Article

A High School Teaching/Learning Path on Electromagnetic Induction: a Case Study

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Abstract: We present here a five hours experimentation of a didactical path about the electromagnetic induction addressed to students of the last year of an Italian scientific high school and oriented to better understand the physical origin of the induced electromotive force. The expression of the induced electromotive force as the sum of the term linked to the time variation of the magnetic field and of the motional one has been obtained in a detailed way, still suitable for presentation at high school. Many examples have been proposed to the students in order to clarify the conceptual physical knots. The students' responses to a 6-questions multiple choice questionnaire have been analyzed. It emerged that our approach is concretely feasible although we find the well-known difficulties in calculating flux and circulation of a vector field. Furthermore, it emerged that an integral approach to the problem masks the understanding of the nature of the forces acting locally on the charges. Hence our proposal of a "redefinition" of the induced electric field in terms of the magnetic vector potential is also presented.

Keywords: Innovation in teaching, Electromagnetic induction, High school students, Qualitative methods

1. Introduction

The difficulties faced by students in dealing with and understanding Electro-Magnetic Induction (EMI) are well known and extensively discussed in the literature. Great efforts have been recently carried on (see [1-9] and references therein) by a number of physics education research groups about the teaching/learning problems related to EMI. In recent papers [1,2], we presented a somewhat novel study that compares some conceptual difficulties in understanding the various aspects of the electromagnetic induction phenomenon at various educational degrees (high school students, graduate students and physics teachers) and that shows that these difficulties seem to be independent of the level of education. Almost regardless of the results of the educational research, the Italian ministerial indications about the final exam for scientific high school require knowledge/skills/competences [10] to solve complicated exercises and discuss theoretical questions about EMI and, therefore, request for a robust disciplinary scaffolding at school. Moreover, differently from most European and American upper secondary schools, mathematics in the Italian upper secondary school curriculum is, at least in principle, relatively "high-level" and include calculus.

All considered, we decided to build an educational path about EMI addressed to students of the last year of an Italian scientific high school. In the developed path, particular attention has been paid to the induced electromotive force (emf) as composed of two terms: a transformer term and a motional one. In fact, many works [1-3,5] suggest the didactic importance of this fact, but, at the best of our knowledge, there are no documented experimentations of a high school path that explicitly takes into consideration these two terms. Taking into account that the level of scientific knowledge of students who have chosen a university course in scientific disciplines other than Physics, Mathematics, Chemistry or Engineering, we believe that the proposed path is also suited form freshmen in not hard scientific disciplines.

Before pandemic, we made a (in classroom) pilot feasibility study directed to a class of 23 students of the fifth year (13th grade) of the Liceo delle Scienze Applicate "G. Natta" of Bergamo, where the math and physics teacher was one of us (MS). After a negotiation with the class teacher, we decided that the classroom intervention had to last 5 hours (that means 2 weeks), plus 45 minutes for the final test. The final test was a 6-questions multiple choice questionnaire, the same as that discussed in our previous papers [1,2]. Lectures have been held by MG; notes have been taken by MS while all the lessons have been recorded. Students involved had not been previously exposed to any teaching of EMI.

In what follows, in section 2 we present a brief and schematic summary of the lessons and in section 3 we discuss the results of the questionnaire. These results led us to the conviction of the opportunity of giving also a local approach, based on a precise definition of the induced electric field through the use of the magnetic vector potential, next to the more usual integral one. This will be discussed in section 4. Finally, in section 5, our conclusions are drawn.

2. Classroom intervention

In this work we are primarily interested to understand which are the fundamental conceptual knots for an effective didactic reconstruction of EMI. We have let the choice of the didactic modalities to be determined by the available time and therefore, after a negotiation with the class teacher, it has been decided the lessons had to be delivered at the level of demonstrated Inquiry.

As prerequisites, students had to know: basic electrostatics and magnetostatics, dot and cross product, the notions of circulation and flux of a vector field, Gauss law, solenoidality of the magnetic field, electromotive force as the circulation of an electric field, definition and very basic notions about the derivative of one-variable function. Besides, students also knew the Lorentz force.

Each lesson began with an "engage" moment with a simple, but intriguing experiment that would have usually been explained later on, during the lesson itself. A large use of qualitative exercises was done, in which students had to discuss and solve the problems by themselves. In the following, a brief chronological description of the lessons' contents is presented.

2.1. Lesson 1 - 1.45 h

Engage 1 - Two rings, one made of aluminum and the other being a strong magnet are, one after the other, dropped around a copper pipe approximately one meter long (Figure 1). The very different dropping time is manifest [11]. A cylindrical magnet with polarities along its axis is dropped inside the same copper pipe. A very long (a few seconds) dropping time is observed.

Figure 1. A ring dropping around a copper pipe.

 doi:10.20944/preprints202206.0332.v1

Experiment - A solenoid is connected to an electroscope: when a bar magnet moves going inside or outside the solenoid, the electroscope charges or discharges (Figure 2).

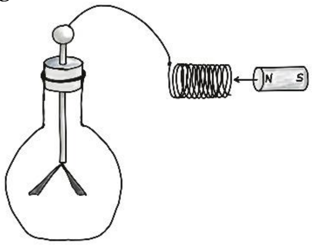

Figure 2. A bar magnet is moving towards a solenoid connected to an electroscope.

Video - Video of the usual Faraday's experiment [12].

Formalization - A formula is proposed for the emf as the circulation $C_\gamma(\vec{E})$ of the induced electric field $\vec{E}$ along the line γ:

$$emf = -\frac{\Delta\phi_{S_\gamma}(\vec{B})}{\Delta t} = -\frac{\phi_{S_{\gamma final}}(\vec{B}) - \phi_{S_{\gamma initial}}(\vec{B})}{t_{final} - t_{initial}}. \quad (1)$$

Where $\phi_{S_\gamma}(\vec{B})$ represents the magnetic field flux through the surface S bounded by γ.

<u>Revisions</u> - Concept of flux of a vector field; Gauss theorem for the electrostatic field recalling also the correct terminology (flux "through" a surface). Observation: the flux in equation (1) is not through a closed surface but through a surface S lying on a closed curve. Concept of derivative and its definition as the limit of the incremental ratio. Therefore equation (1) can be written as:

$$emf = -\frac{d\phi_{S_\gamma}(\vec{B})}{dt}. \quad (2)$$

Question - How do we calculate this flux variation?

<u>Qualitatively</u> (with straightforward symbology):

$$\phi \sim BS. \quad (3)$$

Therefore:

$$\frac{d\phi}{dt} \sim \frac{dB}{dt}S + B\frac{dS}{dt}. \quad (4)$$

That can be intepreted as the flux of the rate of change of B plus B times the rate of change of S, also illustrated by MG moving by hand a circuit and a bar magnet. CLASS DISCUSSION.

<u>Formally</u>:

$$\frac{d\phi_S(\vec{B})}{dt} = \lim_{\Delta t \to 0} \frac{\phi_{S(t+\Delta t)}\left(\vec{B}(t+\Delta t)\right) - \phi_{S(t)}\left(\vec{B}(t)\right)}{\Delta t}. \quad (5)$$

Interlude

$$\vec{B}(t+\Delta t) \sim \vec{B}(t) + \frac{d\vec{B}(t)}{dt}\Delta t. \quad (6)$$

Therefore:

$$\frac{d\phi_S(\vec{B})}{dt} = \lim_{\Delta t \to 0} \frac{\phi_{S(t+\Delta t)}\left(\vec{B}(t) + \frac{d\vec{B}(t)}{dt}\Delta t\right) - \phi_{S(t)}\left(\vec{B}(t)\right)}{\Delta t} = \lim_{\Delta t \to 0} \phi_{S(t+\Delta t)}\left(\frac{d\vec{B}(t)}{dt}\right) +$$

$$\lim_{\Delta t \to 0} \frac{1}{\Delta t}\left(\phi_{S(t+\Delta t)}\left(\vec{B}(t)\right) - \phi_{S(t)}\left(\vec{B}(t)\right)\right). \quad (7)$$

The first term at the RHS of the previous equation is immediately:

$$\phi_{S(t)}\left(\frac{d\vec{B}(t)}{dt}\right). \tag{8}$$

For the calculation of the second term, we refer to Figure 3.

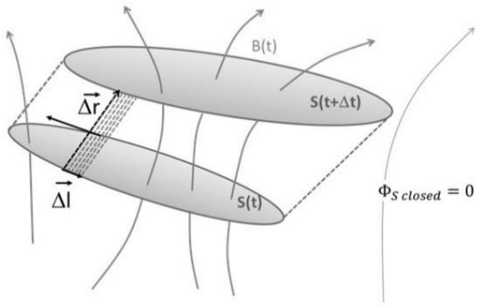

Figure 3. A surface moving in a magnetic field in the time interval $(t, t + \Delta t)$.

From the solenoidality of $\vec{B}$ at a given time t, we have:

$$\phi_{S(t+\Delta t)}\left(\vec{B}(t)\right) - \phi_{S(t)}\left(\vec{B}(t)\right) = -\phi_{side(t,t+\Delta t)}\left(\vec{B}(t)\right), \tag{9}$$

where with $\phi_{side(t,t+\Delta t)}\left(\vec{B}(t)\right)$ we mean the flux through the lateral surface swept by the entire circuit (cut flux) in the time interval $(t, t + \Delta t)$, hereafter called $\phi_{cut}\left(\vec{B}(t)\right)$.

In conlusion:

$$emf = -\phi_{S(t)}\left(\frac{d\vec{B}(t)}{dt}\right) + \frac{d\phi_{cut}\left(\vec{B}(t)\right)}{dt}. \tag{10}$$

Video - To come back to the initial video and explain it with the use of the previous formula, we referred to a second video [13] about the Faraday's experiment with field lines added. There is no more time to explain also the engage experiments. They will therefore be afforded at the beginning of the second lesson.

2.2. Lesson 2 - 0.45 h

Engage 2 - A small cylindrical magnet is dropped into a Plexiglas pipe with four small coils rolled around it, each connected to a LED (see Figure 4).

Figure 4. A magnet dropping into a Plexiglas pipe with four coils rolled around it, each connected to a LED.

When the magnet falls, the LEDs light on. With the help of this experiment and the use of equation (10), we explain the experiments performed as the engage of lesson 1.

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Question - A rectangular conducting loop is moving in a plane, orthogonal to a uniform and constant magnetic field (see Figure 5). Is there an induced current in the loop? CLASS DISCUSSION.

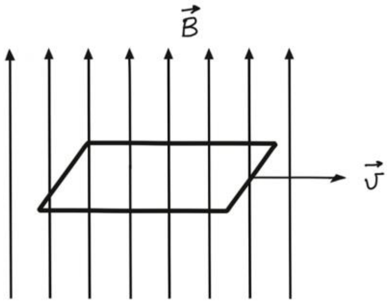

Figure 5. A rectangular conducting loop moving in a plane, orthogonal to a uniform and constant magnetic field.

Using equation (10) students immediately saw that the first term at the RHS is null, but the second term is difficult to be understood. In fact, students reasoned that there was a non null cut flux and, therefore, an induced current. Guiding questions had to be asked, before they realized that the forward flux was at every instant equal to the backward one. It seems that the use of equation (10) is not simple to understand while in the presence of a motional emf.

Question - Does the fact that no emf is present imply that nothing is happening in the loop? CLASS DISCUSSION.

The answer is that, on the two sides of the loop perpendicular to the velocity, the magnetic part of the Lorentz force gives rise to the same potential difference.

Question - Are we able to perform the derivative given by the second term at the RHS of equation (10) in a way so that its meaning is more easily understandable?

Preamble: The volume of the parallelepiped with edges $\vec{a}, \vec{b}, \vec{c}$ is given by: $Vol(\vec{a}, \vec{b}, \vec{c}) = \vec{a} \cdot (\vec{b} \times \vec{c})$

Revision: Dot and cross product, especially the right hand rule.

2.3. Lesson 3 - 0.45 h

Engage 3 - Video describing the experiment of the Thomson's ring where a ring is thrown upward by an electromagnet. Since the power supply was out of sight, students were asked to explain what was happening [14]. CLASS DISCUSSION.

Formalization - Coming back to the calculation of the second term at the RHS of equation (10), and having in mind Figure 3, and indicating with ΔS_i the i-th elemental area given by $\Delta\vec{l}_i \times \Delta\vec{r}_i$ we can write:

$$\phi_{cut}\left(\vec{B}(t)\right) \equiv \phi_{side(t,t+\Delta t)}\left(\vec{B}(t)\right) = \sum_{i=1}^{n} \vec{B}_i \cdot \Delta\vec{S}_i = \tag{11}$$

$$= \sum_{i=1}^{n} \vec{B}_i \cdot (\Delta\vec{l}_i \times \Delta\vec{r}_i) = \sum_{i=1}^{n} (\Delta\vec{r}_i \times \vec{B}_i) \cdot \Delta\vec{l}_i = C_\gamma(\Delta\vec{r} \times \vec{B}).$$

Therefore:

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$$\frac{d\phi_{cut}\left(\vec{B}(t)\right)}{dt} = C_\gamma\left(\vec{v}\times\vec{B}\right), \tag{12}$$

v

where $\vec{v}$ is the velocity of that part of γ subjected to the magnetic field $\vec{B}$.
In conclusion:

$$emf = C_\gamma\left(\vec{E}\right) = -\phi_{S(t)}\left(\frac{d\vec{B}(t)}{dt}\right) + C_\gamma\left(\vec{v}\times\vec{B}\right). \tag{13}$$

Example 3.1 - With the help of equation (13) we re-discussed the example illustrated in Figure 5.

Example 3.2 - A rectangular conducting loop is rotating around one of its axis with constant angular velocity within a uniform and constant magnetic field. The numerical values of the quantities of interest are given and the induced current is evaluated using the second term at the RHS of equation (13). Therefore, with a procedure different from the standard one generally adopted in textbooks. The generation of a. c. currents is briefly numerically discussed. An estimation of the current, induced in the metal edge of the tilting aluminum windows of the classroom moving in the earth's magnetic field, has been proposed and evaluated in a CLASS DISCUSSION.

2.4. Lesson 4 - 1.45 h

Engage 4 - An experimental apparatus is shown: a couple of cylindrical magnets are suspended over a copper disk (see Figure 6).

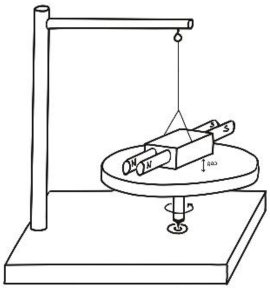

Figure 6. Two cylindrical magnets suspended over a copper disk.

Question - What do you expect to see if the disk is put in rotation? CLASS DISCUSSION. Most of the students think that nothing special will happen.

The experiment is performed: when the disk is put in rotation, the magnets start to rotate in the same direction as the disk until they reach the same angular velocity. The result is explained in terms of equation (13). A clarification about the reference frame in which the calculation of the time variation of magnetic field and the velocity at the RHS of equation (13) refer to has been necessary.

Experiment - A powerful magnet is sliding on an inclined track of different materials: plastic, aluminum, copper (Figure 7 and ref. [15]). Different effects are observed. Students are given tracks and magnets to experiment on their own. The task was to find an explanation of the behaviour of the magnets. CLASS DISCUSSION.

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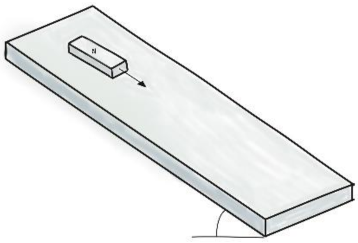

Figure 7. A magnet sliding on an inclined track.

A simple model has been found in terms of the *emf* inducing currents in the metal tracks and nothing in the plastic one. The explanation was judged correct since it foresaw the right behaviour of the magnet at large inclinations of the track; i.e., the front part of the magnet tended to rise up while the rear part remained well attached to the track. Taking into account the above experiment, in a CLASS DISCUSSION students were able to interpret the experiment shown in the engage 1 of the first lesson (magnet falling in a copper tube).

Example 4.1 - A cylindrical iron magnet is rotating with a constant angular velocity. A conducting wire connects the center of one basis of the magnet with its lateral surface through a sliding contact (see Figure 8).

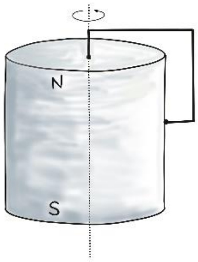

Figure 8. A rotating magnet with a conducting wire connecting the center of one basis with its lateral surface through a sliding contact.

Question - Is there an induced current in the wire? CLASS DISCUSSION. The question appeared difficult to the students since neither the circuit is moving nor the magnetic field is changing. Students would, therefore, be inclined to answer that there is no current, but this fact seemed strange to them.

Students are suggested to change the reference frame. In fact, the presence of the current cannot depend on the chosen reference frame and the problem is much simpler to be solved in a reference frame in which the magnetic field is known. Therefore, in the reference frame of the magnet, it is clear that an induced current is flowing due to the motion of the wire.

Example 4.2 - The Blondel experiment: a solenoidal coil, coaxial with a static magnetic field that is present only in its inner region, forms a closed conducting circuit (Figure 9 and ref. [8]).

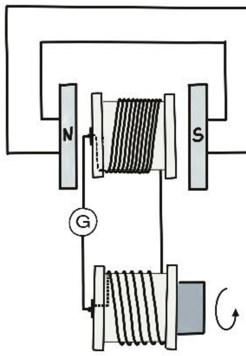

Figure 9. Blondel's experiment apparatus.

Question - If the coil is unwound, will there be an induced current in it? CLASS DISCUSSION. At first glance, there seems to be a time-varying magnetic flux due to the changing of the number of windings of the solenoidal coil and, therefore, an induced current.

Since the magnetic field is point by point orthogonal to the velocity of the wire, even $\vec{v} \times \vec{B}$ is orthogonal to the wire, therefore, the second term at the RHS of equation (13) is zero. Since the magnetic field is stationary, we immediately can say that there is no EMI and no induced current at all.

Example 4.3 - Around an "infinite" cylindrical iron magnet there is an elastic, conductive wire with its ends touching to form a closed ring circuit. While extracting the magnet from the ring, the ends of the wire open, but always remaining in electrical contact with the magnet itself which thus closes the circuit. When the magnet is completely extracted, the ends of the elastic wire continue to be touching, still forming a closed circuit. Therefore, we have always a conductive, closed circuit before, during and after the extraction process of the magnet (see Figure 10).

Figure 10. Top view of a cylindrical magnet and an elastic, conductive wire with its ends touching to form a closed ring circuit, before, during and after the extraction process of the magnet.

Question - During the extraction process of the magnet, will there be an induced current? CLASS DISCUSSION.

A student, referring to the considerations developed in the examples 4.1 and 4.2, and having in mind equation (13), immediately said "where there is $\vec{v}$, there is no $\vec{B}$".

Example 4.4 - Corbino disk: a thick conductive ring with two sliding contacts (one inside and the other outside) is placed in a uniform and constant magnetic field parallel to its axis (see Figure 11).

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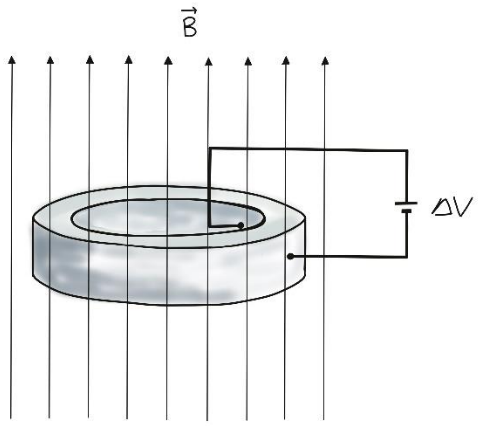

Figure 11. Corbino disk.

Question - What does it happen if a potential difference is applied between the two contacts? CLASS DISCUSSION.

In the absence of a magnetic field there is a radial current. If a magnetic field is switched on, the magnetic part of the Lorentz force adds a circular component to the radial current.

Exercise 4.1 - A circular conductive coil of radius b is coaxial to a linear indefinite solenoid with radius $a < b$. A current flows in the solenoid with intensity $i(t) = kt$, where k is a positive constant. Is there an induced current flowing in the circuit? CLASS DISCUSSION.

Nearly all students immediately realized to use the first term at the RHS of equation (13), but showed difficulties in understanding that the flux of the magnetic field confined into the solenoid is also the flux through the external coil. Numerical calculations have been made by MG at the whiteboard.

Exercise 4.2 - A plane rectangular circuit consists of two vertical parallel conductive tracks connected to each other and of a mobile conductive side of length l falling without friction, due to the acceleration of gravity g. The circuit is immersed in an orthogonal, uniform and constant magnetic field $\vec{B}$. Which is the expression of the induced emf? CLASS DISCUSSION.

MG suggested students to neglect the effects of the magnetic field generated by the induced current. Through the second term at the RHS of equation (13), students calculated $emf = Blgt$.

3. Results from the 6-questions multiple choice questionnaire

The administered 6-questions multiple choice questionnaire consists of: questions D1, D2 and D3 which are similar to questions 29, 30 and 31 of the area X (Faraday's law) of the "Conceptual Survey in Electricity and Magnetism" [5]; question D4 which is essentially question Q4 of [4]; questions D5 and D6 which are close to questions Q1 and Q3 from [7]. Independently of their correctness, the explanations were grouped into categories according to the type of reasoning used. From each question, categories were obtained through a "negotiation phase", starting from the individual analysis made by the authors, paying attention that they do not overlap. This procedure implies that the categories were not given a priori. The explanations that we were not able to categorize (mostly because we did not clearly understand the meaning of the written sentences) and the questions unanswered or without explanation have been put into the general group "Other".

In the following, in subsections from 3.1 to 3.6, Tables 1-6 present our categorization giving the number of explanations that fall into each category (one for each row). Some examples of the students' explanations together with few comments are also reported.

3.1. Question D1 (Fig. 1A in the Appendix)

The results of Question D1, concerning the current that could be induced in a conductor coil when the sources of $\vec{B}$ are in motion and the coil is stationary or the sources of $\vec{B}$ are fixed and the coil moves, are presented in Table 1.

Table 1. Answers to D1.

Cathegory	**Number of answers**
Flux	6
B	2
Motion/Deformation	1
Non appropriate use of equations	10
Appropriate use of equations	2
Other	2

Here and in what follows, we put in the "Flux" category all those explanations that attribute the origin of the induced current to a variation of the magnetic field flux. Two examples:

• *"The magnet enters the coil generating a magnetic flux variation in a dt time, fem is generated [...] which creates a current."*.

• The student puts the cross on the right answer and her/his motivation is: "*I: there is a flux variation due to a variation of the magnetic field that crosses the surface of the coil; II: there is a variation of flux due to a variation of the coil area; IV: there is a variation of flux due to a variation of the field that crosses the surface of the coil. Therefore, in these cases there is a fem.*".

In the "B" category we place the two explanations which relate the cause of EMI to the magnetic field time variation. One of them is:

• *"In case I, the magnet moving away creates a variation in the magnetic field and therefore the current varies in the circuit, [...]"*.

One student gives an explanation that can be put in what we call the "Motion/Deformation" category: EMI is uniquely ascribed to a general motion of a circuit or of a magnet or to a circuit deformation. For instance:

• *"The correct answer is (c) because in the first case the magnet is moving, in the second case there is a variation of the area, in the fourth case the circuit is moving."*.

The "Non appropriate use of equations" category groups students who use equations (13) or (10), but confuse or mix the two parts of them or do not correctly interpret the meaning of the symbols. More than 43% of the students' explanations are to be placed in this category. Two very clear examples are the following:

• The student selects the answer (d) and justifies this choice by writing that "*In the answer I selected, we can observe an induced current given by a change in the magnetic field and by the presence of a velocity.*".

• *"In case I, a current is present because an induced emf is generated; the magnet moves and the loop remains still, the magnetic field $\vec{B}$ changes, but not the surface, so I can calculate the circulation of $\vec{v} \times \vec{B}$ that allows me to find the emf [...]".*

The "Appropriate use of equations" category refers to explanations given by students who use equations (13) or (10), giving the two contributions their proper meaning, regardless of whether the calculation is correct. Only two students' explanations fall into this category:

• *"In the IV case we have the induced emf. The magnet generates a field B which, however, does not undergo a change because it is still. But we have the velocity v with which the loop moves".*

• The student is wrong in the choice of the answer but seems to have understood which part of the equation (13) must be used, writing *"In case III, an induced current is present, the field is stationary, but there is a circulation $C(\vec{v} \times \vec{B})$ due to the loop rotation [...]".*

3.2. Question D2 (Fig. 2A in the Appendix)

Question D2 concerns the current that could be induced in a conductor coil when it moves nearby a very long current carrying wire. The results of our questionnaire are shown in Table 2.

Table 2. Answers to D2.

Cathegory	**Number of answers**
Current-velocity angle	13
Non appropriate use of equations	6
Oher	4

As we have already observed in [1, 2], a significant number of students (in this case as many as 56%) think that the presence / absence of an induced current is to be connected with the angle between the direction of the current and the velocity of the coil ("Current-velocity angle" category). From some students' observations it seems evident that they are not able to correctly visualize the pattern of the field lines surrounding a very long current carrying wire. Some examples of this category:

• *"An induced current will result in cases I and II because the velocity of the loop is not parallel to the current.".*

• *"A straight current carrying wire generates a magnetic field. In case III, electricity and velocity are parallel and this is not good.".*

• *"A current carrying wire generates a magnetic field and the only case in which the field has no effect is the case III, when the velocity is parallel to the field.".*

In the "Non appropriate use of equations" category we find:

• *"In case I, the magnetic field is out of the page. [...] B and v are perpendicular to each other, so the circulation is maximum. In this case there exists an emf that originates an induced current. [...] In the case III, v and B are parallel so the circulation is null and there is no emf and induced current.".*

• *"Case I: the decrease of the field causes currents that create a field which opposes the change of the field; case III: B is constant, it does not change because the vector product $\vec{v} \times \vec{B}$ is equal to zero; case II: if the movement is broken up, we are again in case I.".*

3.3. Question D3 (Fig. 3A in the Appendix)

Question D3, whose results are displayed in Table 3, deals with the presence of EMI when a neutral conductive bar moves in a uniform and stationary magnetic field. This question is a bit outside the context of the topics discussed during the experimentation, where the second term at the RHS of equation (13) explicitly refers to a circulation along a given curve, while the one of equation (10) refers to a flux cut by a given surface. We have however included this question here for completeness, since it has been already proposed in the questionnaire previously administered in [1, 2]. However, the analysis of the students' explanations does make it clear if they have understood the action of the magnetic part of the Lorentz force which they had already studied before our experimentation.

Table 3. Answers to D3

Cathegory	Number of answers
No separation because the bar is neutral	2
Compensation	3
Flux	2
Non appropriate use of equations	2
Lorentz	5
Other	9

In the "No separation because the bar is neutral" category, we find (for example):

• "*There is no charge distribution, because the bar is neutral.*".

Three students ascribe the separation process to a sort of compensation ("Compensation" category), saying:

• "*The charges are distributed in this arrangement because they try to compensate for the magnetic field coming out of the sheet.*".

An example from the "Flux" category is:

• "*When the bar is moved within a uniform magnetic field, there is a velocity which in this case is perpendicular to the field. Moving [the bar], a field is created that opposes to the variation of the field flux and therefore tends to make it return to the previous situation.*".

The "Non appropriate use of equations" category includes, for instance:

• "*An emf originates to oppose the magnetic field change. The circulation $C(\vec{v}\times\vec{B})$ produces a current that contrasts the field [...]*".

The explanations which describe the charge distributions on the surface of the bar as due to the magnetic part of the Lorentz force are grouped together in the "Lorentz" category. An example:

• "*According to the Lorentz's law, the negative charges will move upwards leaving an excess of positive charges at the bottom.*".

3.4. Question D4 (Fig. 4A in the Appendix)

Table 4 summarizes the results relating to Question D4: a loop, connected to an ammeter, is rotating in a uniform and stationary magnetic field. Students are asked to explain the origin of the forces that make the charges move in the loop; we observe that this is the only question that asks about the precise nature of the microscopic force acting.

Table 4. Answers to D4.

Cathegory	Number of answers
Flux	5

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Appropriate use of equations	15
Lorentz	2
Other	1

In the "Flux" category, two explanations:

• *"When the coil is rotated upwards, the angle of the coil with respect to the magnetic field changes. Since the flux is given by the product of the magnetic field with the area and with the angle between them, there is a flux variation that generates an induced* emf *that generates the induced current.".*

• *"The force that moves the charges in the coil is the electromotive force, which generates induced current since while the coil is rotated upwards there is a flux variation given by the variation of the angle.".*

About 65% of the students' answers can be put in the "Appropriate use of equations" category, for instance:

• *"Taking into consideration the circulation along the loop* $C(\vec{v} \times \vec{B})$ *we note that there is a velocity* $\vec{v}$ *(in this case angular velocity) which causes a variation in the circulation itself, giving rise to an* emf*.".*

• The student writes the equation (13) and comments *"In this case the field B is fixed, ergo the first term at the RHS of the formula is zero, however the loop moves with velocity* $\vec{v}$ *not parallel to the field* $\vec{B}$*, so I have a non-zero circulation* $C_\gamma(\vec{v} \times \vec{B})$ *which gives rise to the forces that move the charges generating the induced current.".*

• *"When in a plane I have a magnetic flux and a moving coil, during each instant it generates an* emf*, because the area on which the field acts with its rotational motion is varying [...]".*

• *"A passage of electric current is recorded because, by turning the loop, the area of the circuit is modified.".*

Only two students were able to explain that the force acting on the moving charges is the (local) magnetic part of the Lorentz force ("Lorentz" category). In the following the two answers:

• *"The charges in the loop move because an* emf *is generated, the magnetic field* $\vec{B}$ *is uniform*

and does not vary and the charges are affected by the Lorentz force that causes them to move.".".

• *"Since the loop moves upwards, the electric charges are subjected to a displacement and a velocity. For this reason, the Lorentz force acts on them and makes them move [...]".*

3.5. Question D5 (Fig. 5A in the Appendix)

Question D5 concerns the current which could be induced in a C-shaped conductive wire sliding inside the air gap of a magnet, always remaining parallel to the magnetic field lines. As already observed in papers [1, 2], in order to answer correctly, students must know and be able to use equation (13), realizing that the induced current is due to the action of the magnetic part of the Lorentz force. From the results presented in Table 5 it can be deduced that most of the students use the equation (13) correctly, even if about 22% of them wrongly believe that no electromagnetic induction phenomenon occurs in the wire because there is no flux variation, since the magnetic field lines are parallel to the circuit surface.

Table 5. Answers to D5.

Cathegory	Number of answers
Flux	5

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Appropriate use of equations	15
Other	3

An example in the "Flux" category:

• *"An induced current is not present in this wire since the magnetic field does not affect the area of the wire [...] and the magnetic field B is not variable. So, there is no flux and no* emf*."*.

More than 65% of the students give answers indicating an appropriate use of the equations (10) and (13) ("Appropriate use of equations" category):

• *"The wire velocity is perpendicular to the magnetic field. Here, a circulation of* $\vec{v} \times \vec{B}$ *is created which generates an* emf *which in turn generates a current."*.

• *"Yes, there is a passage of induced current because there is a velocity and a magnetic field, therefore, a circulation."*.

• *"In the wire an induced current occurs because the field* B *does not change, while the wire has a displacement* v*, therefore the flux through the area varies."*.

• *"An induced current flows in the wire because even if* B *is constant, the surface of the wire that affects the lines of force of the field undergoes a displacement."*.

3.6. Question D6 (Fig. 6A in the Appendix)

Table 6 summarizes the results pertaining to Question D6: a circuit, immersed in a constant and uniform magnetic field, is equipped with a switch that is switched from one position to another, thus varying the area of the circuit itself. The problem is to understand if, due to this variation of area, a current will flow in the circuit or not. It is clear that, also in this case, an appropriate use of equation (13) is of fundamental importance.

Table 6. Answers to D6.

Cathegory	**Number of answers**
Area/Flux	15
Open/Closed circuit	1
B changes since the area changes	2
Appropriate use of equations	2
Other	3

Examples of "Flux/Area" category:

• *"No current flows because there is neither flux variation nor area variation as the field is uniform in both situations."*

We can note that the previous answer is the only correct one in this category, where a typical answer is:

• *"An induced current is generated because the area varies and therefore there is an increase in the flux of the magnetic field."*.

In the "Open/Closed circuit" category, we find:

• *"In the circuit we have induced current when the switch passes from position A to position C because to have induced current the circuit must be closed."*.

Two students believe that by moving the switch from "A" to "C" the magnetic field changes and then an emf is generated ("B changes since the area changes" category). One of them:

• *"By moving the switch, an induced current passage occurs because there is a variation in the magnetic field and consequently the production of electromotive force."*.

Finally, in the "Appropriate use of equations" category, we can read:

- *"No, no induced current passes because there is neither a variation of B nor a velocity."*.

4. A useful definition of EMI

The educational path presented above allows to interpret the phenomenon of EMI at a microscopic level only when it is linked to the motion of a conductor in a reference frame in which the field sources are constant. Understanding a physical phenomenology often involves the understanding of the forces acting punctually. In the present situation, this fact only occurs with clarity in the expression of the motional part of EMI: the force acting point by point on the charges is the magnetic part of the Lorentz force. Regarding the transformer part of the emf, it is more difficult to understand and to visualize which are the forces at stake on the charges. The statement, found in many textbooks, that a time-varying magnetic field generates an induced electric field does not help much in understanding, because there is no a mathematical expression that gives the induced electric field in terms of the temporal variation of the magnetic one.

This asymmetry of treatment between the two parts that constitute the emf (see equation (13)) is not typical of our path, in fact even in the literature one often finds questions related only to the microscopic interpretation of the motional part of the emf. We believe that the restoration of this symmetry can only be achieved by providing a definition of induction (which, at the best of our knowledge is lacking in textbooks) that recovers the meaning of electrostatic induction in the limit of a static electric field and that explicitly refers to an electric field defined point by point.

Therefore, we propose the following definition: "Induction is the separation/movement of charges in a conductor due to the presence of an electric field. The general expression of an electric field (see [16], Arts. 598, 599) is given by:

$$\vec{E} = \vec{E}_c - \frac{\partial \vec{A}}{\partial t} + \vec{v} \times \vec{B}. \quad (14)$$

If at the RHS of equation (14) there is only the first term, we have the electrostatic induction due to the Coulombian field $\vec{E}_c$; if there is are only the other two terms, we have EMI". The Coulombian part $\vec{E}_c$and the term $\vec{v} \times \vec{B}$ are well known to students, while the second term is more problematic since its explicit expression implies the use of the magnetic vector potential $\vec{A}$.

The restoration of the symmetry cited above comes from the term $-\frac{\partial \vec{A}}{\partial t}$ that describes the transformer part of EMI. Already other authors (see for instance [8]) propose the use of the vector potential to give the general law for EMI, but, usually, they address to university professors and students. We believe, instead, that the teaching of the vector potential might be useful not only at university level [17], but also for secondary school students [18]. We think that the expression (14) allows a clearer conceptualization of the problem. Once the field has been written in local terms, it should be clear that the velocity that appears in the formula is that of one point of the conductor and that the temporal variation of the magnetic vector potential is the part of the electric field linked to the variation of the magnetic one. Furthermore, the use of (14) allows also to separate the problem of identifying an induction phenomenon from that of explicitly calculating the emf and therefore facing with an explicit calculation of flux and circulation, in general difficult for students [19,20].

5. Conclusions

This work concerns a pilot experimentation for the presentation of EMI at high school, keeping in mind the suggestions coming from the didactic literature [1–9] and the ministerial indications for the Italian high school [10]. A well detailed path has been developed obtaining the main formulas in quite a rigorous way and presenting numerous applications and examples; moreover, a feasibility analysis has been carried out. The experimentation took place in 5 hours plus the final test of 45 minutes, a period of time that is, in our opinion, too short - but, nonetheless, the only one granted by the school structure - to be able to draw definitive conclusions about the learning of such a complex topic.

However, the results obtained undoubtedly indicate that our approach seems concretely feasible and reasonably manages to make students understand which are the two mechanisms underlying the electromagnetic induction. Nevertheless, the well-known difficulty presented by the students when they have to calculate flux and/or circulation of a vector field [19, 20] clearly emerges from our experimentation. Referring to the EMI phenomenon from an integral point of view, as is common at school, is difficult for students and, furthermore, masks the physical origin of the emf, in particular of the transformer part of the emf. Therefore, we propose a definition of induction explicitly linked to a local electric field (see [16], Arts. 598, 599), with the belief that a local approach is simpler to visualize. To do this we need the notion of the magnetic vector potential already at high school. This fact adds a new motivation to our belief in the importance of the magnetic vector potential for the understanding of electromagnetism even at high school [18].

Appendix A

QUESTION D1

The four figures below represent a cylindrical magnet and a small copper coil lying on a plane perpendicular to the NS axis of the magnet. The motion states of the magnet and the coil are shown ($\vec{v}$ represents the velocity). In what situation will the coil be crossed by an induced current? (a) I, III, IV – (b) I, IV – (c) I, II, IV – (d) IV – (e) No one.

Motivate your answer.

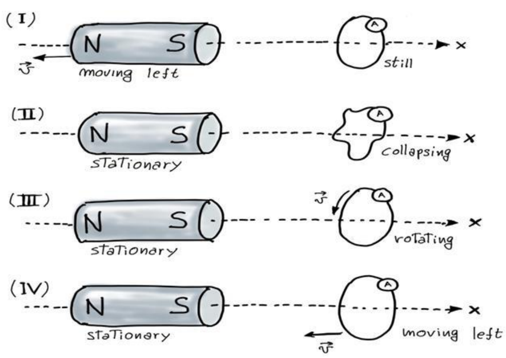

Figure 1A.

QUESTION D2

A very long rectilinear wire carries a DC current i. A metallic loop moves with velocity $\vec{v}$ in the same plane of the wire, in the direction indicated in the three figures. In which cases will the coil be crossed by an induced current?

(a) Only I and II – (b) Only I and III – (c) Only II and III – (d) In all the cases – (e) In none of the cases.

Motivate your answer.

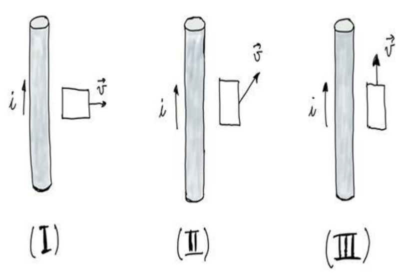

Figure 2A.

QUESTION D3

A neutral metal bar moves at a constant velocity $\vec{v}$ in a uniform and stationary magnetic field directing into the page (and therefore perpendicular to the velocity of the bar). Which of the diagrams on the right best describes the charge distribution on the surface of the metal bar?

Motivate your answer.

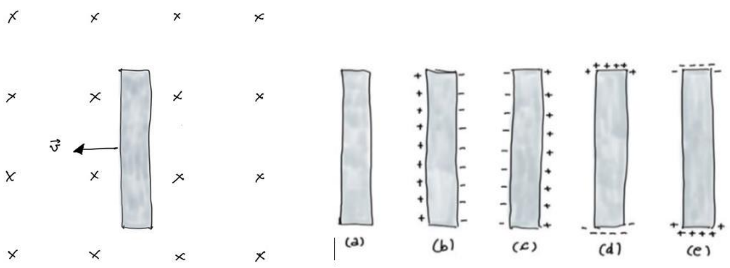

Figure 3A.

QUESTION D4

A circular conductive loop, placed in a steady, uniform, vertical magnetic field, is connected to an ammeter. When the loop is rotated upwards, the ammeter measures a current. Explain the origin of the forces that move the charges in the loop.

Motivate your answer.

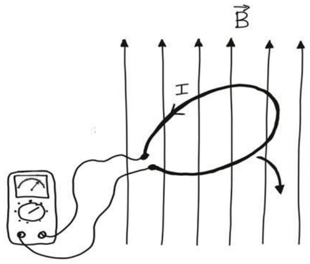

Figure 4A.

QUESTION D5

A C-shaped wire is slipping inside the air gap of a magnet, keeping the contact with one of the poles and remaining constantly on a plane parallel to the magnetic field inside the air gap. Keeping in mind that both the wire and the magnet are conductors, determine whether an induced current flows in the wire.

Motivate your answer.

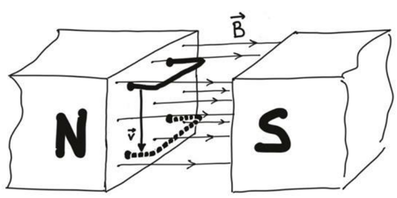

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Figure 5A.

QUESTION D6

The figure below shows a conductive circuit in a steady and uniform magnetic field coming out of the sheet. Determine whether an induced current will flow in the circuit when the switch passes from position A to position C.

Motivate your answer.

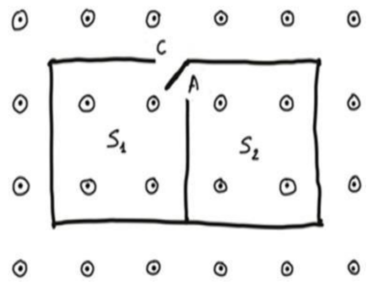

Figure 6A.

References

References must be numbered in order of appearance in the text (including citations in tables and legends) and listed individually at the end of the manuscript. We recommend preparing the references with a bibliography software package, such as EndNote, ReferenceManager or Zotero to avoid typing mistakes and duplicated references. Include the digital object identifier (DOI) for all references where available.

Citations and references in the Supplementary Materials are permitted provided that they also appear in the reference list here.

In the text, reference numbers should be placed in square brackets [ ] and placed before the punctuation; for example [1], [1–3] or [1,3]. For embedded citations in the text with pagination, use both parentheses and brackets to indicate the reference number and page numbers; for example [5] (p. 10), or [6] (pp. 101–105).

1. Giliberti M., Giliberti E. and Cavinato M. A case study about the comprehension of electromagnetic induction in Northern Italy. *J. Phys.: Conf. Ser.* **2019**. 1287, 012006.
2. Cavinato M., Giliberti E. and Giliberti M. Conceptualization of Electromagnetic Induction at various Educational Levels: a Case Study. *Can. J. of Phys.* **2022**, https://doi.org/10.1139/cjp-2019-0328.
3. Galili I., Kaplan D. and Lehavi Y. Teaching Faraday's law of electromagnetic induction in an introductory physics course. *Am. J. Phys*. **2006**, 74, 337-43.
4. Guisasola J., Almudi J. M. and Zuza K. University Students' Understanding of Electromagnetic Induction. *Int. J. of Sci. Ed.* **2013,** 35 (16), 2692-717.
5. https://www.physport.org/assessments/assessment.cfm?A=CSEM. Last access: 20/06/2022.
6. Maloney D. P., O'Kuma T. L., Hieggelke C. J. and Van Heuvelen A. Surveying students' conceptual knowledge of electricity and magnetism. *Am. J. Phys*. **2001**, 69, S12-S23.
7. Zuza K., Guisasola J., Michelini M. and Santi L. Rethinking Faraday's law for teaching motional electromotive force. *Eur. J. Phys*. **2012**, 33, 397-406.
8. Giuliani G. Vector potential, electromagnetic induction and 'physical meaning'. *Eur. J. Phys.* **2010**, 31, 871–80.
9. Jelicic K., Planinic M. and Planinsic G. Analyzing high school students' reasoning about electromagnetic induction. *Phys. Rev. Phys. Ed. Res.* **2017**. 13, 010112(18).
10. https://www.miur.gov.it/il-quadro-di-riferimento-della-seconda-prova-di-fisica-per-gli-esami-di-stato-dei-licei-scientifici (15-12-2015) (Translated by the authors). Last access: 20/06/2022.
11. Ruiz M. J. Lenz's Law Magic Trick. *Phys. Teach.* **2006**, 44 96-8.
12. https://www.youtube.com/watch?v=Hu7dyPONKmE. Last access: 09/06/2022.
13. https://www.youtube.com/watch?v=48Z3PRO5L3E. Last access: 20/06/2022.
14. The video is no more available in internet. For a similar video look at https://www.youtube.com/watch?v=L1ojoSgo-lU. Last access: 10/06/2022.
15. Barbieri S., Carpineti M. and Giliberti M. Teacher Participants in the European Project TEMI Practice the Enquiry Methodology in Their Classroom. Proceedings of the "International Conference GIREP EPEC 2015 July 6-10, Wrocław, Poland. **2016**, 103-108.
16. Maxwell J. C. *A Treatise on Electricity and Magnetism*, Vol. 2; Dover Publications: New York; **1873**. Arts. 598, 599.

 doi:10.20944/preprints202206.0332.v1

17. Barbieri S., Cavinato M. and Giliberti M. An educational path for the magnetic vector potential and its physical implications. Eur. J. Phys. **2013**. 34, 1209-19.
18. Cavinato M., Barbieri S. and Giliberti M. Vector potential at high school: a proposal for its introduction. *Eur. J. Phys*. **2018**, 39 055703(20).
19. Cavinato M., Giliberti M. and Barbieri S. R. Some didactical suggestions for a deeper embedding of DC circuits into electromagnetism. *Eur. J. Phys*. **2017**, 38 055707(10).
20. Guisasola J., Almudi J. M., Salinas J., Zuza K. and Ceberio M. The Gauss and Ampère laws: different laws but similar difficulties for student learning. *Eur. J. Phys.* **2008**, 29 1005-16.