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Mónica Rico Martínez [1], Juan Carlos Vesga Ferreira [2,*], Joel Carroll Vargas [3], María Consuelo Rodríguez [4] and Andres Alejandro Diaz Toro [5], William A. Cuevas [6]

1 Universidad Nacional Abierta y a Distancia; monica.rico@unad.edu.co
2 Universidad Nacional Abierta y a Distancia; juan.vesga@unad.edu.co
3 Universidad Nacional Abierta y a Distancia; joel.carroll@unad.edu.co
4 Universidad Nacional Abierta y a Distancia; mariac.rodriguez@unad.edu.co
5 Universidad Nacional Abierta y a Distancia; andres.diaz@unad.edu.co
6 Universidad Nacional Abierta y a Distancia; william.cuevas@unad.edu.co
* Correspondence: juan.vesga@unad.edu.co; Tel.: +57 3212759948

Abstract: The hybrid in-band on-channel (IBOC) transmission system as a digital audio radio scheme could transmit analog FM and digital audio simultaneously. In IBOC systems, broadcasters transmit signals within the allocated channel bands, so the bandwidth allocated for digital audio transmission is a big challenge. In view of the above, the use of cooperative game theory, supported by the use of the bankruptcy game and Shapley's value, is proposed as a strategy to optimize the bandwidth allocation at each node or station, according to the demand in the service, the number of stations and the radio channel conditions in the FM band. The paper proposes a scenario under saturated traffic conditions, in order to evaluate the degree of optimization that the Shapley value can perform under clearly established traffic and channel conditions. The evaluation process yielded a quite favorable result, where it is observed that the use of cooperative game theory supported by the Shapley value can be considered as an excellent alternative when performing bandwidth optimization processes for digital broadcasting over IBOC in FM and with the possibility of being implemented in other digital radio broadcasting systems.

Keywords: IBOC; FM; Shapley; optimization; game theory

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1. Introduction

Due to inadequate spectrum resources and increasing demand for high quality multimedia services, traditional analog audio broadcasting is migrating to digital radio worldwide. In recent years, several digital audio transmission standards have been developed, such as digital audio transmission under different technologies, among which one of the most relevant is IBOC (NRSC-5). IBOC technology provides an uninterrupted evolution path from analog amplitude modulation (AM) and frequency modulation (FM) radio to a fully digital radio transmission system. The evolution allows listeners to migrate to digital reception of their favorite IBOC-capable analog stations. The history and state of the art of IBOC technologies are well documented in [1]. In addition to the implementation advantage, IBOC digital radio can provide sound quality close to that of a compact disc (CD), as well as multimedia services such as data, images, and even video.

The coexistence of digital and analog radio in the AM and FM bands poses some unique challenges in the design of digital transmission systems. First, the insertion of digital broadcasting into the established analog radio spectrum may cause interference to existing stations. Therefore, the power of the digital signal must be much lower (a power reduction of more than 20 dB is recommended [2]) than that of the analog signal, resulting in tolerable interference in the reception of the analog signal. On the other hand, due to the huge difference in power level, the use of low power digital radio technology with strong resistance to analog interference is a design mandate. Secondly, the gaps between existing analog radio bands are fragmented in most cases. An ideal

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digital radio system must possess the spectral efficiency and flexibility to deliver high data rates without the use of a large amount of continuous bandwidth. Third, the FM/AM spectra available for digital transmission are narrowband in nature. It is well documented that, in wireless communications, stationary and low mobility users are likely to suffer from prolonged poor reception due to the lack of frequency diversity. However, DRM and NRSC-5 effectively address the co-channel problem with sets of innovations, neither adequately addresses data rate and fading performance issues; however, it may not be sufficient against slow fading cases.

The in-band on-channel (IBOC) system is designed to operate in two specific modes: hybrid mode and fully digital mode. In the hybrid mode, the analog and digital components are transmitted simultaneously, where both signals make use of the channel assigned to the analog signal under the traditional radio model. On the other hand, in the fully digital mode, the transmitted signal makes use of the entire bandwidth assigned to the station and its component is purely digital.

IBOC adopts a hierarchical modulation scheme to further enrich BDR services. The concept of hierarchical transmission is based on providing layered services to mobile users within different coverage areas, all through the same radio channel. For example, a *basic layer* signal is used to transmit essential information (normal quality audio/video); however, the layer that is added on top of the basic layer is called the *secondary layer*, which would serve as a transport medium for complementary multimedia services. Therefore, hierarchical streaming offers an attractive solution for terminals with different screen sizes and different video display resolutions. However, the advantages of hierarchical streaming come at a cost. Most notably, the secondary layer not only reduces the effective transmission power of the basic layer, promoting in turn, an additional performance loss in the reception of the basic layer [3].

As a conclusion, it could be said that, although there are several digital radio systems, with specific advantages for each case, all offer great benefits compared to analog radio. Using perceptual audio coding, digital radio can offer better sound quality than AM or FM radio. Based on the OFDM multi-carrier transmission technique, it is able to overcome the mobile reception distortion caused by multipath propagation and other interference inherent in analog radios. Unlike analog radio, digital radio allows the inclusion of additional services such as the transmission of text and/or multimedia information. In fact, almost anything that can be digitized could be transmitted over a digital radio platform. A very important aspect is that these systems use MPEG-2 audio coding, which facilitates the process of transmitting up to five programs with near CD quality in a set. In addition, by synergistically combining LDPC codes, band aggregation and frequency hopping techniques, as well as an efficient hierarchical modulation scheme, IBOC offers performance and service advantages in various application scenarios, accompanied by a wide range of transmission rates (up to 2.53 Mbps) and with the ability to offer high quality radio and rich multimedia services. In view of the above, it is proposed to make use of cooperative game theory, supported by the use of Shapley value as a strategy for bandwidth allocation to each of the nodes or stations that are part of the broadcasting network over IBOC FM, in order to improve its performance, by allowing multiple nodes to transmit simultaneously and under clearly defined distribution policies.

2. Materials and Methods

2.1. Game theory and Shapley value

Game theory, conceptualized by John Von Neumann in 1928, offers a mathematical framework for assessing individual decisions within competitive contexts of gain or loss in comparison to decisions made by other competitors. This competitive scenario is called "Game" and the individuals who are part of this scenario are called "Players"[4].

Game theory provides three models for real scenario representation: extensive, strategic, and coalition. While the first two are applicable to non-cooperative games, emphasizing individual player interests, the coalition model pertains to cooperative games. In cooperative games, players collaborate to achieve common goals, increasing the likelihood of higher gains than individual

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actions (Ramírez R., 2008). Cooperative game analysis avoids the need to scrutinize player strategies, focusing on coalition utility and payment vectors [5].

Within cooperative game theory, a recurring question involves equitable distribution of a good's net value among players, especially when resources are insufficient. This situation, known as the "bankruptcy" problem, is analyzed either as a transferable utility game or through axiomatic methods [6][7][8][9] [10][11][7]. This paper adopts the transferable utility approach.

Definition [5]: A Transferable Utility (TU) cooperative game is a pair (N, v), where N is the set of players {1, 2, ..., n} and v: $2N \rightarrow \mathbb{R}$ is the characteristic function, with $v(\emptyset) = 0$. Each coalition $S \subset N$ associates with (S), representing the assured payoff for its players, irrespective of others' actions.

Definition [12]: A bankruptcy game, represented as (N, d, E), comprises creditors (N), claims vector (d), and net value (E). A cooperative game (N, v) is defined from each bankruptcy problem. The value of coalition S is the property not claimed by non-S members, calculated as:

$$v(S) = max\{0, E - d(N \setminus S)\} \; \forall S \subset N \tag{1}$$

One significant challenge in TU cooperative game theory is fairly allocating the total gain among players. The Shapley value, introduced by Shapley in 1953, addresses this by considering each player's marginal contributions to all possible coalitions, meeting established axioms [13][14]. The Expression corresponding to the Shapley value is:

$$\varphi_i(v) = \sum_{S \subseteq N: i \in S} \frac{(s-1)!\,(n-s)!}{n!}[v(S) - v(S - \{i\})] \;\; where\; n = |N| \; y \; s = |S| \tag{2}$$

The Shapley value ensures efficient, symmetric, passive, and additive resource allocations. O'Neill (1982) demonstrated the correspondence between the recursive partitioning process in a bankruptcy problem and Shapley's value. For the solution to be adequate it is necessary that the vector of payoffs complies with the efficiency principle where [15]:

$$\sum_{i \in N} \varphi_i(v) = v(N) \tag{3}$$

$$\varphi_i(v) \geq (v\{i\}) \;\; \forall i \in N \tag{4}$$

In the context of equitable bandwidth distribution in the digital radio spectrum for IBOC FM, represented as a TU game (N, v), Shapley value emerges as a strategy for fair allocation among nodes under saturation conditions. This scenario involves N nodes as players, λ_i obeys the width demand of each node i (d_i) y $E = BW_T$.

2.2. IBOC FM spectrum for hybrid mode

In the hybrid model, the digital signal is transmitted in primary main sidebands on both sides of the FM signal. In this model, each sideband of the IBOC FM system is composed of 10 frequency divisions allocated between carriers 356 to 545 and between - 356 to - 545. Carriers 546 and - 546 are reference carriers. In turn, the level of the digital subcarriers describes a total power 23 dB below the nominal power of the analog FM carrier [2]. Figure 1 shows the carrier distribution for the hybrid mode.

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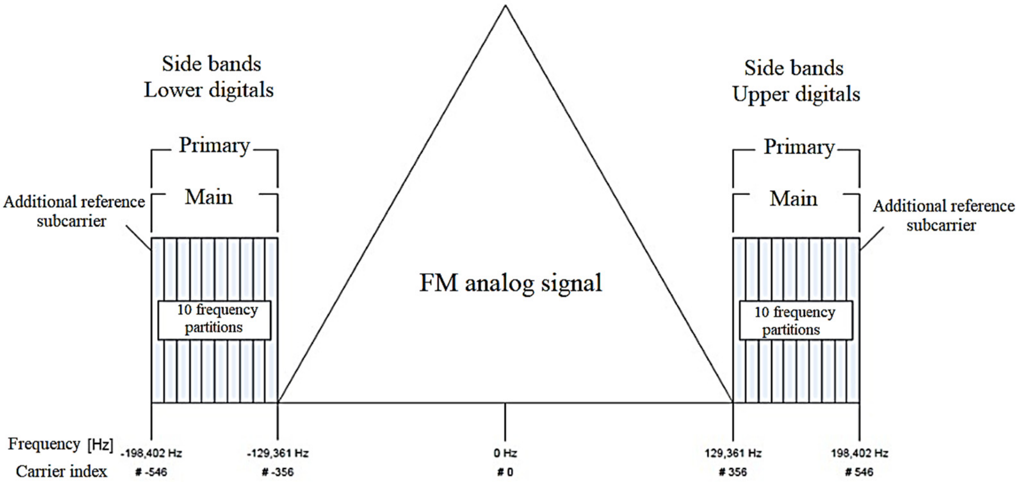

Figure 1. IBOC FM hybrid waveform spectrum. Source: https://www.rfwireless-world.com/.

2.3. Spectrum for the extended hybrid mode

In the extended hybrid mode, sidebands are added to the primary sidebands of the hybrid mode, causing the bandwidth of the hybrid sidebands to be extended towards the central area, where the analog FM component is located. This modification increases the digital capacity of the system. However, the analog coverage is affected, because the analog bandwidth is reduced. Depending on the mode of service, one, two or four frequency divisions are added at the inner edge of the primary main sideband to the detriment of the bandwidth allocated to the analog FM signal. In addition, to the subcarrier structure described in hybrid mode, the extended sidebands comprising the subcarriers from - 356 to 280 and from 280 to 356 must be added. The amplitude of the subcarriers of the extended sidebands has the same level as that of the main primary subcarriers. Figure 2 shows the spectrum corresponding to the IBOC FM extended hybrid IBOC model [3].

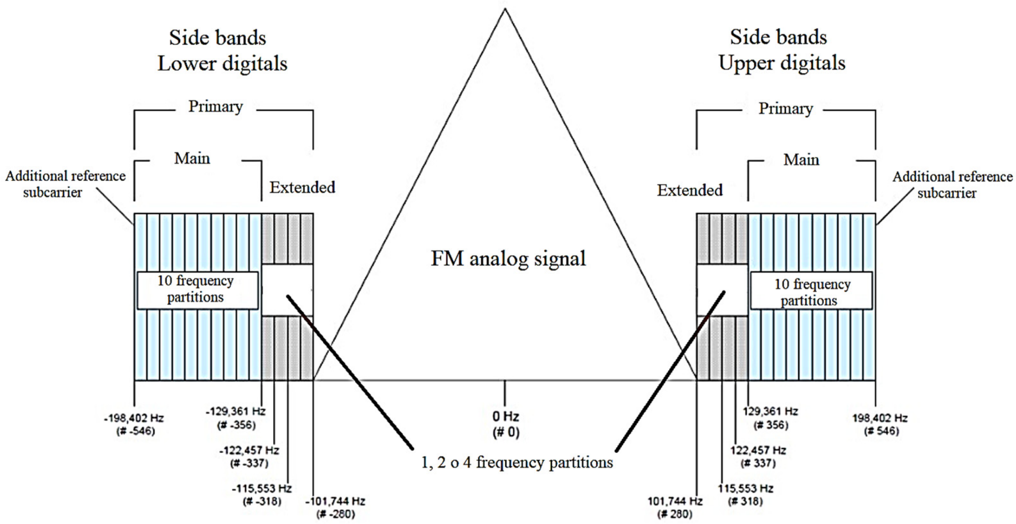

Figure 2. Spectrum corresponding to the IBOC FM extended hybrid model. Source: https://www.rfwireless-world.com/.

2.4. Spectrum for all-digital mode

In the all-digital mode, the spectrum is constructed by eliminating the analog signal in its entirety, allowing the full bandwidth to be used for secondary sidebands. The scheme allows for an improved mode of operation, where broadcasters will switch from the hybrid system to the all-digital system, thus achieving better quality levels. Figure 3 shows the carrier structure for the all-digital

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mode. Each of the secondary sidebands consists of 10 main frequency divisions and 4 extended frequency divisions.

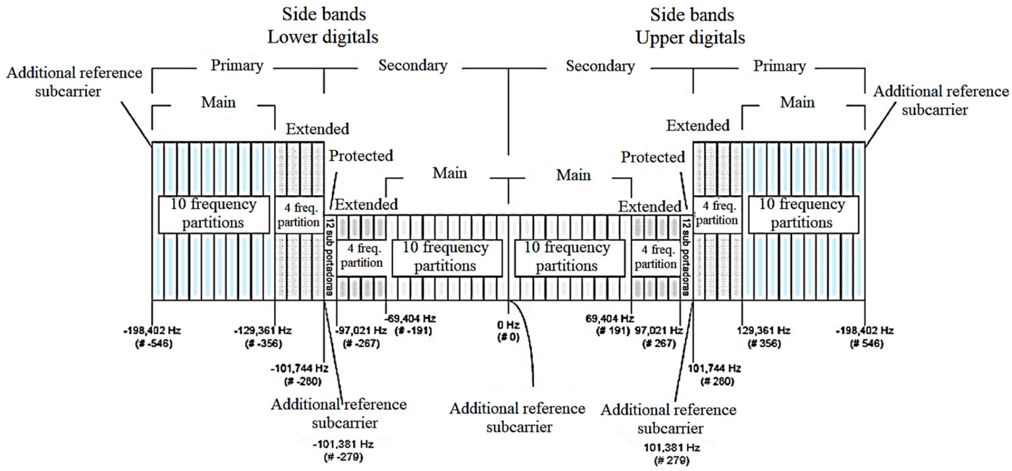

Figure 3. Spectrum of the fully digital IBOC FM waveform. Source: https://www.rfwireless-world.com/.

Two reference carriers are transmitted, those numbered - 279 and 279 and 12 protected OFDM subcarriers that are in that area of the spectrum least likely to be interfered with by analog and digital interference. Each secondary sideband comprises carriers 1 to 190 and - 1 to 190. The extended secondary sidebands comprise subcarriers 191 to 266 and - 191 to - 266. The protected sidebands comprise subcarriers - 267 to - 278 and 267 to 278. The average power of the secondary subcarriers will be between 5 and 20 dB below the primary subcarriers [16].

In the IBOC FM system, different service modes associated with the operating modes and logical channels are defined. For the hybrid mode, the operating modes range from MP1 to MP7. In turn, for the all-digital operating mode, service modes ranging from MS1 to MS4 are available. IBOC has 4 main logic channels (P1, P2, P3 and PIDS) and 6 secondary logic channels (S1, S2, S3, S4, S5 and SIDS). A logical channel is a signal path carrying data frames with a given quality of service. Channels P1, P2, P3 and P4 are used to configure the different primary audio services and the PIDS channel provides the primary data service (IDS). Channels S1, S2, S3, S4 and S5 are used only in the all-digital system for the transmission of data or ambient sound (supplementary audio). The SIDS provides the secondary data service.

The system control channel (SCCH) carries the control and status information relating to the operating mode and configuration parameters. In addition, the OFDM subcarriers are arranged in groups called frequency divisions. Each frequency division consists of 18 data subcarriers and one reference subcarrier, with a subcarrier spacing of 363.4 Hz. Table 1 describes the most important parameters of the IBOC FM system [17]. Tables 2 and 3 show the characterization of the primary and secondary logical channels, respectively, according to the service mode established for transmission.

Table 1. IBOC FM system parameters.

Parameter	Symbol	Units	Exact value	Approximate value
OFDM subcarrier spacing	Δf	Hz	$\frac{1488375}{4096}$	363.4
Predefined cyclic width	α	-	$\frac{7}{128}$	5.469x10^{-2}
OFDM symbol duration	T_s	s	$\frac{1+\alpha}{\Delta f}$	2.902x10^{-3}

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OFDM symbol rate	R_s	Hz	$\frac{1}{T_s}$	344.5
L1 frame duration	T_f	s	$512.T_s$	1.486
L1 frame speed	R_f	Hz	$\frac{1}{T_f}$	$6.729 x 10^{-1}$
L1 block duration	T_b	s	$32.T_s$	$9.288 x 10^{-2}$
L1 block speed	R_b	Hz	$\frac{1}{T_b}$	10.77
Duration of even block L1	T_p	s	$64.T_s$	$1.858 x 10^{-1}$
Torque block speed L1	R_p	Hz	$\frac{1}{T_p}$	5.383

Table 2. Characterization of primary logic channels according to service mode (IBOC FM).

Service mode	**logical channel**	**Transfer**			**L1 latency [s]**	**Relative robustness**
		Frame size [bits]	Frame speed [Hz]	Frame module		
MP1	P1	146.176	R_f	1	T_f	2
	PIDS	80	R_b	16	T_b	3
MP2	P1	146.176	R_f	1	T_f	2
	P3	2.304	R_p	8	$2T_f$	3
	PIDS	80	R_b	16	T_b	3
MP3	P1	146.176	R_f	1	T_f	2
	P3	4.608	R_p	8	$2T_f$	3
	PIDS	80	R_b	16	T_b	3
MP4	P1	176.176	R_f	1	T_f	2
	P3	9.216	R_p	8	$2T_f$	3
	P4	9.216	R_p	8	$2T_f$	3
	PIDS	80	R_b	16	T_b	3
MP5	P1	4.608	R_p	8	$T_p + T_{dd}$	1
	P2	109.312	R_f	1	T_f	2
	P3	4.608	R_p	8	$2T_f$	3
	PIDS	80	R_b	16	T_b	3
MP6	P1	9.216	R_p	8	$T_p + T_{dd}$	1
	P2	72.448	R_f	1	T_f	2
	PIDS	80	R_b	16	T_b	3

Table 3. Characterization of the secondary logic channels according to the service mode (IBOC FM).

Service mode	**logical channel**	**Transfer**			**L1 latency [s]**	**Relative robustness**
		Frame size [bits]	Frame speed [Hz]	Frame module		
MS1	S4	18.272	R_p	8	T_p	7

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	S5	512	R_b	16	T_b	6
	SIDS	80	R_b	16	T_b	8
MS2	S1	4.608	R_p	8	$T_p + T_{dd}$	5
	S2	109.312	R_f	1	T_f	9
	S3	4.608	R_p	8	T_p	11
	S5	512	R_b	16	T_b	6
	SIDS	80	R_b	16	T_b	10
MS3	S1	9.216	R_p	8	$T_p + T_{dd}$	5
	S2	72.448	R_f	1	T_f	9
	S5	512	R_b	16	T_b	6
	SIDS	80	R_b	16	T_b	10
MS4	S1	4.608	R_p	8	T_p	11
	S2	146.176	R_f	1	T_f	9
	S3	4.608	R_p	8	T_p	11
	S5	512	R_b	16	T_b	6
	SIDS	80	R_b	16	T_b	10

2.5. Audio source encoding and compression in IBOC

The human voice (analog), to be transmitted efficiently through any medium in a digital format, requires a transformation process known as "digitization", which is performed by a device called CoDec (Codec / Decoder), where each transmission mechanism may have several alternatives of Codecs, as appropriate. Considering that, for the transport of voice information it is necessary to assemble the information in the form of packets or PDUs, the required bandwidth will depend on the overhead generated by these packets [18].

IBOC uses a PAC encoding algorithm from Lucent Technologies, which is a coding method that uses advanced signal processing techniques and the use of psychoacoustic models, through which it is possible to obtain a high compression of the source signal. It uses a sampling frequency of 44.1 kHz, 16 bits of resolution and by means of a bank of special filters, it obtains a complete description of the audio signal whose processing facilitates an auditory perception analysis for its quantification and subsequent coding, optimizing the transmitted information to the extreme. One of the main benefits of PAC is that it allows audio compression at different bit rates, even as low as 6 kbps. When the channel quality is 6 to 8 kbit/s, the result is similar to an AM signal. For a channel capacity between 16 to 24 kbit/s, the result is similar to an FM signal and when a capacity of 32 or 64 kbps is available, the audio quality is considered very similar to what can be obtained on a compact disc. In the particular case of IBOC FM, it is possible to perform transmission processes of high-quality audio channels (comparable to compact disc) with transmission speeds of only 96 kb/s. In turn, the IBOC FM system (standardized as NRSC-5) does not propose a specific source encoder, but makes use of nominal and minimum rates of the source encoders for each of the operating modes [3].

2.6. Estimation of the transmission rate required by the IBOC FM system

To estimate the values corresponding to the calculation of the transmission velocity, the following expression is used according to the recommendations of the standard [NRSC-5B] and the values recorded in Tables 1, 2 and 3, using the following expression:

$$V_{tx}[bps] = \text{frame size [bits]}.\text{frame speed [Hz]} \tag{5}$$

For example, if you want to estimate the baud rate at IBOC FM, for service mode MP1, and output through logic channel P1:

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$$V_{tx}[bps] = 146176\left[\frac{44100}{65536}\right] \approx 98.4[kbps] \quad (6)$$

Tables 4 and 5 show the required transmission rates of the primary and secondary logical channels respectively as a function of the service mode.

Table 4. Transmission rates of primary logic channels.

Service mode	Approximate transfer speed [kbps]					Waveform
	P1	P2	P3	P4	PIDS	
MP1	98	N/A	N/A	N/A	1	Hybrid
MP2	98	N/A	12	N/A	1	Extended hybrid
MP3	98	N/A	25	N/A	1	Extended hybrid
MP4	98	N/A	25	25	1	Extended hybrid
MP5	25	74	25	N/A	1	All-digital extended hybrid
MP6	50	49	N/A	N/A	1	All-digital extended hybrid

Table 5. Transmission rates of secondary logic channels.

Service mode	Approximate transfer speed [kbps]						Waveform
	S1	S2	S3	S4	S5	SIDS	
MS1	0	0	0	98	6	1	Hybrid
MS2	25	74	25	0	6	1	Extended hybrid
MS3	50	49	0	0	6	1	Extended hybrid
MS4	25	98	25	0	6	1	Extended hybrid

3. Results

3.1. Description of the proposed scenario

For understand the use of the Shapley value as a strategy for resource optimization under an IBOC broadcasting scheme in FM, a scenario is proposed with twelve (12) nodes or digital broadcasting stations perform IBOC hybrid transmission processes over FM, under an RF channel which for the particular case will be considered to offer a total bandwidth of 1150 kbps, due to conditions that affect the maximum performance of the channel.

Recorded in Table 6 each class of traffic per node, where it can be observed that the total BW required is 1387 kbps, which is higher than the total BW available in the RF channel, which indicates that the operation mode will be under a saturation state. To calculate the value of BW_T it to use the expression given by (7):

$$BW_T = \frac{1}{T_s}\sum_{k=1}^{N_{sp}} log_2\left[1 + \frac{SNR_k}{\Gamma}\right] \quad (7)$$

N_{sp}: Number of subcarriers
T_s: Time of an OFDM symbol ($T_s = 2.902ms$) for the particular case of IBOC FM.
SNR_k: Signal to Noise Ratio present on the subcarrier k.
Γ : This is known as SNR gap, which represents the loss in SNR incurred by using a specific discrete coding scheme. At [19] it is suggested that the value of Γ can be calculated for practical purposes by (8):

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$$\Gamma = -\frac{1}{1,6} ln\left[\frac{BER_{obj}}{0,2}\right] \tag{8}$$

Where the BER_{obj}, corresponds to the BER value to be sustained. For the particular case a value of 10^{-6}. For the proposed scenario we have considered $N=12$ nodes that make up the RF system in saturated state ($BW_T \leq \sum_{i=1}^{N} d_i$) and a total available Bit-rate $BW_T = E = 1150kbps$

Table 6. BW required at each node according to service mode.

Node i	Service mode	Logic channels BW required [kbps].				
		P1	P2	P3	P4	PIDS
1	MP1	98				
2	MP2	98		12		
3	MP3	98		25		1
4	MP3	98		25		
5	MP5	25	74	25		1
6	MP1	98				
7	MP4	98		25	25	1
8	MP6	50	49			1
9	MP2	98		12		
10	MP5	25	74	25		1
11	MP5	25	74	25		1
12	MP6	50	49			1

As mentioned above, a bankrupt set will be considered consistent with the saturation state of the RF channel, in order to calculate the transferable utility value for each of the coalitions. The algorithm to perform the calculation of each of the transferable utility values is as follows:

Algorithm: CalculateCoalitions

```
1. Initialize Z as a vector containing numbers from 1 to Nj.
2. Set n_coal to 0.
3. For each coalition size i from 1 to Nj:
   a. Increment n_coal by n choose i, where n is Nj.
4. Initialize M_Coalitions as a matrix of zeros with dimensions (n_coal x Nj).
5. Initialize c (counter) to 0.
6. For each coalition size i from 1 to Nj:
   a. Generate all combinations S of Z taken i at a time.
   b. For each combination S:
      i. Increment the counter.
      ii. Initialize Suma_d to 0.
      iii. For each member k in S:
         - Suma_d += V(k)
         - M_Coalitions(c, k) = k
```

iv. Calculate Suma_dT as BW_T minus (Total_V minus Suma_d).
v. Set V_Coalition(c) to the maximum of 0 and Suma_dT.
End of Algorithm.

Table 7 presents the transferable utility value calculated for each of the possible coalitions in the saturation state, under the use of the bankruptcy game. The values $v(S)$ consider that the principle of rational individuality must be satisfied (N, v) and the principle of efficiency $\sum_{i\in S}\varphi_i(v) \geq v(S) \quad \forall S \subseteq N, \sum_{i\in N}\varphi_i(v) = v(N)$.

Table 7. Transferable utility value for each of the coalitions $v(S)$.

Coalition	Value [*1E+3]	Coalition	Value [*1E+3]
{1}	0.00	{7}	0.00
{2}	0.00	{8}	0.00
{3}	0.00	{9}	0.00
{4}	0.00	{10}	0.00
{5}	0.00	{11}	0.00
{6}	0.00	{12}	0.00
Grand coalition		{1,2,3,4,5,6,7,8,9,10,11,12}	1150

Therefore, the equations that describe each one of the imputations for the game are (N, v) are:

$$\begin{gathered}\varphi_1(v) \geq 0\\ \varphi_2(v) \geq 0\\ \varphi_3(v) \geq 0\\ \varphi_4(v) \geq 0\\ \varphi_5(v) \geq 0\\ \varphi_6(v) \geq 0\\ \varphi_7(v) \geq 0\\ \varphi_8(v) \geq 0\\ \varphi_9(v) \geq 0\\ \varphi_{10}(v) \geq 0\\ \varphi_{11}(v) \geq 0\\ \varphi_{12}(v) \geq 0\end{gathered} \quad (9)$$

The efficiency equation for the proposed scenario is:

$$\begin{gathered}\varphi_1(v) + \varphi_2(v) + \varphi_3(v) + \varphi_4(v) + \varphi_5(v) + \varphi_6(v) + \varphi_7(v) + \varphi_8(v) +\\ \varphi_9(v) + \varphi_{10}(v) + \varphi_{11}(v) + \varphi_{12}(v) = 1150 * 10^3\end{gathered} \quad (10)$$

Based on the above expressions, the core of the game would be as follows:

$$C(v) = \left\{\begin{gathered}\varphi(v) \in \mathbb{R}^n | \sum_{i\in S}\varphi_i(v) \geq v(S) \quad \forall S \subseteq N, \varphi_1(v) + \varphi_2(v) + \varphi_3(v) + \varphi_4(v)\\ +\varphi_5(v) + \varphi_6(v) + \varphi_7(v) + \varphi_8(v) + \varphi_9(v) + \varphi_{10}(v) + \varphi_{11}(v) + \varphi_{12}(v) = v(N)\end{gathered}\right\} \quad (11)$$

The algorithm to calculate the Shapley value $\varphi_k(v)\ \forall k \in N$ is as follows:

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```
Algorithm: ShapleyValueCalculation
1. For each player k in [1, Nj]:
   a. For each coalition size lg in [1, Nj]:
      i. Initialize nMk to 0.
      ii. For each coalition i in [1, n_coal]:
          - For each player j in [1, Nj]:
              * If M_Coalitions(i, j) is equal to k:
                  - Set N_ceros to 0.
                  - For each player g in [1, Nj]:
                      + If M_Coalitions(i, g) > 0:
                          * Increment N_ceros by 1.
                  - If N_ceros is equal to lg:
                      * Increment nMk by 1.
                      * Copy the coalition M_Coalitions(i, :) to Mk_Coalitions(nMk, :).
                      * Copy the coalition value V_Coalition(i) to Vk_Coalition(nMk).
                      * Break the loop for j.
      iii. Initialize Sub_coal as a matrix of zeros with dimensions (nMk, Nj).
      iv. For each coalition i in [1, nMk]:
          - Set j to 0.
          - For each player g in [1, Nj]:
              * If Mk_Coalitions(i, g) > 0 and Mk_Coalitions(i, g) is not equal to k:
                  + Increment j by 1.
                  + Set Sub_coal(i, j) to Mk_Coalitions(i, g).
      v. For each coalition i in [1, nMk]:
          - If lg is equal to 1:
              * Set Vk_Sub_coal(i) to 0.
          - Else:
              * For each coalition j in [1, n_coal]:
                  + If Sub_coal(i, :) is equal to M_Coalitions(j, :):
                      * Set Vk_Sub_coal(i) to V_Coalition(j).
      vi. Set M_Shapley(k, lg) to the sum of (Vk_Coalition - Vk_Sub_coal).
      vii. Clear Vk_Coalition, Vk_Sub_coal, Mk_Coalitions, and Sub_coal.

2. For each player S in [1, Nj]:
   - Set B(S) to (factorial(S-1) * factorial(Nj - S)) / factorial(Nj).

3. For each player i in [1, Nj]:
   - Set Z to M_Shapley(i, :) element-wise multiplied by B.
   - Set Peso(i) to the sum of Z.
```

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Table 8 shows the result corresponding to the Shapley matrix in the saturation state for the proposed scenario. The last column shows the Shapley value calculated for each of the players. Additionally, it can be seen that $\sum_{i\in N}\varphi_i(v) = v(N) = BW_T$

Table 8. Shapley Matrix for the proposed scenario.

Player	Contribution to the coalition containing j players												$\varphi_j(v)$
j	1	2	3	4	5	6	7	8	9	10	11	12	[*1E+3]
1	0	10	4867	16170	32340	45276	45276	32340	16170	5390	1078	98	80.95
2	0	22	5419	18150	36300	50820	50820	36300	18150	6050	1210	110	90.88
3	0	82	6109	20460	40920	57288	57288	40920	20460	6820	1364	124	102.88
4	0	78	6060	20295	40590	56826	56826	40590	20295	6765	1353	123	102.02
5	0	86	6158	20625	41250	57750	57750	41250	20625	6875	1375	125	103.73
6	0	10	4867	16170	32340	45276	45276	32340	16170	5390	1078	98	80.95
7	0	270	7422	24585	49170	68838	68838	49170	24585	8195	1639	149	125.04
8	0	12	4959	16500	33000	46200	46200	33000	16500	5500	1100	100	82.60
9	0	22	5419	18150	36300	50820	50820	36300	18150	6050	1210	110	90.88
10	0	86	6158	20625	41250	57750	57750	41250	20625	6875	1375	125	103.73
11	0	86	6158	20625	41250	57750	57750	41250	20625	6875	1375	125	103.73
12	0	12	4959	16500	33000	46200	46200	33000	16500	5500	1100	100	82.60
$P(j)$	0,0833	0,0076	0,0015	0,0005	0,0003	0,0002	0,0002	0,0003	0,0005	0,0015	0,0076	0,0833	1150

Table 9 shows the values for each node corresponding to the bandwidth requested and the bandwidth assigned, considering that the total bandwidth demanded by the RF network is greater than the total bandwidth available in the channel and therefore, the values assigned for each node should be lower than the demanded value. The use of the Shapley value allocates in an equitable and proportional manner to the needs of each node, favoring the use of the Shapley value as a resource optimization strategy for an IBOC FM broadcasting system.

Table 9. BW requested, BW assigned (Shapley) for a channel saturation state.

Node i	Service mode	Logic channels BW [kbps]	
		BW Requested d_i	BW Assigned $\varphi_i(v)$
1	MP1	98	80.95
2	MP2	110	90.88
3	MP3	124	102.88
4	MP3	123	102.02
5	MP5	125	103.73
6	MP1	98	80.95
7	MP4	149	125.04
8	MP6	100	82.60
9	MP2	110	90.88
10	MP5	125	103.73
11	MP5	125	103.73
12	MP6	100	82.60

3.2. Evaluation of Optimal BW-PL vs. BW-Shapley approaches

To assess the level of optimization achieved through the application of the Shapley value, an alternative optimization method was devised for computing the Bandwidth (BW) for each node (denoted as i). Subsequently, a comparative analysis of treatments was conducted. In addressing this,

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the Bandwidth Allocation problem was formulated as a Linear Programming (LP) problem, stated as follows:

$$Maximize \sum_{i=1}^{n} x_i \tag{12}$$

Subject to:

$$0 \leq x_i \leq d_i$$

$$\sum_{i=1}^{n} x_i \leq BW_T$$

Where n, d_i y x_i correspond to the number of nodes ($n=12$), the bandwidth requested by node i and the bandwidth assigned to node i respectively. The LP problem was solved using the Optimization Toolbox in Matlab, employing various optimization methods. The objective function, constraints, and initial iteration point were organized in matrix form. The values were set as follows:
F: Vector of coefficients of the Objective function
A,b: Inequality restrictions ($Ax \leq b$).
BW_T : 1150 Kbps.
lb, ub: Set the lower and upper limits allowed for each of the nodes respectively.

Finally, the following expression is used to calculate the optimal solution to the problem, using the "interiorpoint-legacy" algorithm, which yielded the best results compared to the "dual-simplex" and "interior-point" algorithms);:

```
options=optimoptions("linprog","Algorithm","interior-point-legacy");
[x,fval] = linprog(F,A,b,[],[],lb, ub, options)
```

Where *x and fval* correspond to the solution vector and the maximum value that the objective function can reach. Table 10 presents the results obtained for the proposed optimization model as a function of the BW requested by each node.

Table 10. BW requested, BW assigned by Linear Programming for a channel saturation state.

Node i	Service mode	Logic channels BW [kbps]	
		BW Requested d_i	BW Assigned Model PL
1	MP1	98	77.8238
2	MP2	110	89.9117
3	MP3	124	104.3908
4	MP3	123	103.3451
5	MP5	125	105.4379
6	MP1	98	77.8238
7	MP4	149	130.8483
8	MP6	100	79.8156
9	MP2	110	89.9117
10	MP5	125	105.4379

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11	MP5	125	105.4379
12	MP6	100	79.8156

Table 11 shows the values to the optimal BW-PL (BWO), BW-Shapley ($BWSh$), $X1$ y $X2$ are the difference between the bandwidth requested by each node (di) and the bandwidth assigned through the PL and Shapley respectively.

Table 11. Optimal BW-PL vs BW-Shapley and differences in BW allocation according to channel state.

Node i	BW Requested d_i	Logic channels BW [kbps]			
		BW_O	$BWSh$	X_1 $d_i - BW_O$	X_2 $di - BWSh$
1	98	77.8238	80.95	20.1762	17.05
2	110	89.9117	90.88	20.0883	19.12
3	124	104.3908	102.88	19.6092	21.12
4	123	103.3451	102.02	19.6549	20.98
5	125	105.4379	103.73	19.5621	21.27
6	98	77.8238	80.95	20.1762	17.05
7	149	130.8483	125.04	18.1517	23.96
8	100	79.8156	82.60	20.1844	17.4
9	110	89.9117	90.88	20.0883	19.12
10	125	105.4379	103.73	19.5621	21.27
11	125	105.4379	103.73	19.5621	21.27
12	100	79.8156	82.60	20.1844	17.4

To assess whether employing the Shapley value as an optimization strategy in an RF network over IBOC FM leads to a superior resource allocation process compared to the PL-optimization method, the following hypotheses are proposed:

$$H_o: \mu_x \leq \mu_y \rightarrow \mu_x - \mu_y \leq 0 \rightarrow \mu_z \leq 0$$
$$H_a: \mu_x > \mu_y \rightarrow \mu_x - \mu_y > 0 \rightarrow \mu_z > 0 \quad (13)$$

Here, μ_y and μ_x represent the means for the difference between requested bandwidth and assigned bandwidth, through the PL-optimization methods and the Shapley value respectively. Hypothesis H_o suggest a significant difference between means, with μx being lower than μ_y, indicating that the Shapley value leads to a more effective optimization process. Hypothesis H_a suggest the opposite. Additionally, a new variable Z is introduced to refine the hypotheses.

To evaluate these hypotheses, a paired t-test is employed, commonly used for assessing the statistical validity of differences between two random samples [20]. The process involves the following steps:

Step 1 : Define a new random variable $Z = X - Y$ and calculate the mean ($\bar{Z}$) and standard deviation (S_z) for Z. The resulting values are 0.0008 and 2.7544 respectively.

Step 2 : Calculate the test statistic (d) using the expression :

$$d = \frac{\bar{Z}}{S_z}\sqrt{n} = \frac{0.0008}{2.7544}\sqrt{12} = 1.006\text{x}10^{-3} \quad (14)$$

Where d is the statistic value, and n is the number of samples for both proposed models.

Step 3: Establish the acceptance range for H_o $\{t: t < T_{(\alpha;n-1)}\}$ at 5% significance level ($\alpha = 0.05$) with $n - 1$ degrees of freedom. For this case, $T(0.05; 11) = 1.7959$ value, defining the acceptance interval for H_o as $(-\infty, 1.67959)$. Evaluating the statistic d reveals that it falls within the acceptance interval, leading to the non-rejection of H_o. Consequently, it can be concluded with 95% confidence that the Shapley value stands as an excellent alternative for executing high-quality optimization processes.

4. Conclusions

In response to the need for equitable resource distribution among nodes in an RF network over IBOC FM, cooperative game theory with transferable utility is proposed as an optimization strategy. The goal is to maximize the allocated bandwidth for each node based on the service class, incorporating Shapley Value in a saturated channel state. This approach optimizes resource allocation by recognizing cooperation among nodes and ensuring adequate Quality of Service (QoS) levels. In the proposed scenario, the cooperative game is framed as a bankruptcy game, characterized by a ternary structure (N, d, C). Here, $N = \{1,2,\dots, n\}$ denotes the set of players, $d = \{d1, d2, \dots, dn\}$ with $di \geq 0$ for all $i \in N$ represents the vector of demands, and C signifies the net value to be distributed among the elements of N. This formulation allows for the establishment of game imputations, leading to the estimation of the resulting payment vector using four proposed techniques. The outcomes of employing the Shapley value, demonstrated outstanding results in equitable resource allocation, closely aligning with the specified service class requirements, with a confidence level of 95%. Furthermore, it was observed that the total sum of bandwidth assigned to each player aligns precisely with the total bandwidth available in the RF channel. This reinforces the effectiveness of the Shapley Value as an optimal strategy for resource allocation in cooperative RF network scenarios.

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