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Article

About Stability of One System of Stochastic Difference Equations with Exponential Nonlinearity

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Abstract: A system of two nonlinear difference equations under stochastic perturbations is considered. Nonlinearity of the exponential type in each equation of the system under consideration depends on all variables of the system. The stability in probability of a positive equilibrium of the system is studied via the general method of Lyapunov functionals construction and the method of linear matrix inequalities (LMIs). The obtained results are illustrated via examples and figures with numerical simulations of solutions of a considered system of stochastic difference equations. The proposed way of investigation can be applied to nonlinear systems of higher dimension and with other types of nonlinearity, for both difference equations and for delay differential equations.

Keywords: nonlinear difference equations; positive equilibrium; stochastic perturbations; asymptotic mean square stability; stability in probability; linear matrix inequality (LMI); numerical simulations; MATLAB

MSC: 39A30; 39A50

1. Introduction

Systems of both difference and differential equations with different forms of exponential nonlinearities are very popular in research and various applications (see, for instance, [1–10] and references therein).

Here, similarly to [9], the stability of the positive equilibrium of a system with exponential nonlinearity is investigated under stochastic perturbations via the general method of Lyapunov functionals construction [10–13] and the method of linear matrix inequalities (LMIs) [14–22]. However, unlike, for instance, [1,9], where the exponential nonlinearity in each equation depends on only one variable, here each equation exponentially depends on all variables of the system under consideration. The obtained results are illustrated via examples and figures with numerical simulations of solutions of a considered system of difference equations. Numerical analysis of the considered LMIs is carried out using MATLAB.

Consider the system of two nonlinear difference equations

$$\begin{aligned}x_1(n+1) &= a_1 + b_1x_1(n-1) + c_1x_1(n-1)e^{-p_1x_1(n)-q_1x_2(n)}, \\x_2(n+1) &= a_2 + b_2x_2(n-1) + c_2x_2(n-1)e^{-p_2x_1(n)-q_2x_2(n)}, \\n \in N &= \{0, 1, \dots\},\end{aligned}\tag{1}$$

with positive parameters, $b_i < 1$, $i = 1, 2$, and positive initial conditions $x_i(j) = \phi_i(j)$, $i = 1, 2$, $j \in N_0 = \{-1, 0\}$.

The equilibrium (x_1, x_2) of system (1) is defined by the system of two algebraic equations

$$\begin{aligned}x_1 &= a_1 + b_1x_1 + c_1x_1e^{-p_1x_1-q_1x_2}, \\x_2 &= a_2 + b_2x_2 + c_2x_2e^{-p_2x_1-q_2x_2}.\end{aligned}\tag{2}$$

Theorem 1. *The system (2) has a unique positive solution.*

Proof. Presenting the first equation (2) in the form

$$e^{q_1 x_2} \left(1 - b_1 - \frac{a_1}{x_1}\right) = c_1 e^{-p_1 x_1} \quad (3)$$

and calculating the logarithm, we get

$$x_2 = \frac{1}{q_1} \left(\ln c_1 - p_1 x_1 - \ln \left(1 - b_1 - \frac{a_1}{x_1}\right) \right), \quad x_1 > \frac{a_1}{1 - b_1}. \quad (4)$$

Similarly, from the second equation (2) we have

$$e^{p_2 x_1} \left(1 - b_2 - \frac{a_2}{x_2}\right) = c_2 e^{-q_2 x_2} \quad (5)$$

and

$$x_1 = \frac{1}{p_2} \left(\ln c_2 - q_2 x_2 - \ln \left(1 - b_2 - \frac{a_2}{x_2}\right) \right), \quad x_2 > \frac{a_2}{1 - b_2}. \quad (6)$$

It is clear that the function $x_2 = f_1(x_1)$ given by (4) is defined and positive if $x_1 \in (x_{1min}, x_{1max})$, where $x_{1min} = \frac{a_1}{1 - b_1}$ and x_{1max} is a unique root of the equation

$$1 - b_1 - \frac{a_1}{x_1} = c_1 e^{-p_1 x_1}, \quad (7)$$

which follows from (3) by $x_2 = 0$.

Calculating the derivative

$$x_2' = \frac{1}{q_1} \left(-p_1 - \frac{a_1}{((1 - b_1)x_1 - a_1)x_1} \right) < 0,$$

it is easy to see that $x_2 = f_1(x_1)$ is strictly decreasing function. Moreover, $\lim_{x_1 \rightarrow x_{1min}} f_1(x_1) = +\infty$ and $f_1(x_{1max}) = 0$.

Calculating the derivative of the function $x_2 = f_2(x_1)$ given by (6), we have

$$p_2 = - \left(q_2 + \frac{a_2}{((1 - b_2)x_2 - a_2)x_2} \right) x_2' > 0,$$

i.e., $x_2' < 0$. It means that $x_2 = f_2(x_1)$ is strictly decreasing function for $x_1 \geq 0$. Moreover, $\lim_{x_1 \rightarrow \infty} f_2(x_1) = \frac{a_2}{1 - b_2}$ and $x_2 = f_2(0)$ is a unique root of the equation

$$1 - b_2 - \frac{a_2}{x_2} = c_2 e^{-q_2 x_2}, \quad (8)$$

which follows from (5) by $x_1 = 0$. It is easy to see that the root x_2 of this equation satisfies the condition $x_2 > \frac{a_2}{1 - b_2}$.

Two decreasing functions $x_2 = f_1(x_1)$ and $x_2 = f_2(x_1)$ has one general point, which is a solution of the system (2) and is the unique equilibrium (x_1^*, x_2^*) of the system (1). The proof is completed. \square

Corollary 1.1. *The equilibrium (x_1^*, x_2^*) of the system (1) satisfies the conditions*

$$x_1^* \in \left(\frac{a_1}{1 - b_1}, x_{1max} \right), \quad x_2^* \in \left(\frac{a_2}{1 - b_2}, x_{2max} \right), \quad (9)$$

where x_{1max} and x_{2max} are roots of the equations (7) and (8) respectively.

Example 1. Consider the system (1) with

$$a_1 = 0.3, \quad a_2 = 0.4, \quad b_1 = 0.7, \quad b_2 = 0.6, \quad c_1 = 0.2810, \quad c_2 = 0.4215, \quad p_1 = p_2 = 0.1, \quad q_1 = q_2 = 0.01.$$

Then the solution of the system (2) is $(x_1^*, x_2^*) = (3, 4)$, from (9) and (7), (8) it follows that $x_{1min} = 1$, $x_{2min} = 1$, $x_{1max} = 3.156$, $x_{2max} = 13.147$. In Figures 1 and 2 the graphs of the functions $x_2 = f_1(x_1)$ (green), $x_2 = f_2(x_1)$ (red) and the equilibrium (x_1^*, x_2^*) are shown.

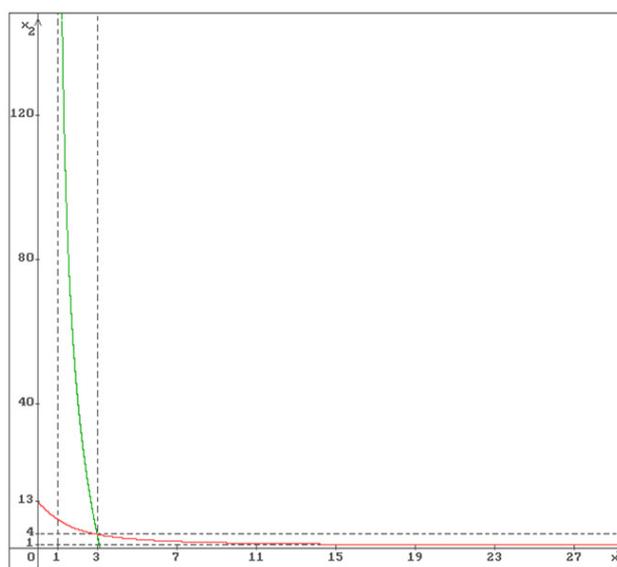


Figure 1. The graphs of the functions $x_2 = f_1(x_1)$ (green) and $x_2 = f_2(x_1)$ (red).

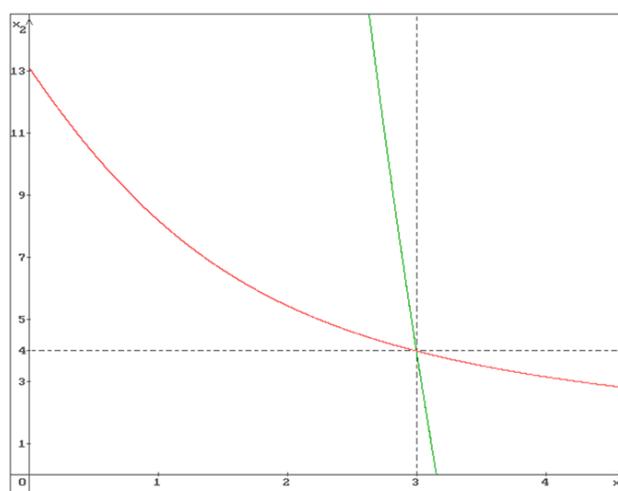


Figure 2. The intersection point of the graphs of the functions $x_2 = f_1(x_1)$ (green) and $x_2 = f_2(x_1)$ (red) is the equilibrium $(x_1^*, x_2^*) = (3, 4)$.

2. Stochastic Perturbations and the System Transformation

Let $\{\Omega, \mathfrak{F}, \mathbf{P}\}$ be a basic probability space, $\mathfrak{F}_n \in \mathfrak{F}$, $n \in N$, be a nondecreasing family of sub- σ -algebras of \mathfrak{F} , i.e., $\mathfrak{F}_{n_1} \subset \mathfrak{F}_{n_2}$ for $n_1 < n_2$, \mathbf{E} be the mathematical expectation with respect to the

measure \mathbf{P} , $\xi_1(n)$ and $\xi_2(n)$, $n \in N$, be two mutually independent sequences with \mathfrak{F}_n -adapted mutually independent random values such that [10]

$$\mathbf{E}\xi_i(n) = 0, \quad \mathbf{E}\xi_i^2(n) = 1, \quad \mathbf{E}\xi_i(n)\xi_j(m) = 0 \quad \text{if } i \neq j \quad \text{or } n \neq m, \quad i, j = 1, 2. \quad (10)$$

Let us assume that the system (1) is exposed to stochastic perturbations that are directly proportional to the deviation of the system state $(x_1(n), x_2(n))$ from the equilibrium (x_1^*, x_2^*) . Then the system (1) takes the form

$$\begin{aligned} x_1(n+1) &= a_1 + b_1x_1(n-1) + c_1x_1(n-1)e^{-p_1x_1(n)-q_1x_2(n)} + \sigma_1(x_1(n) - x_1^*)\xi_1(n+1), \\ x_2(n+1) &= a_2 + b_2x_2(n-1) + c_2x_2(n-1)e^{-p_2x_1(n)-q_2x_2(n)} + \sigma_2(x_2(n) - x_2^*)\xi_2(n+1). \end{aligned} \quad (11)$$

Remark 1. Note that the such type of stochastic perturbations was firstly proposed in [23] for a system of Ito's stochastic delay differential equations and was later used in many other works for both differential and difference equations (see, for instance, [10,13] and references therein). With this type of stochastic perturbations, the equilibrium of the original deterministic system remains also a solution of the stochastically perturbed system.

Presenting the solution of the system (11) in the form $(x_1(n), x_2(n)) = (y_1(n) + x_1^*, y_2(n) + x_2^*)$, we get

$$\begin{aligned} y_1(n+1) + x_1^* &= a_1 + b_1(y_1(n-1) + x_1^*) \\ &\quad + c_1(y_1(n-1) + x_1^*)e^{-p_1y_1(n)-q_1y_2(n)}e^{-p_1x_1^*-q_1x_2^*} + \sigma_1y_1(n)\xi_1(n+1), \\ y_2(n+1) + x_2^* &= a_2 + b_2(y_2(n-1) + x_2^*) \\ &\quad + c_2(y_2(n-1) + x_2^*)e^{-p_2y_1(n)-q_2y_2(n)}e^{-p_2x_1^*-q_2x_2^*} + \sigma_2y_2(n)\xi_2(n+1). \end{aligned} \quad (12)$$

Substituting (2) into (12), from the first equation we have

$$\begin{aligned} y_1(n+1) &= a_1 + b_1x_1^* - x_1^* + b_1y_1(n-1) \\ &\quad + c_1(y_1(n-1) + x_1^*)e^{-p_1y_1(n)-q_1y_2(n)}e^{-p_1x_1^*-q_1x_2^*} + \sigma_1y_1(n)\xi_1(n+1) \\ &= b_1y_1(n-1) - c_1x_1^*e^{-p_1x_1^*-q_1x_2^*} \\ &\quad + c_1(y_1(n-1) + x_1^*)e^{-p_1y_1(n)-q_1y_2(n)}e^{-p_1x_1^*-q_1x_2^*} + \sigma_1y_1(n)\xi_1(n+1) \\ &= \left(b_1 + c_1e^{-p_1x_1^*-q_1x_2^*}e^{-p_1y_1(n)-q_1y_2(n)} \right) y_1(n-1) \\ &\quad - c_1x_1^*e^{-p_1x_1^*-q_1x_2^*} \left(1 - e^{-p_1y_1(n)-q_1y_2(n)} \right) + \sigma_1y_1(n)\xi_1(n+1). \end{aligned}$$

Similarly, for the second equation (12) we get

$$\begin{aligned} y_2(n+1) &= a_2 + b_2x_2^* - x_2^* + b_2y_2(n-1) \\ &\quad + c_2(y_2(n-1) + x_2^*)e^{-p_2y_1(n)-q_2y_2(n)}e^{-p_2x_1^*-q_2x_2^*} + \sigma_2y_2(n)\xi_2(n+1) \\ &= b_2y_2(n-1) - c_2x_2^*e^{-p_2x_1^*-q_2x_2^*} \\ &\quad + c_2(y_2(n-1) + x_2^*)e^{-p_2y_1(n)-q_2y_2(n)}e^{-p_2x_1^*-q_2x_2^*} + \sigma_2y_2(n)\xi_2(n+1) \\ &= \left(b_2 + c_2e^{-p_2x_1^*-q_2x_2^*}e^{-p_2y_1(n)-q_2y_2(n)} \right) y_2(n-1) \\ &\quad - c_2x_2^*e^{-p_2x_1^*-q_2x_2^*} \left(1 - e^{-p_2y_1(n)-q_2y_2(n)} \right) + \sigma_2y_2(n)\xi_2(n+1). \end{aligned}$$

As a result we obtain the nonlinear system

$$\begin{aligned} y_1(n+1) &= \left(b_1 + c_1 e^{-p_1 x_1^* - q_1 x_2^*} e^{-p_1 y_1(n) - q_1 y_2(n)} \right) y_1(n-1) \\ &\quad - c_1 x_1^* e^{-p_1 x_1^* - q_1 x_2^*} \left(1 - e^{-p_1 y_1(n) - q_1 y_2(n)} \right) + \sigma_1 y_1(n) \xi_1(n+1), \\ y_2(n+1) &= \left(b_2 + c_2 e^{-p_2 x_1^* - q_2 x_2^*} e^{-p_2 y_1(n) - q_2 y_2(n)} \right) y_2(n-1) \\ &\quad - c_2 x_2^* e^{-p_2 x_1^* - q_2 x_2^*} \left(1 - e^{-p_2 y_1(n) - q_2 y_2(n)} \right) + \sigma_2 y_2(n) \xi_2(n+1), \end{aligned} \quad (13)$$

with the zero solution.

Remark 2. Note that stability of the zero solution of the system (13) is equivalent to stability of the equilibrium (x_1^*, x_2^*) of the system (11).

Using (2) and the linear approximation $e^{-x} = 1 - x + o(x)$, where $\lim_{x \rightarrow 0} \frac{o(x)}{x} = 0$, we obtain the linear part of the system (13)

$$\begin{aligned} z_1(n+1) &= \left(1 - \frac{a_1}{x_1^*} \right) z_1(n-1) - ((1 - b_1)x_1^* - a_1)(p_1 z_1(n) + q_1 z_2(n)) + \sigma_1 z_1(n) \xi_1(n+1), \\ z_2(n+1) &= \left(1 - \frac{a_2}{x_2^*} \right) z_2(n-1) - ((1 - b_2)x_2^* - a_2)(p_2 z_1(n) + q_2 z_2(n)) + \sigma_2 z_2(n) \xi_2(n+1). \end{aligned} \quad (14)$$

Representing the linear system (14) in the matrix form, we get

$$z(n+1) = -Az(n) + Bz(n-1) + \sum_{i=1}^2 C_i z(n) \xi_i(n+1), \quad (15)$$

where

$$z(n) = \begin{pmatrix} z_1(n) \\ z_2(n) \end{pmatrix}, \quad A = \begin{pmatrix} \alpha_1 p_1 & \alpha_1 q_1 \\ \alpha_2 p_2 & \alpha_2 q_2 \end{pmatrix}, \quad B = \begin{pmatrix} \beta_1 & 0 \\ 0 & \beta_2 \end{pmatrix}, \quad C_1 = \begin{pmatrix} \sigma_1 & 0 \\ 0 & 0 \end{pmatrix}, \quad C_2 = \begin{pmatrix} 0 & 0 \\ 0 & \sigma_2 \end{pmatrix}, \quad (16)$$

and

$$\alpha_i = (1 - b_i)x_i^* - a_i, \quad \beta_i = 1 - \frac{a_i}{x_i^*}, \quad i = 1, 2.$$

3. Stability

3.1. Some Necessary Definitions and Statements

Let ' be the transposition sign. Put now

$$\begin{aligned} y(n) &= (y_1(n), y_2(n))', \quad z(n) = (z_1(n), z_2(n))', \quad n \in N, \\ \phi(j) &= (\phi_1(j), \phi_2(j))', \quad j \in N_0. \end{aligned}$$

Definition 3.1. ([10]). The zero solution of the system (13) is called stable in probability if for any $\varepsilon > 0$ and $\varepsilon_1 \in (0, 1)$ there exists a $\delta > 0$ such that the solution $y(n) = y(n, \phi)$ of the system (13) satisfies the inequality $\mathbf{P}\{\sup_{n \in N} |y(n)| > \varepsilon\} < \varepsilon_1$ for any initial function $\phi(j)$ such that $\mathbf{P}\{\|\phi\|_0 < \delta\} = 1$, where $\|\phi\|_0 = \max_{j \in N_0} |\phi(j)|$.

Definition 3.2. ([10]). The zero solution of the system (14) is called mean square stable if for each $\varepsilon > 0$ there exists a $\delta > 0$ such that $\mathbf{E}|z(n)|^2 < \varepsilon$, $n \in N$, for any initial function $\phi(j)$ such that $\|\phi\|^2 = \max_{j \in N_0} \mathbf{E}|\phi(j)|^2 < \delta$;

asymptotically mean square stable if it is mean square stable and for each initial function $\phi(j)$ such that $\|\phi\|^2 < \infty$ the solution $z(n)$ of the system (14) satisfies the condition $\lim_{n \rightarrow \infty} \mathbf{E}|z(n)|^2 = 0$.

Let $\mathbf{E}_n = \mathbf{E}\{\cdot / \mathfrak{F}_n\}$ be the conditional expectation with respect to the σ -algebra \mathfrak{F}_n , $U_\varepsilon = \{y : |y| \leq \varepsilon\}$, $\varepsilon > 0$, and $\Delta V(n) = V(n+1) - V(n)$.

Theorem 2. ([10]). Let for the system (13) there exists a functional $V(n) = V(n, y(-1), \dots, y(n))$ which satisfies the conditions

$$\begin{aligned} V(n, y(-1), \dots, y(n)) &\geq c_0 |y(n)|^2, \\ V(0, \phi(-1), \phi(0)) &\leq c_1 \|\phi\|_0^2, \\ \mathbf{E}_n \Delta V(n, y(-1), \dots, y(n)) &\leq 0, \quad y(j) \in U_\varepsilon, \quad -1 \leq j \leq n, \quad n \in N, \end{aligned} \quad (17)$$

where $\varepsilon > 0$, $c_0 > 0$, $c_1 > 0$. Then the zero solution of the system (13) is stable in probability.

Theorem 3. ([10]). Let for the system (14) there exists a nonnegative functional $V(n) = V(n, z(-1), \dots, z(n))$ which satisfies the conditions

$$\begin{aligned} \mathbf{E}V(0, \phi(-1), \phi(0)) &\leq c_1 \|\phi\|^2, \\ \mathbf{E}\Delta V(n) &\leq -c_2 \mathbf{E}|z(n)|^2, \quad n \in N, \end{aligned} \quad (18)$$

where $c_1 > 0$, $c_2 > 0$. Then the zero solution of the system (14) is asymptotically mean square stable.

Remark 3. . Note that the system (13) has an order of nonlinearity higher than one. It is known [10] that in this case sufficient conditions for asymptotic mean square stability of the zero solution of the linear system (14) are also sufficient conditions for stability in probability of the zero solution of the nonlinear system (13).

3.2. Stability conditions

Theorem 4. . Let there exist positive definite 2×2 -matrices P and R such that the following linear matrix inequality (LMI)

$$\begin{pmatrix} A'PA + S_0 + R - P & A'PB \\ B'PA & B'PB - R \end{pmatrix} < 0 \quad (19)$$

holds, where

$$S_0 = \sum_{i=1}^2 C_i' P C_i = \begin{pmatrix} \sigma_1^2 p_{11} & 0 \\ 0 & \sigma_2^2 p_{22} \end{pmatrix}, \quad (20)$$

the matrices C_1, C_2 are defined in (16) and p_{11}, p_{22} are diagonal elements of the matrix P . Then the equilibrium (x_1^*, x_2^*) of the system (11) is stable in probability.

Proof. Following the general method of Lyapunov functionals construction [10–13], consider the functional $V(n)$ in the form $V(n) = V_1(n) + V_2(n)$, where $V_1(n) = z'(n)Pz(n)$, $P > 0$, $z(n)$ is defined in (16) and the additional functional $V_2(n)$ will be chosen below. For the functional $V_1(n)$ via (15) we have

$$\begin{aligned} \mathbf{E}\Delta V_1(n) &= \mathbf{E}[V_1(n+1) - V_1(n)] \\ &= \mathbf{E}[z'(n+1)Pz(n+1) - z'(n)Pz(n)] \\ &= \mathbf{E}\left[\left(-z'(n)A' + z'(n-1)B' + \sum_{i=1}^2 z'(n)C_i'\xi_i(n+1) \right) \right. \\ &\quad \left. * P \left(-Az(n) + Bz(n-1) + \sum_{i=1}^2 C_i z(n)\xi_i(n+1) \right) - z'(n)Pz(n) \right]. \end{aligned}$$

From here via (10) and (20) it follows that

$$\begin{aligned} \mathbf{E}\Delta V_1(n) = & \mathbf{E}[z'(n)(A'PA + S_0 - P)z(n) + z'(n)A'PBz(n-1) \\ & + z'(n-1)B'PAz(n) + z'(n-1)B'PBz(n-1)] \end{aligned} \quad (21)$$

or in the matrix form

$$\mathbf{E}\Delta V_1(n) = \mathbf{E} \begin{pmatrix} z(n) \\ z(n-1) \end{pmatrix}' \begin{pmatrix} A'PA + S_0 - P & A'PB \\ B'PA & B'PB \end{pmatrix} \begin{pmatrix} z(n) \\ z(n-1) \end{pmatrix}.$$

Using the additional functional $V_2(n) = z'(n-1)Rz(n-1)$, $R > 0$, with $\Delta V_2(n) = z'(n)Rz(n) - z'(n-1)Rz(n-1)$, for the functional $V(n) = V_1(n) + V_2(n)$ we obtain

$$\mathbf{E}\Delta V(n) = \mathbf{E} \begin{pmatrix} z(n) \\ z(n-1) \end{pmatrix}' \begin{pmatrix} A'PA + S_0 + R - P & A'PB \\ B'PA & B'PB - R \end{pmatrix} \begin{pmatrix} z(n) \\ z(n-1) \end{pmatrix}. \quad (22)$$

From (22) and the LMI (19) for some $c > 0$ we have $\mathbf{E}\Delta V(n) \leq -c\mathbf{E}|z(n)|^2$, i.e., the constructed functional $V(n)$ satisfies the conditions of Theorem 3. Therefore, the zero solution of the linear equation (15) is asymptotically mean square stable. Via Remarks 3 and 2 it means that the equilibrium (x_1^*, x_2^*) of the system (11) is stable in probability. The proof is completed. \square

Remark 4. Note that instead of the LMI (19) for definition of stability some other LMIs also can be used. Using, for instance, the additional functional $V_2(n)$ in the form $V_2(n) = z'(n-1)(R + B'PB)z(n-1)$ instead of the LMI (19) we obtain the LMI

$$\begin{pmatrix} A'PA + B'PB + S_0 + R - P & A'PB \\ B'PA & -R \end{pmatrix} < 0. \quad (23)$$

If at least one from the LMIs (19) and (23) holds then the equilibrium (x_1^*, x_2^*) of the system (11) is stable in probability. Other ways to get appropriate LMIs are shown also in [9].

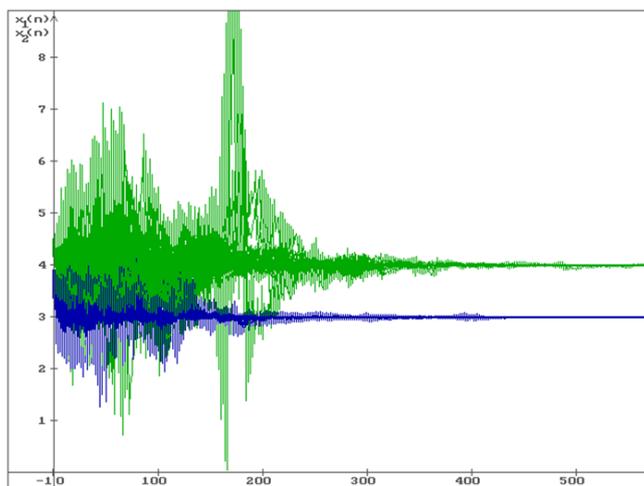


Figure 3. 50 trajectories of the system (11) solution: $x_1(n)$ (blue) and $x_2(n)$ (green). The solution converges to the stable equilibrium $(x_1^*, x_2^*) = (3, 4)$. The first implementation.

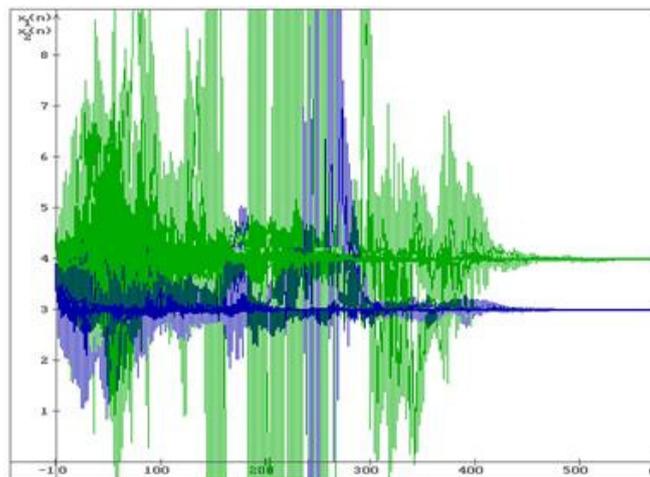


Figure 4. 50 trajectories of the system (11) solution: $x_1(n)$ (blue) and $x_2(n)$ (green). The solution converges to the stable equilibrium $(x_1^*, x_2^*) = (3, 4)$. The second implementation.

Example 2. Consider the system (11) with the values of the parameters given in Example 1. Via MATLAB it was obtained that by $\sigma_1 = 0.23$, $\sigma_2 = 0.38$ the LMIs (19) and (23) hold. In Figures 3 and 4 two different implementations of the solution of the system (11) are shown. In both cases, all trajectories of the solution converge to the stable equilibrium $(x_1^*, x_2^*) = (3, 4)$.

4. Conclusions

Stability of a system of nonlinear difference equations under stochastic perturbations is investigated. The nonlinearity of exponential form in each equation depends on all variables of the system under consideration. The conditions of stability in probability for positive equilibrium of the considered system are obtained using the general method of Lyapunov functionals construction, are formulated in terms of linear matrix inequalities (LMIs) and are illustrated by numerical examples and figures. The method of stability investigation, used in the paper, can be applied to many other types of nonlinear systems for both difference and differential equations in various applications.

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