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Proposed Test for Complex versus Quaternion Quantum Theory via Quantum Superdense Coding

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Abstract

While complex numbers form the foundational number system of quantum mechanics, the theoretical possibility of higher-dimensional hypercomplex alternatives persists. In this paper, we present a test protocol designed to distinguish between complex and quaternionic quantum mechanics. The protocol is founded upon the quantum superdense coding scheme and functions by detecting the presence of multiple distinct quantum phase gates. We also provide the corresponding quantum circuit required to perform this test.

Keywords: hypercomplex quantum theory; complex quantum theory; quaternion quantum mechanics; quantum phase gate; quantum superdense coding

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1. Introduction

Complex numbers are widely regarded as the foundational number system underpinning quantum mechanics; nevertheless, the theoretical framework remains open to exploration with higher-dimensional hypercomplex algebras. In early 1936, Birkhoff and von Neumann [5] discussed the story of hypercomplex numbers in quantum mechanics on the use of quaternions and octonions. What started as a theoretical curiosity soon evolved into structured fields of inquiry, leading to the development of quaternionic quantum mechanics (QQM) [1,9,10,13,23], octonionic quantum mechanics (OQM) [17], and sedenionic quantum mechanics (SQM) [19]. This theoretical momentum has inspired a new wave of models, including quaternionic quantum automata [7,8], quaternionic quantum walks [15,16], and quaternionic quantum neural networks [2,3]. Collectively, this progress points to an increasingly urgent question: the need for specialized experiments that can probe the very validity of hypercomplex quantum mechanics, a necessity noted in Ref.[26].

In 1979, Peres [21] proposed the first experimental test to distinguish between complex quantum mechanics (CQM) and quaternion quantum theory (QQM), based on single-particle interference in a three-path setup. Subsequent experimental efforts with neutrons [14] and photons [22] were limited to testing a simplified version of Peres's original scheme. It was only in recent years that the complete Peres test was successfully conducted in the optical [11] and microwave [25] domains. Nevertheless, these landmark experiments, despite their technical achievement, did not yield sufficient evidence to exclude the possibility of QQM. Subsequently, the Peres test has been generalized to multi-path and multi-particle interference [26]. In particular, Renou et al. [24] introduced a Bell-like experiment demonstrating the inadequacy of real-valued quantum mechanics. This prediction was quickly tested and confirmed in experiments with photons [18] and superconducting qubits [6].

With the advancements in quantum computation and quantum information theory, an intriguing possibility is to utilize these frameworks to devise tests that can distinguish between CQM and QQM. Quantum superdense coding [4] is a quantum communication protocol that allows a sender to transmit two classical bits of information to a receiver by sending only one qubit. This remarkable feat is possible through the use of quantum entanglement and is a foundational protocol in quantum information science [20].

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We present a test method based on quantum superdense coding. This approach is designed to discern whether the physical reality is governed by CQM or a hypercomplex extension, through the detection of higher-dimensional hypercomplex quantum phase gates. Furthermore, we detail the construction of the quantum circuits necessary for the implementation of this test.

2. Preliminaries

2.1. Quaternions

Quaternions [12] are a 4-dimensional extension of complex numbers. Let $\mathbb{H}$ be the set of quaternions. A quaternion $q \in \mathbb{H}$ can be expressed as

$$q = a + bi + cj + dk, \tag{1}$$

where $a, b, c, d \in \mathbb{R}$ are real numbers and i, j, k are imaginary units obeying the relations

$$i^2 = j^2 = k^2 = ijk = -1, \tag{2}$$

and

$$ij = k, jk = i, ki = j, ji = -k, kj = -i, ik = -j. \tag{3}$$

Quaternions form a non-commutative algebra, meaning the order of multiplication matters $qr \neq rq$.

2.2. Quantum Superdense Coding

The quantum superdense coding protocol proceeds as follows:

1. Alice and Bob share the Bell state $\frac{|00\rangle+|11\rangle}{\sqrt{2}}$, with Alice holding the first qubit and Bob the second.

2. Alice encodes her two classical bits (00, 01, 10, or 11) by applying one of four unitary operations to her qubit:

- 00: Apply $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ gate,
- 01: Apply $X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ gate,
- 10: Apply $Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ gate,
- 11: Apply $Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ gate.

And then Alice sends her single qubit to Bob.

3. Bob applies $(H \otimes I) \cdot CNOT$ to the pair and measures, recovering Alice's two-bit message.

From the perspective of a unified system encompassing both Alice and Bob, the entire quantum superdense coding protocol can be described by the quantum circuit shown in Figure 1. The corresponding measurement outcomes for different choices of the unitary gate U within this circuit are summarized in Table 1.

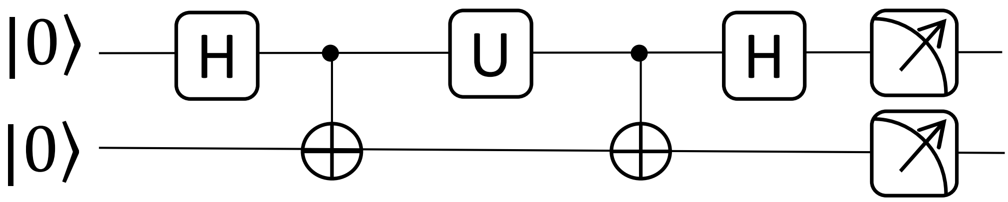

Figure 1. Quantum circuit of superdense coding.

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Table 1. Outcomes and associated encoding gates of Figure 1.

Encoding gates	Outcomes
$U = I$	00
$U = X$	01
$U = Z$	10
$U = Y$	11

3. Proposed Test

Our fundamental idea is that if quantum mechanics operates in higher dimensions, then there must exist hypercomplex quantum gates. To explore this concept, let us consider the quantum phase gate, commonly known as the S gate. The S gate is a fundamental single-qubit gate in quantum computing, essential for manipulating the phase information of quantum states—a key resource in quantum algorithms and error correction. The standard S gate is represented by the unitary matrix:

$$S_i = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \tag{4}$$

where i is the imaginary unit satisfying $i^2 = -1$. For clarity and to distinguish it from subsequent generalized gates, we refer to this gate as S_i.

By extending quantum gates into the hypercomplex domain, such as the quaternionic framework, we can define generalized versions of the S gate that incorporate alternative imaginary units. In QQM, which is characterized by three distinct imaginary units, this yields two additional variants of the quantum phase gate:

$$S_j = \begin{bmatrix} 1 & 0 \\ 0 & j \end{bmatrix} \tag{5}$$

$$S_k = \begin{bmatrix} 1 & 0 \\ 0 & k \end{bmatrix} \tag{6}$$

where j, k are the two quaternion units distinct from i, each satisfying $j^2 = k^2 = -1$.

Our main goal is to explore whether generalized S gates exist. If so, this could indicate that quantum mechanics is inherently describable with hypercomplex numbers, not complex numbers.

Our test method is described as follows. First, a large set of quantum phase gates is generated. The following steps are then executed iteratively:

- Step 1: Select two distinct quantum phase gates, denoted as S_1 and S_2, and construct the quantum circuit shown in Figure 2;
- Step 2: Execute the quantum circuit from Figure 2 and record the measurement outcomes.

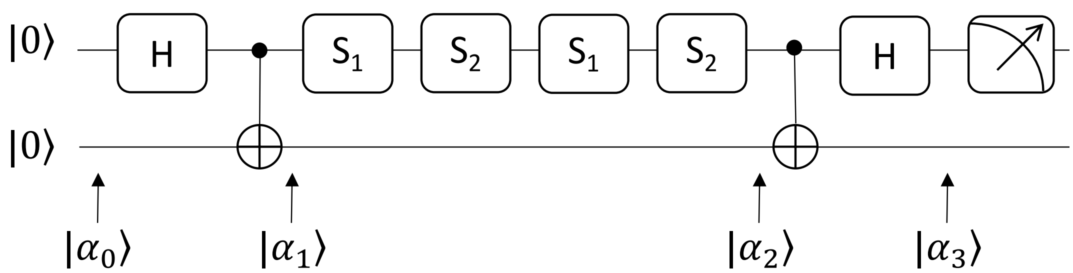

Figure 2. Quantum circuit of our test.

This procedure is repeated for each new pair of distinct quantum phase gates (S_1, S_2), followed by repeated execution of the circuit and observation of the results. It is important to note that S_1 and S_2 in the pair are implemented using a separate quantum phase gate device, rather than applying the same device four times. For example, in photonic quantum computing, the S gate can be physically realized using a quarter-wave plate (QWP). Consequently, the sequence $S_1S_2S_1S_2$ can be implemented by sequentially employing two distinct QWP devices. Alternatively, this can be achieved using two distinct phase shifters. The theoretical predictions for QQM and CQM are fundamentally distinct: the former yields measurement outcome 0 with 100%, while the latter yields outcome 1 with 100%. As a result, the discriminatory power of the test remains robust. Angular errors in wave plates or birefringence dispersion in phase shifters would not alter the qualitative distinction between the binary outcomes, thus preserving the validity of the test.

3.1. Analysis of Our Test

Case I: If both phase gates are same, i.e., $S_1 = S_2$, their combined effect is equivalent to the I gate:

$$S_1S_2S_1S_2 = S_t^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I, \quad \forall\, t \in \{i, j, k\}. \tag{7}$$

In this case, the final state $|\alpha_3\rangle$ is $|00\rangle$. Consequently, the probability of measuring the first qubit in state $|0\rangle$ is exactly 100%.

Table 2. Outcomes at Each Stage of Figure 2.

Case	$\|\alpha_0\rangle$	$\|\alpha_1\rangle$	$\|\alpha_2\rangle$	$\|\alpha_3\rangle$	Measurement results
$S_1 = S_2$	$\|00\rangle$	$\frac{\|00\rangle+\|11\rangle}{\sqrt{2}}$	$\frac{\|00\rangle+\|11\rangle}{\sqrt{2}}$	$\|00\rangle$	0
$S_1 \neq S_2$	$\|00\rangle$	$\frac{\|00\rangle+\|11\rangle}{\sqrt{2}}$	$\frac{\|00\rangle-\|11\rangle}{\sqrt{2}}$	$\|10\rangle$	1

Case II: If the two phase gates are distinct, i.e., $S_1 \neq S_2$, their combined effect is equivalent to the Pauli Z gate:

$$S_1S_2S_1S_2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = Z, \qquad \forall S_1, S_2 \in \{S_i, S_j, S_k\} \text{ and } S_1 \neq S_2. \tag{8}$$

In this case, the final state $|\alpha_3\rangle$ is $|10\rangle$. Consequently, the probability of measuring the first qubit in state $|1\rangle$ is exactly 100%.

Clearly, Case I and Cases II correspond to $U = I$ and $U = Z$ in Figure 1, respectively.

3.2. Discussion

Our method rests on two mutually exclusive facts:

(1) If quantum mechanics is fundamentally based on the complex-number system, only one phase gate exists.

(2) If it is built on a hyper-complex system, additional, distinct phase gates must exist.

Crucially, our test method can discriminate between these two scenarios: a system with a single phase gate yields a measurement outcome of 0, whereas a system with two distinct phase gates yields an outcome of 1.

Mathematically, symmetry ensures that no imaginary unit (for example i) is privileged over others (for example j, k), implying an equal probability for all three gates in QQM. This means that repeated sampling of phase gate pairs with an independent quantum phase gate device will, with high probability, reveal the existence of multiple distinct gates.

In general, for CQM, let

$$U_i(\theta) = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{bmatrix}. \tag{9}$$

Note that $U_i(\pi/2) = S_i$, $U_i(\pi) = Z$, and $U_i(2\pi) = I$.

For QQM, let

$$U_j(\theta) = \begin{bmatrix} 1 & 0 \\ 0 & e^{j\theta} \end{bmatrix}, \tag{10}$$

$$U_k(\theta) = \begin{bmatrix} 1 & 0 \\ 0 & e^{k\theta} \end{bmatrix} \tag{11}$$

Obviously, $U_i(\theta)U_j(\theta) \neq U_k(\theta)$.

The general phase gate of QQM can be presented as

$$\begin{aligned} U(\theta_1, \theta_2, \theta_3) &= \begin{bmatrix} 1 & 0 \\ 0 & e^{i\theta_1}e^{j\theta_2}e^{k\theta_3} \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & e^{i\theta_1} \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & e^{j\theta_2} \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & e^{k\theta_3} \end{bmatrix} \\ &= U_i(\theta_1)U_j(\theta_2)U_k(\theta_3). \end{aligned} \tag{12}$$

Note that due to the non-commutativity of quaternion multiplication, different orderings of $U_i(\theta_1), U_j(\theta_2)$, and $U_k(\theta_3)$ yield distinct phase gates. For instance, $U_j(\theta_2)U_i(\theta_1)U_k(\theta_3) \neq U(\theta_1, \theta_2, \theta_3)$. For simplicity, we restrict our discussion to the case of $U(\theta_1, \theta_2, \theta_3)$, as the results for other orderings are analogous.

In general, for CQM, setting $U = U_i(\theta)$ in Figure 1, the evolution of the state is given by:

$$\begin{aligned} &(H \otimes I) \cdot \text{CNOT} \cdot (U_i(\theta) \otimes I) \cdot \text{CNOT} \cdot (H \otimes I)\,|00\rangle \\ =&(H \otimes I) \cdot \text{CNOT} \cdot (U_i(\theta) \otimes I) \cdot \text{CNOT} \left(\frac{|00\rangle + |10\rangle}{\sqrt{2}}\right) \\ =&(H \otimes I) \cdot \text{CNOT} \cdot (U_i(\theta) \otimes I) \left(\frac{|00\rangle + |11\rangle}{\sqrt{2}}\right) \\ =&(H \otimes I) \cdot \text{CNOT} \left(\frac{|00\rangle + e^{i\theta}|11\rangle}{\sqrt{2}}\right) \\ =&(H \otimes I) \left(\frac{|00\rangle + e^{i\theta}|10\rangle}{\sqrt{2}}\right) \\ =&\frac{1+e^{i\theta}}{2}|00\rangle + \frac{1-e^{i\theta}}{2}|10\rangle. \end{aligned} \tag{13}$$

Similarly, for QQM, setting $U = U(\theta_1, \theta_2, \theta_3)$ in Figure 1, the result is

$$\begin{aligned} &(H \otimes I) \cdot \text{CNOT} \cdot (U(\theta_1, \theta_2, \theta_3) \otimes I) \cdot \text{CNOT} \cdot (H \otimes I)\,|00\rangle \\ =&(H \otimes I) \cdot \text{CNOT} \cdot (U(\theta_1, \theta_2, \theta_3) \otimes I) \cdot \text{CNOT} \left(\frac{|00\rangle + |10\rangle}{\sqrt{2}}\right) \\ =&(H \otimes I) \cdot \text{CNOT} \cdot (U(\theta_1, \theta_2, \theta_3) \otimes I) \left(\frac{|00\rangle + |11\rangle}{\sqrt{2}}\right) \\ =&(H \otimes I) \cdot \text{CNOT} \left(\frac{|00\rangle + e^{i\theta_1}e^{j\theta_2}e^{k\theta_3}|11\rangle}{\sqrt{2}}\right) \\ =&(H \otimes I) \left(\frac{|00\rangle + e^{i\theta_1}e^{j\theta_2}e^{k\theta_3}|10\rangle}{\sqrt{2}}\right) \\ =&\frac{1+e^{i\theta_1}e^{j\theta_2}e^{k\theta_3}}{2}|00\rangle + \frac{1-e^{i\theta_1}e^{j\theta_2}e^{k\theta_3}}{2}|10\rangle. \end{aligned} \tag{14}$$

For CQM, the phase gates $U(\theta)$ are cumulative, satisfying $U(\theta_1)U(\theta_2) = U(\theta_1 + \theta_2)$. In contrast, for QQM, the phase gates $U(\theta_1, \theta_2, \theta_3)$ generally lack this cumulative property, i.e.,

$$U(\phi_1, \phi_2, \phi_3) \cdot U(\psi_1, \psi_2, \psi_3) \neq U(\phi_1 + \psi_1, \phi_2 + \psi_2, \phi_3 + \psi_3).$$

Fundamentally, our test exploits this critical distinction: the cumulativity of complex phases in CQM stands in sharp contrast to the non-cumulativity of quaternionic phases in QQM. By selecting appropriate phase angles, we amplify this difference to generate clearly contrasting observational outcomes. In these special cases, one outcome yields the state $|0\rangle$ with 100% probability, while the other yields $|1\rangle$ with 100% probability.

It is crucial to acknowledge a potential limitation regarding physical realizability. The test relies on the existence of physically distinct, independently implementable phase gates corresponding to different quaternion units (i, j, k). If a specific physical realization of QQM fails to provide the necessary mechanisms for generating or selecting these distinct gate devices, it is entirely possible that the required gates S_i, S_j, S_k could not be independently accessed. In such a scenario, the inability to physically implement distinct sequences involving S_i, S_j, S_k would prevent the test from exhibiting its discriminatory power, thereby rendering the theoretical distinction between QQM and Complex QM empirically indistinguishable based on this specific protocol.

Such a constraint leads to an alternative approach. For QQM, let us consider the following phase gate:

$$U_Q(\theta) = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\theta}e^{j\theta}e^{k\theta} \end{bmatrix}. \tag{15}$$

This definition is also motivated by symmetry considerations. Just as before, symmetry ensures that no imaginary unit (for instance, i) is privileged over the others (such as j or k). Consequently, if i, j, k are to appear simultaneously, they possess equal "weights," which dictates the form $e^{i\theta}e^{j\theta}e^{k\theta}$ rather than $e^{i\theta_1}e^{j\theta_2}e^{k\theta_3}$ with distinct values for θ_1, θ_2, and θ_3. It is important to note that distinct values for θ_1, θ_2, and θ_3 would imply that S_i, S_j, S_k could be independently accessed, which contradicts the physical constraint discussed above.

Note that the phase gates do not satisfy the cumulative property, i.e., $U_Q(\theta_1)U_Q(\theta_2) \neq U_Q(\theta_1 + \theta_2)$.

Then, we have

$$\begin{aligned} U_Q(\pi/2) &= \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/2}e^{j\pi/2}e^{k\pi/2} \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & ijk \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \\ &= Z, \end{aligned} \tag{16}$$

$$\begin{aligned} U_Q(\pi) &= \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi}e^{j\pi}e^{k\pi} \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & (-1)\cdot(-1)\cdot(-1) \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \\ &= Z, \end{aligned} \tag{17}$$

and

$$\begin{aligned} U_Q(2\pi) &= \begin{bmatrix} 1 & 0 \\ 0 & e^{2i\pi}e^{2j\pi}e^{2k\pi} \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= I. \end{aligned} \tag{18}$$

In CQM: If the gate U in the circuit shown in Figure 1 is set to S, the probability of measuring the first qubit in state $|0\rangle$ is 50%, and for state $|1\rangle$ it is also 50%; a large number of shots would reveal an approximately equal ratio between $|0\rangle$ and $|1\rangle$. If U consists of two sequential S gates (SS), the probability of measuring the first qubit in state $|1\rangle$ is exactly 100%. If U consists of four sequential S gates (i.e., $SSSS$), the probability of measuring the first qubit in state $|0\rangle$ becomes exactly 100%.

In QQM: If the gate U in the Figure 1 circuit is a single $U_Q(\pi/2)$ gate, the probability of measuring the first qubit in state $|1\rangle$ is exactly 100%. Similarly, if U consists of two sequential $U_Q(\pi/2)$ gates (i.e., $U_Q(\pi/2)U_Q(\pi/2)$), the probability of measuring the first qubit in state $|1\rangle$ remains exactly 100%. Finally, if U consists of four sequential $U_Q(\pi/2)$ gates, the probability of measuring the first qubit in state $|0\rangle$ is exactly 100%.

It is evident that stacking phase gates from one to two and then to four yields significantly distinct probability variations. Exploiting this characteristic, we adjust the number of S gates in Figure 2 and analyze the measurement outcome ratios for 1, 2, and 4 gates. This methodology provides a clear distinction between CQM and QQM.

4. Conclusions

Inspired by the quantum superdense coding protocol, this paper proposes a test to distinguish between complex and quaternionic quantum mechanics. Our method achieves this by perfectly discriminating between systems with a single phase gate and those with multiple distinct phase gates.

Future theoretical work should focus on developing new quantum protocols to further probe this distinction. If quantum mechanics is founded on a hypercomplex number system, distinct quantum phase gates must exist. In QQM, for example, three different phase gates S_i, S_j and S_k exist corresponding to the imaginary units i, j, k. Similarly, OQM and SQM feature seven and fifteen phase gates, respectively. Moreover, our proposed test method can detect the number of distinct phase gates present, thereby allowing discrimination between QQM, OQM, and SQM. Ultimately, however, it will be up to future experiments to determine which number system truly underlies our quantum reality.

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