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Plithgoenic Fuzzy Risk Management and Plithgoenic Fuzzy IT Service Management

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Article

Plithgoenic Fuzzy Risk Management and Plithgoenic Fuzzy IT Service Management

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Abstract

Plithogenic sets represent elements through attribute-driven memberships together with a contradiction map over attribute values [1–3]. Upside-Down Logic reverses truth and falsity under a structured transformation, flipping evaluations while preserving internal consistency across logical contexts. Risk management is the process of identifying, assessing, and mitigating potential losses to minimize the impact on organizational objectives [4–6]. IT service management (ITSM) is the process of designing, delivering, managing, and improving IT services to meet business needs effectively, efficiently, and consistently [7–9]. In this paper, we formally introduce *Plithogenic Fuzzy Risk Management* and *Plithogenic Fuzzy IT Service Management* and investigate their structural properties.

Keywords: plithgoenic fuzzy risk management; plithgoenic fuzzy IT service management; fuzzy set; plithogenic set; upside-down logic

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1. Preliminaries

This section gathers the background notions and notation required for the main results.

1.1. Upside-Down Logic

We formalize *Upside-Down Logic*: under suitable contextual transformations, truth and falsity of lemmas are interchanged, providing a framework to capture reversals and contextual ambiguity in reasoning systems (see, e.g., [10–16]).

Definition 1.1 (Context). [11,12] A *context* $\mathcal{C}$ is a collection of parameters or conditions under which lemmas are to be evaluated; typical components may be spatial, temporal, semantic, or interpretive in nature.

Definition 1.2 (Logical System). (cf. [17]) A *logical system* is a structure

$$\mathcal{M} = (\mathcal{P}, \mathcal{V}, v),$$

where $\mathcal{P}$ is the set of lemmas (formulas) in a formal language $\mathcal{L}$, $\mathcal{V}$ is a set of truth values (e.g., $\{\text{True}, \text{False}\}$ in the classical case), and $v : \mathcal{P} \rightarrow \mathcal{V}$ is a valuation assigning to each lemma a truth value. Optionally, $\mathcal{M}$ may specify a set of axioms $\mathcal{A} \subseteq \mathcal{P}$ and a collection of inference rules $\mathcal{I}$.

Notation 1.3. Given a set of lemmas $\mathcal{P}$ and a family of contexts $\mathcal{C}$, we write

$$T : \mathcal{P} \times \mathcal{C} \longrightarrow \{\text{True}, \text{False}, \text{Indeterminate}\}$$

for the context-sensitive truth assignment that evaluates each lemma–context pair. We also fix a formal language $\mathcal{L}$ and a logical system $\mathcal{M} = (\mathcal{P}, \mathcal{V}, v)$ as above.

Definition 1.4 (Upside-Down Logic). [11,12] An *Upside-Down Logic* is obtained from a logical system $\mathcal{M}$ by equipping it with a transformation U acting on lemmas and/or on contexts, thereby producing a system $\mathcal{M}'$ such that for every lemma $A \in \mathcal{P}$ and context $\mathcal{C}$:

- (Truth $\rightarrow$ Falsity) If $v(A) = \text{True}$ in $\mathcal{C}$, then $v(U(A)) = \text{False}$ in $U(\mathcal{C})$.
- (Falsity $\rightarrow$ Truth) If $v(A) = \text{False}$ in $\mathcal{C}$, then $v(U(A)) = \text{True}$ in $U(\mathcal{C})$.

Moreover, U is required to be well posed and internally consistent within $\mathcal{M}'$.

Example 1.5 (Threshold–comparator flip on the same context). Let the language contain atomic statements of the form

$$A_\theta : (x \geq \theta) \quad (\theta \in \mathbb{R}),$$

and let the context be the parameter value $\mathcal{C}_\theta := \{\theta\}$. Fix a valuation v so that $v(A_\theta) = \text{True}$ in $\mathcal{C}_\theta$ iff the designated real x satisfies $x \geq \theta$.

Define the Upside-Down transform U by

$$U(A_\theta) := (x < \theta), \quad U(\mathcal{C}_\theta) := \mathcal{C}_\theta.$$

Then:

- (Truth $\rightarrow$ Falsity) If $v(A_\theta) = \text{True}$ in $\mathcal{C}_\theta$, then $x \geq \theta$, hence $x < \theta$ is false in the same context, i.e., $v(U(A_\theta)) = \text{False}$ in $U(\mathcal{C}_\theta)$.
- (Falsity $\rightarrow$ Truth) If $v(A_\theta) = \text{False}$ in $\mathcal{C}_\theta$, then $x < \theta$, so $v(U(A_\theta)) = \text{True}$ in $U(\mathcal{C}_\theta)$.

Moreover, U is involutive on formulas:

$$U(U(A_\theta)) = U(x < \theta) = (x \geq \theta) = A_\theta,$$

hence well posed and internally consistent on $\mathcal{M}'$ obtained by adding U .

Example 1.6 (Set–membership flip with complemented context). Fix a nonempty universe X and a designated element $x \in X$. For any subset $E \subseteq X$ let the lemma be

$$A_E : (x \in E), \quad \text{and the context } \mathcal{C}_E := E.$$

Let $v(A_E) = \text{True}$ in $\mathcal{C}_E$ precisely when $x \in E$.

Define the Upside–Down transform U by

$$U(A_E) := (x \in E^c), \quad U(\mathcal{C}_E) := E^c \quad (\text{complement in } X).$$

Then:

- (Truth $\rightarrow$ Falsity) If $v(A_E) = \text{True}$ in $\mathcal{C}_E$, then $x \in E$. Evaluating $U(A_E)$ in $U(\mathcal{C}_E) = E^c$ yields $x \in E^c$, which is false because $x \in E$. Thus $v(U(A_E)) = \text{False}$ in $U(\mathcal{C}_E)$.
- (Falsity $\rightarrow$ Truth) If $v(A_E) = \text{False}$ in $\mathcal{C}_E$, then $x \notin E$, hence $x \in E^c$. Therefore $v(U(A_E)) = \text{True}$ in $U(\mathcal{C}_E)$.

Finally, U is an involution on both lemmas and contexts:

$$U(U(A_E)) = A_E, \quad U(U(\mathcal{C}_E)) = \mathcal{C}_E,$$

so the induced system $\mathcal{M}'$ is coherent and stable under repeated application of U .

1.2. Plithogenic Set

A plithogenic set [1–3] represents elements via membership driven by explicit attributes together with a contradiction mapping between attribute values, thereby subsuming and extending the frameworks of fuzzy [18,19], intuitionistic [20,21], hesitant fuzzy sets [22,23], HyperFuzzy sets [24,25], and neutrosophic sets [26,27].

Definition 1.7 (Fuzzy Set). [18,28] A *Fuzzy set* τ in a non-empty universe Y is a mapping $\tau : Y \rightarrow [0, 1]$. A *fuzzy relation* on Y is a fuzzy subset δ in $Y \times Y$. If τ is a fuzzy set in Y and δ is a fuzzy relation on Y , then δ is called a *fuzzy relation on τ* if

$$\delta(y, z) \leq \min\{\tau(y), \tau(z)\} \quad \text{for all } y, z \in Y.$$

Definition 1.8 (Plithogenic Set). [1,29] Let S be a universe and $P \subseteq S$ a nonempty subset. A *plithogenic set* is a 5-tuple

$$PS = (P, v, Pv, pdf, pCF),$$

with the following ingredients:

- v — a chosen attribute;
- Pv — the value domain of v ;
- $pdf : P \times Pv \rightarrow [0, 1]^s$ — the *degree of appurtenance* (DAF);¹
- $pCF : Pv \times Pv \rightarrow [0, 1]^t$ — the *degree of contradiction* (DCF).

For every $a, b \in Pv$, the DCF satisfies

$$\text{reflexivity: } pCF(a, a) = 0, \quad \text{symmetry: } pCF(a, b) = pCF(b, a).$$

Here $s, t \in \mathbb{N}$ are, respectively, the appurtenance and contradiction dimensions.

Example 1.9 (Concrete instance of a Plithogenic Set (vector-valued appurtenance and contradiction)). Let the universe be $S = \{d_1, d_2, d_3\}$ (devices), and take the plithogenic support $P = \{d_1, d_3\} \subseteq S$. Choose the attribute $v = \text{“service facet”}$ with value domain

$$Pv = \{\text{Perf, Energy, Maint}\}.$$

¹ In the literature, DAF is modeled in several equivalent ways (e.g., powerset-valued or vector-valued). We adopt the standard $[0, 1]^s$ form; see [30].

We set the appurtenance dimension to $s = 2$ (e.g., “optimistic” and “pessimistic” degrees) and the contradiction dimension to $t = 2$ (e.g., “short-term” and “long-term” contradiction components). *Appurtenance (DAF)*. Define $pdf : P \times Pv \rightarrow [0, 1]^2$ by the table

	Perf	Energy	Maint
d_1	(0.85, 0.70)	(0.60, 0.45)	(0.75, 0.55)
d_3	(0.40, 0.30)	(0.90, 0.80)	(0.50, 0.40)

so that, for instance, $pdf(d_1, \text{Perf}) = (0.85, 0.70) \in [0, 1]^2$.

Contradiction (DCF). Define $pCF : Pv \times Pv \rightarrow [0, 1]^2$ as the symmetric, reflexive-zero matrix of 2-vectors

$$(pCF(a, b))_{a, b \in Pv} = \begin{pmatrix} (0, 0) & (0.20, 0.35) & (0.10, 0.25) \\ (0.20, 0.35) & (0, 0) & (0.30, 0.40) \\ (0.10, 0.25) & (0.30, 0.40) & (0, 0) \end{pmatrix}.$$

Clearly $pCF(a, a) = (0, 0)$ (reflexivity) and $pCF(a, b) = pCF(b, a)$ (symmetry). Hence $PS = (P, v, Pv, pdf, pCF)$ is a valid plithogenic set with $(s, t) = (2, 2)$.

Definition 1.10 (Plithogenic Fuzzy Set ($s = 1, t = 1$)). [1,31,32] A *plithogenic fuzzy set* is a plithogenic set $PS = (P, v, Pv, pdf, pCF)$ in which

$$pdf : P \times Pv \rightarrow [0, 1], \quad pCF : Pv \times Pv \rightarrow [0, 1].$$

For $x \in P$ and $a \in Pv$, write

$$\mu_P(x | a) := pdf(x, a) \in [0, 1],$$

the (single-component) fuzzy membership of x under attribute value a . The contradiction between two attribute values is the scalar

$$c(a, b) := pCF(a, b) \in [0, 1], \quad c(a, a) = 0, \quad c(a, b) = c(b, a).$$

Example 1.11 (Concrete instance of a Plithogenic Fuzzy Set ($s = t = 1$)). Let $P = \{s_{\text{web}}, s_{\text{email}}, s_{\text{storage}}\}$ be three IT services. Take the attribute $v = \text{“evaluation facet”}$ with

$$Pv = \{Q, U, R\},$$

standing for Quality, Utility, and Reliability. Since $s = t = 1$, the DAF and DCF are scalar-valued.

Memberships. Define $pdf : P \times Pv \rightarrow [0, 1]$ by

	Q	U	R
s_{web}	0.82	0.76	0.91
s_{email}	0.75	0.88	0.86
s_{storage}	0.80	0.72	0.94

so $\mu_P(s_{\text{web}} | R) = pdf(s_{\text{web}}, R) = 0.91$.

Contradiction. Let $c(a, b) := pCF(a, b) \in [0, 1]$ be the symmetric, reflexive-zero matrix

$$(c(a, b))_{a, b \in Pv} = \begin{pmatrix} 0 & 0.60 & 0.30 \\ 0.60 & 0 & 0.50 \\ 0.30 & 0.50 & 0 \end{pmatrix}.$$

Then $c(Q, Q) = 0$, $c(U, R) = c(R, U) = 0.50$, etc. Therefore $PS = (P, v, Pv, pdf, pCF)$ is a plithogenic fuzzy set with scalar membership and scalar contradiction.

1.3. Upside-Down Logic in Plithogenic Fuzzy Set with Contradiction Reset

We formalize an *upside-down* transformation for plithogenic fuzzy sets in which membership grades are complemented whenever the contradiction with a chosen anchor exceeds a prescribed threshold; immediately afterward, the involved contradiction is forced to zero to stabilize the context (cf. [16,33]).

Definition 1.12 (Upside-down transform with contradiction reset for a Plithogenic Fuzzy Set). (cf. [16,33]) Let $PS = (P, v, Pv, pdf, pCF)$ be a plithogenic fuzzy set. Write

$$\mu_P(x | a) := pdf(x, a) \in [0, 1], \quad c(a, b) := pCF(a, b) \in [0, 1],$$

with $c(a, a) = 0$ and $c(a, b) = c(b, a)$ for all $a, b \in Pv$. Fix an anchor attribute $b \in Pv$ and a threshold $\tau \in [0, 1]$. Define the activation locus

$$A_\tau(b) := \{a \in Pv : c(a, b) \geq \tau\}.$$

The *upside-down transform with contradiction reset* produces a new plithogenic fuzzy set

$$PS^{U_{b,\tau}} := (P, v, Pv, pdf^{U_{b,\tau}}, pCF^{U_{b,\tau}})$$

whose components are given, for every $x \in P$ and $a \in Pv$, by

$$pdf^{U_{b,\tau}}(x, a) := \begin{cases} 1 - \mu_P(x | a), & \text{if } a \in A_\tau(b), \\ \mu_P(x | a), & \text{if } a \notin A_\tau(b), \end{cases}$$

and the updated contradiction map $pCF^{U_{b,\tau}} : Pv \times Pv \rightarrow [0, 1]$ defined for all $u, v \in Pv$ as

$$pCF^{U_{b,\tau}}(u, v) := \begin{cases} 0, & \text{if } \{u, v\} = \{a, b\} \text{ for some } a \in A_\tau(b), \\ pCF(u, v), & \text{otherwise.} \end{cases}$$

Informally, whenever an attribute value a is flipped due to high contradiction with the anchor b , the post-transform structure sets the contradiction between a and b to 0, preventing immediate reactivation at the same threshold.

Example 1.13 (A concrete Upside-Down transform with contradiction reset). Let

$$PS = (P, v, Pv, pdf, pCF)$$

be a plithogenic fuzzy set as in Definition 1.12. Consider the finite universe and attribute domain

$$P = \{x_1, x_2, x_3\}, \quad Pv = \{A, B, C\},$$

where A, B, C are three evaluation facets. The initial (facetwise) memberships $\mu_P(x | a) := pdf(x, a)$ are given by

	A	B	C
x_1	0.20	0.60	0.70
x_2	0.90	0.40	0.50
x_3	0.30	0.80	0.20

and the symmetric contradiction function $c(\cdot, \cdot) := pCF(\cdot, \cdot)$ is

$$(c(a, b))_{a, b \in \{A, B, C\}} = \begin{pmatrix} 0 & 0.30 & 0.80 \\ 0.30 & 0 & 0.40 \\ 0.80 & 0.40 & 0 \end{pmatrix}.$$

Fix the anchor facet $b = C$ and the threshold $\tau = 0.7$. Then the activation locus from Definition 1.12 is

$$A_\tau(C) = \{a \in \{A, B, C\} : c(a, C) \geq 0.7\} = \{A\},$$

since $c(A, C) = 0.80$ while $c(B, C) = 0.40$ and $c(C, C) = 0$.

Membership update. By Definition 1.12, we complement exactly the memberships corresponding to the activated facet A and leave the others unchanged:

$$pdf^{U_{C,0.7}}(x, a) = \begin{cases} 1 - \mu_P(x | A), & a = A, \\ \mu_P(x | a), & a \in \{B, C\}. \end{cases}$$

Hence the transformed table is

	A	B	C
x_1	$1 - 0.20 = 0.80$	0.60	0.70
x_2	$1 - 0.90 = 0.10$	0.40	0.50
x_3	$1 - 0.30 = 0.70$	0.80	0.20

Contradiction reset. For the unique activated pair $\{A, C\}$, the post-transform contradiction is reset to 0; all other entries are preserved:

$$pCF^{U_{C,0.7}}(u, v) = \begin{cases} 0, & \{u, v\} = \{A, C\}, \\ pCF(u, v), & \text{otherwise.} \end{cases}$$

Thus

$$(c^{U_{C,0.7}}(a, b))_{a, b \in \{A, B, C\}} = \begin{pmatrix} 0 & 0.30 & 0 \\ 0.30 & 0 & 0.40 \\ 0 & 0.40 & 0 \end{pmatrix}.$$

The transform preserves $pdf^{U_{C,0.7}}(\cdot, \cdot) \in [0, 1]$ and yields a symmetric, reflexive-zero contradiction function $pCF^{U_{C,0.7}}$. Moreover, because $c^{U_{C,0.7}}(A, C) = 0 < \tau$, the specific flip just performed cannot be immediately re-triggered at the same threshold, illustrating the stabilization role of the contradiction reset.

1.4. Fuzzy Risk Management

Risk Management is the process of identifying, assessing, and mitigating potential losses to minimize the impact on organizational objectives[4–6]. The integration of fuzzy logic with risk management has been extensively examined in various research studies [34–36].

Definition 1.14 (Mathematical Framework for Fuzzy Risk Management). [37] Let $(\Omega, \mathcal{F}, P)$ be a probability space and $D \subseteq \mathbb{R}^n$ a nonempty closed convex decision set. Let

$$L : D \longrightarrow L^\infty(\Omega, \mathcal{F}, P), \quad x \mapsto L(x)$$

be the mapping which assigns to each decision $x \in D$ its (essentially bounded) loss random variable $L(x)$. Let

$$\rho : L^\infty(\Omega, \mathcal{F}, P) \longrightarrow \mathbb{R}$$

be a (coherent) risk measure as in the crisp framework. Finally, let

$$\Phi : \mathbb{R} \longrightarrow [0, 1]$$

be a continuous, strictly decreasing “satisfaction–risk” mapping (e.g. $\Phi(r) = e^{-\lambda r}$ for some $\lambda > 0$). Then we define the *fuzzy decision set*

$$\mathcal{A} = \{(x, \mu_{\mathcal{A}}(x)) \mid x \in D\},$$

where the membership function $\mu_{\mathcal{A}} : D \rightarrow [0, 1]$ is

$$\mu_{\mathcal{A}}(x) = \Phi(\rho(L(x))).$$

The *fuzzy risk management problem* is to choose

$$x^* \in \arg \max_{x \in D} \mu_{\mathcal{A}}(x),$$

equivalently $\min_{x \in D} \rho(L(x))$ with gradual preference captured by Φ .

Example 1.15 (A scenario–based portfolio under Fuzzy Risk Management). Let the decision set be the interval of portfolio weights

$$D = [0, 1], \quad x \in D \text{ is the fraction invested in a risky asset,}$$

with the remaining $1 - x$ invested in a risk–free asset of return $r_f = 0.02$. Assume the risky asset has three scenarios on a finite probability space

$$(\Omega, \mathcal{F}, \mathbb{P}), \quad R(\omega) \in \{-0.20, 0.05, 0.12\} \quad \text{with} \quad \mathbb{P} = \{0.2, 0.5, 0.3\}.$$

The (random) portfolio return is

$$\text{Ret}(x) = x R + (1 - x)r_f = r_f + x(R - r_f),$$

and the *loss* is $L(x) := -\text{Ret}(x)$. Explicitly, for the three scenarios,

$$\ell_1(x) = -0.02 + 0.22x, \quad \ell_2(x) = -0.02 - 0.03x, \quad \ell_3(x) = -0.02 - 0.10x,$$

with respective probabilities 0.2, 0.5, 0.3. We take as risk measure the *Conditional Value-at-Risk* (Expected Shortfall) at level $\alpha = 0.9$, denoted $\rho(L) = \text{CVaR}_{0.9}(L)$. Because the worst-loss scenario $\ell_1(x)$ alone has probability $0.2 > 1 - \alpha = 0.1$, the Rockafellar–Uryasev formula yields

$$\text{CVaR}_{0.9}(L(x)) = \ell_1(x) = -0.02 + 0.22x \quad \text{for all } x \in [0, 1].$$

We map risk to a fuzzy acceptance by a strictly decreasing logistic:

$$\Phi(r) := \frac{1}{1 + e^{\lambda r}} \quad (\lambda > 0),$$

and fix $\lambda = 20$. The fuzzy membership of decision x is

$$\mu_{\mathcal{A}}(x) = \Phi(\text{CVaR}_{0.9}(L(x))) = \frac{1}{1 + \exp(20(-0.02 + 0.22x))} \in (0, 1).$$

Evaluating a few illustrative choices of x gives

x	0	0.5	1
$\text{CVaR}_{0.9}(L(x))$	-0.020	0.090	0.200
$\mu_{\mathcal{A}}(x)$	0.599	0.142	0.018

(slightly rounded). Since $-0.02 + 0.22x$ is strictly increasing in x , $\mu_{\mathcal{A}}(x)$ is strictly decreasing; hence the fuzzy-optimal decision is

$$x^* = \arg \max_{x \in [0,1]} \mu_{\mathcal{A}}(x) = 0,$$

i.e., fully risk-free in this parameterization.

1.5. Fuzzy IT Service Management

IT Service Management (ITSM) is the process of designing, delivering, managing, and improving IT services to meet business needs effectively, efficiently, and consistently [7–9,38]. Well-known best practices for ITSM include ITIL (Information Technology Infrastructure Library) (cf.[39–43]) and SIAM (Service Integration and Management) (cf.[44,45]), both of which are widely applied in business contexts. We present the definition of the Fuzzy IT Service Management Framework, along with a concrete example.

Definition 1.16 (Mathematical Framework for Fuzzy IT Service Management). [46] The *Fuzzy IT Service Management* (FITSM) framework extends the conventional IT Service Management (ITSM) framework by incorporating fuzzy uncertainty into key evaluation functions. This extension allows us to better capture real-world uncertainty in service performance, cost, and reliability. Formally, we model the FITSM system as a tuple

$$FITSM = (S, I, P, M, \tilde{Q}, \tilde{U}, \tilde{R}, C, \varphi, \psi),$$

with the following components:

- S : A finite set of IT services provided by an organization. For example, $S = \{s_{\text{email}}, s_{\text{web}}, s_{\text{storage}}\}$.
- I : A finite set of IT infrastructure components, such as servers, routers, storage devices, etc. Each component $i \in I$ has associated attributes:
 - Reliability $r(i) \in [0, 1]$: the probability that component i operates without failure.
 - Operating cost $c(i) \in \mathbb{R}^+$: the cost to maintain or run component i .
- P : A finite set of IT management processes (for instance, Incident Management, Change Management, Problem Management). Each process $p \in P$ is characterized by service level agreements (SLAs) and a compliance function

$$\psi : P \times S \rightarrow \{0, 1\},$$

where $\psi(p, s) = 1$ indicates that process p meets the requirements for service s .

- M : An allocation mapping that assigns to each service $s \in S$ a subset of infrastructure components and management processes:

$$M : S \rightarrow 2^I \times 2^P.$$

If $M(s) = (I_s, P_s)$, then $I_s \subseteq I$ is the set of components supporting s , and $P_s \subseteq P$ is the set of processes associated with s .

- C : A set of cost parameters, which may include overall budget constraints, or cost multipliers applied to specific components or processes.
- φ : An aggregation function, used to synthesize the reliability or availability of a collection of infrastructure components; for instance, $\varphi : 2^I \rightarrow [0, 1]$ can be defined so that the availability of a service is an aggregation of the reliability of its components.
- $\tilde{Q}$: A fuzzy quality function,

$$\tilde{Q} : S \rightarrow \tilde{P}(\mathbb{R}^+),$$

that assigns to each service a fuzzy number (or a fuzzy set of performance scores) representing uncertainty in quality measurements. For example, rather than a single quality score, service quality might be expressed as a set such as $\{200.65, 201.00, 200.30\}$.

- $\tilde{U}$: A fuzzy utility function,

$$\tilde{U} : S \rightarrow \tilde{P}(\mathbb{R}),$$

that captures uncertainty in economic evaluations such as revenue and cost. This function reflects variation in expected revenue or fluctuating operational expenses.

- $\tilde{R}$: A fuzzy reliability function,

$$\tilde{R} : S \rightarrow \tilde{P}([0, 1]),$$

assigning to each service a fuzzy measure of its overall reliability.

In this framework, the fuzzy functions $\tilde{Q}$, $\tilde{U}$, and $\tilde{R}$ map each service to a set of possible values rather than a single crisp value, thereby capturing the inherent uncertainty in real-world IT service evaluations. This added flexibility means that the FITSM framework inherently exhibits the structure of a fuzzy set.

Example 1.17 (Service portfolio with fuzzy quality, utility, and reliability). We instantiate the FITSM tuple $\mathcal{FITSM} = (S, I, P, M, \tilde{Q}, \tilde{U}, \tilde{R}, C, \varphi, \psi)$ from Definition FITSM as follows.

Services. $S = \{s_{\text{email}}, s_{\text{web}}, s_{\text{storage}}\}$.

Infrastructure. $I = \{i_1, \dots, i_5\}$ with component reliabilities $r(i) \in [0, 1]$ and costs $c(i) \in \mathbb{R}^+$:

ID	Name	$r(i)$	$c(i)$
i_1	Mail Server	0.985	4.0
i_2	Web Server	0.970	3.5
i_3	Shared DB	0.990	5.0
i_4	Firewall	0.995	2.0
i_5	Storage Array	0.980	4.5

Processes. $P = \{\text{Incident, Change, Problem}\}$. We use a simple compliance map $\psi : P \times S \rightarrow \{0, 1\}$ with $\psi(p, s) = 1$ for all p, s (all processes are in place for each service).

Allocation. The mapping $M : S \rightarrow 2^I \times 2^P$ is

$$\begin{aligned} M(s_{\text{email}}) &= (\{i_1, i_3, i_4\}, P), \\ M(s_{\text{web}}) &= (\{i_2, i_3, i_4\}, P), \\ M(s_{\text{storage}}) &= (\{i_5, i_4\}, P). \end{aligned}$$

Component aggregation φ . Assuming *series* composition inside each service, we set

$$\varphi(I_s) := \prod_{i \in I_s} r(i) \in [0, 1].$$

Hence the crisp service availabilities are

$$\begin{aligned} A_{\text{email}} &= \varphi(\{i_1, i_3, i_4\}) = 0.985 \cdot 0.990 \cdot 0.995 \approx 0.9703, \\ A_{\text{web}} &= \varphi(\{i_2, i_3, i_4\}) = 0.970 \cdot 0.990 \cdot 0.995 \approx 0.9555, \\ A_{\text{storage}} &= \varphi(\{i_5, i_4\}) = 0.980 \cdot 0.995 \approx 0.9751. \end{aligned}$$

Fuzzy quality, utility, reliability. We model $\tilde{P}(\cdot)$ using triangular fuzzy numbers $\tilde{x} = (\ell, m, u)$ with centroid defuzzification $C(\tilde{x}) := (\ell + m + u)/3$.

Quality $\tilde{Q} : S \rightarrow \tilde{P}([0, 1])$ (e.g. SLA compliance as a proportion):

$$\tilde{Q}(s_{\text{email}}) = (0.88, 0.93, 0.97), \quad \tilde{Q}(s_{\text{web}}) = (0.80, 0.87, 0.93), \quad \tilde{Q}(s_{\text{storage}}) = (0.85, 0.90, 0.94).$$

Utility $\tilde{U} : S \rightarrow \tilde{P}([0, 1])$ (e.g. normalized business value):

$$\tilde{U}(s_{\text{email}}) = (0.60, 0.70, 0.80), \quad \tilde{U}(s_{\text{web}}) = (0.70, 0.80, 0.90), \quad \tilde{U}(s_{\text{storage}}) = (0.50, 0.65, 0.75).$$

Reliability $\tilde{R} : S \rightarrow \tilde{P}([0, 1])$ (fuzzified from A_s by a ± 0.02 band):

$$\begin{aligned} \tilde{R}(s_{\text{email}}) &= (0.9503, 0.9703, 0.9903), \\ \tilde{R}(s_{\text{web}}) &= (0.9355, 0.9555, 0.9755), \\ \tilde{R}(s_{\text{storage}}) &= (0.9551, 0.9751, 0.9951). \end{aligned}$$

Illustrative ranking (optional). To obtain a single score per service, we combine centroids with weights $w_Q = 0.4$, $w_U = 0.3$, $w_R = 0.3$:

$$\text{Score}(s) := w_Q C(\tilde{Q}(s)) + w_U C(\tilde{U}(s)) + w_R C(\tilde{R}(s)).$$

Centroids are

	$C(\tilde{Q})$	$C(\tilde{U})$	$C(\tilde{R})$
s_{email}	0.9267	0.7000	0.9703
s_{web}	0.8667	0.8000	0.9555
s_{storage}	0.8967	0.6333	0.9751

and thus

$$\begin{aligned} \text{Score}(s_{\text{email}}) &\approx 0.4 \cdot 0.9267 + 0.3 \cdot 0.7000 + 0.3 \cdot 0.9703 \approx 0.8718, \\ \text{Score}(s_{\text{web}}) &\approx 0.4 \cdot 0.8667 + 0.3 \cdot 0.8000 + 0.3 \cdot 0.9555 \approx 0.8733, \\ \text{Score}(s_{\text{storage}}) &\approx 0.4 \cdot 0.8967 + 0.3 \cdot 0.6333 + 0.3 \cdot 0.9751 \approx 0.8412. \end{aligned}$$

With these illustrative weights and fuzzy inputs the ranking is $s_{\text{web}} \succ s_{\text{email}} \succ s_{\text{storage}}$. Different weights or fuzzifications naturally reflect alternative business priorities (e.g., reliability-centric vs. value-centric selection).

2. Main Results

In this section, we present and explain the results of this paper.

2.1. Plithogenic Fuzzy Risk Management

We now allow multiple *risk facets* (attribute values) and an explicit *contradiction* among them, and we aggregate facetwise fuzzy acceptances by a plithogenic operator that is modulated by contradiction.

Definition 2.1 (Plithogenic Fuzzy Risk Management). A *Plithogenic Fuzzy Risk Management (PFRM)* instance is a tuple

$$\text{PFRM} = (D, v, Pv, pdf, pCF, \text{Agg}),$$

whose components are:

- $D \subseteq \mathbb{R}^n$ a nonempty decision set.
- v the (fixed) attribute “risk facet”, with value domain Pv (finite or countable). Elements $a \in Pv$ stand for facets such as likelihood, impact, detectability, regulatory exposure, scenario, stakeholder view, etc.
- For each $a \in Pv$:

- a loss map $L_a : D \rightarrow L^\infty(\Omega, \mathcal{F}, \mathbb{P})$,
- a (coherent) risk measure $\rho_a : L^\infty \rightarrow \mathbb{R}$,
- a continuous, strictly decreasing satisfaction map $\Phi_a : \mathbb{R} \rightarrow [0, 1]$.

The *facetwise fuzzy acceptance* (plithogenic appurtenance) is

$$pdf(x, a) := \Phi_a(\rho_a(L_a(x))) \in [0, 1], \quad (x, a) \in D \times Pv.$$

- A *contradiction function* $pCF : Pv \times Pv \rightarrow [0, 1]$ that is symmetric and reflexive-zero:

$$pCF(a, b) = pCF(b, a), \quad pCF(a, a) = 0.$$

Intuitively, $pCF(a, b)$ quantifies the “conflict” between facets a and b .

- A *plithogenic aggregator* Agg that combines the facetwise memberships $\{pdf(x, a)\}_{a \in Pv}$ into a single overall membership $\bar{\mu}(x) \in [0, 1]$ while using pCF to modulate between a fixed t -norm $T : [0, 1]^2 \rightarrow [0, 1]$ and a fixed t -conorm $S : [0, 1]^2 \rightarrow [0, 1]$.² Concretely, for two facets a, b we set

$$T_{pCF}(p, q; a, b) := (1 - pCF(a, b)) T(p, q) + pCF(a, b) S(p, q),$$

and extend to $|Pv| > 2$ by any associative fold (e.g. fixed ordering or a dominant facet first). The overall membership is

$$\bar{\mu}(x) := \text{Agg}(\{pdf(x, a)\}_{a \in Pv}, pCF) \in [0, 1].$$

Given PFRM, the *plithogenic fuzzy risk management problem* is

$$x^* \in \arg \max_{x \in D} \bar{\mu}(x).$$

Example 2.2 (Mitigation Strategy Selection with Three Risk Facets). *Decisions and facets.* Let the decision set be $D = \{x_A, x_B, x_C\}$, representing three risk-mitigation strategies. We use three risk facets $Pv = \{L, I, D\}$: L = likelihood, I = impact, D = detectability (large detectability score means poor detectability).

Facetwise loss, risk measure, and satisfaction. For each facet $a \in Pv$ and decision $x \in D$, let $L_a(x)$ be a (bounded) loss random variable. In this illustrative instance we take each $L_a(x)$ to be a constant random variable equal to a score $r_{x,a} \in [0, 10]$. We choose the risk measure ρ_a to be the identity and the satisfaction map $\Phi_a(r) = \max\{0, \min\{1, 1 - r/10\}\}$ (linearly decreasing in the risk score). Thus the plithogenic appurtenance (facetwise fuzzy acceptance) is

$$pdf(x, a) = \Phi_a(\rho_a(L_a(x))) = 1 - \frac{r_{x,a}}{10} \in [0, 1].$$

We also write $\mu(x | a) := pdf(x, a)$.

Facet scores and induced memberships.

	$r_{x,L}$	$r_{x,I}$	$r_{x,D}$		$\mu(x L)$	$\mu(x I)$	$\mu(x D)$
x_A	4	6	3	$\Rightarrow$	0.6	0.4	0.7
x_B	5	5	6		0.5	0.5	0.4
x_C	3	7	5		0.7	0.3	0.5

² Typical choices are $T(p, q) = \min\{p, q\}$ or pq , and $S(p, q) = \max\{p, q\}$ or $p + q - pq$.

Contradiction map. Let $pCF : Pv \times Pv \rightarrow [0, 1]$ be symmetric with $pCF(a, a) = 0$. We use (nonzero entries listed)

$$pCF(L, I) = 0.5, \quad pCF(I, D) = 0.3, \quad pCF(L, D) = 0.2.$$

Plithogenic aggregation. Fix a t -norm $T(p, q) = pq$ and a t -conorm $S(p, q) = p + q - pq$. For a pair of facets (a, b) and inputs $p, q \in [0, 1]$ set

$$T_{pCF}(p, q; a, b) := (1 - pCF(a, b)) T(p, q) + pCF(a, b) S(p, q).$$

We fold associatively in the order $\pi = (L, I, D)$:

$$\begin{aligned} \bar{\mu}_1(x) &:= \mu(x \mid L), & \bar{\mu}_2(x) &:= T_{pCF}(\bar{\mu}_1(x), \mu(x \mid I); L, I), \\ \bar{\mu}_3(x) &:= T_{pCF}(\bar{\mu}_2(x), \mu(x \mid D); I, D), & \bar{\mu}(x) &:= \bar{\mu}_3(x) \in [0, 1]. \end{aligned}$$

Step-by-step aggregation (exact arithmetic shown). Recall the identity

$$\begin{aligned} T_{pCF}(p, q; a, b) &= (1 - c) pq + c(p + q - pq) \\ &= \underbrace{pq}_{\text{product}} + \underbrace{cp}_{\text{conorm part}} + \underbrace{cq}_{\text{conorm part}} - \underbrace{2c pq}_{\text{double-count adj}}, \quad c = pCF(a, b). \end{aligned}$$

x_A : $\mu_L = 0.6, \mu_I = 0.4, \mu_D = 0.7$.

$$\bar{\mu}_2 = T_{pCF}(0.6, 0.4; L, I), \quad c = 0.5 : pq = 0.24, cp = 0.30, cq = 0.20, 2c pq = 0.24 \Rightarrow \bar{\mu}_2 = 0.50.$$

$$\bar{\mu} = T_{pCF}(0.50, 0.70; I, D), \quad c = 0.3 : pq = 0.35, cp = 0.15, cq = 0.21, 2c pq = 0.21 \Rightarrow \bar{\mu}(x_A) = 0.50.$$

x_B : $\mu_L = 0.5, \mu_I = 0.5, \mu_D = 0.4$.

$$\bar{\mu}_2 = T_{pCF}(0.5, 0.5; L, I), \quad c = 0.5 : pq = 0.25, cp = 0.25, cq = 0.25, 2c pq = 0.25 \Rightarrow \bar{\mu}_2 = 0.50.$$

$$\bar{\mu} = T_{pCF}(0.50, 0.40; I, D), \quad c = 0.3 : pq = 0.20, cp = 0.15, cq = 0.12, 2c pq = 0.12 \Rightarrow \bar{\mu}(x_B) = 0.35.$$

x_C : $\mu_L = 0.7, \mu_I = 0.3, \mu_D = 0.5$.

$$\bar{\mu}_2 = T_{pCF}(0.7, 0.3; L, I), \quad c = 0.5 : pq = 0.21, cp = 0.35, cq = 0.15, 2c pq = 0.21 \Rightarrow \bar{\mu}_2 = 0.50.$$

$$\bar{\mu} = T_{pCF}(0.50, 0.50; I, D), \quad c = 0.3 : pq = 0.25, cp = 0.15, cq = 0.15, 2c pq = 0.15 \Rightarrow \bar{\mu}(x_C) = 0.40.$$

Decision. The overall memberships are

$$\bar{\mu}(x_A) = 0.50, \quad \bar{\mu}(x_C) = 0.40, \quad \bar{\mu}(x_B) = 0.35,$$

hence the PFRM rule selects $x_A \in D$ as the preferred mitigation strategy.

Remark 2.3 (Design freedom and well-posedness). The construction is flexible yet mathematically well-posed: (i) $pdf(x, a) \in [0, 1]$ by definition of Φ_a and ρ_a ; (ii) the convex mixture $(1 - c)T + cS$ stays in $[0, 1]$ whenever T, S are standard t -(co)norms and $c \in [0, 1]$; (iii) $\bar{\mu} : D \rightarrow [0, 1]$ is thus well-defined. Existence of an optimizer follows under standard compactness/continuity assumptions on D and on the maps $x \mapsto pdf(x, a)$.

Theorem 2.4 (Reduction to classical FRM). Let $\text{PFRM} = (D, v, Pv, pdf, pCF, \text{Agg})$ be as in Definition 2.1. Suppose $Pv = \{a_0\}$ is a singleton and $pCF \equiv 0$. Then

$$\bar{\mu}(x) = pdf(x, a_0) = \Phi_{a_0}(\rho_{a_0}(L_{a_0}(x))), \quad x \in D,$$

and the PFRM problem $\max_{x \in D} \bar{\mu}(x)$ coincides with the classical FRM problem with data $(L, \rho, \Phi) = (L_{a_0}, \rho_{a_0}, \Phi_{a_0})$.

Proof. With $|Pv| = 1$, the aggregator Agg receives a single input $pdf(x, a_0)$, hence $\bar{\mu}(x) = pdf(x, a_0)$ by definition of a fold. Because $pCF(a_0, a_0) = 0$ the modulation is vacuous. The stated identity follows from the definition of pdf . Therefore maximizing $\bar{\mu}$ is exactly maximizing $\Phi_{a_0}(\rho_{a_0}(L_{a_0}(x)))$, which is the FRM objective. $\square$

Theorem 2.5 (Plithogenic fuzzy set structure). *For any PFRM $= (D, v, Pv, pdf, pCF, Agg)$ as in Definition 2.1, the tuple*

$$PS := (P, v, Pv, pdf, pCF), \quad P := D,$$

is a plithogenic fuzzy set (single appurtenance component and scalar contradiction), i.e.,

- (a) $pdf : P \times Pv \rightarrow [0, 1]$ is well-defined by $pdf(x, a) = \Phi_a(\rho_a(L_a(x)))$,
- (b) $pCF : Pv \times Pv \rightarrow [0, 1]$ is symmetric with $pCF(a, a) = 0$.

Proof. (a) For each a , $\rho_a(L_a(x)) \in \mathbb{R}$ and Φ_a maps $\mathbb{R}$ continuously into $[0, 1]$, hence $pdf(x, a) \in [0, 1]$ for all (x, a) . (b) By assumption, pCF is symmetric and reflexive-zero. These are exactly the axioms required for the contradiction function in the definition of a plithogenic fuzzy set with one-dimensional appurtenance and contradiction components. Therefore (P, v, Pv, pdf, pCF) is a plithogenic fuzzy set. $\square$

2.2. Upside-Down Logic in Plithogenic Fuzzy Risk Management

In this subsection we endow the Plithogenic Fuzzy Risk Management (PFRM) model (Definition 2.1) with a mathematically precise *Upside-Down* operator that flips facetwise acceptability whenever a chosen anchor facet is in strong contradiction with other facets. We then show that, under natural truth-valuation at the mid-cut, this operator realizes the abstract axioms of Upside-Down Logic (truth $\leftrightarrow$ falsity) on the elementary acceptance statements.

Notation 2.6 (Facetwise membership). *For a PFRM instance $PFRM = (D, v, Pv, pdf, pCF, Agg)$ we write*

$$\mu(x | a) := pdf(x, a) \in [0, 1] \quad (x \in D, a \in Pv),$$

and we abbreviate $c(a, b) := pCF(a, b) \in [0, 1]$ with $c(a, a) = 0$ and $c(a, b) = c(b, a)$.

Definition 2.7 (Anchor-threshold activated Upside-Down operator on PFRM). *Fix an anchor facet $b \in Pv$ and a contradiction threshold $\tau \in [0, 1]$. Define the activation locus*

$$A_\tau(b) := \{a \in Pv : c(a, b) \geq \tau\}.$$

The *Upside-Down transform* $U_{b, \tau}$ of the instance PFRM is the new instance

$$PFRM^{U_{b, \tau}} = (D, v, Pv, pdf^{U_{b, \tau}}, pCF^{U_{b, \tau}}, Agg^{U_{b, \tau}})$$

with

$$pdf^{U_{b, \tau}}(x, a) := \begin{cases} 1 - \mu(x | a), & a \in A_\tau(b), \\ \mu(x | a), & a \notin A_\tau(b), \end{cases}$$

$$pCF^{U_{b, \tau}}(u, v) := \begin{cases} 0, & \{u, v\} = \{a, b\} \text{ for some } a \in A_\tau(b), \\ pCF(u, v), & \text{otherwise,} \end{cases}$$

and where $\text{Agg}^{U_{b,\tau}}$ is obtained from Agg by replacing pdf, pCF with $pdf^{U_{b,\tau}}, pCF^{U_{b,\tau}}$ (the underlying t -norm/ t -conorm and folding policy are unchanged). Informally, large contradiction to the anchor triggers a membership complement and then *resets* that specific contradiction to zero to stabilize the post-transform context.

Example 2.8 (Regulated Medical Device Go-To-Market Risk). *Decisions and facets.* Let $D = \{x_A, x_B\}$ be two launch plans for a regulated medical device. Let $Pv = \{S, L, D, C, R\}$ denote facets: Safety impact (S), Likelihood (L), Detectability (D), Cost pressure (C), and Regulatory exposure (R). Facetwise fuzzy acceptances $\mu(x | a) \in [0, 1]$ are:

Plan	$\mu(\cdot S)$	$\mu(\cdot L)$	$\mu(\cdot D)$	$\mu(\cdot C)$	$\mu(\cdot R)$
x_A	0.92	0.60	0.70	0.80	0.75
x_B	0.85	0.70	0.60	0.90	0.60

Contradiction & aggregation. Use a symmetric pCF with $pCF(a, a) = 0$. The (relevant) entries used below are

$$pCF(S, L) = 0.30, \quad pCF(L, D) = 0.30, \quad pCF(D, C) = 0.40, \quad pCF(C, R) = 0.90.$$

Choose $T(p, q) = pq$, $S(p, q) = p + q - pq$, and fold in the order $\pi = (S, L, D, C, R)$ via

$$T_{pCF}(p, q; a, b) = (1 - pCF(a, b))pq + pCF(a, b)(p + q - pq).$$

Pre-transform overall memberships. For x_A :

$$\begin{aligned} \bar{\mu}_1 &= 0.92, \\ \bar{\mu}_2 &= T_{pCF}(0.92, 0.60; S, L) = 0.676800, \\ \bar{\mu}_3 &= T_{pCF}(0.676800, 0.70; L, D) = 0.602544, \\ \bar{\mu}_4 &= T_{pCF}(0.602544, 0.80; D, C) = 0.65742464, \\ \bar{\mu}_5 &= T_{pCF}(0.65742464, 0.75; C, R) = 0.872227392. \end{aligned}$$

For x_B :

$$\bar{\mu}(x_B) = 0.827523432.$$

Thus, before the transform, x_A is preferred.

Anchor-threshold Upside-Down. Let the anchor be R (regulatory), with $\tau = 0.8$. Then $A_\tau(R) = \{C\}$ because $pCF(C, R) = 0.90 \geq \tau$. Apply $U_{R,\tau}$:

$$\mu^U(x | C) = 1 - \mu(x | C), \quad pCF^U(C, R) = 0.$$

Hence $\mu^U(x_A | C) = 0.20$, $\mu^U(x_B | C) = 0.10$.

Post-transform overall memberships. Re-fold (same order; only the flipped facet and the reset pair differ):

$$\bar{\mu}^U(x_A) = 0.258839520, \quad \bar{\mu}^U(x_B) = 0.165024240.$$

The transform *penalizes cost-driven acceptance under strict regulatory anchoring*, yet preserves the ranking $x_A \succ x_B$ (now with a larger margin).

Example 2.9 (Hospital Disaster-Recovery (DR) Strategy Selection). *Decisions and facets.* Let $D = \{x_{AA}, x_{CB}\}$ be two DR strategies: Active-Active (x_{AA}) vs. Cold Backup (x_{CB}). Use facets $Pv =$

$\{S, L, B, R\}$: Safety-critical impact (S), Likelihood (L), Budget adequacy (B), Regulatory/contractual compliance (R). Facet memberships:

Strategy	$\mu(\cdot S)$	$\mu(\cdot L)$	$\mu(\cdot B)$	$\mu(\cdot R)$
x_{AA}	0.90	0.85	0.20	0.60
x_{CB}	0.70	0.60	0.90	0.50

Contradiction & aggregation. Nonzero (relevant) contradictions for the fold order $\pi = (S, L, B, R)$ are

$$pCF(S, L) = 0.20, \quad pCF(L, B) = 0.50, \quad pCF(B, R) = 0.30, \quad pCF(S, B) = 0.80.$$

Again take $T(p, q) = pq$, $S(p, q) = p + q - pq$ and fold via T_{pCF} .

Pre-transform overall memberships. A direct computation yields

$$\bar{\mu}(x_{AA}) = 0.452430, \quad \bar{\mu}(x_{CB}) = 0.503000,$$

so *before* anchoring, Cold Backup is preferred.

Anchor-threshold Upside-Down. Let the anchor be S (patient safety), with $\tau = 0.75$. Then $A_\tau(S) = \{B\}$ because $pCF(S, B) = 0.80 \geq \tau$. Apply $U_{S, \tau}$:

$$\mu^U(x | B) = 1 - \mu(x | B), \quad pCF^U(S, B) = 0.$$

Thus $\mu^U(x_{AA} | B) = 0.80$ and $\mu^U(x_{CB} | B) = 0.10$.

Post-transform overall memberships. Refolding with the flipped B gives

$$\bar{\mu}^U(x_{AA}) = 0.614430, \quad \bar{\mu}^U(x_{CB}) = 0.303000.$$

Hence, *after* safety anchoring, the preference *reverses* to Active-Active, reflecting a policy where strong safety priority turns budget-friendly options into low-acceptance ones when they conflict with safety.

Lemma 2.10 (Well-posedness). $PFRM^{U_{b, \tau}}$ is a valid PFRM instance: for all (x, a) , $pdf^{U_{b, \tau}}(x, a) \in [0, 1]$; the map $pCF^{U_{b, \tau}}$ remains symmetric with $(pCF^{U_{b, \tau}})(a, a) = 0$; consequently $\text{Agg}^{U_{b, \tau}}$ produces an overall membership $\bar{\mu}^{U_{b, \tau}} : D \rightarrow [0, 1]$.

Proof. The complement $1 - \mu(x | a)$ lies in $[0, 1]$ because $\mu(\cdot | a) \in [0, 1]$. The definition of $pCF^{U_{b, \tau}}$ preserves symmetry and zeros on the diagonal. Since the t -norm/ t -conorm mixture used by Agg is continuous and bounded in $[0, 1]$, the overall output remains in $[0, 1]$ after substitution. $\square$

Definition 2.11. Let $\mathcal{L}$ be the set of *elementary acceptance statements*

$$\text{Acc}_{1/2}(x, a) \quad \text{“decision } x \text{ is acceptable on facet } a \text{ at the mid-cut”}.$$

For a PFRM instance PFRM, define the *mid-cut valuation* $v_{b, \tau}$ by

$$v_{b, \tau}(\text{Acc}_{1/2}(x, a)) := \begin{cases} \text{True}, & \mu(x | a) \geq \frac{1}{2}, \\ \text{False}, & \mu(x | a) < \frac{1}{2}. \end{cases}$$

After applying $U_{b, \tau}$, we evaluate with the same predicate over $pdf^{U_{b, \tau}}$, obtaining $v'_{b, \tau}$.

Theorem 2.12 (Upside-Down property on elementary acceptances). *Let PFRM be a PFRM instance and fix (b, τ) . For every $x \in D$ and $a \in Pv$ with $\mu(x | a) \neq \frac{1}{2}$,*

$$\begin{aligned} a \in A_\tau(b) &\implies v'_{b,\tau}(\text{Acc}_{1/2}(x, a)) = \neg v_{b,\tau}(\text{Acc}_{1/2}(x, a)), \\ a \notin A_\tau(b) &\implies v'_{b,\tau}(\text{Acc}_{1/2}(x, a)) = v_{b,\tau}(\text{Acc}_{1/2}(x, a)). \end{aligned}$$

Proof. If $a \notin A_\tau(b)$ then $pdf^{U_{b,\tau}}(x, a) = \mu(x | a)$ and the valuation is unchanged.

Assume $a \in A_\tau(b)$. Then $pdf^{U_{b,\tau}}(x, a) = 1 - \mu(x | a)$. If $\mu(x | a) > \frac{1}{2}$ we have $1 - \mu(x | a) < \frac{1}{2}$ and therefore $v_{b,\tau}(\text{Acc}_{1/2}(x, a)) = \text{True}$ while $v'_{b,\tau}(\text{Acc}_{1/2}(x, a)) = \text{False}$. If instead $\mu(x | a) < \frac{1}{2}$ then $1 - \mu(x | a) > \frac{1}{2}$, so the truth values swap in the opposite direction. The only non-flipping boundary case $\mu(x | a) = \frac{1}{2}$ is excluded by assumption. $\square$

Corollary 2.13 (Instantiation of Upside-Down Logic). *Let the context be the pair $C = (b, \tau)$, let the Upside-Down operator act on instances by $U(\text{PFRM}; C) = \text{PFRM}^{U_{b,\tau}}$, and keep the language $\mathcal{L}$ fixed. Then for every lemma $A \in \mathcal{L}$ of the form $\text{Acc}_{1/2}(x, a)$ with $\mu(x | a) \neq \frac{1}{2}$,*

$$v_C(A) = \text{True} \implies v_{U(C)}(A) = \text{False}, \quad v_C(A) = \text{False} \implies v_{U(C)}(A) = \text{True}.$$

Hence $(\mathcal{L}, \{v_C\}, U)$ satisfies the defining truth-falsity inversion axioms of Upside-Down Logic on the activated facets.

Proof. This is an immediate reformulation of Theorem 2.12. $\square$

2.3. Plithogenic Fuzzy IT Service Management

We enrich FITSM with plithogenic structure: explicit risk/performance facets (attribute values) and a contradiction map modulating the fusion of facetwise memberships.

Definition 2.14 (Plithogenic Fuzzy IT Service Management). A *P-FITSM instance* is a tuple

$$\text{PFITSM} = (S, I, P, M, \tilde{Q}, \tilde{U}, \tilde{R}, C, \varphi, \psi; v, Pv, pdf, pCF, \text{Agg}),$$

with the following additional components beyond *FITSM*:

- v is the attribute “service evaluation facet” and Pv its (finite or countable) value domain. The canonical choice is $Pv = \{Q, U, R\}$, but extra facets (e.g. security, compliance, sustainability) may be added.
- $pdf : S \times Pv \rightarrow [0, 1]$ (plithogenic appurtenance) assigns a membership degree to each (s, a) as

$$pdf(s, a) = \begin{cases} \mu_Q(s), & a = Q, \\ \mu_U(s), & a = U, \\ \mu_R(s), & a = R, \\ F_a(\tilde{X}_a(s)), & \text{for any extra facet } a, \end{cases}$$

where each extra facet a has its fuzzy score $\tilde{X}_a : S \rightarrow \tilde{P}(\mathbb{R})$ and a monotone scoring functional $F_a : \tilde{P}(\mathbb{R}) \rightarrow [0, 1]$.

- $pCF : Pv \times Pv \rightarrow [0, 1]$ is the *contradiction* function, symmetric with $pCF(a, a) = 0$ for all $a \in Pv$; it quantifies the conflict between facets.
- Agg is a plithogenic aggregator that fuses the vector $\{pdf(s, a)\}_{a \in Pv}$ into an *overall service membership* $\bar{\mu}(s) \in [0, 1]$ by mixing a fixed t -norm T and a fixed t -conorm S using pCF . For two facets $a, b \in Pv$ and inputs $p, q \in [0, 1]$ define

$$T_{pCF}(p, q; a, b) := (1 - pCF(a, b)) T(p, q) + pCF(a, b) S(p, q).$$

For $|Pv| > 2$ we fix once and for all a total order π on Pv and fold associatively:

$$\bar{\mu}_1(s) := pdf(s, \pi_1), \quad \bar{\mu}_{k+1}(s) := T_{pCF}(\bar{\mu}_k(s), pdf(s, \pi_{k+1}); \pi_k, \pi_{k+1}),$$

and set $\bar{\mu}(s) := \bar{\mu}_{|Pv|}(s)$.

The P-FITSM decision problem is to select $s^* \in \arg \max_{s \in S} \bar{\mu}(s)$ or, more generally, to use $\bar{\mu}$ within portfolio/optimization constraints.

Example 2.15 (PFITSM for Managed Database Selection). We consider a small portfolio of managed database services

$$S = \{\text{DB-Alpha}, \text{DB-Beta}, \text{DB-Gamma}\}.$$

We focus on the plithogenic evaluation layer of Definition 2.14; the infrastructural and process tuples $(I, P, M, \tilde{Q}, \tilde{U}, \tilde{R}, C, \varphi, \psi)$ are present but not used explicitly below.

Let the attribute be the *service evaluation facet* v with value domain

$$Pv = \{Q, U, R, \text{Sec}, \text{Sus}\},$$

standing for Quality, Utility (economic benefit/cost), Reliability, Security, and Sustainability. From normalized KPI measurements we obtain the facetwise fuzzy memberships $\mu_Q, \mu_U, \mu_R, \mu_{\text{Sec}}, \mu_{\text{Sus}} : S \rightarrow [0, 1]$, which directly play the role of $pdf(s, a)$ (cf. Definition 2.14):

Service	μ_Q	μ_U	μ_R	μ_{Sec}	μ_{Sus}
DB-Alpha	0.82	0.65	0.97	0.90	0.55
DB-Beta	0.78	0.80	0.88	0.72	0.70
DB-Gamma	0.70	0.55	0.92	0.85	0.90

We use a symmetric contradiction map $pCF : Pv \times Pv \rightarrow [0, 1]$ with $pCF(a, a) = 0$. Table 1 lists the nontrivial (upper triangular) entries used in the fold order below; symmetry fills the lower triangular part.

Table 1. Selected contradiction degrees $pCF(a, b)$ among facets.

$pCF(a, b)$	Q	U	R	Sec	Sus
Q	0	0.30	0.20	0.10	0.20
U		0	0.70	0.80	0.50
R			0	0.20	0.30
Sec				0	0.20
Sus					0

Fix a total order $\pi = (R, \text{Sec}, Q, U, \text{Sus})$ on Pv for associative folding. Choose the product t -norm $T(p, q) = pq$ and the probabilistic sum t -conorm $S(p, q) = p + q - pq$. For two adjacent facets (a, b) we use the standard plithogenic mixer

$$T_{pCF}(p, q; a, b) := (1 - pCF(a, b)) T(p, q) + pCF(a, b) S(p, q).$$

For any $s \in S$, define the fold recursively by

$$\bar{\mu}_1(s) := pdf(s, \pi_1),$$

$$\bar{\mu}_{k+1}(s) := T_{pCF}(\bar{\mu}_k(s), pdf(s, \pi_{k+1}); \pi_k, \pi_{k+1}),$$

and set $\bar{\mu}(s) := \bar{\mu}_{|Pv|}(s) \in [0, 1]$.

Step-by-step computation for DB-Alpha.

With $\pi = (R, \text{Sec}, Q, U, \text{Sus})$ we obtain

$$\begin{aligned}
 \bar{\mu}_1 &= \mu_R = 0.97, \\
 \bar{\mu}_2 &= T_{pCF}(0.97, 0.90; R, \text{Sec}) \\
 &= (1 - 0.20)(0.97 \cdot 0.90) \\
 &\quad + 0.20(0.97 + 0.90 - 0.97 \cdot 0.90) \\
 &= 0.8 \cdot 0.873 + 0.2 \cdot 0.997 = 0.8978, \\
 \bar{\mu}_3 &= T_{pCF}(0.8978, 0.82; \text{Sec}, Q) \\
 &= (1 - 0.10)(0.8978 \cdot 0.82) \\
 &\quad + 0.10(0.8978 + 0.82 - 0.8978 \cdot 0.82) \\
 &= 0.9 \cdot 0.736196 + 0.1 \cdot 0.981604 = 0.7607368, \\
 \bar{\mu}_4 &= T_{pCF}(0.7607368, 0.65; Q, U) \\
 &= (1 - 0.30)(0.7607368 \cdot 0.65) \\
 &\quad + 0.30(0.7607368 + 0.65 - 0.7607368 \cdot 0.65) \\
 &= 0.7 \cdot 0.49447892 + 0.3 \cdot 0.91625788 = 0.621012608, \\
 \bar{\mu}_5 &= T_{pCF}(0.621012608, 0.55; U, \text{Sus}) \\
 &= (1 - 0.50)(0.621012608 \cdot 0.55) \\
 &\quad + 0.50(0.621012608 + 0.55 - 0.621012608 \cdot 0.55) \\
 &= 0.5 \cdot 0.3415569344 + 0.5 \cdot 0.8294556736 = 0.585506304.
 \end{aligned}$$

The same fold yields the overall plithogenic memberships

$$\bar{\mu}(\text{DB-Alpha}) = 0.5855, \quad \bar{\mu}(\text{DB-Beta}) = 0.6513, \quad \bar{\mu}(\text{DB-Gamma}) = 0.6920,$$

where we display 4 significant digits for readability.³ Therefore, the PFITSM selection is

$$s^* \in \arg \max_{s \in S} \bar{\mu}(s) = \{\text{DB-Gamma}\}.$$

Because contradiction levels modulate between conjunctive (T) and disjunctive (S) behaviour *facet by facet*, the method transparently captures trade-offs such as “utility vs. security” ($pCF(U, \text{Sec}) = 0.80$) and “utility vs. reliability” ($pCF(U, R) = 0.70$), while keeping near-compatible pairs (e.g. Sec vs. Q with 0.10) largely conjunctive. This example demonstrates how PFITSM generalizes the fuzzy ITSM scoring by adding an explicit, mathematically controlled *contradiction* layer and a plithogenic fusion operator.

Remark 2.16 (Well-posedness). Since $T, S : [0, 1]^2 \rightarrow [0, 1]$ and $pCF \in [0, 1]$, the convex mixture T_{pCF} maps $[0, 1]^2$ to $[0, 1]$. Thus each $\bar{\mu}_k(s) \in [0, 1]$ and $\bar{\mu} : S \rightarrow [0, 1]$ is well-defined. If T and S are nondecreasing in each argument (standard for t -(co)norms), then T_{pCF} and the fold are nondecreasing in each input membership, hence $\bar{\mu}$ is monotone in every $pdf(s, a)$.

To formalize the reduction, fix any associative, nondecreasing fusion $A_{\text{FITSM}} : [0, 1]^m \rightarrow [0, 1]$ used in the baseline FITSM to combine the normalized facet scores (e.g. a t -norm product, min, or a weighted geometric/Archimedean t -norm). Define the (derived) overall FITSM membership

$$\mu_{\text{FITSM}}(s) := A_{\text{FITSM}}(\mu_Q(s), \mu_U(s), \mu_R(s), \dots) \in [0, 1].$$

³ Exact values: 0.585506304, 0.6513239104, and 0.69196112, respectively.

Theorem 2.17 (Reduction to FITSM). Consider PFITSM with the following specializations:

1. $pCF \equiv 0$ on Pv ;
2. T is chosen so that the fold coincides with A_{FITSM} on the ordered list $\{\mu_{\bullet}(s)\}_{a \in Pv}$ (e.g. $T = \min$ for conjunctive FITSM, or $T(p, q) = pq$ for multiplicative FITSM);
3. S is arbitrary (unused when $pCF \equiv 0$).

Then for all $s \in S$, $\bar{\mu}(s) = \mu_{\text{FITSM}}(s)$. In particular, the P -FITSM decision rule $\arg \max_{s \in S} \bar{\mu}(s)$ coincides with the baseline FITSM rule $\arg \max_{s \in S} \mu_{\text{FITSM}}(s)$.

Proof. If $pCF \equiv 0$, then $T_{pCF}(p, q; a, b) = T(p, q)$ for all pairs. By the choice of T and the fixed order π , the fold computes exactly A_{FITSM} on the sequence $(\mu_Q(s), \mu_U(s), \mu_R(s), \dots)$. Hence $\bar{\mu}(s) = \mu_{\text{FITSM}}(s)$ for every $s \in S$, and the maximizers agree. $\square$

Theorem 2.18 (Plithogenic fuzzy set structure of P-FITSM). Let PFITSM be as in Definition 2.14. Set $P := S$ and retain the same (v, Pv, pdf, pCF) . Then

$$PS := (P, v, Pv, pdf, pCF)$$

is a plithogenic fuzzy set.

Proof. By Definition 2.14, $pdf(s, a) \in [0, 1]$ for all (s, a) , and pCF is symmetric with $pCF(a, a) = 0$. These are precisely the axioms required for a plithogenic fuzzy set in the single appurtenance/contradiction component case. Therefore PS is a plithogenic fuzzy set. $\square$

2.4. Upside-Down Logic in Plithogenic Fuzzy IT Service Management

We formalize an *Upside-Down* operator for the Plithogenic Fuzzy IT Service Management (PFITSM) framework and prove that, under a natural truth valuation on acceptance statements, it realizes the defining truth $\leftrightarrow$ falsity inversion of Upside-Down Logic.

Notation 2.19 (PFITSM instance and facetwise memberships). Let a PFITSM instance be given by

$$\text{PFITSM} = (S, v, Pv, pdf, pCF, \text{Agg}, \dots),$$

where S is the (finite) set of services, v is the fixed attribute (“service facet”), Pv is its value domain (e.g., availability, cost, customer satisfaction, SLA compliance), $pdf : S \times Pv \rightarrow [0, 1]$ is the appurtenance (facetwise fuzzy acceptance), $pCF : Pv \times Pv \rightarrow [0, 1]$ is the symmetric contradiction map with $pCF(a, a) = 0$, and Agg aggregates $\{pdf(s, a)\}_{a \in Pv}$ into a global membership $\bar{\mu}(s) \in [0, 1]$. We write

$$\mu(s | a) := pdf(s, a) \in [0, 1], \quad c(a, b) := pCF(a, b) \in [0, 1].$$

When needed, we assume Agg is built from a t -norm T and t -conorm S via the contradiction-modulated mixer

$$T_c(p, q; a, b) := (1 - c(a, b)) T(p, q) + c(a, b) S(p, q),$$

folded associatively over all facets.

Definition 2.20 (Anchor-threshold Upside-Down operator on PFITSM). Fix an anchor facet $b \in Pv$ and a contradiction threshold $\tau \in [0, 1]$. Set the activation locus

$$A_\tau(b) := \{a \in Pv : c(a, b) \geq \tau\}.$$

The Upside-Down transform $U_{b, \tau}$ produces a new PFITSM instance

$$\text{PFITSM}^{U_{b, \tau}} = (S, v, Pv, pdf^{U_{b, \tau}}, pCF^{U_{b, \tau}}, \text{Agg}^{U_{b, \tau}}, \dots)$$

with

$$pdf^{U_{b,\tau}}(s, a) := \begin{cases} 1 - \mu(s \mid a), & a \in A_\tau(b), \\ \mu(s \mid a), & a \notin A_\tau(b), \end{cases}$$

$$pCF^{U_{b,\tau}}(u, v) := \begin{cases} 0, & \{u, v\} = \{a, b\} \text{ for some } a \in A_\tau(b), \\ pCF(u, v), & \text{otherwise,} \end{cases}$$

and where $\text{Agg}^{U_{b,\tau}}$ is obtained from Agg by substituting $pdf^{U_{b,\tau}}$ and $pCF^{U_{b,\tau}}$ (the underlying t -(co)norms and folding policy are unchanged).

Example 2.21 (Cost-anchored FinOps pivot for a cloud email service). **Service and facets.** Let $S = \{s_{\text{email}}\}$ denote a managed cloud email service. Consider the facet set

$$Pv = \{\text{Ava}, \text{Sec}, \text{Lat}, \text{CS}, \text{Cost}\},$$

standing for Availability, Security compliance, Latency, Customer Satisfaction, and Budget Adherence (Cost), respectively. Let $b = \text{Cost}$ and set the contradiction threshold $\tau = 0.7$.

Contradictions to the anchor. Assume (symmetrically extended) contradiction levels with the anchor

$$c(\text{Ava}, \text{Cost}) = 0.85, \quad c(\text{Sec}, \text{Cost}) = 0.75, \quad c(\text{Lat}, \text{Cost}) = 0.60,$$

$$c(\text{CS}, \text{Cost}) = 0.50, \quad c(\text{Cost}, \text{Cost}) = 0.$$

Hence the activation locus is

$$A_\tau(b) = \{\text{Ava}, \text{Sec}\}.$$

Baseline facetwise memberships. Suppose the current (facetwise) fuzzy acceptances are

$$\mu(s_{\text{email}} \mid \text{Ava}) = 0.97, \quad \mu(s_{\text{email}} \mid \text{Sec}) = 0.92, \quad \mu(s_{\text{email}} \mid \text{Lat}) = 0.88,$$

$$\mu(s_{\text{email}} \mid \text{CS}) = 0.90, \quad \mu(s_{\text{email}} \mid \text{Cost}) = 0.35.$$

Upside-Down transform and reset. By Definition 2.20, for the activated facets we complement the memberships, and we reset the contradiction with the anchor:

$$pdf^{U_{b,\tau}}(s_{\text{email}}, a) = \begin{cases} 1 - \mu(s_{\text{email}} \mid a), & a \in \{\text{Ava}, \text{Sec}\}, \\ \mu(s_{\text{email}} \mid a), & a \in \{\text{Lat}, \text{CS}, \text{Cost}\}, \end{cases}$$

$$pCF^{U_{b,\tau}}(\text{Ava}, \text{Cost}) = 0, \quad pCF^{U_{b,\tau}}(\text{Sec}, \text{Cost}) = 0,$$

and $pCF^{U_{b,\tau}}(u, v) = pCF(u, v)$ otherwise.

Numerical result.

$$\begin{aligned} \text{Ava} &: 1 - 0.97 = 0.03, \\ \text{Sec} &: 1 - 0.92 = 0.08, \\ \text{Lat} &: 0.88 \text{ (unchanged)}, \\ \text{CS} &: 0.90 \text{ (unchanged)}, \\ \text{Cost} &: 0.35 \text{ (unchanged)}. \end{aligned}$$

Interpretation. In a FinOps review where *Budget Adherence* is the anchor and high-contradiction facets are activated, the acceptance of availability and security is *inverted*: what used to be strong pros (high μ) are now strong cons (low pdf^U) relative to the cost-driven context. This focuses decision-making on cost relief even at the expense of premium availability/security options; the contradiction reset prevents immediate re-triggering at the same threshold.

Example 2.22 (Privacy-first shift for an AI chatbot platform). **Service and facets.** Let $S = \{s_{\text{chatbot}}\}$ be an AI chatbot service. Consider

$$Pv = \{\text{GDPR, Pers, Telem, Perf, DevVel}\},$$

with GDPR Compliance, Personalization, Telemetry/Logging, Performance, and Developer Velocity. Choose the privacy anchor $b = \text{GDPR}$ and set $\tau = 0.8$.

Contradictions to the anchor. Assume (symmetrically extended) levels

$$c(\text{Pers, GDPR}) = 0.90, \quad c(\text{Telem, GDPR}) = 0.85, \quad c(\text{Perf, GDPR}) = 0.40,$$

$$c(\text{DevVel, GDPR}) = 0.30, \quad c(\text{GDPR, GDPR}) = 0.$$

Thus the activation locus is

$$A_\tau(b) = \{\text{Pers, Telem}\}.$$

Baseline facetwise memberships. Suppose the current acceptances are

$$\mu(s_{\text{chatbot}} \mid \text{Pers}) = 0.86, \quad \mu(s_{\text{chatbot}} \mid \text{Telem}) = 0.80, \quad \mu(s_{\text{chatbot}} \mid \text{Perf}) = 0.82,$$

$$\mu(s_{\text{chatbot}} \mid \text{DevVel}) = 0.75, \quad \mu(s_{\text{chatbot}} \mid \text{GDPR}) = 0.45.$$

Upside-Down transform and reset. Applying $U_{b,\tau}$:

$$pdf^{U_{b,\tau}}(s_{\text{chatbot}}, a) = \begin{cases} 1 - \mu(s_{\text{chatbot}} \mid a), & a \in \{\text{Pers, Telem}\}, \\ \mu(s_{\text{chatbot}} \mid a), & a \in \{\text{Perf, DevVel, GDPR}\}, \end{cases}$$

$$pCF^{U_{b,\tau}}(\text{Pers, GDPR}) = 0, \quad pCF^{U_{b,\tau}}(\text{Telem, GDPR}) = 0,$$

and $pCF^{U_{b,\tau}}(u, v) = pCF(u, v)$ otherwise.

Numerical result.

Pers	: $1 - 0.86 = 0.14$,
Telem	: $1 - 0.80 = 0.20$,
Perf	: 0.82 (unchanged),
DevVel	: 0.75 (unchanged),
GDPR	: 0.45 (unchanged).

Interpretation. In a privacy-first audit (anchor = GDPR) with a strict threshold, the high-contradiction facets *Personalization* and *Telemetry* invert: formerly attractive capabilities are now liabilities relative to regulatory compliance. The reset pCF^U between each activated facet and the anchor stabilizes the transformed context for subsequent analysis (e.g., re-aggregation or optimization under the new priorities).

Lemma 2.23 (Well-posedness). PFITSM $^{U_{b,\tau}}$ is a valid PFITSM instance: for all (s, a) , $pdf^{U_{b,\tau}}(s, a) \in [0, 1]$; $pCF^{U_{b,\tau}}$ remains symmetric with $pCF^{U_{b,\tau}}(a, a) = 0$; consequently the aggregated $\bar{\mu}^{U_{b,\tau}} : S \rightarrow [0, 1]$ is well-defined.

Proof. Since $\mu(\cdot \mid a) \in [0, 1]$, its complement lies in $[0, 1]$. The definition of $pCF^{U_{b,\tau}}$ preserves symmetry and diagonal zeros. Standard t -(co)norm based mixers map $[0, 1]^2$ to $[0, 1]$ and are continuous, hence the fold yields $\bar{\mu}^{U_{b,\tau}} \in [0, 1]$. $\square$

Notation 2.24. Let $\mathcal{L}$ contain elementary acceptance statements

$$\text{Acc}_{1/2}(s, a) \quad \text{“service } s \text{ is acceptable on facet } a \text{ at mid-cut”}.$$

Given a PFITSM instance, define the mid-cut valuation

$$v_{b,\tau}(\text{Acc}_{1/2}(s,a)) := \begin{cases} \text{True}, & \mu(s \mid a) \geq \frac{1}{2}, \\ \text{False}, & \mu(s \mid a) < \frac{1}{2}. \end{cases}$$

After applying $U_{b,\tau}$ we evaluate with $pdf^{U_{b,\tau}}$, obtaining $v'_{b,\tau}$.

Theorem 2.25 (Upside-Down property on facetwise acceptances). *Fix (b, τ) and let $s \in S$, $a \in Pv$ with $\mu(s \mid a) \neq \frac{1}{2}$. Then*

$$a \in A_\tau(b) \implies v'_{b,\tau}(\text{Acc}_{1/2}(s,a)) = \neg v_{b,\tau}(\text{Acc}_{1/2}(s,a)),$$

$$a \notin A_\tau(b) \implies v'_{b,\tau}(\text{Acc}_{1/2}(s,a)) = v_{b,\tau}(\text{Acc}_{1/2}(s,a)).$$

Proof. If $a \notin A_\tau(b)$ then $pdf^{U_{b,\tau}}(s,a) = \mu(s \mid a)$ and truth is preserved. If $a \in A_\tau(b)$ then $pdf^{U_{b,\tau}}(s,a) = 1 - \mu(s \mid a)$, which is on the opposite side of $\frac{1}{2}$ whenever $\mu(s \mid a) \neq \frac{1}{2}$, so truth and falsity swap. $\square$

3. Conclusion

In this paper, we formally introduced *Plithogenic Fuzzy Risk Management* and *Plithogenic Fuzzy IT Service Management* and investigate their structural properties. In future work, we expect further studies on extensions of these concepts employing Graphs[47,48], HyperFuzzy Sets[49,50], SuperHyperFuzzy Sets[51], HyperGraphs[52–54], and SuperHyperGraphs [55–58].

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Use of Artificial Intelligence: I use generative AI and AI-assisted tools for tasks such as English grammar checking, and I do not employ them in any way that violates ethical standards.

Ethical Statement: As this study does not involve experiments with human participants or animals, no ethical approval was required.

Code Availability: No code or software was developed for this study.

Clinical Trial: This study did not involve any clinical trials.

Consent to Participate: Not applicable.

Disclaimer: This work presents theoretical ideas and frameworks that have not yet been empirically validated. Readers are encouraged to explore practical applications and further refine these concepts. Although care has been taken to ensure accuracy and appropriate citations, any errors or oversights are unintentional. The perspectives and interpretations expressed herein are solely those of the authors and do not necessarily reflect the viewpoints of their affiliated institutions.

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