

Brief Report

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Brief Report

A Note About the Cosmological Constant

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Abstract

In this note, we explain dark energy and the cosmological constant, and present formulas for both. These predictions are part of our exact solution of Einstein’s equation for a universe in which the curvature varies with time, and the vacuum energy acts upon itself. The predicted value of the cosmological constant agrees with the currently accepted value exactly.

Keywords: cosmology; cosmological constant; accelerated expansion; scaling of the universe; dark energy

1. A Formula for the Cosmological Constant

The problems of the nature of cosmological constant, and its value has been around for a long time. The approaches to the problem usually involve matter in some form (quantum field theory), but we will show that it is entirely a property of the (classical) vacuum. Instead of being an independent entity, the cosmological constant is the infinite time limit of our time-varying solution for the vacuum energy and pressure.

Until recently, dark energy has been routinely considered a synonym for the idea of a cosmological constant which seemed to work as long as one believed that dark energy is constant in both time and location. During the past few years, DESI researchers have begun to suspect that dark energy varies with time [1] which means that the interpretation of dark energy must change since it can’t both vary and be a constant. The cosmological constant has always been considered to be a property of the vacuum, so a reasonable extension is to interpret the term dark energy to be synonymous with time-varying vacuum energy. The cosmological constant remains but is now reduced to being a particular of vacuum energy at some specific point in time.

Recently, we published a new model of cosmology that predicts the present-day exponential expansion of the universe. This model is based on two ideas. The first is that the curvature of spacetime varies with time. The second is that gravity acts upon itself, so the vacuum energy must appear in the energy-momentum (EM) tensor of Einstein’s equations. This model, including the metric, is described in several of our papers [2,3].

Given that starting point, we found the exact solution of the resulting equations [3]¹. The scaling has the form

$$a(ct) = a_* \left(\frac{ct}{ct_0} \right)^{\gamma_*} e^{\frac{ct}{ct_0} c_1} \tag{1}$$

where $a_* = a_0 e^{-c_1}$. This model has just two adjustable parameters. The value of the first is $\gamma_* = 0.5$, and with $H_0 = 73$, the second is $c_1 = 0.53$. With this value of the Hubble constant, the scaling of Equation (1) matches the luminosity distance measurements exactly over the full range of redshifts [4]. Note that this formula demands an exponential expansion for any reasonable pair of parameters, and

¹ The development of the solution takes some explaining so please refer to the reference. Also, in [3], we used a value of $H_0 = 67.3$ so the values of some of the parameters will be slightly different from those in the more recent publications. The differences, however, do not affect the general conclusions.

that it is entirely a consequence of the vacuum. Neither ordinary matter nor radiation has anything to do with it.

The time-varying curvature is given by

$$k(ct) = \bar{k}_0 \left(\frac{a(ct)}{ct} \right)^2 = \frac{1}{2} \gamma_h a(ct)^2 \kappa (\rho_{vac} c^2(ct) + p_{vac}(ct)) \quad (2)$$

where the two new parameters are fixed by the solution together with a third constraint that states that the curvature must always have its maximum possible value. The resulting values are $\bar{k}_0 = 1/8$, and $\gamma_h = 1/3$. The curvature was initially very large. After the initial inflation, it decreased with time reaching a minimum at $t = 0.94 t_0 = 4.11 \times 10^{17} \text{s}$, and now is increasing. Its present-day value is $k(ct_0) = 1.414$.

The formulas for the energy and pressure are

$$\rho c^2[ct] = \frac{3}{\kappa(ct_0)^2} \left(\frac{c_1^2}{(1 - \gamma_h)^2} + \left(\frac{2c_1 \bar{k}_0}{\gamma_h} \right) \frac{ct_0}{ct} + \bar{k}_0 \left(1 + \bar{k}_0 \frac{(1 - \gamma_h)^2}{\gamma_h^2} \right) \frac{(ct_0)^2}{(ct)^2} \right) \quad (3a)$$

$$p[ct] = \frac{-3}{\kappa(ct_0)^2} \left(\frac{c_1^2}{(1 - \gamma_h)^2} + \left(\frac{2c_1 \bar{k}_0}{\gamma_h} \right) \frac{ct_0}{ct} + \bar{k}_0 \left(1 - \frac{2}{3\gamma_h} + \bar{k}_0 \frac{(1 - \gamma_h)^2}{\gamma_h^2} \right) \frac{(ct_0)^2}{(ct)^2} \right). \quad (3b)$$

These formulas, which express the reality of dark energy, certainly vary with time which explains the recent DESI hint. From these, the so-called equation of state can be calculated. It is positive at early times, becomes negative at $t = 5.7 \times 10^{16} \text{s}$, and has an infinite-time limit of $w_{t \rightarrow \infty} = -1$.

Finally, the total energy is given by

$$\rho c^2[ct] + p[ct] = \frac{2\bar{k}_0}{\kappa(ct_0)^2 \gamma_h} \frac{(ct_0)^2}{(ct)^2}. \quad (4)$$

which vanishes at infinite time because the constant values of the energy and pressure cancel in the sum.

It is significant that our new model gives formulas (not curve fits) for all these quantities. The FRW model, on the other hand, does not predict any of these. In the FRW model, one has to guess the value of the (constant) curvature, and it doesn't have anything at all to say about the vacuum energy or pressure beyond an *ad hoc* cosmological constant whose value the FRW model fails to predict.

Getting now to the cosmological constant, our EM tensor is given by

$$\mathbf{T}^{\mu\nu} = (\rho_{vac} c^2(ct) + p_{vac}(ct)) \delta_0^\mu \delta_0^\nu + p_{vac}(ct) \mathbf{g}^{\mu\nu}, \quad (5)$$

and we see that it does not contain a cosmological constant. To turn this into something that does appear to have a cosmological constant, we make a simple rearrangement of the solution. The solution energy and pressure have non-zero values in the limit that $t \rightarrow \infty$. We now introduce two new definitions for the energy and pressure that vanish at infinite time.

$$\bar{\rho}_{vac} c^2 = \rho_{vac} c^2 - \rho_\infty c^2 \quad (6a)$$

$$\bar{p}_{vac} = p_{vac} + \rho_\infty c^2 \quad (6b)$$

The total energy is unaffected by the change, but the EM tensor becomes

$$\mathbf{T}^{\mu\nu} = (\bar{\rho}_{vac} c^2(ct) + \bar{p}_{vac}(ct)) \delta_0^\mu \delta_0^\nu + (\bar{p}_{vac}(ct) - \rho_\infty c^2) \mathbf{g}^{\mu\nu} \quad (7)$$

which does contain what appears to be a cosmological constant. From either the energy or pressure equation, the formula for the constant is

$$\rho_\infty c^2 = \frac{3}{\kappa(ct_0)^2} \frac{c_1^2}{(1 - \gamma_h)^2} = 5.33 \times 10^{-10} \text{J m}^{-3} \quad (8)$$

The standard model value is [5]

$$\rho c^2_{FRW} = 5.36 \times 10^{-10} \text{J m}^{-3} \quad (9)$$

which is the same.

This is the first time that anyone has not only discovered a formula for the cosmological constant, but discovered a formula whose value agrees exactly with the recently observed value.

2. Conclusion

We show that time-varying curvature together with vacuum self-interaction result in vacuum energy and pressures that vary with time. Our solution formulas for these explain the time variation of dark energy, and another formula gives a value of the cosmological constant that agrees exactly with the currently accepted value of the constant. We thus resolve both the dark energy and cosmological constant questions.

Declaration of AI use

No AI-assisted technologies were used in the development of this article.

Code Availability Statement.

This manuscript has no associated code/software.

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