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Posted Date: 18 March 2025

doi: 10.20944/preprints202503.1261.v1

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Bianchi Type-V Cosmological Model in *f(R,T)* Gravity: Exact Solutions and Dynamical Analysis

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Abstract: Here we investigated modified f(R, T) gravity for Bianchi type-V metric in the presence of Lyra Geometry To acquire the deterministic solution of the field equations with f(R, T), we have considered the conditions: (i) $f(R, T)=R^{m_1}+T^{m_2}$, R is the Ricci scalar, T is the energy momentum tensor And m_1, m_2 are constants (ii) The equation of state $P=\omega\rho$,where p is the pressure, ρ is the energy density and ω is the equation of state parameter that depends on the type of matter. Some Physical and Geometrical features of the model are also be discussed.

Keywords: f(R,T) gravity; cosmological constant; Lyra Geometry

1. Introduction

Cosmology is the mathematical study of the universe (or Cosmos) as a whole. Cosmology is the study of structure, origin and development of the universe. The word "cosmology" derived from the combination of two greek words, Kosmos (meaning harmony or order) and logia (means words or disclosures) and cosmology deals with very large distances, objects that are very big and timescales that are very long. In addition, cosmology also deals with very small things.

A cosmological model describes the origin, structure and evolution of the universe. And by introducing or modifying the laws of gravity to get a phenomenon, which is observed as better fit in universe, that the standard cosmological model might not fully explained. In cosmology, f(R, T) gravity model is a modified theory of gravity which is an extension of general relativity by introducing action, which is a general term, depends on Ricci scalar R and the trace of stress energy tensor T. Modified theory explains certain issues, like accelerated expansion of the universe, dark energy and dark matter. f(R, T) gravity explains the late time accelerated expansion of the universe, effect of dark energy and also used to study inflationary scenarios and other early-universe phenomena.

Lyra geometry (which is a modified version of Riemannian geometry used in cosmology) is also an important theory in modified theories of gravity, in which displacement field can be deemed a building block of the total energy that can play the role of dark energy. Lyra geometry is introduced by Lyra [1] which is a function known as gauge function ϕ_i into the structure less geometry. Various researchers constructed cosmological models universe in the context of Lyra geometry [2–9]. The behavior of the cosmological model with variable deceleration parameter, the scenario of two-fluid dark energy models in the Bianchi type-III Universe, and FLRW Cosmological Models with Dynamic Cosmological Term in Modified Gravity are all discussed by Tiwari, Beesham, and Shukla [10–13]. Hyperbolic Scenario of Accelerating Universe with Modified Gravity was covered by Khan et al.[14].We have studied the various deceleration parameter forms proposed by various researchers

[15–18]. By altering the geometrical component of Einstein's equation of motion, Starobinsky [19] and Kerner [20] proposed an alternative method known as modified gravity theory; f(R), where R is represented by scalar curvature [21,22]; f(T) theory, where T is represented by torsion scalar [23–25]; f(G) theory, where G is represented by Gauss Bonnet [26,27]; f(R,T) theory [28], where R is the Ricci scalar and T is the trace of the energy–momentum tensor and Different scalar-tensor theories [29,30] and modified theories of gravity include extra-dimensional theories [31]. In this paper, we have discussed Bianchi type-V cosmological model Modified f(R, T) Gravity Cosmological Model in the presence of Lyra geometry with certain forms of varying deceleration parameter.

2. Bianchi Type Metric and Field Equations

Bianchi type-V metric is

$$ds^2 = -dt^2 + M^2 dx^2 + N^2 e^{2x}(dy^2 + dz^2) \tag{1}$$

where M, N are functions of cosmic time t only.

The action S of f(R, T) gravity is

$$S = \int \sqrt{-g}\left\{\frac{1}{16\pi} f(R,T) + L_m\right\} d^4x \tag{2}$$

where R is the Ricci scalar, T is the Trace of stress energy tensor and L_m be the Lagrangian density of matter, where the value of stress-energy tensor of the matter is defined as

$$T_{ij} = -\frac{2}{\sqrt{-g}}\frac{\delta\sqrt{-g}L_m}{\delta g^{ij}} \tag{3}$$

The effective energy-momentum tensor T_{ij} in f(R, T) which is an arbitrary function of the trace of stress tensor, is

$$T_{ij}^{eff} = \left(\rho + p + \frac{\partial f}{\partial T}\right)u_i u_j + \left(p + \frac{\partial f}{\partial T}\right)g_{ij} \tag{4}$$

where ρ is the energy density, p is the pressure of the matter and u^i=(1, 0, 0, 0) is the four velocity vector in comoving coordinate system satisfy the condition

$$u_i u^i = -1 \tag{5}$$

The field equations obtained by modifying the action S in Equation (2) in f(R, T) theory with respect to metric tensor g_{ij} is

$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + (g_{ij}\nabla^i\Delta_j - \nabla_i\nabla_j)f_R(R,T) = 8\pi T_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\Theta_{ij} \tag{6}$$

where

$$\Theta_{ij} = -2T_{ij} + g_{ij}L_m - 2g^{lm}\frac{\partial^2 L_m}{\partial g^{ij}\partial g^{lm}} \tag{7}$$

And $f_R(R,T) = \frac{\partial f(R,T)}{\partial R}$, $f_T(R,T) = \frac{\partial f(R,T)}{\partial T}$ and ∇_i is the covariant derivative.

Here, we are using $L_m = -p$, since, there is not any unique definition of matter lagrangian. And using this definition of L_m in Equation (7), we get the value of Θ_{ij}, on which the physical nature of the matter field depends.

$$\Theta_{ij} = -2T_{ij} - pg_{ij} \tag{8}$$

Harko et al. [28] have considered three possible forms for the f(R, T), which are

$$f(R,T)=\begin{cases} R+2f_1(t) \\ f_1(R)+f_2(t) \\ f_1(R)+f_2(R)f_3(T) \end{cases} \tag{9}$$

In this discussion, we are going to consider the second condition of Equation (9) i.e.

$$f(R,T)=f_1(R)+f_2(T)=R^{m_1}+T^{m_2} \tag{10}$$

where m_1 and m_2 are constants.

Now from Equation (6), we get

$$R_{ij}-\frac{1}{2}Rg_{ij}=(8\pi+3)T_{ij}+\Lambda(t)g_{ij} \tag{11}$$

where $\Lambda(t)=p+\frac{T}{2}$ is the cosmological constant, depends upon the trace of the energy-momentum tensor T, which is proposed by Poplawski [32] where the cosmological constant in the gravitational lagrangian is a function of the trace of energy-momentum tensor. And Ahmed and Pradhan [33] proposed Bianchi type-V cosmology in f(R, T) gravity with Λ(t). If the pressure of matter is neglected, Λ(t) gravity, which is more general than Palatini f(R), might be reduced to it.[34,35].For the sake of simplicity, here we will take $m_1=m_2=1$.

The field equations in Lyra manifold with 8πG=1 and c=1, given by Sen [36] is

$$R_{ij}-\frac{1}{2}Rg_{ij}+\frac{3}{2}\phi_i\phi_j-\frac{3}{4}\phi_i\phi^j g_{ij}=(8\pi+3)T_{ij}+\Lambda(t)g_{ij} \tag{12}$$

where ϕ_i is the displacement vector field in Lyra manifold defined as

$$\phi_i(0,0,0,\beta(t)) \tag{13}$$

For the metric (1), Einstein field equations obtained by taking $8\pi+3=b$ are

$$2\frac{\dot{M}\dot{N}}{MN}+\frac{\dot{N}^2}{N^2}-\frac{3}{M^2}-\frac{3}{4}\beta^2-\Lambda(t)=b(p+1) \tag{14}$$

$$2\frac{\ddot{N}}{N}+\frac{\dot{N}^2}{N^2}-\frac{1}{M^2}+\frac{3}{4}\beta^2-\Lambda(t)=b(\rho+1) \tag{15}$$

$$\frac{\ddot{M}}{M}+\frac{\ddot{N}}{N}+\frac{\dot{M}\dot{N}}{MN}-\frac{1}{M^2}+\frac{3}{4}\beta^2-\Lambda(t)=b(p+1) \tag{16}$$

$$2\frac{\dot{M}}{M}-2\frac{\dot{N}}{N}=0 \tag{17}$$

Here an overhead dot indicates the differentiation w.r.t. cosmic time t.

Equations (14)–(16) are three equations in five unknowns M, N, β, p, ρ. Therefore, to found the solution, two more conditions are required

(i) We assume the solution of system of equations in the form

$$\frac{\dot{M}}{M}=\frac{\dot{N}}{N}=\frac{a}{t^n} \tag{18}$$

where a, n are constants.

Integrating Equation (18), we get

$$M = N = M_1 \exp\left\{\frac{a}{1-n} t^{1-n}\right\} \tag{19}$$

(ii) We assume the equation of state

$$p = \omega\rho \tag{20}$$

For cosmological constant (Dark energy), $\omega = -1$, i.e.

$$p = -\rho \tag{21}$$

Volume,

$$V = M_1 \exp\left\{\frac{a}{1-n} t^{1-n}\right\} \tag{22}$$

Deceleration Parameter,

$$q = -1 \tag{23}$$

i.e. dark energy dominated universe is there.
Shear Scalar,

$$\sigma^2 = \frac{1}{2}\sigma^{ij}\sigma_{ij} = 0 \tag{24}$$

i.e shear scalar does not exists.
Expansion Scalar,

$$\theta = \frac{3a}{t^n} \tag{25}$$

Also

$$\lim_{t\to\infty}\frac{\sigma}{\theta} = 0 \tag{26}$$

Therefore, our model describes a non-shearing, non-rotating, continuously expanding universe.
For the sake of simplicity, taking n=1, we get
Pressure of matter,

$$p = \frac{a}{t^2}(1-3a) \tag{27}$$

Energy density,

$$\rho = \frac{a}{t^2}(3a-1) \tag{28}$$

Cosmological constant,

$$\Lambda = 3\left\{\frac{1}{2} - \frac{a}{t^2}(3a-1)\right\} \tag{29}$$

Displacement vector,

$$\beta = \frac{2}{\sqrt{3}}\sqrt{\frac{1}{t^2}\left\{a(1-3a)(b+3) + (2-a^2)\right\} + b + \frac{3}{2}} \tag{30}$$

Ricci scalar,

 doi:10.20944/preprints202503.1261.v1

$$R = \frac{2a}{t^2}(9 - 11a) \tag{31}$$

Trace of stress-energy tensor,

$$T = -4\left\{\frac{a}{t^2}(3a - 1)\right\} + 3 \tag{30}$$

$$f(R,T) = \frac{2}{t^2}(11a - 17a^2) + 3 \tag{31}$$

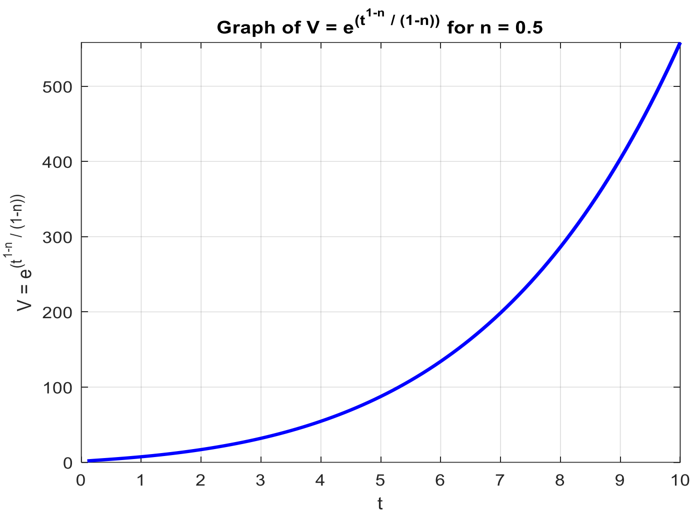

Figure 1. Variation of Spatial volume V versus cosmic time t taking constant M_1=1, a=1.

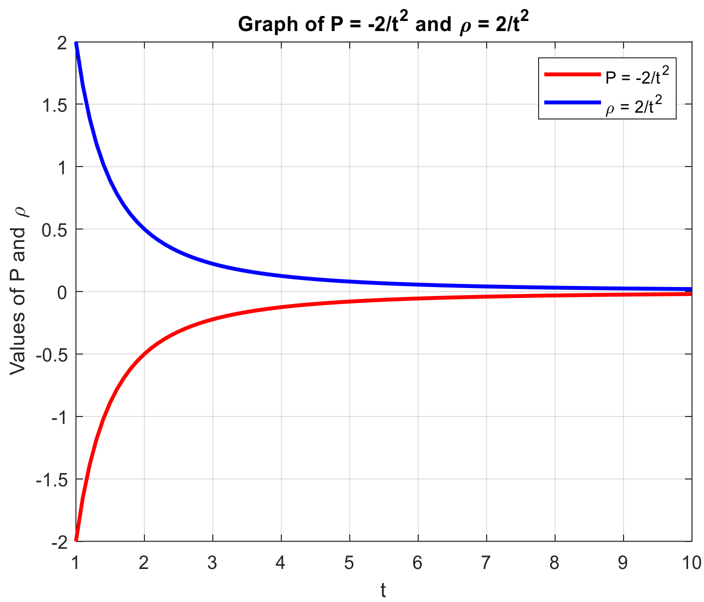

Figure 2. Variation of Pressure of matter and energy density versus cosmic time t taking a=1.

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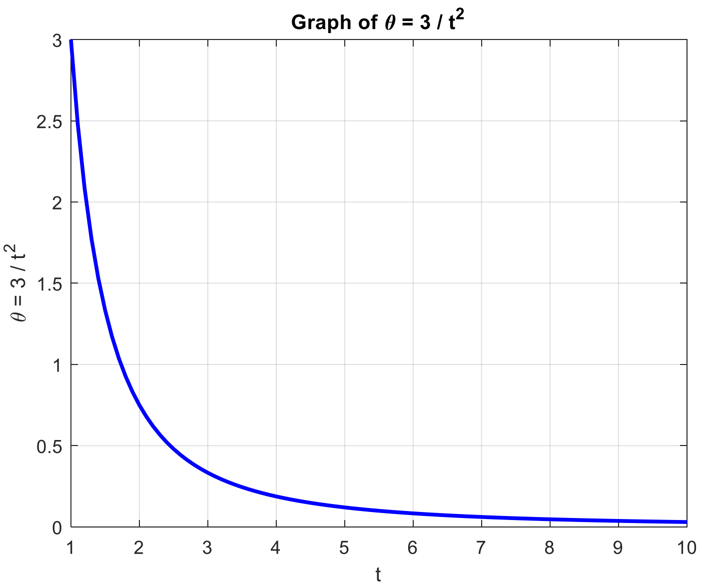

Figure 3. Variation of expansion scalar θ versus cosmic time t taking n=1.

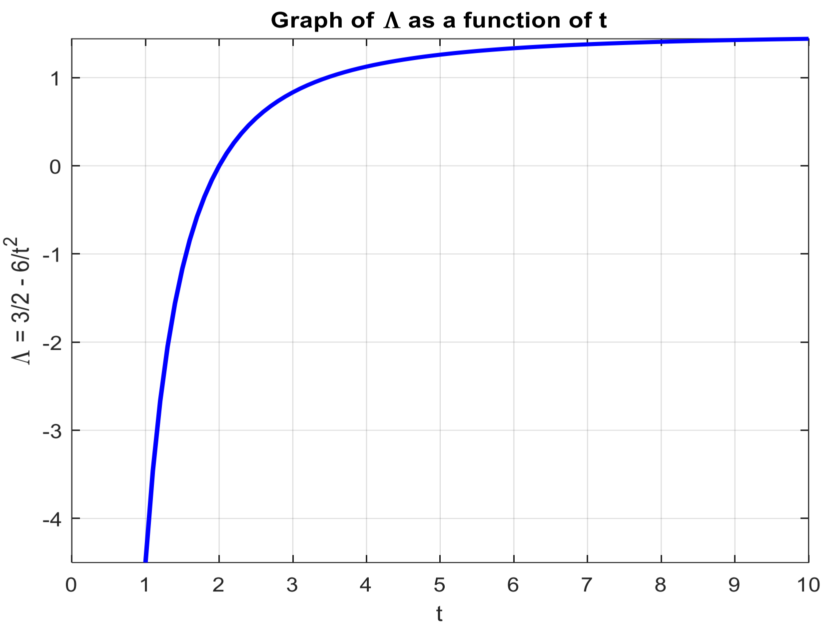

Figure 4. Variation of Cosmological constant Λ versus cosmic time t taking a=1.

3. Results

Here we discussed Bianchi type-V cosmological model in f(R, T) theory of gravity with deceleration parameter. It is observed that the shear scalar σ is zero i.e. shear scalar does not exists. Due to shear no distortion is there. Hence $\lim_{t\to\infty}\frac{\sigma}{\theta}=0$, therefore model describes a non-shearing, non-rotating, continuously expanding universe. Also it is observed that, as the time t increases, expansion scalar θ decreases.

The energy density of the vacuum is linked to the cosmological constant Λ, which affects the universe's pace of expansion. When cosmic time t>2, cosmological constant Λ>0, then universe experience accelerated expansion similar to dark energy model, when t<2, Λ<0, our universe experience deceleration and a negative value of cosmological constant act as attractive force, which opposes the expansion and if t=2, we get Λ=0, i.e. there is no vacuum energy is contributed at that time, expansion rate is governed by matter and radiations only. This point is known as transition point where universe shifts from deceleration to acceleration.

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