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Geometric Interpretation of Mass-Energy Equivalence in Weyl Spacetime

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Abstract: This paper presents a novel interpretation of the mass-energy equivalence $E = mc^2$ within the framework of Weyl geometry, a generalization of Riemannian geometry that incorporates local conformal invariance. By introducing a scalar field to break this invariance, we derive particle masses and recover the standard relativistic energy relation in a specific gauge. The model is formulated in natural units ($\hbar = c = 1$), ensuring dimensional consistency across all equations, and offers testable predictions, including deviations in particle trajectories and potential cosmological effects. This approach provides a geometric perspective on mass generation without requiring additional fields beyond the metric and a scalar field, bridging classical relativity with conformal gauge theories.

Keywords: cosmological effects; gauge theories; particle masses; mass-energy equivalence; scalar field; Weyl geometry; conformal invariance

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Note: This preliminary manuscript, though thoroughly reviewed, may contain minor errors. The final section discusses authorship, ongoing research, and future directions

1. Introduction

The mass-energy equivalence $E = mc^2$ stands as one of the most profound discoveries in modern physics, fundamentally altering our understanding of matter and energy [1]. This relation, derived within the framework of special relativity, describes the intrinsic connection between a particle's rest mass and its energy content. In Einstein's general relativity, spacetime is modeled as a Riemannian manifold where particle masses appear as fundamental parameters in the theory [2,6].

However, the geometric foundation of mass itself remains an open question in theoretical physics. Weyl geometry, originally proposed by Hermann Weyl in 1918 [3], offers an intriguing alternative geometric framework that extends general relativity by incorporating local conformal invariance. This extension introduces a gauge field A_μ alongside the metric $g_{\mu\nu}$, allowing for scale transformations of the metric tensor. While Weyl's original formulation faced criticism regarding the behavior of physical clocks [7], modern applications in gauge theories and cosmology have renewed interest in this geometric framework [8,9].

Recent developments in theoretical physics have demonstrated the fundamental role of scalar fields in mass generation, most notably through the Higgs mechanism [4,5]. This mechanism shows how spontaneous symmetry breaking via a scalar field can generate masses for gauge bosons and fermions. Building upon this insight, we propose a framework where the mass-energy equivalence emerges naturally within Weyl geometry through the introduction of a mass-generating scalar field.

Our approach constructs a covariant model that accomplishes several key objectives. First, it defines particle masses through a scalar field ϕ that breaks the conformal invariance of Weyl geometry. Second, it recovers the standard $E = mc^2$ relation in a specific gauge choice. Third, it maintains dimensional consistency throughout all calculations by working in natural units. Fourth, it provides concrete testable predictions that could distinguish this framework from standard general relativity.

The significance of this work lies in its potential to provide a geometric understanding of mass itself, rather than treating mass as a fundamental parameter. By embedding mass generation within the geometric structure of spacetime, we open new avenues for exploring the deep connections between geometry, gauge invariance, and the fundamental constants of nature.

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All calculations in this paper are performed in natural units ($\hbar = c = 1$), where mass and energy have dimensions $[M]$, length and time have dimensions $[M^{-1}]$, and the action is dimensionless. This choice simplifies the mathematical expressions while maintaining complete physical generality.

2. Geometric Framework: Weyl Geometry

Weyl geometry represents a natural generalization of Riemannian geometry that preserves the essential features of Einstein's theory while introducing additional geometric structure [3,10]. The fundamental innovation lies in the replacement of the metric compatibility condition with a weaker constraint that allows for conformal rescalings.

In Weyl geometry, the metric tensor $g_{\mu\nu}$ and a gauge field A_μ transform under conformal rescalings according to:

$$g_{\mu\nu} \to e^{2\sigma(x)} g_{\mu\nu}, \tag{1}$$

$$A_\mu \to A_\mu - \partial_\mu \sigma, \tag{2}$$

where $\sigma(x)$ is an arbitrary scalar function. This transformation law ensures that physical observables remain invariant under local scale changes, a property known as conformal invariance [11].

The connection in Weyl geometry incorporates both the geometric information from the metric and the gauge field:

$$\Gamma^\lambda_{\mu\nu} = \{\lambda\mu\nu\} + \delta^\lambda_\mu A_\nu + \delta^\lambda_\nu A_\mu - g_{\mu\nu} A^\lambda, \tag{3}$$

where $\{\lambda\mu\nu\}$ denote the Christoffel symbols of the metric $g_{\mu\nu}$. This connection reduces to the standard Christoffel symbols when $A_\mu = 0$, recovering the Riemannian limit.

2.1. Dimensional Analysis and Consistency

A rigorous dimensional analysis ensures the mathematical consistency of our framework. In natural units, we have:

- $[g_{\mu\nu}] = 1$ (dimensionless, as it defines the causal structure)
- $[x^\mu] = [M^{-1}]$, implying $[\partial_\mu g_{\nu\sigma}] = [M]$
- $[\{\lambda\mu\nu\}] = [M]$ for Christoffel symbols
- $[A_\mu] = [M]$ from the gauge transformation property

Each term in the Weyl connection (3) carries dimension $[M]$:

$$[\delta^\lambda_\mu A_\nu] = [1] \times [M] = [M], \tag{4}$$

$$[\delta^\lambda_\nu A_\mu] = [1] \times [M] = [M], \tag{5}$$

$$[g_{\mu\nu} A^\lambda] = [1] \times [M] = [M]. \tag{6}$$

This dimensional consistency ensures that the Weyl connection is mathematically well-defined and physically meaningful.

2.2. Non-Metricity and Geometric Structure

The defining characteristic of Weyl geometry is its non-metric compatibility, expressed through:

$$\nabla_\lambda g_{\mu\nu} = -2A_\lambda g_{\mu\nu}. \tag{7}$$

This equation encodes the failure of the metric to be covariantly constant, with the gauge field A_μ measuring the rate of conformal change.

The dimensional consistency of equation (7) can be verified:

$$[\nabla_\lambda g_{\mu\nu}] = [M] \quad \text{(left-hand side)}, \tag{8}$$

$$[-2A_\lambda g_{\mu\nu}] = [M] \times [1] = [M] \quad \text{(right-hand side)}. \tag{9}$$

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This non-metricity distinguishes Weyl geometry from both Riemannian geometry (where $\nabla_\lambda g_{\mu\nu} = 0$) and more general metric-affine theories [12].

3. Particle Dynamics in Weyl Spacetime

The motion of test particles in Weyl geometry follows geodesics determined by the Weyl connection. This leads to a modified geodesic equation that incorporates both gravitational and conformal effects.

3.1. Geodesic Equation

Particle trajectories in Weyl geometry satisfy:

$$\frac{d^2x^\lambda}{d\tau^2} + \Gamma^\lambda_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = 0, \tag{10}$$

where τ represents the proper time with dimension $[\tau] = [M^{-1}]$.

The dimensional homogeneity of this equation can be established:

$$\left[\frac{d^2x^\lambda}{d\tau^2}\right] = \frac{[M^{-1}]}{[M^{-2}]} = [M], \tag{11}$$

$$\left[\Gamma^\lambda_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}\right] = [M] \times [1] \times [1] = [M]. \tag{12}$$

When $A_\mu = 0$, equation (10) reduces to the standard geodesic equation of general relativity, ensuring compatibility with established physics in appropriate limits.

3.2. Physical Interpretation of Modified Geodesics

The presence of non-zero A_μ introduces additional terms in the geodesic equation that can be interpreted as effective forces acting on test particles. In the weak-field limit where $g_{\mu\nu} \approx \eta_{\mu\nu}$ (Minkowski metric), the geodesic equation becomes:

$$\frac{d^2x^\lambda}{d\tau^2} + \left(\delta^\lambda_\mu A_\nu + \delta^\lambda_\nu A_\mu - \eta_{\mu\nu}A^\lambda\right)\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = 0. \tag{13}$$

This modification represents a departure from standard general relativity that could, in principle, be observed in high-precision gravitational experiments [13].

4. Scalar Field Dynamics and Mass Generation

To break the conformal invariance of Weyl geometry and generate particle masses, we introduce a scalar field ϕ with carefully chosen dynamics. This field plays a role analogous to the Higgs field in particle physics but operates within the geometric framework of Weyl spacetime.

4.1. Scalar Field Action

The action for the scalar field is given by:

$$S_\phi = \int d^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right], \tag{14}$$

where the potential takes the form:

$$V(\phi) = \frac{\lambda}{4}(\phi^2 - \phi_0^2)^2. \tag{15}$$

Here, λ is a dimensionless coupling constant, and ϕ_0 represents the vacuum expectation value of the scalar field.

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4.2. Dimensional Analysis of the Scalar Sector

The dimensional consistency of the scalar field action requires careful analysis:

- Volume element: $[d^4x] = [M^{-4}]$, $[\sqrt{-g}] = 1$
- Scalar field: $[\phi] = [M]$
- Kinetic term: $[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi] = [1] \times [M] \times [M] \times [M] \times [M] = [M^4]$
- Potential term: $[V(\phi)] = [1] \times [M^4] = [M^4]$
- Action: $[S_\phi] = [M^{-4}] \times [M^4] = 1$ (dimensionless)

This dimensional structure ensures that the scalar field action is mathematically consistent and physically meaningful.

4.3. Mass Generation Mechanism

The key insight of our approach lies in connecting particle masses to the vacuum expectation value of the scalar field:

$$m = g\phi_0, \tag{16}$$

where g is a dimensionless coupling constant. This relation ensures that $[m] = [1] \times [M] = [M]$, consistent with the dimension of mass.

The scalar field ϕ breaks the conformal invariance of Weyl geometry by providing a preferred scale. When ϕ acquires a non-zero vacuum expectation value ϕ_0, it selects a particular conformal frame and generates masses for particles coupled to this field.

5. Recovery of Mass-Energy Equivalence

Having established the geometric framework and mass generation mechanism, we now demonstrate how the famous relation $E = mc^2$ emerges naturally from our theory.

5.1. Gauge Choice and Simplification

In the gauge where $A_\mu = 0$, the Weyl connection (3) reduces to the standard Christoffel symbols:

$$\Gamma^\lambda_{\mu\nu} = \{\lambda\mu\nu\}. \tag{17}$$

This gauge choice effectively selects a conformal frame where the non-metric effects of Weyl geometry are minimized, and the theory approximates general relativity.

5.2. Energy Derivation for Particles at Rest

For a particle at rest in this gauge, we have $\frac{dx^i}{d\tau} = 0$ and $\frac{dx^0}{d\tau} = 1$. The action for such a particle is:

$$S = -m \int d\tau, \tag{18}$$

which yields the Lagrangian $L = -m$ and the corresponding energy $E = m$ in natural units.

The dimensional consistency of this result can be verified:

$$[S] = [M] \times [M^{-1}] = 1 \quad \text{(dimensionless action)}, \tag{19}$$

$$[E] = [m] = [M] \quad \text{(energy dimension)}. \tag{20}$$

Converting back to conventional units where $c \neq 1$, we recover the familiar relation:

$$E = mc^2. \tag{21}$$

This derivation shows that Einstein's mass-energy equivalence emerges naturally from the geometric structure of Weyl spacetime when conformal invariance is broken by the scalar field.

6. Physical Predictions and Experimental Tests

Our geometric reinterpretation of $E = mc^2$ leads to several testable predictions that could distinguish this framework from standard general relativity. These predictions arise from the residual effects of Weyl geometry even when A_μ is small but non-zero.

6.1. Modified Particle Trajectories

When $A_\mu \neq 0$, the geodesic equation (10) predicts deviations from standard general relativistic trajectories. These deviations could be observable in several contexts:

High-precision tests of the equivalence principle using torsion balances or drop tower experiments could reveal anomalous accelerations proportional to A_μ [14]. Gravitational lensing observations, particularly those involving the Event Horizon Telescope, might detect subtle deviations in light ray trajectories around massive objects [15]. Planetary motion within the solar system could exhibit small perturbations detectable through lunar laser ranging or spacecraft tracking [16].

6.2. Cosmological Implications

The scalar field ϕ and gauge field A_μ may influence cosmological evolution in observable ways. The scalar field could contribute to dark energy, potentially explaining the observed acceleration of cosmic expansion [17,18]. Variations in the gauge field A_μ across cosmic scales might leave imprints in the cosmic microwave background radiation, detectable by current and future missions [19].

The equation of state parameter for the scalar field component would differ from that of a cosmological constant, providing a potential observational signature through supernova distance measurements and baryon acoustic oscillation surveys [20].

6.3. Particle Physics Signatures

High-energy particle physics experiments could probe the interactions mediated by ϕ and A_μ. The Large Hadron Collider and future colliders might detect new interaction channels or modifications to standard model processes [21,22]. Precision measurements of particle masses and their ratios could reveal the geometric origin of mass through correlations predicted by equation (16).

Neutrino oscillation experiments might be particularly sensitive to the effects of Weyl geometry, as the small neutrino masses could be especially susceptible to geometric modifications [23].

7. Theoretical Limitations and Future Directions

While our framework provides a compelling geometric interpretation of mass-energy equivalence, several limitations and areas for future development must be acknowledged.

7.1. Global Consistency Issues

The gauge choice $A_\mu = 0$ may not be globally consistent in spacetimes with non-trivial topology. Near black holes or in cosmological contexts, maintaining this gauge might require singular behavior in the scalar field ϕ, potentially limiting the theory's applicability [24].

Future work must address the global structure of solutions and investigate whether alternative gauge choices can resolve these issues while preserving the physical predictions of the theory.

7.2. Quantum Corrections and Renormalization

The classical theory presented here requires extension to the quantum regime. Quantum corrections to the scalar field dynamics could significantly modify the mass generation mechanism, particularly at high energies where loop effects become important [25].

A complete quantum treatment would need to address the renormalization of the theory, ensuring that divergences can be controlled and that the classical limit is properly recovered. The conformal properties of Weyl geometry might provide advantages in this regard, as conformal invariance often improves the ultraviolet behavior of quantum field theories [26].

7.3. Connection to Established Physics

Future theoretical work should establish clearer connections between our Weyl geometric framework and established physics. This includes understanding how the standard model of particle physics emerges from the geometric structure and how the various coupling constants relate to fundamental geometric quantities.

The relationship between our scalar field ϕ and the Higgs field deserves particular attention, as both play similar roles in mass generation despite operating in different conceptual frameworks [4].

8. Conclusions

This paper has presented a novel geometric interpretation of Einstein's mass-energy equivalence $E = mc^2$ within the framework of Weyl geometry. By introducing a scalar field to break conformal invariance, we have demonstrated how particle masses can emerge from the geometric structure of spacetime itself, rather than being treated as fundamental parameters.

Our key achievements include the construction of a dimensionally consistent framework in natural units, the derivation of $E = mc^2$ from geometric principles, and the identification of testable predictions that could distinguish this approach from standard general relativity. The theory naturally incorporates both gravitational and conformal effects while reducing to established physics in appropriate limits.

The geometric perspective on mass generation offered by this work opens new avenues for understanding the deep connections between spacetime structure, gauge invariance, and fundamental physics. While challenges remain in extending the theory to quantum regimes and addressing global consistency issues, the framework provides a promising foundation for future developments in theoretical physics.

The testable predictions identified here, ranging from modified particle trajectories to cosmological signatures, offer concrete pathways for experimental validation. As precision in gravitational experiments and cosmological observations continues to improve, these predictions may soon be within reach of observational confirmation or refutation.

Ultimately, this work contributes to the broader program of geometrizing physics, following in the tradition established by Einstein's general relativity. By showing how mass-energy equivalence can emerge from geometric principles, we take another step toward a unified understanding of the fundamental forces and particles that constitute our universe.

Author and Paper Context and Future Implications

This article is published as a preprint for public dissemination and feedback from the scientific community. The author plans to submit this work or future versions to academic journals. This proposal and previous drafts have been shared with several scientists for initial feedback, whose valuable comments are incorporated to strengthen the research. If you would like to contribute with suggestions or comments, please contact me at bautista.baron@proton.me. Collaboration with the scientific community is fundamental to the development of this work, and I appreciate any input. Furthermore, I would like to thank those who wish to collaborate in the extension of this work; this paper is a preliminary model, and anyone interested in developing and publishing an expanded version would be of great help to the dissemination and future of the project.

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References

1. A. Einstein, "Does the inertia of a body depend upon its energy content?" *Annalen der Physik* **18**, 639–641 (1905).
2. A. Einstein, "Die Feldgleichungen der Gravitation," *Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften* **1915**, 844–847 (1915).

 doi:10.20944/preprints202506.0539.v1

3. H. Weyl, "Gravitation and electricity," *Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften* **1918**, 465–480 (1918).
4. P. W. Higgs, "Broken symmetries and the masses of gauge bosons," *Physical Review Letters* **13**, 508–509 (1964).
5. F. Englert and R. Brout, "Broken symmetry and the mass of gauge vector mesons," *Physical Review Letters* **13**, 321–323 (1964).
6. C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (W.H. Freeman, San Francisco, 1973).
7. W. Pauli, "Zur Theorie der Gravitation und der Elektrizität von Hermann Weyl," *Physikalische Zeitschrift* **22**, 742–744 (1921).
8. E. Scholz, "Weyl geometry in late 20th century physics," arXiv:1111.3220 [math-ph] (2011).
9. C. Romero, J. B. Fonseca-Neto, and M. L. Pucheu, "Conformally flat spacetimes and Weyl frames," *Foundations of Physics* **42**, 224–240 (2012).
10. L. O'Raifeartaigh and N. Straumann, "Gauge theory: Historical origins and some modern developments," *Reviews of Modern Physics* **72**, 1–23 (2000).
11. C. Brans and R. H. Dicke, "Mach's principle and a relativistic theory of gravitation," *Physical Review* **124**, 925–935 (1961).
12. F. W. Hehl, J. D. McCrea, E. W. Mielke, and Y. Ne'eman, "Metric-affine gauge theory of gravity: Field equations, Noether identities, world spinors, and breaking of dilation invariance," *Physics Reports* **258**, 1–171 (1995).
13. C. M. Will, "The confrontation between general relativity and experiment," *Living Reviews in Relativity* **17**, 4 (2014).
14. E. G. Adelberger, B. R. Heckel, and A. E. Nelson, "Tests of the gravitational inverse-square law," *Annual Review of Nuclear and Particle Science* **53**, 77–121 (2003).
15. Event Horizon Telescope Collaboration, "First M87 Event Horizon Telescope results. I. The shadow of the supermassive black hole," *Astrophysical Journal Letters* **875**, L1 (2019).
16. J. G. Williams, S. G. Turyshev, and D. H. Boggs, "Progress in lunar laser ranging tests of relativistic gravity," *Physical Review Letters* **93**, 261101 (2004).
17. S. Perlmutter *et al.* (Supernova Cosmology Project), "Measurements of Ω and Λ from 42 high-redshift supernovae," *Astrophysical Journal* **517**, 565–586 (1999).
18. A. G. Riess *et al.* (Supernova Search Team), "Observational evidence from supernovae for an accelerating universe and a cosmological constant," *Astronomical Journal* **116**, 1009–1038 (1998).
19. Planck Collaboration, "Planck 2018 results. VI. Cosmological parameters," *Astronomy & Astrophysics* **641**, A6 (2020).
20. DESI Collaboration, "The DESI Experiment Part I: Science, targeting, and survey design," *Astronomical Journal* **164**, 207 (2022).
21. G. Aad *et al.* (ATLAS Collaboration), "Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC," *Physics Letters B* **716**, 1–29 (2012).
22. S. Chatrchyan *et al.* (CMS Collaboration), "Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC," *Physics Letters B* **716**, 30–61 (2012).
23. A. Gando *et al.* (KamLAND Collaboration), "Precision measurement of neutrino oscillation parameters with KamLAND," *Physical Review Letters* **100**, 221803 (2008).
24. S. W. Hawking, "Particle creation by black holes," *Communications in Mathematical Physics* **43**, 199–220 (1975).
25. S. Weinberg, "The cosmological constant problem," *Reviews of Modern Physics* **61**, 1–23 (1989).
26. G. 't Hooft and M. Veltman, "One-loop divergencies in the theory of gravitation," *Annales de l'Institut Henri Poincaré A* **20**, 69–94 (1974).