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Disentagling the Cosmic/Comoving Duality: The Cognitive Stability and Typicality Tests

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Abstract

Cosmological scenarios wherein the cumulative number of spontaneously-formed, cognitively impaired, disembodied transient observers is vastly larger than the corresponding number of *atypical* 'ordinary observers' (OO's) formed in the conventional way – essentially via cosmic evolution and gravitational instability – are disqualified in modern cosmology on the grounds of Cognitive Instability, let alone the atypicality of OO's like us. According to the concordance ΛCDM cosmological model – when described in the (expanding) 'cosmic frame' – the cosmological expansion is future-eternal, and we are *atypical* OO's, who just happen to observe the future-eternal Universe at a finite time after its beginning – before dark energy (DE) completely takes over the expansion – and vastly outnumbered by typical Boltzmann Brains (BB's) that spontaneously form via sheer thermal fluctuations in the future-eternal asymptotic de Sitter spacetime. In case that DE ultimately decays a finite time after the Big Bang and the Universe empties out the cumulative number of transient 'Freak Observers' (FO's) formed and destroyed spontaneously by virtue of the quantum uncertainty principle ultimately overwhelms that of OO's. Either possibility is unacceptable. We argue that these unsettling conclusions are artifacts of employing the (default) cosmic frame description in which space expands; when analyzed in the comoving frame, OO's overwhelmingly outnumber both BB's and FO's. This suggests that the dual comoving description where space is globally static, masses monotonically increase, and the space describing gravitationally bounded objects contracts, is the cognitively stable preferred framework for describing our evolving Universe, assuming that the latter is indeed future-eternal.

Keywords: FRW; comoving frame; Boltzmann Brains; Freak Observers

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1. Introduction

The concordance flat ΛCDM cosmological model successfully and efficiently describes our Universe from very early after the Big Bang until the present time, and from the Hubble scale down to galaxy cluster scales, and it does so with only a handful of free parameters. Yet, it has a certain disturbing feature that has long been recognized and acknowledged.

According to the concordance ΛCDM model – that contains cold dark matter (CDM) and a cosmological constant, Λ – the current accelerated expansion phase of the observable Universe will last indefinitely, and we appear to be very 'special,' i.e., *atypical*, observers, since we make our observations at a finite time after the beginning of a future-eternal Universe. This somewhat counter intuitive conclusion arises naturally within the concordance model and is closely related to another well-known puzzle – the 'why now?' problem. The latter revolves around the question of how likely is it for us to observe the Universe at the unique era in which dark energy (DE) and non-relativistic (NR) matter contribute comparably to the cosmic energy budget, in spite of their very different evolution histories. This is the Cosmic Coincidence Problem (CCP) [1–3].

Whereas the common expectation that we are typical observers has been challenged [4] counter compelling arguments in favor of our typicality, namely that our observations are typical within our

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reference class, e.g. [5,6], touch upon the bedrock of the scientific method; when experimentally or observationally tested theory is compared to only a finite number of data points. It is a fundamental underlying assumption that is made by default that these data points (i.e. observations) are 'typical' and randomly selected, in the observationally accessible range. Otherwise, we cannot trust our conclusions that theory provides a good fit to the data for there is always a non-negligible chance in this case that the measurements are carried out specifically where theory matches the data, and fails otherwise.

In a future-eternal Universe, any process with a finite (however small, yet finite) probability to take place will do so an infinite number of times. In $O(10^{110})$ years, after the most massive black holes have been evaporated, and long after the last luminous stars have exhausted their nuclear fuel, the Universe is expected to enter a Dark Era where it is described by a static de-Sitter spacetime containing a heat bath of massless photons, gravitons, and other light degrees of freedom, characterized by a Gibbons-Hawking (GH) temperature $K_B T_{GH} = \frac{\hbar c}{2\pi}\sqrt{\frac{\Lambda}{3}}$ [7], where K_B, $\hbar$ and c are the Boltzmann constant, the reduced Planck constant, and speed of light, respectively. Occasionally, and due to the infinitely long time available for this (otherwise unlikely) event to take place, very rare thermal fluctuations will randomly form a certain type of transient 'Boltzmann Brains' (BB's), for a very brief time period, but with highly disordered memories and cognitive disabilities, e.g., [8,9]. These transient observers might even think that they live 14 Gyrs after the Big Bang in an ordered Universe, where large scale structure is formed via gravitational instability in the conventional way, etc., but this thought would not last for longer than a brief moment. BB's – rather than Ordinary Observers (OO's) like ourselves who formed in the usual way – are expected to be typical observers in an eternal Universe such as the one which is described by the concordance model [10]. This is so because the cosmological constant comes to dominate the expansion a finite time after the beginning. After this transition takes place, the Universe enters an *infinitely* long period of exponential expansion in which essentially no new structure forms that evolves – among other structure – to habitable planets.

Even if DE ultimately decays, eventually leaving behind an empty Milne-like spacetime, one expects yet another type of cognitively impaired 'Freak Observers' (FO's) to form thanks to Heisenberg's Uncertainty Principle, e.g. [11]. FO – essentially an entire self-aware brain – can spontaneously pop into existence, with all memories built in, giving it a false impression of 'reality' as we know it. This channel of FO production typically has a much larger (albeit still stupendously small) production rate than BB's do. Yet, in a future-eternal Universe (either asymptotically Λ-dominated de Sitter or empty Milne) the number of untrustworthy observations made by either type – BB's or FO's – will overwhelm the number of observations made by OO's. Since only a finite narrow time window is available for the latter type to form, basically until DE takes over the expansion rate, it is then paradoxical that we seem to be belonging to the class of OO's rather than to either BB's or FO's.

This paradox has a long history starting already in the wake of the ninetienth century, with Boltzmann conception of an entire expanding Universe that starts off as a thermal fluctuation, through a latter idea by Eddington that it is even much more likely that a 'mathematical physicist' forms via (a much smaller and therefore more probable) thermal fluctuation [12], and the more modern way of thinking about it as the minimal randomly formed organism necessary for observing the world – a BB, e.g., [8,9]. In spite of the significant progress achieved over the years in the field of cosmology, and multiple attempts to address the BB paradox, e.g. [5,11,13–21] – in some cases along with its FO variant – it remains defiant and still largely regarded an open question. It was pointed out that this could simply be a measure problem in the *multiverse* context, e.g. [22–28]. However, the discussion in present work focuses on the non-OO's problem of our own *Universe*, that is independent of whether the multiverse exists or not.

The observation that BB's and FO's abundance overwhelms that of OO's challenges the assumption of typicality in cosmology, as it implies that most observations in an eternal Universe would be made by disembodied brains rather than by OO's like us. Even worse, these (standard model) scenarios are justifiably subject to criticism on the ground of Cognitive Instability; scenarios in which the most likely observers have disordered false memories and incoherent thoughts cannot possibly be trusted and qualify as viable cosmological models, e.g. [9,29–31].

This argument, and possibly others as well, render the presence of BB's and FO's a diagnostic tool, rather than a realistic possibility; a scenario or model that predict an overwhelming abundance of either BB's or FO's over OO's are deemed unfeasible or pathological. This is currently debated in the context of the prevailing inflationary scenario – eternal inflation – and the *multiverse*. The problem with this cognitive instability test is that even our own, ΛCDM *Universe*, is future-eternal and therefore must be plagued by the BB (or FO) problem, even if we ignore inflation and the multiverse [11]; *this problem still afflicts our single ΛCDM Universe*. Therefore, it seems – based on these arguments – that either we are very *atypical* and rare OO's [10] or the Universe must come to an end, i.e. it decays [11], or faces a 'cosmic doomsday', e.g. [32–34]. None of these possibilities is very appealing, especially since they are proposed first and foremost to explain the (otherwise almost inconceivable) existence of OO's like us. In the present work a much more mundane explanation is proposed, that does not by itself recourse to new physics.

The paper is organized as follows. In Section 2 the cosmic/comoving duality in the standard cosmological model is described. In Section 3 the conventional considerations in the cosmic frame that lead to the perplexing BB and FO paradoxes are layed out. In Section 4 it is shown how these paradoxes go away when analyzed in the comoving frame. This work is summarized in Section 5 where it is concluded that at least from the perspective of the BB and FO paradoxes the cognitively stable picture – and a one that does not run into typicality issues – is the given by the comoving frame description.

2. The Cosmic/Comoving Frame Duality in Cosmology

A homogeneous and isotropic spacetime is described in the cosmic (expanding) frame by the Friedmann-Robertson-Walker (FRW) infinitesimal line interval, $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$, where summation convention over the spacetime coordinates applies,

$$\begin{aligned} ds^2 &= -dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)\right] \\ &= a^2(\eta)\left[-d\eta^2 + \frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)\right], \end{aligned} \tag{1}$$

where t and η are the cosmic and conformal time coordinates, respectively, are related via $d\eta \equiv \frac{dt}{a}$, $a(t)$ is the scale factor, and K is the spatial curvature parameter. By convention a_0, the scale factor at present, is set to unity. The scale factor is related to the redshift observable via $a = \frac{1}{1+z}$. Space is expanding in this frame and particle masses are fixed universal constants. The scale factor $a(t)$ is determined by straightforward integration of the Friedmann equation

$$H = H_0\sqrt{\frac{\Omega_r}{a^4} + \frac{\Omega_m}{a^3} + \frac{\Omega_k}{a^2} + \Omega_\Lambda}, \tag{2}$$

where $H \equiv d\ln(a)/dt$ is the expansion rate, H_0 is its current value, and Ω_r, Ω_m, $\Omega_k \equiv -Kc^2/H_0^2$ and $\Omega_\Lambda \equiv \Lambda c^2/(3H_0^2)$ are the (cosmologically averaged) energy densities in critical density units of radiation (relativistic degrees of freedom such as the cosmic microwave background photons), NR matter (baryons and CDM), the effective energy density associated with the global spatial curvature parameterized by K, and the cosmological constant, respectively, subject to the constraint $\Omega_r + \Omega_m + \Omega_k + \Omega_\Lambda = 1$. Here we assume the energy content of either flat ΛCDM or KΛCDM (extension of flat ΛCDM allowing for non-flat spatial geometries, $K \neq 0$).

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Scaling the $a^2(\eta)$ factor out from the FRW metric, we are left with a static metric $d\tilde{s}^2 = \tilde{g}_{\mu\nu}dx^\mu dx^\nu$

$$\begin{aligned} d\tilde{s}^2 &= -dt^2/a^2(t) + \frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) \\ &= -d\eta^2 + \frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2), \end{aligned} \tag{3}$$

which describes spacetime in the comoving frame, where it is understood that the time coordinate here is η. The time-dependence of the scale factor in the cosmic frame is replaced by time-dependent masses when described in the comoving frame, $m(\eta) = m_0 a(\eta)$. This is the (indeed less popular) alternative comoving frame picture that is mostly employed as a computational tool, e.g. in [35,36], and in the Mukhanov-Sasaki equation of quantum perturbations of the inflaton field, as well as in standard Boltzmann solvers, e.g. CAMB. Establishing the formal duality – essentially indistinguishability – between these two alternative descriptions (at the background level as well as at the linear perturbation level) is straightforward, e.g., [37–40]. In both pictures space relatively expands; in the cosmic frame it expands relative to fixed yardsticks – the Planck length and Compton wavelength of, e.g., the electron. In the comoving frame space is static but the yardsticks contract.

In the standard model (SM) of particle physics masses are fixed universal constants *by convention*. This is manifested by the fact that the Higgs vacuum expectation value (VEV) and the chirality breaking scale in quantum chromodynamics (QCD) are fixed constants. The universal constant of gravitation, G, and consequently the Planck mass $m_{pl} = G^{-\frac{1}{2}}$, is fixed as well. Again, this is no more than a 'convenient' *convention*; if all these masses would vary identically is space and time there is practically no experimental or observational way in which the two alternative pictures could be distinguished.

Allowing for masses to vary, in particular multiplying them by a scale function $m_0 \rightarrow m_0 a(t)$ in the presence of the spacetime metric (that is necessarily present in the SM lagrangian), implies that the corresponding VEV of the Higgs field must be locally rescaled to offset that of the metric; masses now carry the time-dependence that would be otherwise carried by the expanding spacetime metric. This implies that the SM of particle physics must be 'conformalized' [41], otherwise no compensation can be introduced to counteract the new contributions from the time derivatives of the Higgs VEV. The QCD sector is similarly conformalized. This framework was established in [37]. It certainly represents a radical departure from the standard *convention* of constant VEV's, but similar concepts have been explored in the literature, e.g., [39,40,42,43], and indeed this is no more and no less than an issue of convention; carefully conformalizing the SM of particle physics and general relaticity (GR) all masses have the same spacetime evolution and so measurable dimensionless mass ratios are identical to those measured based on the standard fixed mass convention.

A point of conceptual (albeit no practical) importance is that in the process of conformalization of the SM of particle physics, the Higgs sector lagrangian density is corrected by a conformal coupling term of the curvature scalar $\frac{1}{6}RH^\dagger H$ where R and H are the Ricci curvature scalar and H is the Higgs field [37]. The purpose of adding this term is to balance spacetime variations of the Higgs VEV in its kinetic term so that overall the action is locally scale-invariant. Comparing this new term to the effective mass term in the Higgs sector, $m_H^2 H^\dagger H$, where m_H is the Higgs mass, and recalling that $R \sim G\rho$, we conclude that the new curvature term essentially does not affect the mass value because its fractional contribution, even in as dense regions as a typical neutron star density, $\rho \sim m_p^4$, where m_p is the proton mass, is $G\rho/m_H^2 \sim m_p^4/(m_{pl}m_H)^2 \sim 10^{-40}$.

Going beyond the homogeneous and isotropic approximation, adding linear scalar metric perturbations in the FRW line element, Equation (1) generalizes to

$$ds^2 = a^2(\eta)[-(1+2\Phi)d\eta^2 + (1-2\Phi)\gamma_{ij}dx^i dx^j], \tag{4}$$

where Φ is the gravitational potential and γ_{ij} is the spatial metric. However, in the cosmic frame description, light emitted at a given time will undergo redshift irrespective of whether the source is gravitationally bound or is at rest at the Hubble frame. Therefore, it is essential to show how light emitted from gravitationally bound objects in the comoving frame description is still redshifted even though space does not expand from this perspective.

Considering nonlinear gravitationally bounded objects, then if the *spatial* coordinates are transformed $dx^i \to dx^i/a$ (where the superscript i runs through all three spatial coordinates), while masses scale as $m = m_0 a$ as in the background of the comoving frame, then emitted light from nonlinear gravitationally bound objects is redshifted, irrespective of whether the emitting atoms, molecules, etc., reside in the background or in gravitationally bounded objects. This is so because the descriptions of both the background and of that of gravitationally bound objects is conformally transformed in going from the cosmic to comoving frame, and null geodesics, subject to the constraint $ds^2 = 0$, are blind to conformal transformations, e.g. [44]. To summarize, in the comoving frame, a nonlinear gravitationally bound object, e.g., a galaxy, star, planet, etc., is described in the comoving frame by a metric of the form

$$d\tilde{s}^2 = a^{-2}(t)[-dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)], \tag{5}$$

where we ignored additional spatio-temporal metric variations via evolving gravitational potentials but rather imprinted the effect of the global cosmic evolution on the spacetime metric that describes these structures (that would be considered static in the cosmic frame picture). The nontrivial lapse function $g_{tt} = a^{-2}$ that is common to both the background spacetime (Equation 3) and that of gravitationally bound objects (Equation 5) is responsible for the gravitational redshift according to the classical general-relativistic redshifting of frequencies $\nu \propto \sqrt{g_{tt}} = a^{-1} = 1 + z$. Thus, the redshift phenomenon that is naturally explained in the cosmic frame is equally well explained in the comoving frame description, but now gravitationally bounded objects actually contract over time relative to the static background, and the static background Universe appears to be expanding only relative to these contracting objects. This includes our own solar system, the Sun, Earth, etc., from within which we observe the Universe to be *relatively* expanding. In any case, both cosmic and comoving frame descriptions are conformally related and null geodesics, subject to the constraint $ds^2 = 0$, are unaffected when switching between the cosmic and comoving frame descriptions. This fact is significant as our cosmological observations are limited to the past *lightcone*.

This difference between the standard (i.e., cosmic) and the comoving descriptions makes a crucial difference for the FO problem, as we see below. The key reason for this difference is the disparity between the roles played by masses and volumes in the production rate of FO's and BB's; whereas the volume appears as an overall multiplicative factor (the number of observers plausibly scales linearly with the volume), the mass appears in a Boltzmann suppression factor in the expression for the production rate, i.e. the likelihood for production decays *exponentially* with the mass. In both thermal and quantum scenarios, the respective BB's and FO's are overwhelmingly produced during the indefinite phases of expansion in the cosmic frame description. In contrast, in the comoving frame description, the corresponding production rates are characterized by growing brain masses, which exponentially outweighs the contraction of brains (considered bound objects in our analysis). This will be discussed in Section 4.

3. The Standard (Cosmic Frame) Argument and the Atypicality of OO's

In the following we closely follow the cosmic frame-based derivation of [11], the idea being that no matter how unlikely is the event of the spontaneous formation of transient disembodied brains, insofar it the probability is finite it will take place an infinite number of times in an indefinitely expanding Universe, whereas the conventional formation of OO's essentially halts once DE takes over the expansion, a finite time after the Big Bang. This 'nightmare scenario' leads to the BB (and FO) paradox.

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Since a FRW spacetime has a horizon it then follows that it has a Hawking temperature, T_H. Specifically, e.g. [45–48]

$$T_H = \frac{\hbar c}{2\pi k_B}\sqrt{H^2 + \frac{K}{a^2}}. \quad (6)$$

Considering ΛCDM, or even the KΛCDM model, then sufficiently long time after the various contributions (including that of spatial curvature) redshifted away $H^2 \to \Lambda/3$ and we arrive at the Gibbons-Hawking (GH) temperature of de Sitter spacetime, $K_B T_{GH} = \frac{\hbar c}{2\pi}\sqrt{\frac{\Lambda}{3}}$ [7].

The cumulative number of BB's of mass m_0 to form is essentially proportional to the product of the Boltzmann factor, $e^{-\alpha_{BB}}$, where $\alpha_{BB} \equiv m_0 c^2/(k_B T_{GH}) = 2\pi m_0 c/(\hbar\sqrt{\Lambda/3})$, and the time-integrated three-dimensional spatial volume of the observable Universe in units of the spacetime brain volume. Other, non-exponential factors, are ignored because the overwhelmingly dominant term is the Boltzmann factor, due to the extraordinarily huge α_{BB}. The cumulative number of such brains throughout the infinite lifetime of the Universe is

$$N_{BB} \sim \int_{t_0}^{\infty} \frac{\exp(-\alpha_{BB}) a^3(t) dt}{V_{br}\tau}, \quad (7)$$

where t_0 is the present time when structure formation essentially ceases and consequently new OO's stop forming, and V_{br} is the three-dimensional brain volume. Assuming flat space the volume is infinite and so the number of BB's and OO's will be infinite as well. To regularize these infinities it is customary to assume a finite comoving volume integrated indefinitely over the eternal lifetime of the ΛCDM Universe. Since the scale factor $a(t) \sim e^{\sqrt{H}t}$ grows exponentially during the Λ-domination era with $H = \sqrt{\frac{\Lambda}{3}}$, then $dt = da/a$ at late times and so $N_{BB} \sim \int_1^{\infty} \frac{\exp(-\alpha_{BB}) a^2 da}{V_{br}\tau} = \frac{\exp(-\alpha_{BB})}{V_{br}\tau} \int_1^{\infty} a^2 da$ diverges, however large (yet finite) α_{BB} is. Here, the lower integration limit is set at $a = 1$ for concreteness although in practice this limit will be higher. In any case, the divergence comes from the upper integration limit so we ignore this subtlety in what follows. Crucially for the current discussion, and in contrast to the corresponding calculation in the comoving frame discussed in the next section, the extremely small (but finite) Boltzmann term decouples from the diverging term $\int_1^{\infty} a^2 da$, thereby giving rise to diverging N_{BB}. This results in the absurd expectation for the thermal fluctuation into and out of existence of an infinite number of BB's over the future-eternal lifespan of the Universe. Estimates of the human population to have ever lived on Earth is in the ballpark of 10^{11} humans. Even if we make the bold and entirely unrealistic assumption that the number of habitable planets that eventually formed self-aware life is as large as the number of luminous stars in the observable Universe, $O(10^{22})$, we still end up with $O(10^{33})$ OO's at best (assuming that the cumulative Earthly population is representative among all habitable planets). This wildly generous upper limit on the number of OO's ever formed via conventional cosmic evolution is still finite and therefore vastly swamped by the diverging N_{BB}.

In case that ΛCDM is only an over-idealized approximation, and DE is not accounted for by a pure gravitational constant, and that it ultimately decays, then after a sufficiently long (but finite) time the Universe approaches an empty spacetime state, essentially a Milne Universe with $a = \sqrt{-K}t$ that is characterized by spatially open geometry, $K < 0$. In this case, $dt = da/\sqrt{-K}$, and the Hawking temperature vanishes, $T_H = 0$, following Equation (6). Whereas this implies that the BB formation channel does not exist in this case, transient deluded brains can still spontaneously form via the Heisenberg Uncertainty Principle. Following a similar rationale to the one that led to the estimated N_{BB} in Equation (7), the Boltzmann factor $\exp(-m_0 c^2/T_{GH})$ is now replaced by $\exp(-m_0 c^2 \tau/\hbar)$, where c is the speed of light, and τ is the brief lifetime of the brain that is assumed to be sufficiently long as to allow for a 'conscious observation' of the Universe.

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In this case we obtain that

$$N_{FO} \sim \int_{t_0}^{\infty} \frac{\exp(-\alpha_{FO}) a^3(t) dt}{V_{br}\tau} = \int_{1}^{\infty} \frac{\exp(-\alpha_{FO}) a^3 da}{\sqrt{-K} V_{br}\tau}, \tag{8}$$

i.e., $N_{FO} \sim \frac{\exp(-\alpha_{FO})}{\sqrt{-K}V_{br}\tau} \int_1^\infty a^3 da$ diverges, similar to the BB case, irrespective of how large α_{FO} is. As in the BB's case, the lower limit of the integration we employ, $a = 1$, is unrealistic – the present Universe is far from being empty. A more plausible lower integration limit is some $a \gg 1$ that correspond to a scenario in which DE decays and the Universe empties out in the remote future. Nevertheless, exactly as in the BB's case the divergence comes from the upper limit on a so our conclusions are unchanged – the cumulative number of FO's integrated along the indefinite expansion diverges and infinitely exceeds that of OO's.

To get a sense of how large α_{BB} and α_{FO} are, we make the following assumptions. The mass of the disembodied brain is $m_0 \sim 1$ Kg, the contribution of DE (assuming that it is indeed accounted for by a cosmological constant) to the present-day cosmic energy budget in critical density units $\Omega_\Lambda = \Lambda/(3H_0^2) = 0.69$ [49], the current expansion rate is $H_0 = 67$ km/sec/Mpc, and the lifetime of the brain is $\tau = 0.1$ seconds. Under these assumptions we obtain

$$\alpha_{BB} = \frac{2\pi Mc}{\hbar H_0 \sqrt{\Omega_\Lambda}} \sim 4.7 \times 10^{68} \tag{9}$$

$$\alpha_{FO} = \frac{Mc^2\tau}{\hbar} \sim 1.4 \times 10^{49}. \tag{10}$$

These specific values were irrelevant to the conclusions arrived at in this section, nor will they be relevant to the next section analysis. What will be important and indeed relevant is that $\alpha_{BB}, \alpha_{FO} \gg 1$.

It is worth mentioning at this point that carbon- soft-tissue-based brains are not the simplest imaginable observers. *A priori*, future self-aware silicon-based human-made observers are not theoretically ruled out, and it is not entirely banned that such artificial-like systems spontaneously pop-up in the future de Sitter vacuum or empty Milne Universe. These are significantly less complex than a human brain and the probability for their spontaneous creation is much larger, e.g. [50,51]. Yet, as discussed in Section 4 below, when considered in the comoving frame, even this type of minimal disembodied conscious observers is overwhelmed by OO's.

4. Typicality Restoration: The Comoving Frame Perspective

The comoving picture is obtained from the cosmic picture practically by setting $a = 1$ in the metric and simultaneously applying the transformation $m_0 \to m_0 a(\eta)$. While the masses (including the Planck mass) change in this picture the Hawking temperature solely depends on the geometry and is not affected by the new dynamics. This implies that in the comoving frame described by the metric of Equation (3)

$$\tilde{T}_H = \frac{\hbar c\sqrt{K}}{2\pi k_B}, \tag{11}$$

and unlike in the cosmic frame, Equation (6), it does not evolve. Following the standard assumption that $K = 0$ we see that the Hawking temperature vanishes, whereas in the case $K < 0$ it is not even defined; in both cases there is no horizon, and consequently no heat bath for BB's to form. Only in the case $K > 0$ cosmic horizon exists and the temperature does not vanish. Even in this latter case, however, BB's are overwhelmingly outnumbered by OO's as we momentarily see. Since the heat bath in this case is determined by K, not Λ, we update Equation (9) $\alpha_{BB} = \frac{2\pi m_0 c^2}{\hbar H_0 \sqrt{-\Omega_K}} \gtrsim 1.6 \times 10^{69}$ where we adopted $\Omega_K = -0.058$ [49] as the lower limit in the case $\Omega_K < 0$. As pointed out below Equation (9) the specific value α_{BB} is of no relevance for the current discussion insofar $\alpha_{BB} \gg 1$.

By virtue of Equation (2) $H_0 dt = (\Omega_r a^{-2} + \Omega_m a^{-1} + \Omega_k + \Omega_\Lambda a^2)^{-\frac{1}{2}} da$, and correspondingly $d\eta = dt/a$. In the comoving frame the volume integrations of Equations (7) and (8) transform as follows $a^3(t)d^3xdt \to d^3xd\eta$, but during DE domination $a \propto e^{Ht}$, where H is constant so $d\eta = da/a^2$. On the other hand, $V_{br}^{(3)} \to V_{br}^{(3)}/a^3$, because the brain volume contracts as is summarized by Equation (5). So, the analog of Equation (7) in the comoving frame is

$$N_{BB} \sim \frac{V^{(4)}}{V_{br}^{(4)}} \int_{a_*}^{\infty} e^{-a\alpha_{BB}} a da \lesssim \frac{V^{(4)}}{V_{br}^{(4)}} \times \frac{\exp(-\alpha_{BB})}{\alpha_{BB}} \sim \exp(-\alpha_{BB}) \tag{12}$$

where $\frac{V^{(4)}}{V_{br}^{(4)}}$ is the comoving spacetime volume $\int d^4x$ (calculated in the relevant region) in units of the four-dimensional brain spacetime volume $V_{br}^{(4)} \sim V_{br}^{(3)}\tau$. Use has been made in Equation (12) in the fact that a stupendously large $\alpha_{BB} \gg 1$ is exponentiated, $a_* \gtrsim 1$, and crucially that masses now vary with the scale function, $m_0 \to m_0 a(\eta)$. As we see, not even a single BB is expected to form in the comoving frame description during the entire eternal history of ΛCDM irrespective of the sign of K. According to our results $N_{BB} \ll 10^{-69}$ at best. Crucially, it is a combination of the fact that $\alpha_{BB} \gg 1$ and the fact that $m_0 \to m_0 a$ that resulted in $N_{BB} \ll 1$.

The FO's case is treated similarly, and as we see it is analogously reasoned that their abundance is overwhelmed by that of OO's. In this empty Universe case $a = \sqrt{-K}t$, where $K < 0$. In this case $d\eta = da/(\sqrt{-K}a)$ and following similar steps to the treatment of the BB case we obtain

$$N_{FO} \sim \frac{V^{(4)}}{V_{br}^{(4)}} \int_{a_*}^{\infty} e^{-a\alpha_{FO}} a^2 da \lesssim \frac{V^{(4)}}{V_{br}^{(4)}} \times \frac{\exp(-\alpha_{FO})}{\alpha_{FO}} \sim \exp(-\alpha_{FO}), \tag{13}$$

where $\alpha_{FO} \equiv m_0 c^2 \tau/\hbar \gg 1$ and τ is the lifetime of the transient FO.

Even if we consider a comoving 4-volume as large as the entire observable Universe, $\sim 10^{27}$ cm wide, and an age of 14 Gyrs, and a brain 10 cm on a side that survives for a mere $\tau \sim 0.1$ seconds, we obtain the prefactor $\frac{V^{(4)}}{V_{br}^{(4)}} \sim 10^{96}$. It then follows that not even a single BB or FO is expected to form in our four-dimensional comoving Hubble scale volume. We conclude that

$$N_{BB} \ll N_{FO} \ll 1 \ll 10^{11} < N_{OO}, \tag{14}$$

and the BB and FO paradoxes clearly go away in the comoving frame presentation. In the last inequality use has been made again in the estimate that the cumulative human population over history on Earth alone is at least ~120 billion OO's. In this analysis we compared the number of *observers* of each type. Had we chosen to compare the number of *observations* instead, and assuming that OO's are much longer lived than BB's or FO's then a similar (and stronger) inequality to Equation (14) would obtain. To illustrate the dramatic change incurred by analyzing the problem in the comoving frame, it should be noted that even a single electron – let alone a 1 kg brain – is not likely to spontaneously form within a volume as large as the comoving Hubble volume over the indefinite age of asymptotically de Sitter or Milne universes [see Equations (12) and (13)].

The BB and FO paradoxes arise in the cosmic frame essentially since the diverging time-integrated three-dimensional comoving volumes in future-eternal de-Sitter and Milne spacetimes decouple from the corresponding Boltzmann factors. In contrast, in the comoving frame picture, masses increase over time, thereby coupling between the Boltzmann factor and the time-integrated three-dimensional comoving volumes. Specifically, whereas BB's contract in this frame, thereby effectively implying three-dimensional volume integration over *conformal* time, late-time contributions are exponentially suppressed; the growing mass BB's and FO's impose natural cutoffs on the time integration that is otherwise not imposed in the cosmic frame description. This is so because the arguments in the exponents in Equations (12) and (13) explicitly depend on the scale factor a.

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5. Summary

The hypothetical cumulative overabundance of Boltzmann Brains (deluded disembodied transient brains thermally fluctuating into and out of existence due to the asymptotic de Sitter heat bath) and Freak Observers (cognitively impaired brains that are expected to spontaneously form and vanish due to the Heisenberg Uncertainty Principle) in the ΛCDM model is a classic example of typicality puzzles in cosmology. In a Universe where Ordinary Observers are vastly outnumbered by Boltzmann Brains or Freak Observers, it becomes exceedingly more likely that we ourselves are of either type – entities that are only deluded into believing that they are Ordinary Observers. Worse still, observations and theoretical predictions made by these cognitively impaired hypothetical observers are untrustworthy and undermine the foundations on which their theories rely. This unsettling scenario is typically dismissed on the basis of unacceptable Cognitive Instability. They are viewed as indicators of pathological models, which are rejected primarily due to these troubling implications.

The problem with the Cognitive Instability and Atypicality arguments is that applying the same standards to the concordance ΛCDM cosmological model leads to its outright disqualification as a trustworthy and viable description of our Universe. Yet, it is an undeniable well-established fact that ΛCDM has been an impressively good and parsimonious fit to a broad spectrum of independent cosmological observations and probes.

Nevertheless, in an indefinitely expanding Universe – asymptotically de Sitter in the case that dark energy is indeed accounted for by a cosmological constant or Milne Universe in case that dark energy eventually decays – the cumulative number of Boltzmann Brains and Freak Observers formed after dark energy takes over the expansion or alternatively its demise – diverges. In contrast, Ordinary Observers have only a finite narrow window in time to form in the conventional way (via primordial gas fragmentation, followed in succession by galaxy, star, and habitable planetary formation) prior to this epoch, and so their cumulative number is finite. The paradox then is how is it that we happen to be of the second type whereas our likelihood for being Boltzmann Brains and Freak Observers is exceedingly – actually infinitely – larger.

As discussed in the present work, this diagnostic tool – together with the fact that the concordance ΛCDM model, described in the cosmic frame, implies that we must be atypical observers – perhaps somewhat unexpectedly suggest that the Universe is best described as spatially static at the background level, while contracting within gravitationally bound objects. All this – with evolving masses.

While the cosmic and comoving frame formulations of the cosmological model are two observationally indistinguishable descriptions of the homogeneous and isotropic cosmological model, the finite conformal lifetime of the Universe – provided that the latter is indeed described by ΛCDM – is a tantalizing property ideally suited to address the Cosmological Constant Problem. This would seem in principle to at least suggest a pathway towards addressing the Boltzmann Brains and Freak Observers paradoxes as they typically form after dark energy comes to dominate the expansion dynamics.

However, as was argued here – independent of the Cosmological Constant Problem – the abundance of Boltzmann Brains and Freak Observers is exponentially suppressed in comparison to Ordinary Observers in the comoving frame description. This trivial fact appears to have been overlooked in past considerations of this foundational and conceptually paradoxical aspect of the standard cosmological model. This by itself tilts the balance in favor of the comoving frame picture over the cosmic frame description; it appears that the cognitively stable picture is that space is globally static, local gravitationally bound objects monotonically contract, and masses monotonically increase. Whereas this dual picture is (perhaps for historical reasons) less popular than the expanding space perspective, it at least seems to pass – unlike when described in the expanding space picture – the typicality and Cognitive Stability tests.

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