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Article

To What Extent Does General Relativity Provide Generalization?

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Abstract: In this paper we discuss whether Einstein's initial expectations before he developed the theory of general relativity were fulfilled and whether general relativity fully satisfied him in this respect. Einstein employed the equivalence principle as a guiding principle in the development of general relativity. We follow Einstein's historical reasoning on the equivalence principle by adopting a dynamical approach to relativity and question the theory of general relativity he arrived at in terms of his initial expectations. The key issue here is related to the interpretation of gravitational and inertial "fields" in the equivalence principle. Einstein spoke of the identity of these two notions. This reveals that his intention was to unify gravitation and inertia. In fact, the issue is also related to the extent of generalisation provided by general relativity. Einstein demands a strong generalization in general relativity that requires the equality of all frames, including accelerating frames, in the same way that the relative character of velocity provides in special relativity. However, general relativity does not provide such a strong generalisation as expected from it. Furthermore, in general relativity, the gravitational field is reduced to inertia, but the two are not fully unified. We therefore argue that Einstein's initial expectations of general relativity were not met and that the theory did not fully satisfy him. Nevertheless, we show that there is a satisfactory way out. On the basis of some heuristic arguments, we demonstrate that if gravitational and non-gravitational interactions are unified into a single elementary interaction at the most fundamental level in nature, then inertia and gravitation are fully unified and the strong generalisation being sought can be obtained. We speculate that Einstein was well aware of this fact and one of his motivations (perhaps the most important one) in his long search for a unified field theory was an attempt to fulfill his initial expectations about general relativity.

Keywords: foundations of general relativity; equivalence principle; unified field theory

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1. Introduction

Einstein's main motivation in developing the theory of general relativity was to generalize the principle of relativity by addressing the incompleteness of special relativity. In special relativity, the principle of relativity applies only to inertial frames of reference and is therefore an incomplete theory. The principle of relativity needs to be generalized to all frames of reference, regardless of their state of motion [1]. On the other hand, it was also known that Newton's theory of gravitation was not compatible with special relativity. Making the theory of gravitation relativistically invariant was another problem to be solved. The equivalence principle hinted that these two problems can be solved together. As a result, the general theory of relativity is both a generalization of special relativity and also a theory of gravitation. If we take a closer look at the generalisation character of the theory, we see that general relativity provides a generalization of special relativity in that the laws of physics written in the suitable form are diffeomorphism invariant [2,3]. This principle is sometimes known as the principle of general covariance. By virtue of this principle, the laws of physics are independent of the choice of coordinate system, whether it belongs to an inertial or an accelerating frame of reference. However, the generalization Einstein sought was stronger than that provided by the principle of general covariance. To grasp the issue, we need to understand the role of the relativity principle in the special theory of relativity. The principle of relativity is a statement of the equal validity of the laws of physics in all inertial frames. On the other hand, the proposition about the equivalence of all inertial frames provided by Einstein's special theory of relativity [4] is more than what the principle

of relativity offers. The special theory of relativity asserts the relativistic character of velocity and shows that the concept of motion at constant velocity has no absolute meaning. Accordingly, there is no preferential frame of reference that we can use to define some kind of 'absolute' velocity relative to itself; only the velocity of one inertial frame of reference relative to another can be meaningfully defined. Thus, an inertial frame of reference cannot be distinguished from another inertial frame, not only in terms of the description of the laws of physics, but in every respect.¹ Einstein chose a special simultaneity convention known as Poincaré-Einstein synchronization when formulating his special theory of relativity. In later years, however, it became clear that it was possible to choose different synchronisations without violating the principle of relativity [5–8]. In 1977 Mansouri and Sexl [9] showed that with an appropriate choice of clock synchronisation, it is possible to choose a preferential frame system equivalent to special relativity in terms of all experimental results. In the preferential frame theory of Mansouri and Sexl, the relative character of velocity would in some sense disappear. Indeed, by taking the observation of this preferential frame of reference absolute in the definition of velocity, we lose its relative character. In line with the arguments of Mansouri and Sexl, one could argue that there is a preferential frame of reference in nature and that velocity is an absolute notion defined with respect to that preferential frame (although we cannot experimentally determine the numerical value of this absolute velocity).² The above discussion has made it clear that the relative/relational character of velocity is not a necessary consequence of the principle of relativity. Einstein demands a strong generalization in general relativity that requires the equality of all frames, including accelerating frames, in the same way that the relative character of velocity provides in special relativity [10]. For this to happen, acceleration, like velocity, needs to be a relative quantity. The main difficulty in achieving this is the lack of reciprocity in accelerated motion. Indeed, there is no reciprocity in accelerated motion and acceleration is not a relative notion in general relativity [11]. It seems that although Einstein achieved his goal of generalisation with the principle of general covariance to a great extent, he could not achieve a generalisation in the strong sense he had initially intended.

In the years following its establishment, general relativity achieved significant success in understanding the universe and the behavior of celestial bodies. It provided an indispensable framework for the standard model of cosmology and was used to explain various gravitational phenomena such as black holes, gravitational lenses and gravitational waves [2,12]. Particle physicists, cosmologists and astrophysicists tend to see general relativity as a theory of gravitation and are less interested in its "generalization" character. The second aspect of the subject is that general relativity is a theory of gravitation. Einstein's expectation of strong generalisation from general relativity is deeply related to his idea of unifying the notions of inertia and gravitation. The equivalence principle guided Einstein in developing the theory of general relativity. This principle also served as a bridge between accelerative motion and the theory of gravitation. Therefore, it is important to follow his historical reasoning on the principle of equivalence in order to understand his real intention. In his 1912 paper Einstein explicitly used the name 'equivalence principle' and in this and earlier papers [13–15] he referred to the physical equivalence of the gravitational and acceleration fields. In his 1921 paper [16] he was more explicit about his intentions and made it clear that gravity and inertia are identical and that this fact is a consequence of the numerical equality of gravitational and inertial masses. We think that when Einstein used the term "identical" (instead of equivalent), he wanted to emphasize that inertia and gravitation are essentially the same. That is, they are not two different concepts that imitate each other under certain conditions, but they are unified as a more fundamental concept. Indeed,

¹ Here we refer to the abstract concept of the frame of reference. We can only distinguish these (abstract) inertial frames of reference by their relative position and velocity, but these are purely relational concepts.

² We should note that it is not important for our argument whether the preferential frame of reference is empirically detectable or not. Although the numerical value of the velocity with respect to the preferential frame cannot be determined, it is sufficient to assume that a certain value exists.

Einstein probably thought that the numerical equality of gravitational and inertial masses was not the result of a coincidence, but the result of a fundamental principle in nature. He used the name equivalence principle, presumably to refer to this fundamental principle. That is, the name equivalence principle should be understood in the sense of the fundamental principle of nature underlying the equivalence of gravitational and inertial masses.³ Therefore, it can be argued that the idea of the identity of gravitational and inertial fields reflects Einstein's original thought and his expectation from general relativity. According to this interpretation, gravitational and inertial "fields"⁴ are identical. They are unified in one field and the distinction between gravitation and inertia is only apparent. On the other hand, in general relativity, gravitational field is reduced to inertia [17], but these two are not fully unified. Indeed, even though a free-falling test body in a gravitational field is inertial and the gravitational field is thus reduced to inertia, the gravitational field alone is not sufficient to determine inertia; the inertia of the test body can be defined in gravity-free space. A second important reason why unification cannot be achieved is that the inertial field cannot be described by a dynamical field, but the gravitational field is dynamical. We will discuss this issue in detail in the following sections.

To summarize, what Einstein expected from general relativity was a strong generalization of special relativity and a unification of the notions of inertia and gravitation. Although these were not explicitly stated by Einstein, we think they emerge from a careful analysis of his writings and his gedankenexperiment on the equivalence principle. On the other hand, his expectations were not fully met, as briefly mentioned above. As we will discuss in detail in this paper a dynamical approach to relativity is useful to better understand the issue and to see that there is a way out, i.e., it could be a way of meeting his expectations. What we mean by a dynamical approach can be summarized as follows: We consider the frame of reference as a dynamic system, not as an abstract mathematical construction. Acceleration and all types of change in the frame of reference occurs as a result of particle interactions (between the particles that make up the reference frame and the force carriers) on the reference frame. Such an approach differs from the kinematic approach, which does not deal at all with the dynamical system of the frame of reference and considers it only as an abstract mathematical construction. The dynamical approach provides two important benefits in our analysis: (1) In following Einstein's original gedankenexperiment on the equivalence principle, we need a well-defined notion of acceleration. However, acceleration as a kinematic quantity $\frac{d^2x^\mu}{dt^2}$ gives an ill-defined vector. On the other hand, in our dynamical approach we define acceleration indirectly as "being subjected to a net interaction on average" using the well-defined notion of interaction. (2) The dynamical approach makes it clear that an active transformation takes place in the clocks and rulers of the accelerating frame by dynamical processes caused by particle interactions, while the accelerating observer's observation of other frames (inertial frames) is generated by passive transformations. Perspectival observations that arise as a result of passive transformations do not obey dynamical laws. This issue is vital to our argument, as we shall see later.

The seeds of the dynamical approach to relativity were planted in the work of FitzGerald, Larmor, Lorentz and Poincaré [5,18]. In his 1919 paper, Einstein divided the theories in physics into two categories: constructive and principle theories [19]. He then stated that the theory of relativity is a principle theory. Principle theories assume observed properties of phenomena as fundamental principles and reveal their consequences. On the other hand, instead of accepting these principles as fundamental assumptions, constructivist theories construct them from simpler propositions. While the approaches of FitzGerald, Larmor, Lorentz, Poincaré to relativity were constructive, Einstein's approach in 1905 [4] was a principle approach [5,18]. One important reason why Einstein's approach

³ Einstein's original thought experiment on equivalence principle (see section 3.2) is considered erroneous or misleading by some contemporary authors. However, if reconsidered in the light of the above perspective, Einstein can be vindicated. We will address this issue later on.

⁴ By the term inertial field we mean the non-dynamical field that an accelerating observer observes as an inertial effect. For example, the centrifugal force field is one such inertial field.

is a principle approach is that he accepted the coordinates of the reference frame (including the timelike coordinate) as fundamental notions. According to this approach, primitive structureless clocks measuring time and structureless rulers measuring length and distance are placed at every point in space. The consequences of transformations (Lorentz transformations) in these fundamental notions of coordinates are revealed and analyzed. On the other hand, a working clock is nothing but a dynamical system in periodic motion, and a ruler is a rigid structure of material points. But then, what is the physical origin of the changes in accelerating clocks and rulers? Einstein's principle approach does not answer these questions because it presupposes them from the beginning. Einstein himself admitted that the primitive notions of the structureless clock and ruler were unsatisfactory [5]. Einstein's principle approach provides a kinematical viewpoint by treating the frame of reference as an abstract construction. On the other hand, the dynamical approach has a constructivist character because it tries to explain relativistic phenomena with dynamical laws. Indeed, the dynamical approach answers questions that Einstein's principle approach fails to answer, explaining what happens to moving rods and clocks on the basis of some dynamical processes at work. Although there are many papers in the literature on the dynamical approach to relativity, see for example [20–22], our approach here is defined in general terms and contains assumptions that may be obvious to many physicists. This is because our aim in this paper is not to describe a dynamical approach in detail, but only to use general assumptions to the extent necessary for our purpose. If it is desired, the assumption that the theory describing particle interactions is Poincaré invariant can be added to our dynamical approach. For our purposes here, however, less restrictive assumptions are sufficient. The idea that changes that occur in the dynamical system of the frame of reference are ultimately the result of elementary particle interactions is a perspective that is compatible with quantum field theory. However, it should be noted that we adopt a realistic approach to quantum field theory. According to this approach, we assume that not only on-shell particles, but also virtual particles have physical reality.⁵ Such an approach is in line with Feynman's path integral formulation of quantum mechanics [23] and his perturbation approach to quantum field theory [24]. Finally, we would like to note that while it may seem obvious to think of reference frames as dynamical physical systems, it causes some serious problems in both classical and quantum reference frames [25]. Therefore, the problem of treating reference frames as dynamical systems is not so straightforward, but rather a thorny one. We ignore such issues in this paper.

2. Dynamical Approach to Accelerating Frames

Consider an inertial frame S and an accelerating frame S' that is initially stationary with respect to S . The acceleration of the S' frame is due to particle interactions at the quantum level. An observer in the S' frame can in principle detect the effects of these interactions, for example with an accelerometer, to the extent allowed by the uncertainty principle. Relative to an observer at S , as the S' frame accelerates, the periods of its clocks dilate, its rulers contract and two spatially separated clocks of S' that were initially synchronous become desynchronized.⁶ From the point of view of the observer in the S' frame, the S frame is also accelerating. That is, the speed of S observed by S' increases with time. But such acceleration is not dynamic. The acceleration of S relative to S' is not due to particle interactions, and the accelerometer of the observer S does not record any acceleration. The observation of the observer S' about S 's clocks and rulers is symmetric with S 's observation about S' . Indeed, relative to an observer at S' , as the S frame accelerates, the periods of its clocks dilate, its rulers contract and two

⁵ By the physical reality of all quantum fields, whether on-shell or off-shell, we mean that they are not epistemic constructions but dynamical fields with ontological reality. Our dynamical approach may not seem correct for those who claim that virtual particles involved in the realization of elementary particle interactions are merely mathematical tools and have no ontological reality.

⁶ In this paper we assume Poincaré-Einstein synchronization and apply Lorentz transformations. For our analysis in this paper, the choice of synchronization is not critical. Therefore, we always adopt the Poincaré-Einstein synchronization without discussing the freedom of synchronization.

spatially separated clocks of S that were initially synchronous become desynchronized. However, the observation of the observer S' about the clocks and rulers of S is not dynamic. It is the clock and ruler of S' that actually accelerate and undergo dynamical change. At this point we make the distinction between *dynamical* and *perspectival*.⁷ A change observed in relative velocities, clocks and rulers may have a dynamical origin. In this case, the relative velocity of a frame we observe and its clocks and rulers change through a dynamical process, i.e., the interactions of quantum fields (particles) caused the change. Or the change we observe in relative velocities, clocks and rulers may be the result of the dynamical change in our own clocks and rulers. What really changes is our frame of reference and the clock and rulers we use. Since we measure the changes in other clocks and rulers we observe compared to our own clocks and rulers, we conclude that they have changed. If the change or effect is of the type we first described, we call it a *dynamical* change or effect. On the other hand, if the change or effect is of the second type, we call it a *perspectival* change or effect. However, there is an important point we would like to point out here. At first glance, one might think that a perspectival change should happen in the opposite way to a dynamical change. For example, the dilation of the period of the clock used by S' does not mean that the observer of S' will observe that the period of S 's clock will shorten. The same can be said for rulers. The point to keep in mind here is that an observer's statement about another observer is a statement that needs to be found by measurement. That is, one should consider a proposition of the form "If an observer S' measures the length of a ruler or the period of a clock in frame S , and if we take into account that the speed of information transmission during this measurement is limited to the speed of light, then the observer S' finds". If all the details of the measurement are handled carefully, it can be shown that the observation of an observer in the inertial frame, momentarily at rest with the accelerating S' frame, about S 's clocks and rulers is symmetric with S 's observation about the clocks and rulers in this momentarily rest frame.

In relativity, the distinction between dynamical and perspectival is also important for understanding some phenomena that at first glance appear to be paradoxes. A historically important one of these so-called paradoxes is the "clock paradox" or the "twins paradox". We will not go into its details in order not to distract from our purpose in this paper. But in summary, we can say that the clock paradox can be resolved by considering whether the accelerations of clocks are dynamical or perspectival. One of the clocks is actually accelerating, the acceleration of the other relative to the first is only apparent, i.e., perspectival. The clock paradox is easily solved by realizing that acceleration is not a relative notion. In Feynman's own words, "This is called a paradox only by the people who believe that the principle of relativity means that all motion is relative." [26]. The distinction between dynamical and perspectival is important in interpreting the equivalence principle. We will interpret the equivalence principle by using these concepts, but first we need to make an analysis of the accelerating frames of reference. Suppose the S' frame is rigidly accelerated by a dynamic process. In rigid acceleration, the proper lengths in the accelerating frame are constant. Rigid acceleration is necessary for us to construct a coordinate system composed of rigid material axes on the accelerating frame. For such a rigid acceleration, the following hyperbolic transformations are applied between the coordinates of the frames S and S' [3]:

$$\begin{aligned} t &= \left[\frac{c}{g} + \frac{x'}{c} \right] \sinh \left(\frac{gt'}{c} \right) \\ x &= \left[\frac{c^2}{g} + x' \right] \cosh \left(\frac{gt'}{c} \right) - \frac{c^2}{g} \\ y &= y' \\ z &= z' \end{aligned} \tag{2.1}$$

⁷ We borrowed this terminology from Miller [20].

Here, (x, y, z, t) are the coordinates of the inertial frame S , (x', y', z', t') are the coordinates of the accelerating frame S' and g is the constant proper acceleration at the origin of frame S' . We assume that the velocity of S' with respect to S is along positive x -axis. With the help of transformations (2.1), the metric used by the accelerating observer S' is found as follows:

$$ds^2 = -\left(1 + \frac{gx'}{c^2}\right)^2 c^2 dt'^2 + dx'^2 + dy'^2 + dz'^2. \quad (2.2)$$

This metric is known as the Kottler-Møller metric [27,28]. The coordinate axes we define on the accelerating S' frame cannot be arbitrarily large. The length ℓ of the axes must satisfy the inequality $\ell < \frac{c^2}{g}$ [3]. The Kottler-Møller metric gives a non-uniform proper acceleration on the x' -axis. Indeed, the magnitude of the proper acceleration at x' is given by

$$g(x') = \frac{g}{1 + \frac{gx'}{c^2}}. \quad (2.3)$$

From the expression (2.3), it can be seen that the proper acceleration diverges for $x' = -\frac{c^2}{g}$. This is because the $x' = -\frac{c^2}{g}$ surface defines a horizon for the accelerating observer. The horizon $x' = -\frac{c^2}{g}$ is called the Rindler horizon and an event in the region $x' < -\frac{c^2}{g}$ cannot be causally related to the accelerating observer, provided that acceleration does not cease, but continues forever [2]. The observer S' cannot observe an event behind the Rindler horizon, but with the help of analytic continuation one can construct a metric of the region behind the horizon. The metric of the region $x' < -\frac{c^2}{g}$ behind the horizon is known in the literature as the Milne metric. To obtain the Milne metric, one moves from the x' coordinate of the Kottler-Møller metric to the x^* coordinate by the following variable substitution:

$$1 + \frac{gx'}{c^2} = \sqrt{1 + \frac{2gx^*}{c^2}} \quad (2.4)$$

We then obtain the following metric describing the spacetime in both regions $x^* > -\frac{c^2}{2g}$ and $x^* < -\frac{c^2}{2g}$ separated by the horizon [29]:

$$ds^2 = -\left(1 + \frac{2gx^*}{c^2}\right) c^2 dt'^2 + \frac{dx^{*2}}{\left(1 + \frac{2gx^*}{c^2}\right)} + dy'^2 + dz'^2. \quad (2.5)$$

If we cross the horizon $x^* = -\frac{c^2}{2g}$ from $x^* > -\frac{c^2}{2g}$ to the region $x^* < -\frac{c^2}{2g}$, we see that the spacelike and timelike coordinates are displaced. In the region $x^* < -\frac{c^2}{2g}$, x^* coordinate becomes timelike and the ct' coordinate becomes spacelike. If we define x^* as the timelike coordinate and ct' as the spacelike coordinate and denote by cT and X respectively, then the metric (2.5) takes the following form:

$$ds^2 = -\frac{c^2 dT^2}{\left(\frac{2gT}{c} - 1\right)} + \left(\frac{2gT}{c} - 1\right) dX^2 + dy'^2 + dz'^2. \quad (2.6)$$

The above procedure is equivalent to crossing the Rindler horizon with analytic continuation [29].

An important result for accelerated motion is that there is *no reciprocity* between observers in inertial and accelerated frames observing each other [11]. The trajectory of the accelerated frame S' with respect to an observer in the inertial frame S can be obtained by taking $x' = 0$ in equations (2.1).

On the other hand, the trajectory of S relative to an observer in the S' frame is found by solving the geodesic equation. Non-zero Christoffel symbols for the Kottler-Møller metric are

$$\Gamma_{00}^1 = \frac{g}{c^2} \left(1 + \frac{gx'}{c^2} \right), \quad \Gamma_{01}^0 = \Gamma_{10}^0 = \frac{g}{c^2 \left(1 + \frac{gx'}{c^2} \right)} \quad (2.7)$$

which gives the following geodesic equation for a free falling particle along the x' -axis:

$$\frac{1}{\left(1 + \frac{gx'}{c^2} \right)} \frac{d^2 x'}{dt'^2} - \frac{2g}{c^2 \left(1 + \frac{gx'}{c^2} \right)^2} \left(\frac{dx'}{dt'} \right)^2 + g = 0 \quad (2.8)$$

The solution of this geodesic equation for the initial condition $x'|_{t'=0} = 0$, $\frac{dx'}{dt'}|_{t'=0} = 0$ is as follows:

$$x'(t') = \frac{c^2}{g} \left(\frac{1}{\cosh\left(\frac{gt'}{c}\right)} - 1 \right) \quad (2.9)$$

We observe from this equation that $x' \rightarrow -\frac{c^2}{g}$ when $t' \rightarrow \infty$. This result confirms that the surface $x' = -\frac{c^2}{g}$ is a horizon for the accelerated observer. From (2.1) and (2.9) we see that no reciprocity exists for accelerated motion; there is no reciprocity between S 's observation of S' and S' 's observation of S . This result was emphasized by Rohrlich in his well-known paper on the equivalence principle [11]. On the other hand, from a purely kinematic point of view, this result is not so obvious. Indeed, from S' 's perspective the origin of S also appears to be accelerating, i.e., the relative velocity of the origin of S changes per unit time. Moreover, since it is possible to apply Lorentz transformations between the instantaneous rest frame at S' and the frame at S using the relative velocity of S at each acceleration instant, one should obtain hyperbolic transformations like (2.1) with respect to the viewpoint of S' . The reason why we cannot apply hyperbolic transformations between S' and S (namely the transformation $S' \Rightarrow S$) is our observation that it is S' that is “really” accelerating; S 's proper acceleration is zero but S' 's is g . However, the origin of our observation about proper acceleration is dynamical; the accelerometer determines the effects of the dynamical processes that lead to acceleration. For example, if acceleration occurs with the thrust of rocket engines, the accelerometer detects the wave created by the particle interactions coming to it from the engines.

The absence of reciprocity in accelerated motion is an obvious fact according to the dynamical approach to relativity. Indeed, the acceleration of S' with respect to S takes place through a dynamical process; S' is subject to accelerating particle interactions. On the other hand, the acceleration of S with respect to S' is perspectival. Therefore, the acceleration of S' with respect to S and the acceleration of S with respect to S' are two distinguishable processes. Here, it is key to understand that S' 's observation of geodesics and hence of the spacetime metric (Kottler-Møller metric) is perspectival. Indeed, while the accelerating-dynamical processes taking place in the reference frame S' perform active transformations on the S' frame (on its clocks and rulers), it is the passive transformations that determine S' 's observation of the trajectories of inertial observers and hence the geodesics of the Kottler-Møller metric. Therefore, the metric observed by the S' cannot be represented by a dynamical field. We will discuss the consequences of this fact in the context of the equivalence principle. But first we need to analyze the dynamical process of acceleration in a little more detail.

Let us now examine rigid acceleration via the dynamical approach. The axes of the S' frame are constructed from rigid material points. The proper lengths do not change on average during acceleration. Therefore, accelerating interactions occur simultaneously on average at different spatial points in the S' frame, or equivalently, we can say that interactions occur simultaneously at different axis points relative to the center of mass frame of the material axes. Interactions should occur simultaneously, at least on *average*, at different axis points in the S' frame. Otherwise, the axes

are deformed and the proper lengths change. As a realistic example, consider that the S' frame is attached to an accelerating rocket. When the rocket engines start to run, the accelerating interactions will first occur at the trailing end of the rocket, but will spread in waves over time to the entire rocket. Consequently, the proper length of the rocket may be subject to small oscillations. However, if we average the interactions over a long time period with respect to the interaction time intervals, then the interactions are on average simultaneous in the rocket's rest frame and the proper length of the rocket does not change. From the point of view of the inertial frame S , the interactions that accelerate the frame S' do not occur simultaneously at different points on the axes of S' , but the interactions at the rear occur before those at the front. This is a consequence of the relativity of simultaneity. As S' accelerates, the number of interactions per unit time at the rear points of the axes increases compared to the front points and the axes contract in the direction of acceleration. Here we have assumed that the acceleration does not change direction. It can be shown that the contraction in S' parallel to the direction of motion gives the Lorentz-FitzGerald contraction [30,31]. We have seen that the hyperbolic transformations (2.1) apply to a non-uniform acceleration with respect to an observer in both the S' frame and the inertial frame S . The non-uniform nature of rigid acceleration can be seen as a consequence of Lorentz-FitzGerald contraction. Since the rear part of an accelerating rigid rod will approach the front part due to contraction, the acceleration of the rear part should be greater than that of the front. This simple fact is not mentioned in many standard textbooks and leads some students and even experienced physicists to get the wrong idea that hyperbolic transformations (2.1) are valid for uniform acceleration and that the Kottler-Møller metric (2.2) is the metric for such a uniformly accelerating observer covering a finite domain.⁸ For example, in ref.[32], the Kottler-Møller metric was considered to describe a rigid frame with uniform acceleration. However, if so, the equivalence principle can be applied in a finite size region of the accelerating frame. On the other hand, the Kottler-Møller metric describes a non-uniform (Newtonian) gravitational field. Therefore, it can be concluded (and has been concluded in ref.[32]) that the equivalence principle is not valid. However, since the Kottler-Møller metric can be obtained from the transformations (2.1), it is the metric of a finite-size frame with *non-uniform* acceleration. Since the frame has a non-uniform acceleration in field-free space, the equivalence principle can only be applied in the infinitesimal neighborhood of a point of the accelerating frame. In such an infinitesimal neighborhood, the equivalence principle is strictly valid.

We have already discussed that rigidly accelerating frame axes cannot cover a very large domain, but must be on a length scale ℓ smaller than $\frac{c^2}{g}$. Otherwise, the magnitude of the acceleration diverges. The condition $\ell < \frac{c^2}{g}$ is automatically fulfilled for a dynamical acceleration process. The acceleration divergence in kinematic relativity is due to Lorentz-FitzGerald contraction; when a very long rod of length $\ell > \frac{c^2}{g}$ contracts, the rear end of the rod approaches the front end and its speed becomes greater than the speed of light in order to accommodate the contraction. But such a situation is dynamically impossible because it requires infinitely intense particle interactions and infinite forces. Therefore, no matter how long the rigid rods on the accelerating frame S' , their contraction never results in an infinite acceleration; the condition $\ell < \frac{c^2}{g}$ is always satisfied. On the other hand, we have seen that there is no reciprocity in acceleration. What about the contraction of a rod in S relative to an observer in S' ? To answer this question let us now take the point of view of an observer in the accelerating S' frame and consider a very long rigid rod of length $\ell \gg \frac{c^2}{g}$ at rest on the inertial frame S . In this case, the acceleration of the rod does not occur dynamically and is therefore not subject to dynamical constraints. If we assume that the observation of the observer in the S' frame at a given moment coincides with

⁸ In some textbooks, for example in [3], the term uniform acceleration is used for an infinitesimal coordinate system (Cartan's repère-moblie) that is accelerating. But later, when reference frames of finite size are considered, it is no longer mentioned that the acceleration in field-free space is not uniform. This leads to the misconception that for rigidly accelerating frames covering a finite region, the acceleration in field-free space is uniform.

the observation of the observer in the momentarily rest inertial frame (comoving inertial frame), then the observer S' observes that the rod in the S frame undergoes Lorentz-FitzGerald contraction. As a result, according to the observer S' , the rear end of the rod exceeds the speed of light. But the rod is actually inertial. It has not been subjected to any dynamical acceleration process. The observation of the observer in S' about the acceleration of frame S and the rod on it is perspectival. Therefore, there is nothing paradoxical here. It is like observing the movement of distant stars when we turn our heads 30 degrees while looking at the night sky. In this case, we see stars traveling across an arc hundreds of light years long in a few seconds. In the case where the velocity changes direction, the observations of the accelerating observer are exactly analogous to this example. In such case, the accelerated rigid axis system undergoes a dynamic Wigner rotation. On the other hand, the accelerating observer's observation of distant stars as they rotate on arcs is perspectival. One might object that an accelerating S' observer cannot observe the faster-than-light contraction of the rear end of a very long inertial rod such that $\ell \gg \frac{c^2}{g}$ because the rear end lies behind the Rindler horizon. This is true, but it does not provide an argument that S' 's observation is not perspectival. Because for the accelerating observer, the horizon exists as long as the acceleration continues. Suppose that the observer S' decides with her free will to end the acceleration at a time t' . Let us also assume that the observer in frame S' measures the length of the rod just before the acceleration begins and just after the acceleration is completed. By comparing these two measured values, she can determine the average approach speed of the rear end of the rod towards the front end. The S' observer finds the following average approach speed:

$$\bar{v} = \frac{2g\ell}{c} \left[1 - \sqrt{1 - \beta_0^2} \right] \ln^{-1} \left(\frac{1 + \beta_0}{1 - \beta_0} \right). \quad (2.10)$$

Here, the initial and final relative velocities of S' with respect to S are zero and v_0 , respectively. For a very long inertial rod, average speed takes values that exceed the speed of light. For instance, for $\ell > 2.61 \times \frac{c^2}{g}$ and $\beta_0 = 0.9$, average speed $\bar{v}$ exceeds the speed of light. This result clearly proves that S' 's observations are perspectival, since faster-than-light motion cannot occur dynamically. However, the observer S' can realize that her observations are perspectival only after she stops accelerating. While acceleration continues, dynamically impossible processes such as faster-than-light motion are hidden behind the Rindler horizon. After the observer S' finishes her acceleration, she will observe that the horizon disappears, the timelike coordinate of an observer in the region behind the horizon becomes spacelike, and most importantly, the average contraction speed of the inertial rods is faster than light. The fact that the observer reaches these observation results after completing her acceleration does not change the perspectival character of her observation. Her observations are perspectival even as her acceleration continues. Indeed, it may be that the S' will decide to stop the acceleration some time in the future. In such a case, S' will realize in the future that her observations are perspectival. If retrocausality is ignored, which is justified in a classical theory, then the observations of the S' must always be perspectival. In this case, however, the observations of the observer S' , the geodesics describing the trajectories of inertial particles and the spacetime metric are not dynamical phenomena. If the equivalence principle is a guiding principle for general relativity, as in Einstein's original reasoning, then it follows that the gravitational field is not dynamical either. To be precise, the issue is this: If the local equivalence of the observations of observers in uniform acceleration and uniform gravitational fields is not a coincidence, but is due to the fact that these two fields (gravitational and inertial fields) are essentially the same, as Einstein believed, then we have to face the problem of the

inertial field being perspectival.⁹ It follows that either the gravitational field must be non-dynamical or the two fields cannot be essentially the same, that is, they cannot be unified.

To summarize, the fact that acceleration is not a relative quantity in the theory of relativity is an evident consequence of the dynamical approach. A frame that is accelerating by a dynamical process can be distinguished from an inertial frame that appears to be accelerating. This distinguishability is the origin of the non-relative character of acceleration. The dynamic process (particle interactions) that leads to acceleration performs an active transformation in the frame of reference, while the apparent change in the inertial frame, which appears to be accelerating, is the result of a passive transformation. Indeed, a dynamically accelerating observer can, by non-local measurements in spacetime, observe faster-than-light contraction of the inertial frame that appears to be accelerating. Since such an observation is the result of passive transformation, it does not contradict the theory of relativity. However, it shows us that the essence of such an observation is not dynamical. Accordingly, inertial effects are perspectival in character, not constrained by dynamical laws. If we interpret the equivalence principle as the *identity* of the inertial and gravitational fields, as Einstein originally did, we come to the conclusion that the gravitational field is not dynamical, a conclusion that is not compatible with current views on gravitation.

3. Einstein's Equivalence Principle and the Unification of Gravitational and Inertial Fields

3.1. Statement of Einstein's Equivalence Principle

The term equivalence principle is used in many different meanings in the literature [10]. In this paper we will follow the definition according to Ref.[33]. Let us give the definitions we use for the sake of precision. First, the *weak equivalence principle* is defined. According to the weak equivalence principle, to an observer in a laboratory free-falling in a uniform gravitational field, the laws of mechanics for free-falling bodies behave as if gravity does not exist. In other words, the existence of gravity cannot be demonstrated in such a laboratory based on the laws of mechanics. The rigorous definition of the weak equivalence principle is given as follows [33]:

The trajectory of a neutral test body dropped at an initial point in spacetime and given an initial velocity is independent of its internal structure and composition.

Einstein's equivalence principle requires two extra conditions in addition to the weak equivalence principle. These extra conditions are the independence of the outcomes of local non-gravitational test experiments from the velocity of the free-falling laboratory and from the spacetime point. The rigorous definition of the Einstein's equivalence principle is given as follows [33]:

- *The weak equivalence principle holds.*
- *The outcome of any local non-gravitational test experiment is independent of the velocity of the free-falling laboratory.*
- *The outcome of any local non-gravitational test experiment is independent of the point in spacetime where it is performed.*

Heuristically, according to Einstein's equivalence principle, the existence of gravity in a free-falling local laboratory cannot be demonstrated not only by laws of mechanics, but also by all non-gravitational laws of physics. The gravitational laws of physics are not taken into account in Einstein's equivalence principle. For example, a Cavendish type experiment in a free-falling laboratory is not included. An important result from Einstein's equivalence principle, which is also crucial for our analysis in this paper, is the *universal coupling* property of gravitation [33,34]. According to the universal coupling

⁹ It could be argued that Einstein's original gedankenexperiment regarding the equivalence principle was misleading in relating acceleration to the gravitational field, and therefore it is erroneous to infer the nature of the gravitational field from this gedankenexperiment. This may be true, but our aim is to follow Einstein's historical reasoning and understand his thinking. We will discuss the original gedankenexperiment in detail in the next section, but for now I would like to note that one of the main reasons why the original gedankenexperiment is considered misleading by some authors is that it was designed in line with Einstein's initial expectations, which were not fulfilled in general relativity.

property, the gravitational field $\mathbf{g}$ (symmetric rank 2 tensor field) is coupled to all non-gravitational fields in the same manner. Consider a frame of reference in which the self-gravitational effects of its constituents are neglected. In this case, we can assume that the reference frame is made entirely of non-gravitational fields. The reference frame consists of measuring instruments, clocks and rulers. But then it is not possible to locally detect the gravitational field, because a measurement is a relational concept with respect to the measuring instruments. Since the gravitational field will couple to every part of the measuring device in the same manner, there is no reference point from which we can determine its presence. Of course, this conclusion does not hold for non-local measurements on a scale large enough to detect inhomogeneities in the gravitational field.

There is a subtle issue in the above discussion regarding the universal coupling property. Here the gravitational field is represented by the metric, that is a symmetric, second rank tensor $\mathbf{g}$ with signature +2 that reduces to Minkowski metric when gravity is turned off; and when $\mathbf{g}$ is replaced by η (Minkowski metric), the action for non-gravitational fields becomes the action of special relativity. However, in general relativity, the gravitational field is associated with the curvature of spacetime, not the metric tensor [2]. Then the question may arise whether it is correct to represent the gravitational field by the metric tensor. This issue is especially important in the analysis regarding the equivalence principle. In fact, the Kottler-Møller metric (2.2) describes a spacetime of zero curvature, and if the gravitational field is seen as curvature, then the spacetime of the Kottler-Møller metric does not contain any gravitational field. But then (as we will see in the next subsection) how can one establish an equivalence between an accelerating frame of reference and a frame in the gravitational field? It should be noted here that the gravitational field mentioned in the gedankenexperiments on the equivalence principle is the Newtonian gravitational field. In the Newtonian limit the metric is indeed proportional to the Newtonian gravitational field. Therefore, representing the gravitational field with the metric tensor facilitates the reasoning regarding the equivalence principle. Moreover, in quantum field theoretical approaches to general relativity, the gravitational field is sometimes represented by a spin-2 particle, called the graviton [2]. The graviton field is a perturbation of the metric tensor. The treatment of the gravitational field as a metric perturbation is also used in various extensions of general relativity (see for example [35–40]). Although there are serious problems regarding the renormalizability of such a quantum field theoretical approach to general relativity, this issue is not important for our purpose in this paper. We can make an ad hoc hypothesis that various mechanisms at the fundamental level in nature guarantee that the theory remains convergent.¹⁰ Our aim here is not to propose some kind of unified field theory or to build on any such theory in the literature, but simply to construct a framework that we can use in our reasoning about the unification of inertial and gravitational fields. In setting up our reasoning, we will think of the gravitational field as if it were a particle (graviton). We will consider its geometric features as a result of its interaction with other fields. According to this approach, the universal coupling property means that the graviton minimally couples to all non-gravitation fields in the same manner (eg. it couples to their energy-momentum tensor). It can be argued that the trajectory of a body subjected to interactions characterized by universal coupling moves in a way that corresponds to the geodesics of some metric. Indeed, the acceleration due to interactions that exhibit universal coupling should be considered inertial, since in such an accelerating local reference frame there is no reference point from which we can determine the existence of these interactions and hence the acceleration. According to this interpretation, geometry emerges as a result of the interactions of particles (or quantum fields representing particles). We do not intend to argue here whether geometry or particle interactions are more fundamental. What is important for us is that such an approach makes our reasoning lucid by liberating the notion of inertia from a kinematic context and linking it to the well-defined notion of interaction.

¹⁰ It can be assumed that ultraviolet divergences are eliminated by some fundamental mechanisms in nature. For example, in the ADD [35,36] and RS [38] models, divergences are assumed to be eliminated by string theory at a fundamental level.

3.2. Einstein's Gedankenexperiment on Equivalence Principle

In his early papers before general relativity, Einstein referred to the physical equivalence between a reference frame with constant acceleration and a reference frame in a uniform gravitational field [13–15]. He discussed the gedankenexperiment expressing this equivalence in detail in his famous book [1] (see sections 18–20). Let us now see this gedankenexperiment and try to interpret it from the dynamical point of view we have described in the previous sections. The gedankenexperiment is as follows [1]: Imagine a large room-like chest with an observer inside, equipped with instruments. The chest is a Galileian reference-body and therefore gravity does not naturally exist for the observer inside the chest. In the center of the lid of the chest there is a hook to which a string is attached from the outside, and it is thought that at a certain moment the string starts to be pulled with a constant force. Under the influence of the force, the chest and the observer make a uniform accelerated motion. Accordingly, the observer should experience inertial effects. Einstein emphasized that the observer in the chest could come up with an alternative explanation for inertial effects and think that he is not actually accelerating uniformly but rather suspended at rest in the gravitational field. This is a necessary consequence of the equality of gravitational and inertial masses. Thus, a gravitational field exists for the observer in the chest, even though no such field exists for the first coordinate system chosen. What this gedankenexperiment leads us to is the physical equivalence between a reference frame with constant acceleration and a reference frame in a uniform gravitational field. On the other hand, does this physical equivalence mean that the gravitational and inertial fields are two things that are equivalent under certain conditions, but different in their essence? Or does it mean that they are identical (i.e., unified in the same essence)? Einstein made his idea clear in the last paragraph of chapter 19 of his book [1]. He said, *A satisfactory interpretation can be obtained only if we recognise the following fact: The same quality of a body manifests itself according to circumstances as "inertia" or as "weight" (lit. "heaviness")*. As we mentioned in the introduction, he expressed a similar idea in his 1921 Nature paper [16], where he said that gravity and inertia are identical. However, the general theory of relativity does not provide a complete unification of the notions of gravitation and inertia in the way Einstein intended. Indeed, as we showed in section 2, the inertial field is perspectival in character and cannot be represented by a dynamical field. Whereas the gravitational field is dynamical and therefore these two notions cannot be identical. It is also known that although the gravitational field is reduced to inertia, the gravitational field alone is not sufficient to determine it. Indeed, the inertia of a test body can also be defined in a gravity-free space. Notice that the gravitational field we are discussing in the context of Einstein's gedankenexperiment is actually the Newtonian gravitational field. Similarly, so is the notion of inertia. One could argue that such Newtonian concepts are meaningless in the context of general relativity. This is a reasonable argument, but it does not limit our reasoning as far as our purpose in this paper is concerned. This is because in this paper we are not discussing what is true according to the general theory of relativity, but whether the theory fulfills Einstein's initial expectations. In fact, the reason why Einstein's gedankenexperiment is considered erroneous or misleading by some contemporary authors is precisely because he invoked such Newtonian concepts in his argument. For example, according to Rohrlich, a principle should be a statement with general validity [11]. However, the identity of gravitational and inertial fields is a locally valid statement under the Newtonian approximation. For this reason, Rohrlich divides the equivalence principle into various statements. According to him, the statement about the identity of gravitational and inertial fields is not a principle but only a definition of the inertial frame. For Rohrlich, it is the independence of the orbits of test particles in the gravitational field from their mass and composition, implying the equivalence of their inertial and gravitational masses, which deserves to be a principle [11]. Some other authors also find Einstein's gedankenexperiment deeply flawed because of its approximate character [41]. Why, then, did Einstein present the equivalence principle through a gedankenexperiment that is, from today's point of view, flawed or at least misleading? Has he failed to see these flaws that contemporary authors have drawn attention to? In our opinion, this is not the case. We believe that Einstein designed his gedankenexperiment in line with his initial expectations regarding general relativity. He expected

that with general relativity, inertial and gravitational fields would unify and special relativity would generalize in a strong sense to all frames of reference, regardless of the state of motion. With the general theory of relativity, his initial expectations were largely fulfilled, but as we have discussed, not completely. From today's perspective, we consider his gedankenexperiment to be flawed because his initial expectations were not fully met; the theory he aimed to achieve was somewhat different from the general relativity he formulated in 1916. His use of the Newtonian approximation in his gedankenexperiment is not a flaw, since continuity of thoughts is necessary to build a bridge to a new theory that he aims to establish.¹¹ On the other hand, the idea that the gravitational field is identical to the inertial field does not comply with general relativity and may be seen as a flaw according to today's views.

To better understand the issue and to see if there is a way out to fulfill Einstein's expectations, let us reconsider the gedankenexperiment based on the dynamical approach. Denote the Galileian observer in gravity-free space by S and the observer with constant acceleration inside the chest by S' . The S frame is not subject to any accelerating particle interactions. Of course, the particles that make up the frame constantly interact with each other, but on average the sum of the forces generated by these interactions is zero. We assume that observers in the S and S' frames can both measure non-gravitational interactions, albeit indirectly. At least they can measure their average effects. Therefore, if we take the condition of not being subject to accelerating particle interactions (in the average sense) as the definition of an inertial frame, this definition would be a generally applicable definition. Indeed, since interaction is an invariant notion, S is an inertial frame with respect to all frames regardless of their state of motion. An important issue here concerns the definition of acceleration. If we take the acceleration as a kinematic quantity $\frac{d^2x^\mu}{d\tau^2}$, it is clear that this does not give a well-defined four-vector. However, in the framework of the dynamical approach, we define acceleration in terms of accelerating interactions. The notion of interaction is well defined in the context of relativity. Thus, we define the notion of inertia with the help of the notion of interaction. After presenting his gedankenexperiment on the equivalence principle in his book, Einstein claimed that the equivalence principle requires the generalization of the principle of relativity to all frames of reference, regardless of the state of motion. However, it is clear from his writings that the generalization he means is a generalization in the strong sense, as we mentioned in the introduction. Just like the role of velocity in special relativity, he intended to give a relative character to acceleration. In the last paragraph of chapter 18, he discussed the observer in a carriage that suddenly braked and decelerated. He discussed whether the observation of inertial effects by such an observer would violate the principle of general relativity, as it would give absolute physical reality to the decelerating frame. He brings this discussion to a conclusion at the end of Chapter 20. In his own words [1]: *It is certainly true that the observer in the railway carriage experiences a jerk forwards....But he is compelled by nobody to refer this jerk to a "real" acceleration....He might also interpret his experience thus: "My body of reference remains permanently at rest. With reference to it however, there exists (during the period of application of the brakes) a gravitational field which is directed forwards..."*. The above expression gives the impression that acceleration can be canceled out by the gravitational field. Perhaps the observer in the carriage is not really accelerating, but is in a gravitational field that imitates the inertial effects. However, this argument does not seem to be compatible with general relativity because according to general relativity, the static observer in the gravitational field is actually being accelerated. Here we realize that Einstein's intention was to give acceleration a relative character with the help of the gravitational field. From a dynamical point of view we could say: The observer in the carriage is accelerating because it is subject to accelerating interactions. But perhaps there are gravitational interactions that balance these accelerating non-gravitational interactions. If we take the

¹¹ In Einstein's gedankenexperiment, although the notion of inertia is used in the Newtonian sense, the existence of a more general notion of inertia is evident; he contacts this general notion of inertia under the Newtonian approach. Indeed, in the case of particle interactions, there is the notion of inertia as a measure of the change in the state of motion of particles with the interaction. Any theory that claims to explain nature at a fundamental level must account for such a notion of inertia.

average of all interactions, the observer is not actually accelerating. Since the existence of gravitational interactions cannot be known locally, one cannot speak of an absolute acceleration. A state of inertia that appears to be accelerating might be non-accelerating when gravitational interactions are taken into account. It seems that in this way Einstein tried to establish the relativistic character of acceleration and to arrive at a stronger generalization than that provided by general covariance. On the other hand, as we discussed in chapter 2, giving a relative character to acceleration is not compatible with general relativity. There seems to be a flaw in Einstein's argument above. This flaw is actually deeply related to the first one (that the notions of gravitation and inertia are identical). If we want to interpret the flaw in Einstein's argument in a dynamical context, we can say that the flaw is the idea that gravitational interactions can balance non-gravitational ones on average, thus canceling the acceleration. This idea is erroneous because gravitational and non-gravitational interactions are intrinsically different. Non-gravitational interactions can be determined by a local measurement and thus distinguished from gravitational interactions. Therefore, the idea that gravitational and non-gravitational interactions can neutralize each other in total is not correct. A dynamical approach to the subject based on particle interactions reveals the following interesting fact: The reason why Einstein's initial expectations were not fully met by the general theory of relativity is that gravitational and non-gravitational interactions are fundamentally different. In general relativity, a frame subject to gravitational interactions is described as inertial (a free-falling frame), whereas a frame subject to non-gravitational interactions is described as accelerating. It is the difference between gravitational and non-gravitational interactions that separates these two states of a body.

3.3. *The Unification of Forces*

The careful reader should have easily noticed from our discussion in the last chapter that there is a way in which Einstein's initial expectations can be fulfilled. This way is what Einstein called the unified field theory. What we understand by this term is that gravitational and non-gravitational interactions are one and the same thing (the same field) at the fundamental level in nature, but on a larger scale an apparent distinction emerges between them. If the fundamental field where gravitational and non-gravitational interaction fields merge has the property of universal coupling, then this fundamental field can be interpreted as a geometric phenomenon. Since his approach to unification is geometric, we understand that Einstein considered such an option. Let us elaborate on the subject to see that such a unification of forces would also enable the unification of inertia and gravity, and would give acceleration a relative character, making possible the strong generalization that Einstein desired.

Inertial and gravitational fields are distinct in that they are of perspectival and dynamical origin. However, the following possibility should be considered: Perhaps the perspectival/dynamical distinction is a distinction that does not exist at a fundamental level in nature and can only be defined for complex systems of observers. According to this view, there is no such distinction in nature at the fundamental level, and gravitational and inertial effects are identical as in the original interpretation of the equivalence principle. Only in the context of complex systems on a large scale does such a distinction emerge. Indeed, it is necessary to use measuring instruments to detect the presence of non-gravitational interactions and to determine the state of inertia. Such a definition of the notion of inertia requires the existence of complex measuring instruments and complex systems that evaluate these results. Therefore, the definitions and concepts we use do not correspond one-to-one to the elements at the fundamental level, but are merely reflections of them. Hence, we need to reconsider the concepts and definitions we use when making inferences about elementary physics. We think that giving the following argument, which some particle physicists claim, as an example will contribute to the understanding of the subject. According to this view, elementary particles gain their mass through their interaction with the Higgs particle that fills the vacuum. When you try to accelerate an elementary particle, it will make collisions with Higgs particles in the vacuum and there will be a resistance to motion. According to this claim, inertia is not a fundamental notion, but a notion that emerges as a result of multiple collisions, just like electrical resistance. Since an observer measuring on

a large scale would average over multiple collisions, she would reach the erroneous conclusion that there is a genuine notion of inertia. We are not defending the truth of this claim in this form, we are just giving it as an example of how inertia may not be a fundamental concept. As this example teaches us, the definition of the notion of inertia can be very different at the fundamental level in nature than what we perceive.

Let us discuss the physical origin of our perception of the inertial field to get a better grasp of the subject. The inertial fields or effects exist because non-gravitational interactions, unlike gravitational ones, do not have the property of universal coupling. Indeed, the inertial field arises when one part of a frame of reference interacts with accelerating interactions but the other part does not. For example, let's imagine a reference frame on which the electric charge is uniformly distributed. But only a small piece on the x-axis is neutral. Let the reference frame be in a uniform electric field and the frame is uniformly accelerated. The (charged) observer in the frame observes an inertial effect for the neutral part located on the x-axis. However, if every component of the frame of reference were universally subject to the same interaction, such an inertial effect would not arise. In line with this view, we can say that the local emergence of inertial effects is related to the fact that gravitational and non-gravitational interactions are locally distinguishable (they are distinguishable on the basis of universal coupling property). On the other hand, it might also be the case that such distinguishability is not valid at the fundamental level. If gravitational and non-gravitational interactions are one and the same thing at the fundamental level, but on a larger scale there is an apparent distinction between them, then gravitational and inertial fields are also identical at the fundamental level, but on a larger scale they appear to be two different but equivalent things. According to this view, we have a single dynamical field at the fundamental level. The inertial field that we perceive at the classical scale, which is perspectival in character, is a spurious field that emerges at a scale beyond the fundamental scale. It follows that, at the fundamental level, *all frames are inertial*, and the concept of inertia arises entirely from the fundamental interaction where gravitational and non-gravitational interactions are unified. The notions of inertia and gravitation are unified, and since all frames are inertial, the strong generalization Einstein originally sought is established.

Consequently, the correctness of the original interpretation of the equivalence principle depends on whether the unified field theory (unification of forces) exists. Perhaps this is why Einstein pursued the idea of unification for many years. We have not found any evidence from Einstein's writings on unified field theory to support this view. Einstein's long and persistent search for a unified field theory is said to have been motivated by formal simplicity and the belief that quantum paradoxes and the unification of forces are fundamentally related [42]. However, although he did not explicitly state it, he may have thought that the unification of gravitational and non-gravitational forces was necessary for the original interpretation of the equivalence principle to be correct and for a satisfactory generalization of relativity to be achieved. Our reasoning in this paper is based on the fact that all dynamical effects are the result of fundamental particle interactions, and this criterion is used to distinguish between dynamical and perspectival. One could argue that Einstein could not have come up with such a dynamical approach and made aforementioned dynamical/perspectival distinction in a period before quantum field theory was developed. Such an approach, however, does not require the details of quantum field theory. A reasoning similar to the one we have made could have been done in Einstein's time. Perhaps Einstein intuitively grasped the implication of the equivalence principle for the unification of forces, but preferred to remain silent on the matter, since the detailed theory of elementary particle interactions was not yet known.

4. Concluding Remarks

In order to identify inertial fields and to distinguish between perspectival and dynamical effects, we need the notion of a frame of reference. However, the frame of reference and the clocks and rulers that constitute it are composite systems. It is not clear, then, to what extent the definitions we make with the help of these large-scale composite systems are legitimate at a fundamental level.

On the other hand, we assume that the notion of interaction also applies at the fundamental level. Accordingly, if we define acceleration as "being subjected to a net interaction on average" using the well-defined notion of interaction, but not as a kinematic quantity $\frac{d^2 x^\mu}{d\tau^2}$ which gives an ill-defined vector, then at the fundamental level we arrive at a relative notion of acceleration in case gravitational and non-gravitational forces are unified. Indeed, if there is only one interaction with the universal coupling property at the fundamental level, then there can be no reference point on the accelerating frame from which the acceleration can be determined. Therefore, we need to think of the accelerating frame as inertial. In this case, the state of inertia is generalized to include accelerated motion. On the basis of this reasoning, a concept like absolute acceleration no longer exists and acceleration becomes a relative concept. Thus, the strong generalization Einstein was looking for is achieved.

Some authors in the literature, have not accepted the principle of equivalence as a fundamental principle. For example, Carroll regards general relativity as an effective theory valid at macroscopic distances and defines the equivalence principle not as a strict principle valid at the fundamental level, but as a useful guiding principle [2]. Some earlier authors, with different reasons, came to a similar conclusion and did not consider the equivalence principle to be a fundamental principle of physics [43,44]. Einstein's use of Newtonian concepts in his original gedankenexperiment on the equivalence principle, and hence the approximate character of his argument, has also been criticized by various authors, leading his argument to be seen as erroneous or misleading [11,41]. Nowadays, Einstein's original argument for the equivalence principle is often regarded either as an approximate statement that is not strictly valid or as a poorly defined statement. However, what is often overlooked in discussions on this topic is the following: Einstein did not consider the numerical equality of gravitational and inertial masses to be the result of a coincidence, but the result of a fundamental principle in nature. We think he used the name equivalence principle to refer to this fundamental principle. That is, the name equivalence principle should be understood in the sense of the fundamental principle of nature underlying the equivalence of gravitational and inertial masses¹². Einstein did not try to define this principle, but rather to understand it and explore it in depth in all its aspects. Therefore, we should not understand the gedankenexperiments he gives when explaining the equivalence principle as a definition of the equivalence principle; we should consider them as ways of revealing this principle in its different aspects. If we want to reveal this principle in its different aspects, one pillar is the identity of gravitational and inertial fields, and the other is a strong generalization of special relativity to all frames, regardless of the state of motion. A dynamical approach to relativity shows that unified field theory is another pillar of this principle. Did Einstein in his late period (1920-1955) realize this fact? Did this idea become an important motivation for his search for a unified field theory? We don't know the answers to these questions. However, although he did not express it explicitly (perhaps because his views on this subject were not clear enough), he may have had an intuition or thought that the principle of equivalence was related to the unification of forces. We admit that the claim that the equivalence principle was one of Einstein's motivations for finding the unified theory is a speculation unless supported by Einstein's writings. In this paper, we have both speculated on a historically possible scenario and tried to bring a new approach to the meaning and interpretation of the principle of equivalence.

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