Article

The Quantum Measurement Problem and Two Famous Questions

Guang-Liang Li *

Posted Date: 12 August 2025

doi: 10.20944/preprints202508.0765.v1

Keywords: Einstein-Bohr debate; quantum measurement problem; point-set topology

Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 12 August 2025 doi:10.20944/preprints202508.0765.v1

Article

The Quantum Measurement Problem and Two Famous Questions

Guang-Liang Li

University of Hong Kong, Hong Kong, China; glli@eee.hku.hk

Abstract

By revisiting two famous questions about the Copenhagen interpretation of quantum mechanics, this article presents a concise analysis of the quantum measurement problem concerning both microscopic and macroscopic objects. The method used here is mainly based on the concept "isolated point" in "point-set topology". The findings reported are as follows. (a) Einstein's argument has been misunderstood; he was opposed to the so-called "inherently probabilistic nature of quantum mechanics" in the Copenhagen interpretation rather than to the use of probability in quantum mechanics. (b) Probability used in Einstein's ensemble interpretation is identical to the quantum-mechanically calculated probability. (c) The wave-functions in Einstein's ensemble interpretation neither describe any single quantum object purportedly possessing mutually exclusive properties simultaneously when nobody looks nor collapse abruptly when an observer performs a measurement.

Keywords: Einstein-Bohr debate; quantum measurement problem; point-set topology

---

1. Introduction

This article revisits two famous questions about the Copenhagen interpretation of quantum mechanics and presents a concise analysis of the quantum measurement problem concerning both microscopic and macroscopic objects [1–3]. The method used here is mainly based on the concept "isolated point" in "point-set topology". Only familiar with real numbers and with the usual distance function defined on the set of real numbers is needed to understand the point-set topological analysis of the measurement problem.

The measurement problem involves experiments with quantum-mechanically described *single* objects, including both microscopic and macroscopic objects. Both kinks of objects are expressed by quantum superpositions. The legitimacy of quantum-mechanical descriptions is the essence of the Einstein-Bohr debate [4,5]. When describing a single microscopic object in general, a quantum superposition, namely, a wave-function, consists of orthonormal vectors spanning an n-dimensional Hilbert space. The orthogonality of superposed vectors is associated with properties possessed by the object. Representing alternative outcomes obtained by measuring the object, the properties are exclusive. There is no limit to the number of superposed orthonormal vectors. For the purpose of the present study, it is sufficient to consider a two-dimensional Hilbert space $\mathcal{H}$.

The Copenhagen interpretation of quantum mechanics, which is defended by Bohr in the Einstein-Bohr debate, may be summarized briefly as follows. Consider a wave-function ψ consisting of orthonormal vectors ψ_1 and ψ_2 spanning $\mathcal{H}$.

$$\psi = c_1\psi_1 + c_2\psi_2$$

where c_1 and c_2 are complex numbers. According to the Copenhagen interpretation, a single microscopic object possesses exclusive properties represented by ψ_1 *and* ψ_2 *simultaneously* when nobody measures (or observes) it. Once an observer performs a measurement on the object, ψ collapses abruptly onto ψ_1 *or* ψ_2 according to the measurement outcome. The outcomes obtained by measurements are inherently probabilistic. The probability of finding the measurement outcome represented by ψ_i is $|c_i|^2, i = 1, 2$.

2. Two Famous Questions

In the Einstein-Bohr debate, two questions concerning the Copenhagen interpretation of quantum mechanics are very famous. The questions are famous, because they bothered Einstein very much in his debate with Bohr.

- Question 1: Why are outcomes obtained by measuring a single quantum object inherently probabilistic rather than deterministic?
- Question 2: Can a single quantum object possess exclusive properties *simultaneously* when nobody looks?

The Copenhagen interpretation does not provide any reasonable answer to Question 1. For a single object described by a quantum superposition, the Copenhagen interpretation gives an *affirmative* answer to Question 2. Einstein disagreed with the Copenhagen interpretation and argued against quantum superpositions used to describe any object in the real world. Regrettably, Einstein's argument has been misunderstood, mainly because the experimental results of testing Bell inequalities are misinterpreted. Einstein was not opposed to the use of probability in quantum mechanics; he was opposed to the so-called "inherently probabilistic nature of quantum mechanics" in the Copenhagen interpretation. The present study concerns only Einstein's argument in the Einstein-Bohr debate. Bell inequalities and the corresponding experiments have been discussed intensively in the existing literature [6–13] and will not be considered here.

Questions mentioned above not only disturbed Einstein; they also disturb some people nowadays. For example, the answer to Question 1 given in the existing literature goes as follows [14]: "this is simply the way the world is. Where is it written that the laws of nature have to be deterministic?" Although this answer is standard and influential, it has not convinced everybody. Nevertheless, because the calculations in quantum mechanics are indeed unbelievably successful, people who used to worry the probabilistic nature of quantum mechanics now just "shut up and calculate." They simply avoid considering any disturbing question and do not bother themselves anymore.

However, the disturbing questions deserve reasonable answers. The study presented in this article has found a reasonable answer to Question 1 and a *negative* answer to Question 2. The basic ideas behind the study are as follows. Experimental verification of the Copenhagen interpretation is based on an unjustified assumption. According to this assumption, time modeled by $\mathbb{R}$, the set of real numbers (or its subsets) endowed with the usual point-set topology, could be measured perfectly precisely. This assumption is not practically meaningful, and the experimental verification of the Copenhagen interpretation is questionable, as indicated by the point-set topological analysis (see below). But the point-set topological analysis alone cannot reveal the origin of randomness observed in experiments with single quantum objects. To observe random phenomena in any experiment with single quantum objects, a further condition is needed, which is a banal fact: a large number of repetitions (i.e., runs) of the experiment in question must be performed. In one repetition of the experiment, only one measurement outcome can be obtained, which makes no sense statistically.

3. The Measurement Problem: Microscopic Objects

Consider first the affirmative answer to Question 2 given by the Copenhagen interpretation. Denote by $E = \{q_n, n \geq 1\}$ an ensemble of single microscopic objects, where q_n is the object to be measured in the n-th repetition of the corresponding experiment. The objects are all described by the wave-function ψ. In other words, each element of the ensemble is characterized by the same pure state, and E is a pure ensemble. Einstein's argument can be better appreciated in this simple, ideal situation. According to the answer given by the Copenhagen interpretation, each object $q_n \in E$ possesses exclusive properties represented by ψ_1 *and* ψ_2 at any time before an observer measures the object. The assumption behind this answer, namely, time can be measured perfectly precisely, does not hold in practice. Thus the answer given by the Copenhagen interpretation is not practically meaningful.

To justify the above claim , denote by $H_n(\psi_1, \tau)$ and $H_n(\psi_2, \tau)$ two propositions, where $H_n(\psi_1, \tau)$ means "ψ_1 is the outcome obtained by measuring q_n at time τ." The meaning of $H_n(\psi_2, \tau)$ is similar. In $H_n(\psi_1, \tau)$ and $H_n(\psi_2, \tau)$, the time τ is fixed. If time could be measured perfectly precisely, $H_n(\psi_1, \tau)$ and $H_n(\psi_2, \tau)$ would hold simultaneously in the corresponding experiment, which requires τ to be an isolated point of $\mathbb{R}$. By definition, if τ is an isolated point of $\mathbb{R}$ equipped with its usual point-set topology, then there exists a number $r > 0$, such that the distance between τ and any other element of $\mathbb{R}$ is at least r. As can be readily verified, τ is not an isolated point of $\mathbb{R}$. Actually, $\mathbb{R}$ does not have any isolated point.

4. The Measurement Problem: Macroscopic Objects

The claim concerning microscopic objects can also be justified when a macroscopic quantum object is considered. For example, let Q be such an object with two macroscopically distinguishable states [1,2]: Q^+ and Q^-. For this macroscopic object, denote by $(Q_n)_{n\geq 1}$ a sequence of outcomes obtained by measuring Q at time τ in the n-th repetition. Let $H_n(Q^+, \tau)$ and $H_n(Q^-, \tau)$ be two propositions; $H_n(Q^+, \tau)$ states "Q^+ is observed by measuring Q at time τ in the n-th repetition." The meaning of $H_n(Q^-, \tau)$ is similar. Analyzing the quantum measurement problem concerning this macroscopic object is exactly the same as analyzing the measurement problem concerning the microscopic objects: if time could be measured perfectly precisely, both $H_n(Q^+, \tau)$ and $H_n(Q^-, \tau)$ would hold simultaneously in the corresponding experiment, which requires τ to be an isolated point of $\mathbb{R}$. However, $\mathbb{R}$ does not have any isolated point as shown by the point-set topological analysis. Therefore, for both microscopic and macroscopic quantum objects, the experimental verification of the answer to Question 2 given by the Copenhagen interpretation is indeed not practically meaningful, simply because time cannot be measured perfectly precisely in practice.

5. The Origin of Randomness

Now consider the standard answer to Question 1 given in the literature [14]. Any answer like this cannot be considered convincing, because it is actually not a reasonable answer. The point-set topological analysis and the condition needed to observe randomness in experiments with single quantum objects can provide a reasonable answer. Random phenomena can only be observed in *different* repetitions of the corresponding experiment. Needless to say, any single quantum object can possess mutually exclusive properties; it just cannot have such properties simultaneously. Because time cannot be measured perfectly precisely in practice, mutually exclusive properties of a single quantum object are actually observed in *different* repetitions of the experiment and correspond to *different* measurement outcomes associated with *different* objects of the same kind. By no means can the mutually exclusive properties be observed in only one repetition by measuring the object only once. But such properties are attached to an *imaginary* object. The imaginary object does not exist in the real world. Thus the origin of randomness observed in experiments with single quantum objects is concealed by incorrectly interpreted experimental results.

6. Einstein's Argument Grounded on Ensemble Interpretation

In the spirit of Einstein's argument grounded on his ensemble interpretation of wave-functions [15], single quantum objects of the same kind are measured in different repetitions of the corresponding experiment; the objects form an ensemble described by the wave-function in question. Each element of the ensemble possesses mutually exclusive properties; however, none of them possesses such properties simultaneously. By revealing the origin of randomness observed in experiments with single quantum objects, the study presented in this article is helpful to see why Einstein was not opposed to the use of probability in quantum mechanics; he was only opposed to the Copenhagen interpretation of quantum mechanics. In Einstein's ensemble interpretation, the use of probability is still needed to characterize randomness observed in the experiments, and probability used in Einstein's ensemble interpretation is identical to the quantum-mechanically calculated probability. But the wave-functions in Einstein's ensemble interpretation neither describe any single quantum object purportedly possessing mutually exclusive properties simultaneously when nobody looks nor collapse abruptly when an observer performs a measurement.

7. Conclusion

Based on the point-set topological analysis together with the necessary condition needed to observe random phenomena in experiments with single quantum objects, this article revisited two famous questions about the Copenhagen interpretation of quantum mechanics and analyzed the quantum measurement problem concerning both microscopic and macroscopic objects. The findings reported are as follows. (a) Einstein's argument has been misunderstood; he was opposed to the so-called "inherently probabilistic nature of quantum mechanics" in the Copenhagen interpretation rather than to the use of probability in quantum mechanics. (b) Probability used in Einstein's ensemble interpretation is identical to the quantum-mechanically calculated probability. (c) The wave-functions in Einstein's ensemble interpretation neither describe any single quantum object purportedly possessing mutually exclusive properties simultaneously when nobody looks nor collapse abruptly when an observer performs a measurement.

Funding: This research received no funds or grants.

Data Availability Statement: No new data were created or analyzed in this study. Data sharing is not applicable to this article.

Conflicts of Interest: The author declares no conflicts of interest.

References

1. Legget, A. The Quantum Measurement Problem. *Science* **2005**, *307*, 871–872. https://doi.org/10.1126/science.1109541.
2. Leggett, A. The EPR-Bell Experiments: The Role of Counterfactuality and Probability in the Context of Actually Conducted Experiments. *Philosophies* **2024**, *9*, 133.
3. Hance, J.R.; Hossenfelder, S. What does it take to solve the measurement problem? *Journal of Physics Communicacctions* **2022**, *6*. https://doi.org/10.1088/2399-6528/ac96cf.
4. Einstein, A.; Podolsky, B.; Rosen, N. Can quantum-mechanical description of physical reality be considered completed? *Physical Review* **1935**, *47*, 777–780. https://doi.org/10.1103/PhysRev.47.777.
5. Bohr, N. Can quantum-mechanical description of physical reality be considered complete? *Physical Review* **1935**, *48*, 696–702. https://doi.org/10.1103/PhysRev.48.696.
6. Bell, J.S. On the Einstein-Podolsky-Rosen paradox. *Physics* **1964**, *1*, 195–200. https://doi.org/10.1103/PhysicsPhysiqueFizika.1.195.
7. Maudlin, T. What Bell did. *Journal of Physics A: Mathematics and Theory* **2014**, *47*, 424010.
8. Hess, K. A Critical Review of Works Pertinent to the Einstein-Bohr Debate and Bell's Theorem. *Symmetry* **2022**, *14*, 163.
9. Aspect, A. Bell's inequality test: more ideal than ever. *Nature* **1999**, *398*, 189–190. https://doi.org/10.1038/18296.
10. Aspect, A. Closing the door on Einstein and Bohr's quantum debate. *Physics* **2015**, *8*, 123. https://doi.org/10.1103/Physics.8.123.
11. Aspect, A. Bell's theorem: the naive view of an experimentalist. In Proceedings of the Quantum [Un]speakables-From Bell to Quantum information; Bertlmann, R.; Zeilinger, A., Eds., Berlin, 2002.
12. Clauser, J.F.; Shimony, A. Bell's theorem: experimental tests and implications. *Reporting Progress Physics* **1978**, *41*, 1881–1927. https://doi.org/10.1088/0034-4885/41/12/002.
13. Clauser, J.F.; Horne, M.A.; Shimony, A.; Holt, R.A. Proposed experiment to test local hidden variable theories. *Physical Review Letters* **1969**, *23*, 880–84. https://doi.org/10.1103/PhysRevLett.23.880.

doi:10.20944/preprints202508.0765.v1

14. Weinberg, S. The greatest physics discovery of the 20th century. *J. Phys.: Conf. Ser* **2024**, *2877*, 012114.
15. Howard, D. Einstein on Locality and Separability. *Stud. Hist. Phil. Sci* **1985**, *16*, 171–201.