)$ as the potential $V_D(x,k)$:

$$V_D(x,k) = -k\frac{\frac{1}{\lambda_2}}{x \pm \frac{\lambda_1}{\lambda_2}}(\exp(-\xi\int \mu(x)dx + \mu_1) - 1). \tag{23}$$

where $\xi \gneq 0, \lambda_2 \gneq 0, \lambda_1 \geq 0, \mu_1 \geq 0$ are real numbers and $\mu(x) \gneq 0$ is a positive real function. The first derivative $V_D'(x,k)$ is satisfied as follows :

$$\begin{aligned} V_D'(x,k) = k\frac{\frac{1}{\lambda_2}}{(x \pm \frac{\lambda_1}{\lambda_2})^2}(\exp(-\xi\int\mu(x)dx + \mu_1) - 1) \\ + k\frac{\frac{1}{\lambda_2}}{x \pm \frac{\lambda_1}{\lambda_2}}\xi\mu(x)\exp(-\xi\int\mu(x)dx + \mu_1)). \end{aligned} \tag{24}$$

Let $V_D(x,k)$ be named as the potential of $S_D(x,k)$, and $V_D'(x,k)$ be named as the acceleration of $S_D(x,k)$. Namely, we assume as follows :

Assumption 2. *It assume that the potential $V_D(x,k)$ is defined the negative inverse of the partial entropy $S_D(x)$. Therefore, the acceleration $V_D'(x,k)$ is defined the first derivative of $V_D(x,k)$.*

In the next chapter, we will discuss applications of the potential $V_D(x,k)$ and the acceleration $V_D'(x,k)$.

3. Application of $V_D(x,k)$ to Gravity

The constants, variables, and functions in the above formulas can be chosen arbitrarily within the range of conditions. Therefore, we attempt to interpret these constants, variables and functions as gravity. Namely, we attempt to interpret $V_D(x,k)$ as the gravitational potential and $V_D'(x,k)$ as the gravitational acceleration.

3.1. Interpretation to $V(R,G)$

We consider the interpretation of formula $V_D(x,k)$ as follows :

$$\begin{aligned} & x := R \geq 0, && R \text{ is the distance,} \\ & \frac{1}{\lambda_2} := m \gneq 0, && m \text{ is the mass within } R, \\ & k := G, && G \text{ is the gravitational constant,} \\ & \xi := \xi^g, && \xi^g \text{ is a constant,} \\ & \mu(x) := \mu_2^g \gneq 0, && \mu_2^g \text{ is a positive real constant,} \\ & \mu_1 := \mu_1^g \geq 0, && \mu_1^g \text{ is a real constant,} \\ & \lambda_1 := \lambda_1^g \geq 0, && \lambda_1^g \text{ is a real constant,} \end{aligned} \tag{25}$$

where the symbol g in the upper right corner of the alphabet stands for g of gravity.

We assume as follows :

The direction with smaller R is defined as the central direction. The gravitational potential increases away from the center and decreases toward the center. However, when $R = 0$ become $V(R,G) = 0$. Moreover, it assume the constant $1/\lambda_2$ is equal to the mass m within R.

Assumption 3. *Assume the constant $1/\lambda_2$ is equal to the mass m within R.*
Assume that 2-times the inverse of entropy acceleration, $2/S_D''(x,k)$, that is, $1/\lambda_2$ is equal to the mass m within R. In other word, the mass m within R is defined as the inverse of the second-order term of $S_D(x,k)$, that is $1/\lambda_2$. □

According assumption 3, if the entropy acceleration $S_D''(x,k)$ is large, the mass m becomes small, and if the entropy acceleration $S_D''(x,k)$ is small, the mass m becomes large. Doesn't this relationship between entropy acceleration and mass seem intuitive?

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We define $V(R,G)$ as the gravitational potential of G as follows :

$$\begin{aligned} V(R,G) &= -\frac{Gm}{R \pm \lambda_1^g m}(\exp(-\xi^g \mu_2^g \int dR + \mu_1^g) - 1), \\ &= -\frac{Gm}{R \pm \lambda_1^g m}(\exp(-\xi^g \mu_2^g R + \mu_1^g) - 1), \end{aligned} \tag{26}$$

where $\xi^g \gneq 0$, $\mu_2^g \gneq 0$, $\mu_1^g \geq 0$ and $\lambda_1^g \geq 0$.

The first derivative $V'(R,G)$ is satisfied as follows :

$$\begin{aligned} V'(R,G) =& \frac{Gm}{(R \pm \lambda_1^g m)^2}(\exp(-\xi^g \mu_2^g R + \mu_1^g) - 1) \\ &+ \frac{Gm\xi^g \mu_2^g}{R \pm \lambda_1^g m}\exp(-\xi^g \mu_2^g R + \mu_1^g). \end{aligned} \tag{27}$$

The above formula (27) become the gravitational acceleration. The second term of formula (27) becomes like Yukawa potential. (Refer to H.Yukawa[17], R.Feynman[2])

The solution of formula (27) for μ_1^g is satisfied as follows :

$$\mu_1 = \log(\frac{1}{1 + \xi^g \mu_2^g (R \pm \lambda_1^g m)}) + \xi^g \mu_2^g R. \tag{28}$$

Since the following conditions is needed to satisfied :

$$\exp(-\xi^g \mu_2^g R + \mu_1^g) = \frac{1}{1 + \xi^g \mu_2^g (R \pm \lambda_1^g m)} > 0, \tag{29}$$

hence, it is satisfied as follows :

$$1 + \xi^g \mu_2^g (R \pm \lambda_1^g m) > 0. \tag{30}$$

Therefore,

$$\pm \lambda_1^g < \left(R + \frac{1}{\xi^g \mu_2^g}\right)\frac{1}{m}. \tag{31}$$

Namely, the following conditions are satisfied :

$$if\ 1 + \xi^g \mu_2^g (R - \lambda_1^g m) > 0, \quad then \quad \lambda_1^g < \left(R + \frac{1}{\xi^g \mu_2^g}\right)\frac{1}{m}, \tag{32}$$

$$if\ 1 + \xi \mu_2^g (R + \lambda_1^g m) > 0, \quad then \quad \lambda_1^g > -\left(R + \frac{1}{\xi^g \mu_2^g}\right)\frac{1}{m}, \tag{33}$$

where $\xi^g \gneq 0, \mu_2^g \gneq 0$, $m \gneq 0, \lambda_1^g \geq 0$, the distance $R > 0$ and the mass $m > 0$.

3.2. When the Distance R Is Small Enough

If the distance R is small enough, that is, since R approaches 0, hence $\exp(-\xi^g \mu_2^g R)$ approaches 1 infinitely. Therefore, the formula (27) is satisfied as follows:

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$$\begin{aligned} V'(R,G) &= \frac{Gm}{(R \pm \lambda_1^g m)^2}(\exp(\mu_1^g)-1) + \frac{Gm\xi^g \mu_2^g}{R \pm \lambda_1^g m}\exp(\mu_1^g) \\ &\simeq \frac{G}{(\lambda_1^g)^2 m}(\exp(\mu_1^g)-1) + \frac{G\xi^g \mu_2^g}{\pm\lambda_1^g}\exp(\mu_1^g), \\ &(\because R \to 0,\ \exp(-\xi^g \mu_2^g R) \to 1). \end{aligned} \tag{34}$$

If the distance $\lambda_1^g = 0$, then it is satisfied as follows :

$$V'(R,G) = \frac{Gm}{R^2}(\exp(-\xi^g \mu_2^g R + \mu_1^g)-1) + \frac{Gm\xi^g \mu_2^g}{R}\exp(-\xi^g \mu_2^g R + \mu_1^g). \tag{35}$$

The case of the above formula (35), if $R \to 0$, then it become $V'(R,G) \to \infty$. Hence, we consider that it make $\lambda_1^g \neq 0$ and R is small enough, and later consider the case $\lambda_1^g = 0$.

Therefore, if the distance R is small enough, then the acceleration $V'(R,G)$ is approximated by the constant. Namely, the following formula is satisfied :

Suggestion 1. *The acceleration $V'(R,G)$ becomes a constant with small R.*
Let m be a positive real number (the mass). For sufficiently small distance $R > 0$, the following condition is satisfied : the acceleration $V'(R,G)$ becomes a constant.

$$V'(R,G) \simeq \frac{G}{(\pm\lambda_1^g)^2 m}(\exp(\mu_1^g)-1) + \frac{G\xi^g \mu_2^g}{\pm\lambda_1^g}\exp(\mu_1^g), \tag{36}$$

where $\xi^g \gneq 0, \mu_2^g \gneq 0, \lambda_1^g \geq 0$ and $\mu_1^g \geq 0$. □

The solution of formula (36) for μ_1 is satisfied as follows :

$$\mu_1^g = \log\left(\frac{1}{1 \pm \lambda_1^g \mu_2^g \xi^g m}\right) > 0. \tag{37}$$

Since the following conditions is needed to satisfied :

$$\exp(\mu_1^g) = \frac{1}{1 \pm \lambda_1^g \mu_2^g \xi^g m} > 0, \tag{38}$$

hence, it is satisfied as follows :

$$1 \pm \lambda_1^g \mu_2^g \xi^g m > 0. \tag{39}$$

Because $\xi^g \gneq 0, \mu_2^g \gneq 0$, $m \gneq 0$ and $\lambda_1^g \geq 0$, hence, these values condition is satisfied as follows :

$$if \quad 1 - \lambda_1^g \mu_2^g \xi^g m > 0, \quad then \quad \lambda_1^g < \frac{1}{\xi^g \mu_2^g m}, \tag{40}$$

$$if \quad 1 + \lambda_1^g \mu_2^g \xi^g m > 0, \quad then \quad \lambda_1^g > -\frac{1}{\xi^g \mu_2^g m}. \tag{41}$$

According to the sign of plus and minus of λ_1^g and μ_1^g, the formula (36) and its solution for μ_1^g can be classified four patterns as follows :

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Case A) : $+\lambda_1^g, +\mu_1^g$

$$V'(R,G) \simeq \frac{G}{(\lambda_1^g)^2 m}(\exp(\mu_1^g) - 1) + \frac{G\xi^g\mu_2^g}{\lambda_1^g}\exp(\mu_1^g), \tag{42}$$

$$\mu_1^g = \log(\frac{1}{1+\lambda_1^g\mu_2^g\xi^g m}) \quad if\ V'(R,G) = 0. \tag{43}$$

This above case does not occur because $\mu_1^g > 0$ and $1+\lambda_1^g\mu_2^g\xi^g m > 0$. However, if $\mu_1^g \to 0$, then $V'(R,G) = \frac{G\xi^g\mu_2^g}{\lambda_1^g}$. Therefore ξ is also $\xi \to 0$ and the divisions become very small.

Case B) : $-\lambda_1^g, +\mu_1^g$

$$V'(R,G) \simeq \frac{G}{(\lambda_1^g)^2 m}(\exp(\mu_1^g) - 1) + \frac{G\xi^g\mu_2^g}{-\lambda_1^g}\exp(\mu_1^g), \tag{44}$$

$$\mu_1^g = \log(\frac{1}{1-\lambda_1^g\mu_2^g\xi^g m}) \quad if\ V'(R,G) = 0. \tag{45}$$

This above case does not occur because $\mu_1^g > 0$ and $1 \pm \lambda_1^g\mu_2^g\xi^g m > 0$. However, if $\mu_1^g \to 0$, then $V'(R,G) = -\frac{G\xi^g\mu_2^g}{\lambda_1^g}$. Therefore ξ is also $\xi \to 0$ and the divisions become very small.

Suggestion 2. *The classified $V'(R,G)$ with small R.*
According to values of $\xi^g \gtrapprox 0, \mu_2^g \gtrapprox 0$ $\lambda_1^g \geq 0$ and $\mu_1^g \geq 0$, the formula (36) can be classified as follows:

Case 1) If the constant μ_1^g is satisfied as follows :

$$\mu_1^g > \log(1 \pm \lambda_1^g\mu_2^g\xi^g m) \geq 0, \tag{46}$$

then the above formula (36) is positive, that is, it is satisfied as follows :

$$V'(R,G) > 0. \tag{47}$$

Case 2) If the constant μ_1^g is satisfied as follows :

$$0 \leq \mu_1^g \leq \log(1 \pm \lambda_1^g\mu_2^g\xi^g m), \tag{48}$$

then the above formula (36) is negative, that is, it is satisfied as follows :

$$V'(R,G) \leq 0. \tag{49}$$

Case 3) If the constant $\mu_1^g = 0$, then the following condition is satisfied :

$$V'(R,G) \simeq \frac{G\xi^g\mu_2^g}{\pm\lambda_1^g}, \qquad (\because \exp(\pm\mu_1^g) \to 1). \tag{50}$$

□

3.2.1. Summarize the Gravitational Acceleration for Small Enough R

By the above the acceleration $V'(R,G)$, we summarize the gravitational acceleration as follows :

$$g_{\pm\lambda_1^g+\mu_1^g} = \frac{Gm}{(R\pm\lambda_1^g m)^2}(\exp(-\xi^g\mu_2^g R + \mu_1^g) - 1) + \frac{Gm\xi\mu_2^g}{R\pm\lambda_1^g m}\exp(-\xi^g\mu_2^g R + \mu_1^g), \tag{51}$$
$$Adjusted\ Gravitational\ Acceleration\ with\ \xi^g\ and\ \mu_2^g,$$

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If the distance $\lambda_1^g = 0$, then

$$\tilde{g}_{\pm 0+\mu_1^g} = \frac{Gm}{R^2}(\exp(-\xi^g \mu_2^g R + \mu_1^g) - 1) + \frac{Gm\xi^g \mu_2^g}{R}\exp(-\xi^g \mu_2^g R + \mu_1^g), \quad (52)$$
$$\mathit{Adjusted\ Gravitational\ Acceleration\ with}\ \xi^g\ \mu_2^g\ \mathit{and}\ \lambda_1^g = 0,$$

If the distance $R \to 0$, then

$$\tilde{g}_{\pm\lambda_1^g+\mu_1^g} = \frac{G}{(\pm\lambda_1^g)^2 m}(\exp(\mu_1^g) - 1) + \frac{G\xi^g \mu_2^g}{\pm\lambda_1^g}\exp(\mu_1^g), \quad (53)$$
$$\mathit{Adjusted\ Gravitational\ Acceleration\ with}\ \xi^g,\ R\ \mathit{is\ small\ enough}.$$

Therefore, if the distance $R \to 0$, then it is satisfied as follows :

$$\begin{aligned}
&\tilde{g}_{\pm\lambda_1^g+\mu_1^g} = \lim_{R\to 0} \tilde{g}_{\lambda_1^g+\mu_1^g\pm}, \\
&\lim_{\mu_1^g\to 0} \tilde{g}_{\pm\lambda_1^g+\mu_1^g} = \frac{G\xi^g\mu_2^g}{\pm\lambda_1^g}, \\
&\lim_{\mu_1^g\to 0, \lambda_1^g\to 0} \tilde{g}_{\pm\lambda_1^g+\mu_1^g} = \pm\infty, \\
&\lim_{\mu_1^g\to 0, \lambda_1^g\to \infty} \tilde{g}_{\pm\lambda_1^g+\mu_1^g} = 0, \\
&\lim_{\mu_1^g\to \infty} \tilde{g}_{\pm\lambda_1^g+\mu_1^g} = \infty, \quad \because) \quad \lambda_1^g < \frac{1}{\xi^g\mu_2^g m}, \ \mathit{no\ this\ case}, \\
&\lim_{\mu_1^g\to \infty} \tilde{g}_{\pm\lambda_1^g+\mu_1^g} = -\infty, \quad \because) \quad \lambda_1^g > -\frac{1}{\xi^g\mu_2^g m}, \ \mathit{no\ this\ case}.
\end{aligned} \quad (54)$$

From the above, we can suppose as follows :

Suggestion 3. *Within the distance R is small enough, gravity have 5-states.*
Within the distance R is small enough, it is possible that gravity have 5-states such that finite 2-states $\frac{G\xi^g\mu_2^g}{\pm\lambda_1^g}$, *and that infinite 2-states* $\pm\infty$ *and 1-state of* 0. □

The value $-G\xi^g\mu_2^g/\lambda_1^g$ has the same direction as Newton's classical gravity, However, the value $G\xi^g\mu_2^g/\lambda_1^g$ has the opposite direction. Therefore could this represent the existence of anti-gravity?

If the distance R is small enough, hence the acceleration $V'(R,G)$ becomes some finite constants depend on constants $\xi^g, \lambda_1^g, \mu_1^g$ and μ_2^g, not infinite. However, if the constant λ_1^g or μ_1^g approach 0 or ∞, then the acceleration $V'(R,G)$ becomes ∞ or 0. Depending on the value of μ_1 and λ_1^g, the value of $V'(R,G)$ can be positive or negative. When the value of $V'(R,G)$ is the negative, the deceleration acts toward the center. These constants is depended on generalized entropy and the part of partial entropy. Namely, the acceleration depend on generalized entropy. Therefore, there exists 5-states within the distance R is small. The constant λ_1^g is the coefficients of degree one of the approximate generalized entropy $\lambda_2^g x^2 + \lambda_1^g x$, and the constant μ_1^g is the integral constant obtained by integrating the parts of partial entropy $S''_D(x)$. In other word, The acceleration moving away from the center is changed by these constant, that is, simply the acceleration is depended on entropy. (Note : The center direction is defined as the positive direction.) The above discussion can be applied to Coulomb's law (electric field) . By adjusting the value of $\mu_1, \mu_2, \lambda_1, m = 1/\lambda_2$ and ξ, it may be possible to make the argument by replacing the gravitational constant G to the Coulomb constant k_c. (In this paper, the Coulomb constant is defined as k_c.) We will discuss this possibility next.

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3.2.2. Compare $V(R,G)$ and $V(R,k_c)$ for Small R

We attempt to compare $V(R,G)$ and $V(R,k_c)$, and consider its values. Similarly the gravity potential $V(R,G)$, we define the Coulomb potential $V(R,k_c)$ as follows :

$$V(R,k_c) = -\frac{k_c e_q}{R \pm \lambda_1^c e_q}(\exp(-\xi^c \mu_2^c R + \mu_1^c) - 1), \tag{55}$$

where $e_q > 0$ is an elementary charge, and $\xi^c \gneqq 0$, $\mu_2^c \gneqq 0$, $\mu_1^c \geq 0$ and $\lambda_1^c \geq 0$. and the symbol c in the upper right corner of the alphabet stands for c of Coulomb.

For example, we set the value of constants as follows :

$$\begin{aligned}
&G := 6.673E\text{-}11, && G \text{ is the gravitational constant},\\
&\xi^g = \xi^c := h = 6.670E\text{-}34, && h \text{ is Planck's constant},\\
&k_c := 8.987E\text{+}9, && k_c \text{ is Coulomb constant},\\
&e_q := 1.604E\text{-}19, && e_q \text{ is elementary charge},\\
&m := m_p = 2.176E\text{-}8, && m_p \text{ is Planck's mass}(unit : kg),\\
&\mu_2^g := 1, && \mu_1^g \text{ is a real constant},\\
&\mu_2^c := 1, && \mu_1^c \text{ is a real constant},\\
&\lambda_1^g := 1, && \lambda_1^g \text{ is a real constant},\\
&\lambda_1^c := 1, && \lambda_1^c \text{ is a real constant},\\
&R := 1.000E\text{-}6, && \lambda_1 \text{ is a real constant}(unit : meter).
\end{aligned} \tag{56}$$

Calculating with the above constants, the gravitational potential $V(R,G)$ is satisfied as follows :

$$V(R,G) = -2.442E\text{-}12, \quad if\ \mu_1^g = 1, \tag{57}$$

$$V(R,G) = -2.480E\text{-}3, \quad if\ \mu_1^g = 21.28, \tag{58}$$

where Planck mass m_p is used instead of mass m. and the sign of μ_1^g and λ_1^g are $+\mu_1^g$ and $+\lambda_1^g$.

Similarly, Coulomb potential $V(R,k_c)$ is satisfied as follows :

$$V(R,k_c) = -2.474E\text{-}3, \quad if\ \mu_1^c = 1, \tag{59}$$

$$V(R,k_c) = -2.512E\text{+}6, \quad if\ \mu_1^c = 21.28, \tag{60}$$

where elementary charge e_q is used instead of mass m and used k_c instead of G. the sign of μ_1^c and λ_1^c are $+\mu_1^c$ and $+\lambda_1^c$.

The value of $V(R,G)$ and $V(R,k_c)$ changes depending on how the constants μ_1^g and $\mu 1^c$ are selected. The above values (58) and (59) is closed. Therefore, for any the distance R, $\mu_1^g = 21.28$ and $\mu_1^c = 1$, it is satisfied $V(R,G) \simeq V(R,k_c)$. In consequence, it is satisfied as follows :

Suggestion 4. *Let m_p be mass , e_q be elementary charge, G be the gravitational constant and k_c be Coulomb constant. For small distance $R > 0$, there exists constants ξ^g, ξ^c, μ_2^g μ_2^c, μ_1^g μ_1^c λ_1^g and λ_1^c such that the following condition is satisfied :*

$$V(R,G) \simeq V(R,k_c), \tag{61}$$

where $\xi^g, \xi^c, \mu_2^g, \mu_2^c \gneqq 0$ and $\lambda_1^g, \lambda_1^c, \mu_1^g, \mu_1^c \geq 0$. □

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Because, if it is satisfied as follows :

$$\begin{aligned} V(R,G) &= -\frac{Gm_p}{R \pm \lambda_1^g m_p}(\exp(-\xi^g \mu_2^g R + \mu_1^g) - 1) \\ &= -\frac{k_c e_q}{R \pm \lambda_1^c e_q}(\exp(-\xi^c \mu_2^c R + \mu_1^c) - 1) \\ &= V(R,k_c), \end{aligned} \tag{62}$$

then transforming the above formula, it becomes as follows :

$$\exp(-\xi^g \mu_2^g R + \mu_1^g) = 1 + \frac{R \pm \lambda_1^g m_p}{Gm_p}\frac{k_c e_q}{R \pm \lambda_1^c e_q}(\exp(-\xi^c \mu_2^c R + \mu_1^c) - 1). \tag{63}$$

Therefore, if the value of μ_2^c is given, the value of μ_2^g can be found as follows :

$$\mu_2^g = \frac{1}{\xi^g R}\left[\mu_1^g - \log\left(1 + (\frac{R \pm \lambda_1^g m_p}{R \pm \lambda_1^c e_q})(\frac{k_c e_q}{Gm_p})(\exp(-\xi^c \mu_2^c R + \mu_1^c) - 1)\right)\right]. \tag{64}$$

Namely, using the equation for the potential derived from entropy, within small distance, it may be possible to treat Gravity potential and Coulomb potential in the same way by appropriately choosing some constants. In same way, applying the gravitational acceleration $V'(R,G)$ and Coulomb's law (electric field) $V'(R,k_c)$, we can get as follows :

Suggestion 5. *Let m_p be mass , e_q be elementary charge, G be the gravitational constant and k_c be Coulomb constant. For small distance $R > 0$, there exists constants ξ^g, ξ^c, μ_2^g μ_2^c, μ_1^g μ_1^c λ_1^g and λ_1^c such that the following condition is satisfied :*

$$V'(R,G) \simeq V'(R,k_c), \tag{65}$$

where $\xi^g, \xi^c, \mu_2^g, \mu_2^c \gtrneqq 0$ and $\lambda_1^g, \lambda_1^c, \mu_1^g, \mu_1^c \geq 0$. □

3.3. When the Distance R Is Large, However ξ Is Small Enough

Assuming the distance R is large and the constant ξ^g is small like Planck constant, that is $\xi^g \sim h$. The constant h is the Planck constant, 6.626E-34$J \cdot s$ and the constant μ_2^g is also small, that is, $\mu_2^g \leq 1$. Let R be the size of the universe within 13.8 billion light years. Since one light years is 9.461E+15 meter, that is, $R \simeq 1.305E{+}28$ meter. Therefore, the following conditions is satisfied :

$$\xi^g \mu_2^g R \simeq 8.708E\text{-}6 \ll 1. \tag{66}$$

We consider that the function $\exp(-\xi^g \mu_2^g R)$ is approximately equal to 1, that is,

$$\exp(-\xi^g \mu_2^g R) \simeq 1. \tag{67}$$

Therefore, the following formulas are satisfied :

$$\begin{aligned} V'(R,G) &= \frac{Gm}{(R \pm \lambda_1^g m)^2}(\exp(-\xi^g \mu_2^g R + \mu_1^g) - 1) + \frac{Gm\xi^g \mu_2^g}{(R \pm \lambda_1^g m)}\exp(-\xi^g \mu_2^g R + \mu_1^g) \\ &\simeq \frac{Gm}{(R \pm \lambda_1^g m)^2}(\exp(\mu_1^g) - 1) + \frac{Gm\xi^g \mu_2^g}{(R \pm \lambda_1^g m)}\exp(\mu_1^g), \\ &\quad (\because \exp(-\xi^g \mu_2^g R) \to 1). \end{aligned} \tag{68}$$

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When the condition $\xi^g \mu_2^g R \ll 1$ is satisfied, applying to mass M in circular orbit around mass m, the following are satisfied :

$$\frac{GmM}{(R \pm \lambda_1^g m)^2}(\exp(\mu_1^g) - 1) + \frac{GmM\xi^g \mu_2^g}{(R \pm \lambda_1^g m)} \exp(\mu_1^g) = M\frac{v^2}{R}. \quad (69)$$

where the mass m within radius R and the value v is rotation speed of the mass M on radius R. The right side of equation(69) is centrifugal acceleration of mass M. Hence, the following are satisfied :

$$\begin{aligned} v &= \sqrt{\frac{GmR}{(R \pm \lambda_1^g m)^2}(\exp(\mu_1^g) - 1) + \frac{Gm\xi^g \mu_2^g}{(1 \pm \frac{\lambda_1^g m}{R})} \exp(\mu_1^g)} \\ &= \sqrt{\frac{Gm}{(R \pm \lambda_1^g m)(1 \pm \frac{\lambda_1^g m}{R})}(\exp(\mu_1^g) - 1) + \frac{Gm\xi^g \mu_2^g}{(1 \pm \frac{\lambda_1^g m}{R})} \exp(\mu_1^g)} \quad (70) \\ &\simeq \sqrt{Gm\xi^g \mu_2^g \exp(\mu_1^g)}, \quad (\because R \text{ is large enough and } (1 + \frac{\lambda_1^g m}{R}) \to 1). \end{aligned}$$

Therefore, we propose that the following is satisfied :

Suggestion 6. *Let m (the mass) and v (the speed of rotation) be positive real numbers. For distance $R > 0$ within 1.305E+28, the following condition is satisfied :*

$$v \simeq \sqrt{Gm\xi^g \mu_2^g \exp(\mu_1^g)}, \quad (71)$$

where $\xi^g, \mu_2^g \gtrsim 0$ and $\lambda_1^g, \mu_1^g \geq 0$. As a results, the speed of rotation v at the radius R is approximated by the constant $\sqrt{Gm\xi^g \mu_2^g \exp(\mu_1^g)}$, not depend on the radius R. □

Therefore, the speed of rotation v is depended on constants G, m, ξ^g, μ_1^g and μ_2^g, not depend on the radius R. It is noticed that these constants is decided by generalized entropy $S_D(x, k)$ and the distribution function $Q_D(x)$.

According the suggestion 6, let m be equal to the mass of the Milky Way Galaxy , that is, $m \simeq$ 1.989E+30 × 2.0E+12kg. Therefore, if setting $\xi = 1E\text{-}34 \sim h$ (Planck's constant) and $\mu_2^g = 1$, then the speed of rotation is satisfied according the constant μ_1^g as follows :

$$v \simeq 4.208E\text{-}1\sqrt{\exp(\mu_1^g)}\ m/s. \quad (72)$$

For example, let $\mu_1^g = 26.29$, the speed of rotation v became as follows :

$$v \simeq 2.152E\text{+}5\ m/s. \quad (73)$$

In this case, the speed of (73) is close to the rotation speed of Milky Way Galaxy, that is, approximately 2.100E+5 ∼ 2.200E+5 m/s.

3.4. When the Distance R Is Large Enough

If the distance R is large enough, the formula (27) is satisfied as follows:

$$V'(R, G) = -\frac{Gm}{(R \pm \lambda_1^g m)^2}, \quad (\because \exp(-\xi^g \mu_2^g R + \mu_1^g) \to 0). \quad (74)$$

Therefore,

$$-\frac{Gm}{(R-\lambda_1^g m)^2} \lesssim -\frac{Gm}{R^2} \lesssim -\frac{Gm}{(R+\lambda_1^g m)^2}. \tag{75}$$

If the distance R is large enough and the constant λ_1^g is small enough, that is $\lambda_1^g \to 0$, then the gravitational acceleration $V'(R,G)$ becomes classical gravity.

3.4.1. Summarize the Gravitational Acceleration for Large R

By the above the gravitational acceleration $V'(R,G)$, we summarize the gravitational acceleration as follows :

$$\begin{aligned} &\hat{g}_{\pm} = -\frac{Gm}{(R \pm \lambda_1^g m)^2}, \\ &Adjusted\ \ Gravitational\ Acceleration,\ R\ is\ large\ enough, \end{aligned} \tag{76}$$

$$\begin{aligned} &g = -\frac{Gm}{R^2}, \\ &Newton's\ Classical\ Gravitational\ Acceleration, \\ &R\ is\ large\ enough\ and\ \lambda_1^g \to 0. \end{aligned} \tag{77}$$

Classical gravity is satisfied when R is large enough and $\lambda_1^g \to 0$. According to g of (77) and $\hat{g}_{\pm}$ of (76) in the above equation, we propose as follows :

Suggestion 7. *Gravity changes on the value λ_1^g of generalized entropy coefficient.*
Let m be a positive real number (the mass). For large $R > 1$, the following condition are satisfied :

$$\hat{g}_{-} = -\frac{Gm}{(R-\lambda_1^g m)^2} \lesssim g \lesssim -\frac{Gm}{(R+\lambda_1^g m)^2} = \hat{g}_{+}, \tag{78}$$

where $\lambda_1^g \geq 0$ is a real constant. □

The suggestion above is an extension of Newton's classical theory of gravity. For large distance R, it is possible that the adjusted gravity $\hat{g}_{\pm}$ is smaller or larger towards the center than the classical gravity g. In other word, the gravitational acceleration towards the center of a rotating substance can be slightly changed at sufficient large distance. The gravitational acceleration moving away from the center is changed by the constant λ_1^g. The constant λ_1^g is the coefficients of degree one of the approximate generalized entropy $\lambda_2^g x^2 + \lambda_1^g x$. In other word, The gravitational acceleration moving away from the center is changed by the coefficients of degree one of the approximate generalized entropy, that is, the gravitational acceleration is depended on entropy.

4. Conclusion

The idea behind Planck's radiation formula was to apply the number of cases due to the division by the resonator to the entropy. These ideas are similar to the logistic function of a dynamical system. By applying these ideas, we treated the partition of entropy as a function $D(x)$, that is not minimum unit, and the potential $V_D(x,k)$ and the acceleration $V'_D(x,k)$ was derived. We assumed that generalized entropy $S_D(x,k)$ can be expressed by a second-order polynomial, and that the potential $V_D(x,k)$ is defined as the negative inverse of $S_D(x)$. Therefore, by interpreting each variables and constants used in the potential $V_D(x,k)$ and the acceleration $V'_D(x,k)$ as the gravity theory, and the mass is defined the inverse of the second-order coefficient term of $S_D(x,k)$, that is $1/\lambda_2$. We proposed the following three conclusions:

1. If the distance R is small enough, hence the gravitational acceleration $V'(R, G)$ becomes 2-states with finite constants depend on constants $\xi, \lambda_1^g, \mu_1^g$ and μ_2^g, not infinite. However, if the constant $\lambda_1^g \to 0$, then the gravitational acceleration $V'(R, G)$ becomes $\pm\infty$, and if the constant $\lambda_1^g \to \infty$, then the gravitational acceleration $V'(R, G)$ becomes 0. Depending on the value of μ_1^g and λ_1^g, the value of $V'(R, G)$ can be positive or negative. Therefore, it is possible that gravity have 5-states within the distance R is small enough. Furthermore, using the equation for the potential derived from entropy, within small distance, it may be possible to treat Gravity potential and Coulomb potential in the same way by appropriately choosing some constants. Similarly, the same suggestion can be made for the gravitational acceleration and Coulomb's law (electric field).
2. At distances large enough to be within the size of the universe, gravity follows an adjusted inverse law. Within this distances, the rotation speed of a galaxy v follows the gravitational constant G, the mass $m = 1/\lambda_2^g$ and constants ξ^g, μ_2^g and μ_1^g which depend on entropy. Besides, the rotation speed of a galaxy v does not little depend on its radius R, (the galaxy rotation curve problem).
3. At large distances, gravity follows an adjusted inverse square law. Comparing to conventional gravity, the gravitational acceleration towards the center of rotation is slightly weaker or stronger. This means that the gravitational acceleration towards the center of a rotating substance can be slightly changed at distance. (The Pioneer Anomaly)

From the above discussion , it is possible there exists some constants $\xi, \lambda_2, \lambda_1, \mu_2$ and μ_1 which depend on entropy that controls gravity and the speed of galaxies. The constant λ_1 is the coefficients of degree one of the approximate generalized entropy $\lambda_2 x^2 + \lambda_1 x$. Namely, the gravitational acceleration moving away from the center is changed by the coefficients of the approximate generalized entropy. Moreover, the constants μ_2 and μ_1 are defined the part of the partial entropy. Therefore, gravity may depended on entropy.

By developing the concept of the logistic function and combining it with entropy and Planck's ideas, we derived that the potential $V_D(x, k)$ and the acceleration $V'_D(x, k)$. Thereby, we applied these ideas to gravity theory. Similarly, we think that these ideas can be applied to Coulomb's law (electric field), which is the inverse square law, and other natural sciences. In addition, because the potential $V_D(x, k)$ derived in this paper contains an equation similar to Yukawa potential. Therefore, it may also have applications in particle theory and other potential theory. The concept of logistics is applied to population theory and the evolution of life. By combining the concepts of entropy and logistics, we believe it may be possible to understand the evolution of the universe in a similar way to evolution. We hope that the concepts of entropy and logistic functions will explain more things and provide new perspectives.

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