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Article

Dark-Matter and Gravitational-Force Details That Echo Familiar Physics and Explain Cosmic Data

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Abstract

We suggest details about dark matter and gravitational forces. The details explain cosmic data. The data pertain to galaxies, galaxy clusters, cosmic microwave background radiation, the expansion of the universe, and densities of the universe. Possibly, no other work explains the data. Our dark-matter specification reuses a set of known elementary particles. The gravitational-force details extend Newtonian-force concepts, extend a list of two-body gravitational properties of objects, and reuse Lorentz invariance.

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1. Introduction

This unit provides context for our work, previews aspects of our work, and discusses data that our work seeks to explain.

1.1. Context

This unit discusses two 90-year-old challenges that provide context for our work.

Cosmology includes two 90-year-old challenges. Since the 1920s, people have discussed the extent to which large objects separate from one another [1,2]. Recently, people associate with the separation the term rate of expansion of the universe. People have not fully developed an explanation for the separation. For a mechanism that governs some aspects of the separation, people suggest the term dark energy. Since the 1930s, people have discussed phenomena that might not associate with the term ordinary-matter stuff [3,4]. People use the term dark matter when discussing some possible explanations for the phenomena. People have yet to settle on a preferred description of dark matter [5,6].

1.2. Preview of Our Work

This unit previews results and methods of our work.

Our work might offer insight and partial resolution regarding the two 90-year-old challenges. Our work has been ongoing for a decade [7,8]. Our work suggests the following. Dark matter exists. Dark matter elementary particles are like ordinary matter elementary particles. Some dark-matter stuff is like ordinary-matter stuff and comports with popular modeling notions of self-interacting dark matter. Most dark-matter stuff is unlike ordinary-matter stuff and comports with popular modeling notions of collisionless dark matter. A new use of multipole expansion mathematics helps describe two-body gravitation. Our techniques include means to catalog some interaction-properties of objects. Parallels exist between the combination of classical electrodynamics and Lorentz invariance and a combination of an expanded list of two-body gravitational properties of objects and gravitational multipole aspects. Uniting our dark-matter specification and our multipole gravitational expansions requires one new integer-based equation. Our approach offers candidate explanations for data. Our candidate explanations cover enough data for people to possibly find our explanations credible and compelling.

1.3. Seemingly Otherwise Unexplained Cosmic Data

This unit discusses seemingly otherwise unexplained cosmic data that our work seeks to explain.

A key goal for our work is to explain the following observed ratios of not-ordinary-matter effects to ordinary-matter effects.

- 1:1 – Amounts of some depletion of cosmic microwave background radiation [9–11].
- 1:0+ – Amounts of stuff in some individual galaxies [12–20].
- 0+:1 – Amounts of stuff in some individual galaxies. (Popular modeling associates the symbol z with redshift. Popular modeling associates redshifts of zero with the present universe. Popular modeling associates larger redshifts with earlier times in the history of the universe. Various online services associate popular modeling notions of time after a supposed so-called Big Bang and redshifts [21]. For example, $z = 7$ associates with approximately 0.76 billion years after the supposed Big Bang and with approximately 13 billion years before now.)
  - Redshifts of more than approximately seven [22,23].
  - Redshifts of approximately six [24].
  - Redshifts of less than six through redshifts of nearly zero [25–32].
- $\sim$ 4:1 – Amounts of stuff in some individual galaxies [33,34].
- 5+:1 – Amounts of stuff in many individual galaxies [12,35].
- 5+:1 – Amounts of stuff in many individual galaxy clusters [35–39].
- 5+:1 – Densities of the universe [40].

2. Methods

This unit discusses assumptions that our work makes and methods that our work develops and uses. The following statements summarize aspects of our methods.

1. Our methods include an assumption that most dark matter associates with new elementary particles and that the new elementary particles are like elementary particles that underlie ordinary matter. Most dark matter associates with popular modeling notions of collisionless dark matter. Some dark matter associates with popular modeling notions of self-interacting dark matter.
2. Our methods include applications of multipole-expansion mathematics that describe electromagnetic fields and gravitational fields that associate with multiple properties, such as electromagnetic charge and electromagnetic magnetic moment or such as gravitational mass and object-internal angular momentum, of single objects. This contrasts with popular modeling multipole expansions that associate with spatial distributions of single properties such as charge or mass. Our multipole expansions underlie characterizations of two-body electromagnetic interactions and characterizations of two-body gravitational interactions. Our methods suggest that attention to such two-body gravitational interactions can be key to explaining eras in the rate of expansion of the universe.
3. Our methods include one new, with respect to popular modeling, integer-based equation. The equation applies once for each electromagnetic property of objects and once for each gravitational property of objects. For each such application, one of four arithmetically possible solutions to the equation pertains. Our methods suggest a set of solutions that enables explaining cosmic data that pertain to galaxies, galaxy clusters, cosmic microwave background radiation, the expansion of the universe, and densities of the universe. The breadth of the data that we explain might associate with credibility for our methods and explanations.

2.1. Dark-Matter Specification

This unit discusses the candidate specification for dark matter that our work assumes and uses.

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We assume that nature includes dark matter. We assume that not-ordinary-matter effects are effects of dark matter.

We consider the set of all known elementary particles other than the photon. Based on a ratio of five-plus to one for dark-matter density of the universe to ordinary-matter density of the universe, we suggest that nature includes five dark-matter isomers of the set of all known non-photon elementary particles. Based on a ratio of one to one for some depletion of cosmic microwave background radiation, we suggest that stuff that associates with one dark-matter isomer is similar to ordinary-matter stuff. Here, as in other areas of physics, the word isomer can associate with variations with respect to might-be symmetries. Here, possible symmetries or broken symmetries might associate with the handedness of elementary particles and with the non-alignment between neutrino flavour eigenstates and neutrino mass eigenstates [41]. We suggest that one symmetry associates with the handedness of elementary particles (such as fermion elementary particles) that exhibit handedness. We suggest that one might-be symmetry associates with matches between charged-lepton flavours and charged-lepton masses.

Table 1 suggests a numbering scheme for the six isomers, specifications for the one ordinary-matter isomer and the five dark-matter isomers, and aspects of the stuff that associates with each isomer.

Table 1. Matches between masses and flavours, for isomers of elementary fermions. The symbol l_{isomer} denotes the isomer number. The symbol $l_{isomer-pair}$ denotes the isomer-pair number. The masses of counterpart elementary particles are, across the isomers, the same. Handedness associates with whether the relevant handed elementary particles are left-handed or right-handed. For each row, the quark column assigns the three generation numbers in the order of increasing geometric-mean mass, with the geometric mean associating with the masses for the two quarks that are relevant to the generation. (The following pertain for the ordinary-matter isomer. Generation-1 associates with the up quark and the down quark. Generation-2 associates with the charm quark and the strange quark. Generation-3 associates with the top quark and the bottom quark.) For each row, the lepton column assigns the three flavor numbers in the order of increasing mass for the one charged lepton that is relevant to the flavour. (The following pertain for the ordinary-matter isomer. Flavour-1 associates with the electron. Flavour-2 associates with the muon. Flavour-3 associates with the tau.) The stuff column identifies stuff made from the isomer as OM, as in ordinary matter, or DM, as in dark matter. The acronym SEA abbreviates our term significantly-electromagnetically-active. The stuff that associates with DM (SEA) interacts electromagnetically with itself on a par with OM stuff interacting electromagnetically with OM stuff. The acronym MEA abbreviates our term marginally-electromagnetically-active. The stuff that associates with MEA interacts electromagnetically with itself marginally, perhaps mostly via the magnetic moments of zero-net-charge objects. PMN abbreviates the two-element term popular-modeling notion. We suggest that some dark matter associates with popular modeling notions of self-interacting dark matter. The acronym SIDM abbreviates the popular modeling term self-interacting dark matter. We suggest that some dark matter associates with popular modeling notions of CDM. The acronym CDM abbreviates the popular modeling term collisionless dark matter.

l_{isomer}	$l_{isomer-pair}$	Handedness	Quark generations	Lepton flavours	Stuff	PMN
0	0	Left	1, 2, 3	1, 2, 3	OM (SEA)	OM
3	0	Right	1, 2, 3	1, 2, 3	DM (SEA)	SIDM
1	1	Left	1, 2, 3	3, 1, 2	DM (MEA)	CDM
4	1	Right	1, 2, 3	3, 1, 2	DM (MEA)	CDM
2	2	Left	1, 2, 3	2, 3, 1	DM (MEA)	CDM
5	2	Right	1, 2, 3	2, 3, 1	DM (MEA)	CDM

We use the one-element term MEA-isomer to designate an isomer other than isomer-0 and isomer-3. We suggest that the fermion flavour-and-mass pairings for the MEA isomers (that is, isomer-1, isomer-2, isomer-4, and isomer-5) led to stuff that associates with the MEA isomers forming stable counterparts to isomer-0-stuff neutrons and to stuff that associates with the MEA isomers not forming significant numbers of counterparts to isomer-0-stuff atoms. For each one of the six isomers, a ground-state singly-charged baryon that includes exactly three generation-3 quarks would be more massive than the counterpart, within the same isomer, ground-state zero-charge baryon that includes exactly three generation-3 quarks. For example, for isomer-0, a nonzero-charge baryon that includes just two tops and one bottom would have a larger mass than would a ground-state zero-charge baryon that includes just one top and two bottoms. Popular modeling suggests that, for isomer-0, W bosons play key roles regarding the decay of generation-3 baryons, such as possible generation-3 baryons to which the previous sentence alludes, into ground-state generation-1 baryons, namely the neutron and the proton [42]. Per Table 1, MEA-isomer flavour-3 charged leptons would be less massive than isomer-0 flavour-3 charged leptons. When generation-3 quark states are much populated, the stuff that associates with an MEA-isomer would convert more charged baryons to zero-charge baryons than would the stuff that associates with isomer-0. Eventually, regarding the stuff that associates with the MEA-isomer, interactions that entangle multiple MEA-isomer W bosons would result in the stuff that associates with the MEA-isomer having more counterparts to isomer-0 neutrons and fewer counterparts to isomer-0 protons than does the stuff that associates with isomer-0. The sum of the mass of an MEA-isomer-counterpart-to-isomer-0 proton and the mass of an MEA-isomer flavour-1 charged lepton would exceed the mass of a counterpart-to-isomer-0 neutron. Compared to isomer-0 neutrons, MEA-isomer neutrons would scarcely decay.

Regarding DM (SEA), we note that some observational results [43–45] suggest that some dark matter might comport with popular modeling notions of self-interacting dark matter [5,46]. Some popular modeling results [47–50] point to possible benefits of considering that some dark matter is self-interacting dark matter.

We note, as an aside, that similarities between isomer-0 and isomer-3 might provide a basis for popular modeling to better, than now, come to terms with the popular modeling notion of matter-antimatter asymmetry (which is also known as baryon asymmetry) [51].

2.2. Two-Body Gravitational Properties of Objects

This unit develops (by reusing popular modeling aspects regarding Newtonian gravity, electromagnetism, and Lorentz invariance) a list of suggested new two-body gravitational properties of objects.

We discuss interactions between an object-A and an object-P. The A in object-A associates with the two-word term active properties. Popular modeling associates active properties with the notion of properties about which fields, such as electromagnetic fields and gravitational fields, convey information. The P in object-P associates with the two-word term passive properties. Popular modeling associates passive properties with interactions, by object-P, with fields that associate with objects, such as object-A, other than object-P.

The following notions contrast aspects of Newtonian force equations and Lorentz-invariant interpretations of the values of properties of objects.

- In Newtonian force equations, the values of some properties, including mass [52] and charge [53], of an object-A are invariant to the motion of object-A with respect to an observer, which could be object-P, of object-A.
- Based on Lorentz invariance [54], the perceived values of some object-A properties, including mass and charge, can vary based on a choice of an observer, which could be object-P. The magnitudes of the variations, away from values that associate with a frame of reference in which object-A is at rest, can associate with a nonzero velocity and with nonzero values for the object-A property current that associates with a property of object-A. (For example, charge current associates with charge.)

The following themes associate with this discussion. Expand the list of two-body gravitational properties of objects to include properties other than mass. For each one of some two-body gravitational properties of object-A, discuss the extent to which two-body gravitational interactions between that property of object-A and the mass of object-P associate with attraction (or, pull) of object-P toward

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object-A or associate with repulsion (or, push) of object-P away from object-A. Point to situations in which the total (across two-body properties of object-A) gravitational push can exceed the total (across two-body properties of object-A) gravitational pull.

The following remarks characterize aspects of our approach.

We discuss some gravitational-force cases and electromagnetic-force cases that lead to so-called extended Newtonian gravity.

Throughout our discussion of gravity and electromagnetism, we de-emphasize the notion that object-P can experience a torque based on its interactions with object-A. We assume that the gravitational mass of object-P equals the inertial mass of object-P. Regarding gravity, we generally assume that the only adequately relevant property of object-P is mass. Regarding electromagnetism, we generally assume that the only adequately relevant properties of object-P are (the electromagnetic property of) charge and (the inertial property of) mass.

For much of our discussion of gravity and electromagnetism, we de-emphasize the notion that each one of object-A and object-P might change, for example via radiation, its internal state.

Throughout our discussion of gravity and electromagnetism, we de-emphasize the notion that object-A and object-P might collide with each other.

Throughout our discussion of gravity and electromagnetism, we exclude from the list of possible objects-A and from the list of possible objects-P popular modeling zero-mass objects (such as photons and gluons). Throughout our discussion of gravity and electromagnetism, we exclude from the list of possible objects-A and from the list of possible objects-P popular modeling objects (such as quarks and gluons) that popular modeling models as not existing individually.

Equation (1) provides notation for the 3-vector velocity of object-A relative to the position of object-P.

$$v_{oA,rel.to.oP} \tag{1}$$

For much of our discussion of gravity and electromagnetism, we consider aspects for which popular modeling could consider that object-P is at rest. In such contexts, $v_{oA,rel.to.oP}$ also associates with the 3-vector difference between the velocity of object-A and the velocity of object-P.

Our work might have some parallels to work that popular modeling associates with the word gravitoelectromagnetism [55–58].

2.2.1. Two-Body Gravity and Seventeenth Century Modeling

This unit discusses aspects of Newtonian gravity.

Equations (2), (3), and (4) describe aspects regarding the motion of object-P [52]. G is the gravitational constant. m_{oA} is the mass of object-A. Mass is a scalar property. m_{oP} is the mass of object-P. r is the 3-vector distance that object-P is away from object-A. $\bigtriangledown$ is the gradient operator. $\bigtriangledown$ produces a 3-vector from a scalar. F_{oP} is the force that object-P feels. F_{oP} is a 3-vector. Object-P might sense effects of that force via an accelerometer that associates with object-P. In equations such as equation (2), V is a scalar. Popular modeling associates with V the word potential. In equations such as equation (3), r^{n_r} denotes the n_r-th power of the magnitude of the 3-vector r.

$$Gm_{oA}m_{oP}(-\nabla V) = F_{oP} \tag{2}$$

$$V = -1/r^{n_r} \tag{3}$$

$$n_r = 1 \tag{4}$$

Popular modeling suggests that F_{oP} associates with a rate of change, with respect to time, of the momentum of object-P. Relevant popular modeling suggests that the momentum of object-P equals the mass m_{oP} of object-P times the velocity of object-P. Relevant aspects of our work assume that m_{oP} does

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not change. For a constant mass m_{oP}, equation (5) pertains. a_{oP} is the acceleration of object-P. a_{oP} is a 3-vector.

$$F_{oP} = m_{oP} a_{oP} \tag{5}$$

Equations (2), (3), (4), and (5) describe, for example, motion in which one body orbits the other body. Newtonian physics includes the notion that each of the two objects can exhibit nonzero motion.

Equations (2), (3), (4), and (5) refer directly to some properties of objects (that is, the mass of object-A and the mass of object-P), a position of one object relative to and other object, and the notion of acceleration. Equations (2), (3), (4), and (5) do not refer directly to notions of velocity. Equations (2), (3), (4), and (5) do not refer directly to the notion of a velocity $v_{oA,rel.to.oP}$ of object-A with respect to the position of object-P. Equations (2), (3), (4), and (5) do not refer directly to the notion of a velocity of object-A with respect to a velocity of object-P.

2.2.2. Two-Body Electromagnetism and Eighteenth Century Modeling

This unit discusses aspects of eighteenth century two-body electromagnetism.

Equation (6) is an aspect of eighteenth century two-body electromagnetism [53,59]. ε_0 denotes the vacuum electric permittivity. q_{oA} is the charge of object-A. q_{oP} is the charge of object-P. Equation (5) pertains.

$$-(1/(4\pi\varepsilon_0))q_{oA}q_{oP}(-\nabla(V)) = F_{oP} \tag{6}$$

The eighteenth century precedes the notion that object-P-perceived properties of object-A can vary based on a frame of reference that associates with object-P. Each one of q_{oA} and q_{oP} is invariant to a choice of a frame of reference.

We suggest that equation (6) provides an electromagnetic parallel to equation (2). $v_{oA,rel.to.oP}$ is not necessarily relevant regarding equations (3), (4), (5), and (6).

2.2.3. Two-Body Electromagnetism and Nineteenth Century Modeling

This unit discusses and extends aspects, including two-body electromagnetic properties of objects, of nineteenth century two-body electromagnetism.

Compared to equation (6), popular modeling added two two-body electromagnetic properties, magnetic moment and charge current. Popular modeling added Lorentz invariance and notions that perceived values of properties depend on frames of reference.

We propose a way to catalog two-body electromagnetic properties of objects. The way features two integers. One integer is n_r, as in equations (3) and (4). The other integer is n_{3s}. n_{3s} denotes a number of so-called threesomes that appear directly or indirectly on the left-hand sides of equations such as equation (2) and equation (6). A threesome might be a 3-vector.

Table 2 lists some two-body electromagnetic properties that might associate with an object.

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Table 2. Some two-body electromagnetic properties that might associate with an object. In each of the $n_{3s} = n_r$ column and the $n_{3s} = n_r + 1$ column, the table lists a name for a property of the object. Popular modeling suggests the names that do not appear in parentheses. We suggest the properties and the names for the items that appear in parentheses. For each nonzero-valued property, there is a contribution (to an overall potential) that has radial characteristics that associate with $V(r) \propto -r^{-n_r}$. r denotes the distance from the object. For a property for which $n_{3s} > 1$, non-constant angular-dependence pertains regarding that property's contribution to an overall $V(r)$. Uses of the words monopole, dipole, and so forth associate with values of n_r and echo popular modeling uses of the words. Throughout this table, n_{3s} counts sets of three parameters. Regarding charge, one set of three independent parameters associates with position. Regarding magnetic moment, one set of three independent parameters associates with position. Another set of three independent parameters associates with the three components of the magnetic dipole 3-vector. Each $n_{3s} = n_r + 1$ property associates with adding, compared to the counterpart $n_{3s} = n_r$ property, one set of three independent parameters that associate with linear velocity. Because of the notion that the object is an object, one position is common to all properties. One velocity is common to all $n_{3s} = n_r + 1$ properties. The Earth exhibits nonzero precessing magnetic moment. For the Earth, the axis that associates with rotation does not equal the axis that associates with the dipole moment. Precessing magnetic moment differs from Larmor precession.

n_r	**Potential**	$n_{3s} = n_r$	$n_{3s} = n_r + 1$
1	Monopole	Charge	Charge current
2	Dipole	Magnetic moment	(Magnetic-moment current)
3	Quadrupole	(Precessing magnetic moment)	(Precessing-magnetic-moment current)

Popular modeling suggests that relevant quantities include $q_{oP,rest}$, $q_{oA,rest}$, $v_{oA,rel.to.oP}$, q_{oA}, ϕ_{oA}, and A_{oA}. q denotes charge. $q_{oP,rest}$ denotes the charge of object-P that associates with observations in a frame of reference in which object-P is not moving. $q_{oA,rest}$ denotes the charge of object-A that associates with observations in a frame of reference in which object-A is not moving. q_{oA} denotes the charge that object-P associates with object-A. ϕ_{oA} denotes the electromagnetic scalar potential that object-P associates with contributions to the electromagnetic field that associate with object-A. A_{oA} denotes the electromagnetic 3-vector vector potential that object-P associates with contributions to the electromagnetic field that associates with object-A. In popular modeling, ϕ_{oA} and A_{oA} combine to form a Lorentz-invariant 4-vector.

Popular modeling provides the following equations. E_{oA} denotes the electric field that object-P associates with contributions, to the electromagnetic field, that associate with object-A. v_{oP} denotes the velocity of object-P. B_{oA} denotes the magnetic field that object-P associates with contributions, to the electromagnetic field, that associate with object-A. Each one of E_{oA}, v_{oP}, and B_{oA} is a 3-vector. $\partial .../\partial t$ denotes a partial derivative with respect to time. The equations are invariant with respect to a choice of a frame of reference. Values for a variable in an equation can vary, based on the choice of a frame of reference.

$$q_{oP}(E_{oA} + (v_{oP} \times B_{oA})) = F_{oP} \quad (7)$$

$$E_{oA} = -\nabla\phi_{oA} - \partial A_{oA}/\partial t \quad (8)$$

$$B_{oA} = \nabla \times A_{oA} \quad (9)$$

Popular modeling suggests that the following equations pertain. I_{oA} is the charge current that associates with object-A. I_{oA} is a 3-vector. I_{oA} associates with both the charge q_{oA} and the velocity $v_{oA,rel.to.oP}$. $A_{oA,I}$ is a 3-vector. $A_{oA,I}$ contributes to A_{oA}.

$$\phi_{oA} \propto q_{oA}/r \quad (10)$$

$$A_{oA,I} \propto I_{oA}/r \tag{11}$$

In popular modeling, q_{oA} and I_{oA} combine to form a Lorentz-invariant 4-vector.

Equation (7) contrasts with equation (6).

Popular modeling associates equation (12) with Lorentz invariance. The symbol $\cdot$ associates with the notion of dot (or, scalar) product. The constant is invariant to changes in $v_{oA,rel.to.oP}$.

$$|E_{oA}|^2 - c^2|B_{oA}|^2 \equiv E_{oA} \cdot E_{oA} - c^2 B_{oA} \cdot B_{oA} = \text{constant} \tag{12}$$

2.2.4. A Re-look at Two-Body Electromagnetism and Nineteenth Century Modeling

This unit explores relationships between aspects of eighteenth century two-body electromagnetism and aspects of nineteenth century two-body electromagnetism.

Regarding eighteenth century two-body electromagnetism, we assume that equation (13) adequately associates with equation (6).

$$-(1/(4\pi\varepsilon_0))q_{oA,rest}q_{oP,rest}(-\nabla(V)) = F_{oP} \tag{13}$$

We explore relationships between equation (6) and equation (7).

We discuss interpretations with respect to the rest frame that associates with object-P. Equation (14) pertains.

$$v_{oP} = 0 \tag{14}$$

We note, as an aside, that choosing this rest frame associates with standardizing some notions regarding the time t. This paper does not necessarily directly take advantage of such standardization. Choosing this rest frame associates with standardizing notions of lengths, such as distances r away from object-A.

We note that distances r may pertain to present times for object-P and earlier times for object-A. The popular modeling notion of retarded time would pertain. We suggest that, for the purposes of this paper, time delays regarding the propagation of information about object-A are not necessarily adequately important to warrant more careful attention to the delays.

We limit discussions to cases in which q_{oP} is the only nonzero-value electromagnetic property that associates with object-P.

Based on equations (7) and (14), the value of B_{oA} is not relevant regarding F_{oP}. However, equations (7) and (8) suggest that A_{oA} can still have relevance regarding F_{oP}.

We use the symbol μ_{oA} to denote the magnetic moment that object-P associates with object-A. μ_{oA} is a 3-vector.

We discuss a so-called case-e1.

A theme for case-e1 is to try to recover equation (13) from equation (6). Recovery associates with notions that associate with replacing q_{oA} with $q_{oA,rest}$. Recovery associates with notions of reverting to invariances that associate with equation (13). In particular, equation (13) associates with the notion that each one of $q_{oA,rest}$ and $q_{oP,rest}$ is invariant with respect to velocity.

For case-e1, we make the following assumptions. $q_{oA,rest} \neq 0$. $\mu_{oA,rest} = 0$. $q_{oP,rest} \neq 0$. No two-body electromagnetic property, other than $q_{oP,rest}$, of object-P is nonzero.

Popular modeling that associates with equation (7) and with Lorentz invariance suggests that, if object-P would infer that the magnitude of $v_{oA,rel.to.oP}$ is nonzero, object-P would infer that $|\nabla\phi_{oA}|/|\nabla\phi_{oA,rest}|$ exceeds one, that $q_{oA}/q_{oA,rest}$ exceeds one, and that $|E_{oA}|/|E_{oA,rest}|$ exceeds one.

We use the definitions that equations (15) and (16) show. The subscript oA, v associates with the notion that each one of $q_{oA,v}$ and $-\nabla\phi_{oA,v}$ characterizes a contrast between popular modeling based on (essentially eighteenth century) equation (13) and popular modeling based on (nineteenth century) equation (7). In the subscript oA, v, the symbol v associates with the velocity $v_{oA,rel.to.oP}$.

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$$q_{oA,v} \equiv q_{oA} - q_{oA,rest} \tag{15}$$

$$-\nabla\phi_{oA,v} \equiv -\nabla\phi_{oA} - (-\nabla\phi_{oA,rest}) \tag{16}$$

We suggest that equation (8) points to possibilities for popular modeling for which $q_{oA} = q_{oA,rest}$, $-\nabla\phi_{oA} = -\nabla\phi_{oA,rest}$, $E_{oA} = E_{oA,rest}$ and for which equation (17) pertains.

$$E_{oA} = -\nabla\phi_{oA,rest} \tag{17}$$

We suggest adding a new component, $A_{oA,v}$, to the vector potential that associates with object-A. Equation (18) pertains.

$$A_{oA} = A_{oA,I} + A_{oA,v} \tag{18}$$

We require that $A_{oA,v}$ does not impact the magnetic field B_{oA}. That is, we require that $\nabla \times A_{oA,v} = 0$.

The following paragraphs provide an example of such an $A_{oA,v}$ for which $\nabla \times A_{oA,v} = 0$.

We assume that each one of q_{oA}, $v_{oA,rel.to.oP}$, and I_{oA} is a constant with respect to the time t. Equation (19) defines a candidate $A_{oA,v}$.

$$A_{oA,v} = (-\nabla\phi_{oA} - (-\nabla\phi_{oA,rest}))t \tag{19}$$

Equation (20) restates equation (19).

$$A_{oA,v} = -\nabla((\phi_{oA} - \phi_{oA,rest})t) \tag{20}$$

Mathematics suggests that the curl of the gradient of a scalar field is zero. Except at $r = 0$ (which is not necessarily physically relevant), the contributions to B_{oA} are $\nabla \times A_{oA,v}$, which is zero. (We note, as an aside, that the $A_{oA,v}$ that equation (19) suggests associates with 3-vectors that exhibit radial spatial dependencies of $1/|r_{oA}|$, point along radii with respect to the position of object-A, and otherwise do not vary based on angular coordinates. r_{oA} denotes the 3-vector distance from object-A.)

Within and beyond the above example an $A_{oA,v}$ for which $\nabla \times A_{oA,v} = 0$, we suggest that, at least in the rest frame of object-P, equation (13) provides a useful basis for modeling for phenomena that associate with equation (7).

We also suggest that equation (12) associates with the notion that the reversion process does not associate with notions for which $|E_{oA}|/|E_{oA,rest}| < 1$ or with notions that the push or pull sense of a_{oP} changes (either from push to pull or from pull to push).

The following notions summarize discussion above. We suggest that, for the magnitude of $v_{oA,rel.to.oP}$ being nonzero, one can revert Lorentz-invariance-compliant modeling toward equation (13). The amount of reversion associates with $|v_{oA,rel.to.oP}|$ and does not depend on the direction that associates with $v_{oA,rel.to.oP}$. The reversion suggests that, from the perspective of object-P, for a specific value of n_r, effects that associate with $n_{3s} = n_r + 1$ detract from effects that associate with $n_{3s} = n_r$.

We discuss a so-called case-e2.

A theme for case-e2 is to try to understand the extent to which object-P perceived values, that differ from a rest-frame value of object-A charge, of charges that associate with object-A might associate with object-P perceived nonzero values of object-A magnetic moment.

For case-e2, we make the following assumptions. $v_{oA,rel.to.oP} = 0$. $q_{oA,rest} \neq 0$. $\mu_{oA,rest} \neq 0$. Object-A includes sub-objects that have charges that have the same sign as $q_{oA,rest}$. Object-A includes no sub-objects that have charges for which the signs are the opposite of the sign of $q_{oA,rest}$. Motions, within object-A, of charged sub-objects account for the entirety of $\mu_{oA,rest}$. No two-body $oA, rest$

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electromagnetic property, other than $q_{oA,rest}$ and $\mu_{oA,rest}$, of object-A is nonzero. $q_{oP,rest} \neq 0$. No electromagnetic property, other than $q_{oP,rest}$, of object-P is nonzero.

Unlike for case-e1, for case-e2, the two-body relative velocity $v_{oA,rel.to.oP}$, which is zero, is not a variable 3-vector. For case-e2, the two-body 3-vector relative position r is relevant.

We use the symbol $v_{sub.object.of.oA,rel.to.oP}$ to denote the velocity, relative to object-P, of a charged sub-object of object-A.

While $v_{oA,rel.to.oP} = 0$ pertains regarding object-A, $|v_{sub.object.of.oA,rel.to.oP}| > 0$ pertains for each moving charged sub-object of object-A. Popular modeling suggests that charges add. Paralleling case-e1, we suggest that each moving charged sub-object of object-A contributes to the notion that object-P would infer that $|\nabla\phi_{oA}|/|\nabla\phi_{oA,rest}|$ exceeds one, that $q_{oA}/q_{oA,rest}$ exceeds one, and that $|E_{oA}|/|E_{oA,rest}|$ exceeds one. Popular modeling associates the factor $1/r^2$ with the μ_{oA}-related contribution to the overall potential that associates with F_{oP}. We suggest that the amounts that the three ratios exceed one scale as $1/r^2$. We associate this scaling with the two-word phrase dipole effects.

We suggest that, from the perspective of object-P, effects that associate with nonzero magnetic moment μ_{oA} dilute effects that associate with nonzero charge q_{oA}. The magnitude of the dilution scales linearly with the magnitude of μ_{oA}. The magnitude of the dilution does not depend on the direction of μ_{oA}.

Regarding reversion toward equation (13), we suggest that, for case-e2, dipole effects dilute monopole effects.

We note, as an aside, that a case-e2 analog to the case-e1 $A_{oA,v}$ that equation (19) suggests associates with 3-vectors that exhibit radial spatial dependencies of $1/|r|_{oA}|^2$, point along radii with respect to the position of object-A, and otherwise do not vary based on angular coordinates.

Assuming that object-A and object-P are adequately small, there is a range of small r for which object-P accelerates in the direction opposite to the acceleration for case-e1 (for which there are no μ_{oA}-related effects).

We note, as asides, the following notions. Popular modeling includes cases, such as for bar magnets, in which objects can model as having zero charge and nonzero magnetic moment. We do not explore such cases. We do not explore popular modeling notions that there might be more than one popular modeling definition of magnetic moment for such cases and that the definitions might not be equivalent regarding Lorentz-invariant transformations [60]. For an object-A that has more than one nonzero-charge sub-object, there can be many circumstances for which the $n_{3s} = n_r = 2$ property associates with too much physical complexity to have associations just with, or perhaps even adequately meaningfully with, notions of a magnetic moment. For example, one can consider cases in which sub-objects of object-A have same-signed charges and the motions of the sub-objects are such that contributions toward a might-be object-A magnetic moment tend to cancel each other.

We discuss a so-called case-e3.

A theme for case-e3 is the notion that motions of charged sub-objects of object-A might be chaotic.

Compared to case-e2, case-e3 removes the following assumptions. $\mu_{oA,rest} \neq 0$. No two-body $oA, rest$ electromagnetic property, other than $q_{oA,rest}$ and $\mu_{oA,rest}$, of object-A is nonzero.

For case-e3, the following case-e2 assumptions remain. $v_{oA,rel.to.oP} = 0$. $q_{oA,rest} \neq 0$. Object-A includes sub-objects that have charges that have the same sign as $q_{oA,rest}$. $q_{oP,rest} \neq 0$. Object-A includes no sub-objects that have charges for which the signs are the opposite of the sign of $q_{oA,rest}$. Motions, within object-A, of charged sub-objects account for the entirety of $\mu_{oA,rest}$. No electromagnetic property, other than $q_{oP,rest}$, of object-P is nonzero.

For case-e3, the motion of the charged sub-objects might be chaotic. For case-e2, the two-word term magnetic moment describes the $n_r = n_{3s} = 2$ property. For case-e3, we suggest using the two-element term charge whirl-or-jitter to name the $n_r = n_{3s} = 2$ property. For case-e3, we suggest using the two-element term charge-whirl-or-jitter current to name the property for which $n_r = 2$ and $n_{3s} = 3$.

We discuss a so-called case-e4.

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Compared to case-e3, case-e4 removes the assumption that $v_{oA,rel.to.oP} = 0$.

For case-e4, we suggest considering the notion that charge-whirl-or-jitter current corrects for otherwise possible double-counting regarding effects that would associate with the motions of object-A nonzero-charge sub-objects for which the velocities within object-A are not perpendicular to $v_{oA,rel.to.oP}$. Also, in the sense that object-P perceptions of object-A charge increase with object-P perceptions of increasing object-A charge current and with object-P perceptions of increasing object-A charge whirl-or-jitter, object-P perceptions of object-A charge decrease with object-P perceptions of increasing object-A charge-whirl-or-jitter current.

Within and beyond case-e1 through case-e4, we note the following statements. The notion of a binding energy that might be necessary to keep object-A intact is not necessarily relevant. Charges add across sub-objects. While notions that popular modeling might associate with potentials (or with potential energies) appear in discussions above, notions that popular modeling might associate with kinetic energy do not appear explicitly in discussions above.

2.2.5. Two-Body Gravity and Suggested Twenty-First Century Modeling

This unit suggests two-body gravitational properties, of objects, that might extend aspects of popular modeling.

We discuss and extend a so-called case-g1, which is a gravitational analog to case-e1, and a so-called case-g2, which is a gravitational analog to case-e2.

In popular modeling, mass is the gravitational analog to charge in electromagnetism. We note that, paralleling notions above regarding electromagnetism, considering object-A structurally internal energies might not be necessary. Also paralleling notions above regarding electromagnetism, considering object-A kinetic energies might not be necessary. (We note, as an aside, that notions that we are discussing here do not extend to some cases in which gravity does not provide an adequately dominant force. One such case involves quantum-chromodynamics interactions within hadrons. Also, in cases such as ones for which quantum chromodynamics pertain, the notion that mass might add across objects is not necessarily useful.)

Table 3 lists some two-body gravitational properties that might associate with an object.

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Table 3. Some two-body gravitational properties that might associate with an object. In each of the $n_{3s} = n_r$ column and the $n_{3s} = n_r + 1$ column, the table lists a name for a property of the object. For each nonzero-valued property, there is a contribution (to an overall potential) that has radial characteristics that associate with $V(r) \propto -r^{-n_r}$. r denotes the distance from the object. For a property for which $n_{3s} > 1$, non-constant angular-dependence pertains regarding that property's contribution to an overall $V(r)$. Uses of the words monopole, dipole, and so forth associate with values of n_r and echo popular modeling uses of the words. Throughout this table, n_{3s} counts sets of three parameters. Regarding mass, one set of three independent parameters associates with position. Regarding angular momentum, one set of three independent parameters associates with position. Another set of three independent parameters associates with the three components of the angular momentum 3-vector. Regarding moments of inertia, one set of three independent parameters associates with position. Another set of three independent parameters associates with the three on-diagonal components of the 3-by-3 symmetric moments-of-inertia tensor. Another set of three independent parameters associates with three off-diagonal components of the 3-by-3 symmetric moments-of-inertia tensor. The case of rotating moments of inertia adds, compared to the case of moments of inertia, one set of three parameters that associate with the angular velocity 3-vector that associates with the rotation of the object. Each $n_{3s} = n_r + 1$ property associates with adding, compared to the counterpart $n_{3s} = n_r$, one set of three independent parameters that associate with linear velocity. Because of the notion that the object is an object, one position is common to all properties. One velocity is common to all $n_{3s} = n_r + 1$ properties. NNR, as in not necessarily relevant, denotes the notion that we anticipate de-emphasizing the notion of possibly relevant $n_{3s} = n_r$-property hexadecapole potentials.

n_r	**Potential**	$n_{3s} = n_r$	$n_{3s} = n_r + 1$
1	Monopole	Mass	Mass current
2	Dipole	Angular momentum	Angular-momentum current
3	Quadrupole	Moments of inertia	Moments-of-inertia current
4	Octupole	Moments-of-inertia rotation	Moments-of-inertia-rotation current
5	Hexadecapole	NNR	NNR

We suggest that Table 3 associates with a new type of gravitational multipole expansion. In each of popular modeling multipole expansions and our multipole expansions, an expansion features a series of terms. Each term contributes via addition or subtraction to a notion of an overall spatial potential. In popular modeling, a multipole expansion tends to have a basis in a spatial distribution of one property such as charge or mass. Our multipole expansions feature one object that models spatially as somewhat pointlike or as small and somewhat spherically symmetric and physically as having at least one nonzero-valued property.

We note, as an aside, that an n_{3s} that exceeds by two the counterpart n_r might associate with a nonzero acceleration and with the notion that, from the perspective of object-P, object-A would model as part of a system, of objects, for which the system does not necessarily include object-P.

Popular modeling suggests that the object-property of mass is always nonnegative. In the context of two-body interactions, the contribution that associates with the mass of object-A interacting with the mass of object-P associates with a pull component of force. Pull associates with notions of attraction of object-P toward object-A. More generally, we suggest that equations (21) and (22) pertain for two-body gravitation. $n_{3s,oA}$ denotes an n_{3s} for object-A. $n_{3s,oP}$ denotes an n_{3s} for object-P. The symbol $\leftrightarrow$ denotes the two-word phrase associates with. Push associates with notions of repulsion of object-P away from object-A.

$$(n_{3s,oA} + n_{3s,oP}) \text{ is even } \leftrightarrow \text{ pull} \tag{21}$$

$$(n_{3s,oA} + n_{3s,oP}) \text{ is odd } \leftrightarrow \text{ push} \tag{22}$$

Table 4 lists some contributions, by an object-A, to gravitational forces, as perceived by an object-P. Table 4 extends Table 3.

Table 4. Some contributions, by an object-A, to gravitational forces, as perceived by an object-P. The value of each object-A property is the value as perceived by object-P. $n_{3s,oA}$ denotes an n_{3s} for object-A. $n_{3s,oP}$ denotes an n_{3s} for object-P. Pull associates with notions of attraction of object-P toward object-A. Push associates with notions of repulsion of object-P away from object-A. RSD abbreviates the three-word term radial spatial dependence. An RSD associates with the component of force that associates with a row in the table. The values of the exponents for RSD items echo the popular modeling notion that a force can associate with the gradient of a potential. Type associates with relevant $n_{3s} = n_r$ properties. Notions of monopole, dipole, and so forth echo popular modeling use of such terms. For example, a row for which the RSD is r^{-2} associates with a potential, that associates with object-A, for which $n_r = 1$. However, for a current property, the velocity-related 3-vector associates with an extra (compared to for the counterpart $n_{3s} = n_r$ property) notion of angular dependence. For example, for the second row in the table, the force varies based on the angle with respect to the velocity-related 3-vector that associates with the object-A mass current, even though, for the first row, the force does not vary based on angular coordinates. For each row, the RSD has the form $r^{-n_{RSD}}$, in which $n_{RSD} = n_{r,oA} + n_{r,oP}$ with $n_{r,oA}$ denoting the n_r for the object-A property and $n_{r,oP}$ denoting the n_r for the object-P property.

Object-A property	$n_{3s,oA}$	Object-P property	$n_{3s,oP}$	Force	RSD	Type
Mass	1	Mass	1	Pull	r^{-2}	Monopole
Mass current	2	Mass	1	Push	r^{-2}	Monopole
Angular momentum	2	Mass	1	Push	r^{-3}	Dipole
Angular-momentum current	3	Mass	1	Pull	r^{-3}	Dipole
Moments of inertia	3	Mass	1	Pull	r^{-4}	Quadrupole
Moments-of-inertia current	4	Mass	1	Push	r^{-4}	Quadrupole
Moments-of-inertia rotation	4	Mass	1	Push	r^{-5}	Octupole
Moments-of-inertia-rotation current	5	Mass	1	Pull	r^{-5}	Octupole

For a pair of rows, in Table 4, that associate with the same RSD, we suggest that the pull (or push) that associates with an object-A $n_{3s} = n_r$ property dominates the push (or, respectively, pull) that associates with the counterpart object-A $n_{3s} = n_r$-property current (for which $n_{3s} = n_r + 1$ pertains).

For a pair of rows, in Table 4, that associate with two different object-A non-current properties such that one row associates with pull and the other row associates with push, we suggest that dominance with respect to pull or push depends on r. For example, consider the object-A properties of mass and angular momentum. For adequately large values of r, pull dominates. For lesser values of r, push can dominate. (We note, as an aside, that for yet lesser values of r, the notion that the objects are not colliding might no longer pertain.)

We discuss a so-called case-g3, which is analogous to case-e3, and a so-called case-g4, which is analogous to case-e4.

For case-g3 and for case-g4, the motions of nonzero-mass sub-objects within object-A might be chaotic. For case-g2, the two-word term angular momentum describes the $n_r = n_{3s} = 2$ property.

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For case-g3 and for case-g4, we suggest using the two-element term mass whirl-or-jitter to name the $n_r = n_{3s} = 2$ property. For case-g3 and for case-g4, we suggest using the two-element term mass-whirl-or-jitter current to name the property for which $n_r = 2$ and $n_{3s} = 3$.

Within and beyond case-g1 through case-g4, we note the following statements. The notion of energies that might be necessary to keep object-A structurally intact is not necessarily relevant. Masses add across sub-objects. While notions that popular modeling might associate with potentials (or with potential energies) appear in discussions above, notions that popular modeling might associate with kinetic energy do not appear explicitly in discussions above. For one value of n_r, $n_{3s} = n_r + 1$ effects cannot reverse the object-P push or pull sense that associates with $n_{3s} = n_r$ effects. For one value of n_g, $n_{3s} = n_r = n_g + 1$ effects can reverse the object-P push or pull sense that associates with $n_{3s} = n_r = n_g$ effects if object-A and object-P are adequately close to each other and are not colliding with each other.

2.3. Instances of Properties of Objects, Plus Reaches per Instance of Contributions to Interactions Between Objects

This unit introduces a notion of instances of electromagnetic properties and of two-body gravitational properties and a notion of reach per instance. This unit suggests, for some key two-body gravitational properties of objects and some key electromagnetic properties of objects, instances and reaches per instance. This unit suggests that numeric values of instances and reaches per instance can be key to explaining some cosmic data.

Popular modeling suggests that ordinary-matter stuff scarcely, if at all, sees dark-matter stuff. We suggest that nature includes six instances of the property of charge. We suggest that each isomer associates with its own instance of charge. We say that, for each one of the six instances of charge, the reach per instance is one isomer. Similarly, we suggest (based on the notion that ordinary matter does not see dark matter stars) that each isomer associates with its own instance of the property of blackbody temperature and that the reach per instance for blackbody temperature is one isomer.

Popular modeling suggests that each nonzero-mass object can interact with all other nonzero-mass objects. We say that nature includes one instance of the property of mass. We say that the reach per instance for that one instance is six isomers.

We use the symbol n_{in} to denote the number of instances of a property. We use the symbol $R_{/in}$ to denote the reach of an instance of the property. The reach is a number of isomers. Each one of n_{in} and $R_{/in}$ is a positive integer.

We suggest that, for each electromagnetic property and for each gravitational property, equation (23) pertains.

$$n_{in} \cdot R_{/in} = 6 \tag{23}$$

Equation (23) pertains regarding properties, such as properties that Table 3 lists, of individual objects.

For a solution for which $n_{in} = 3$ and $R_{/in} = 2$, we assume that each one of the three instances associates with an isomer-pair and that the reach of two isomers associates with the two isomers that associate with the isomer-pair. We assume that the $n_{in} = 2$ and $R_{/in} = 3$ solution is not relevant for our work.

Table 5 suggests instances and reaches per instance for some two-body gravitational properties of objects and for some electromagnetic properties of objects.

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Table 5. Suggested instances and reaches per instance for some two-body gravitational properties of objects and for some electromagnetic properties of objects. The properties that this table lists do not include currents, such as mass current or charge current. For each property for which the table provides a value for n_r, there is (in Newtonian-like modeling that models an object as somewhat pointlike or as small and somewhat spherically symmetric) a contribution (to an overall gravitational or electromagnetic potential) that has radial characteristics that associate with $V(r) \propto -r^{-n_r}$. r denotes the distance from the relevant object. NNR denotes that a value is not necessarily relevant for this study. Properties that associate with NNR do not associate with the notion of no change of the internal state of the relevant object. G2BF denotes gravitational two-body force. The gravitational instances, reaches, and G2BF information pertain for interactions with the mass of a second object. n_{in} denotes the number of instances of the property. $R_{/in}$ denotes the interaction reach, in number of isomers, per instance. For potentials that associate with somewhat pointlike (or with small and somewhat spherically symmetric) Newtonian modeling for a stationary object, $n_{3s} = n_r = 1$ associates with a spatially monopole potential, $n_{3s} = n_r = 2$ associates with a spatially dipole potential, $n_{3s} = n_r = 3$ associates with a spatially quadrupole potential, and $n_{3s} = n_r = 4$ associates with a spatially octupole potential. (For a current-related object-property, such as mass-current or charge-current, n_{3s} is one plus the n_{3s} for the counterpart non-current object-property, n_r equals the n_r for the counterpart non-current object-property, and the pole-characteristic for the spatial potential for the current-related object-property remains the same as the pole-characteristic for the counterpart non-current object-property.) The choice of a reach per instance of one isomer for blackbody temperature associates with the notion that people do not observe thermal radiation from stars other than ordinary-matter stars. TBD denotes to be determined. We suggest that the reach per instance for magnetic moment might be one. We suggest that the reach per instance of hyperfine interactiveness might be two or six. NR denotes not relevant.

Type of property	Property	n_r	G2BF	n_{in}	$R_{/in}$	n_{3s}
Gravitational	Mass	1	Pull	1	6	1
Gravitational	Internal angular momentum	2	Push	3	2	2
Gravitational	Moments of inertia	3	Pull	6	1	3
Gravitational	Rotating moments-of-inertia	4	Push	TBD	TBD	4
Electromagnetic	Charge	1	NR	6	1	1
Electromagnetic	Magnetic moment	2	NR	TBD (6)	TBD (1)	2
Electromagnetic	Blackbody temperature	NNR	NR	6	1	NNR
Electromagnetic	Hyperfine interactiveness	NNR	NR	TBD (3 or 1)	TBD (2 or 6)	NNR

For a component, such as a component that Table 4 lists, of a force that associates with a two-body interaction, we suggest that one needs to calculate the n_{in} and $R_{/in}$ based on the relevant $n_{in,property.of.oA}$, $R_{/in,property.of.oA}$, $n_{in,property.of.oP}$, and $R_{/in,property.of.oP}$. We suggest the following notions. If at least one of $R_{/in,property.of.oA}$ and $R_{/in,property.of.oP}$ is one, $n_{in} = 6$ and $R_{/in} = 1$. If one of $R_{/in,property.of.oA}$ and $R_{/in,property.of.oP}$ is two and the other one of $R_{/in,property.of.oA}$ and $R_{/in,property.of.oP}$ is at least two, $n_{in} = 3$ and $R_{/in} = 2$. For this case, each one of the three instances of $R_{/in} = 2$ associates with an isomer-pair. If each one of $R_{/in,property.of.oA}$ and $R_{/in,property.of.oP}$ is six, $n_{in} = 1$ and $R_{/in} = 6$.

We suggest that the instances and reaches per instance that Table 5 features seem to be compatible with all the data that this study seeks to explain.

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2.4. Cosmic Clumping of Stuff

This unit suggests that the formation and evolution of smaller cosmic objects occurred earlier and more quickly than did the formation and evolution of larger cosmic objects.

We assume, for discussion purposes, that, at some time in the evolution of the universe, stuff (ordinary matter and dark matter) had a uniform spatial distribution. We assume that, at that time, bunches of stuff were moving away from each other. We de-emphasize notions related to properties that associate with the word current.

We consider Newtonian notions. We consider two spatially non-overlapping, similarly-sized spherical regions of stuff. The amount of stuff in each region scales as the cube of the radius of the region. The monopole contribution to the gravitational force that one region exerts on the other region scales inversely as the square of the distance between the centers of the two regions. If one scales the two radii and the one distance similarly, the monopole aspect of gravitational force scales as $r^3 r^3 r^{-2} = r^4$. Dipole aspects scale as $r^3 r^3 r^{-3} = r^3$. Quadrupole aspects scale as $r^3 r^3 r^{-4} = r^2$. Octupole aspects scale as $r^3 r^3 r^{-5} = r^1$. Hexadecapole aspects scale as $r^3 r^3 r^{-6} = r^0$.

We suggest notions that might associate with observed and possible clumping, at various scale sizes, in the history of the universe. Hexadecapole attraction might be scale-invariant and might not necessarily be adequately relevant for this discussion of clumping. Across a range of sizes, octupole repulsion was dominant at some time. Smaller similar-sized clumps transited a sequence, of octupole repulsion to quadrupole attraction and onward, faster than did larger similar-sized clumps. Typically, pairs of neighboring solar-system-sized clumps transited to dominance by monopole attraction before pairs of neighboring galaxy-sized clumps transited to dominance by monopole attraction. Typically, pairs of neighboring galaxy-sized clumps transited to dominance by monopole attraction before pairs of neighboring galaxy-cluster-sized clumps might transit to dominance by monopole attraction.

3. Results

This unit suggests that Table 5 numeric values of instances and reaches per instance underlie steps forward regarding explaining observed ratios of presumed-dark-matter effects to ordinary-matter effects and regarding explaining eras in the rate of expansion of the universe.

3.1. Hyperfine Depletion of Cosmic Microwave Background Radiation

This unit suggests that our work provides a candidate quantitative explanation for an observation regarding some depletion of cosmic microwave background radiation.

Regarding the observed depletion of cosmic microwave background radiation, popular modeling suggests that the second 1 in the 1:1 ratio associates with hyperfine effects of ordinary-matter hydrogen atoms. We suggest, in Table 5, that a reach per instance of at least two isomers pertains regarding hyperfine interactivity. We suggest that MEA-isomers do not underlie significant numbers of hydrogen-like atoms. We suggest that the first 1 in the 1:1 ratio associates with hyperfine effects of hydrogen-like atoms that associate with isomer-3.

3.2. Galaxy Formation and Galaxy Evolution

This unit suggests that our work provides candidate quantitative explanations for some observations regarding galaxy formation and evolution and that our work adds insight regarding galaxy formation and galaxy evolution.

The solar system that includes the Earth associates with the notion of a solar-system-size clump of stuff that associates essentially with just one isomer. Our solar system is not spatially spherical.

We suggest that many other such one-isomer, not-spatially-spherical, somewhat-solar-system-sized clumps exist and generally date to early in the history of the universe. We suggest that such clumps associate with nonzero moments of inertia. We suggest that, during some period early in the history of the universe, quadrupole gravitational attraction dominated regarding interactions between neighboring same-isomer, solar-system-sized clumps. Table 5 suggests that a reach per instance of one isomer pertains. We suggest that solar-system-sized clumps clumped to form the halos of galaxies.

We suggest that the discussion above explains 1:0+ ratios that pertain to some early galaxies and 0+:1 ratios that pertain to some early galaxies.

We suggest that some later 1:0+ galaxies and some later 0+:1 galaxies retain their ratios from early in the evolution of the universe.

For each of some $\sim$ 4:1 galaxies, we suggest the following scenario. The galaxy started as a 0+:1 galaxy. Table 5 suggests that a reach per instance of two isomers pertains for dipole interactions. Reach-2 dipole push contributions to gravity drove away some ordinary-matter stuff and the stuff that associated with one dark-matter isomer. That one dark-matter isomer is isomer-3. Table 5 suggests that a reach per instance of six isomers pertains for monopole interactions. Reach-6 monopole pull contributions to gravity attracted remaining nearby stuff. The galaxy evolved to a ratio of $\sim$ 4:1.

For each of some $\sim$ 4:1 galaxies, we suggest the following scenario. The galaxy started as a not-isomer-3 1:0+ galaxy. Table 5 suggests that a reach per instance of two isomers pertains for dipole interactions. Reach-2 dipole push contributions to gravity drove away some dark-matter stuff but essentially no ordinary-matter stuff. Table 5 suggests that a reach per instance of six isomers pertains for monopole interactions. Reach-6 monopole pull contributions to gravity attracted remaining nearby stuff. The galaxy evolved to a ratio of $\sim$ 4:1.

Many later galaxies are 5+:1 galaxies. We suggest that many 5+:1 galaxies resulted from mergers of smaller, previous galaxies. We suggest that such mergers associate with monopole gravitational attraction. Table 5 suggests that a reach per instance of six isomers pertains. We suggest that the earliest mergers that led to a 5+:1 galaxy could have been mergers that involved 1:0+ galaxies and 0+:1 galaxies.

3.3. The fives in 5+:1 Ratios of Dark-Matter Effects to Ordinary-Matter Effects

This unit suggests that our work provides a candidate quantitative explanation for the fives in some observed 5+:1 ratios of dark-matter effects to ordinary-matter effects.

Popular modeling suggests that 5+:1 ratios of dark-matter effects to ordinary-matter effects pertain for many galaxies, many galaxy clusters, and for densities of the universe.

We suggest that the notion of five dark-matter isomers explains the fives in such 5+:1 ratios of dark-matter effects to ordinary-matter effects.

3.4. The Pluses in 5+:1 Ratios of Dark-Matter Effects to Ordinary-Matter Effects

This unit suggests that our work provides a candidate qualitative explanation for the pluses in some observed 5+:1 ratios of dark-matter effects to ordinary-matter effects.

Popular modeling suggests that 5+:1 ratios of dark-matter effects to ordinary-matter effects pertain for many galaxies, many galaxy clusters, and for densities of the universe.

We suggest that nature might include electromagnetic properties (of objects) that would associate with reach-6. Table 5 provides a possible example. Based on reach-6 properties, early in the evolution of the universe, more electromagnetic energy would flow from isomer-pair-0 stuff to isomer-pair-1 stuff than would flow from isomer-pair-1 stuff to isomer-pair-0 stuff. Similarly, based on reach-6 properties, early in the evolution of the universe, more electromagnetic energy would flow from isomer-pair-0 stuff to isomer-pair-2 stuff than would flow from isomer-pair-2 stuff to isomer-pair-0 stuff.

We suggest that the electromagnetic energy flow imbalances would result in the existence of more isomer-pair-1 stuff than isomer-pair-0 stuff and in the existence of more isomer-pair-2 stuff than isomer-pair-0 stuff. We suggest that the flows would not disturb a one-to-one ratio of presence of isomer-3 stuff to presence of isomer-0 stuff.

We suggest that these energy flow imbalances might have produced (or, at least, contributed to) the pluses in the 5+:1 ratios of dark-matter presence to ordinary-matter presence.

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Our work does not necessarily rule out the possibility that some portions of (or the entireties of) the pluses in the 5+:1 ratios of dark-matter presence to ordinary-matter presence associate with axions or with other popular modeling suggestions regarding the nature of dark matter.

3.5. *Eras in the Rate of Expansion of the Universe*

This unit suggests that our work provides candidate qualitative explanations for some observations regarding the rate of expansion of the universe and that our work might add insight regarding possible eras, in the evolution of the universe, for which there might not be much data.

Popular modeling suggests two observed multibillion-year eras regarding the so-called rate of expansion of the universe [61–64]. Chronologically, the first multibillion-year era associates with a positive rate of expansion that decreases as time increases. The second multibillion-year era associates with a positive rate of expansion that increases as time increases.

Data and popular modeling might provide hints that the second multibillion-year era might be ending [65,66] and that a new era, which would associate with a positive rate of expansion that decreases as time increases, might be starting.

We suggest that those two or three eras associate with the moving apart from each other of similarly sized neighboring, but not colliding, large objects. We suggest that above-discussed notions regarding cosmic clumping of stuff pertain.

We suggest that, generally, similarly sized neighboring, but not necessarily colliding, galaxies transited long ago from an era of gravitational dipole push to an era of gravitational monopole pull. We suggest that, generally, gravitational monopole pull dominates interactions between non-colliding galaxies within galaxy clusters.

We suggest that the rate of expansion of the universe associates with interactions between neighboring non-colliding galaxy clusters. We suggest that the start of the first multibillion-year era associates with a transition to dominance, regarding interactions between neighboring non-colliding galaxy clusters, by gravitational quadrupole attraction (or, pull). We suggest that the start of the second multibillion-year era associates with a transition to dominance, regarding interactions between neighboring non-colliding galaxy clusters, by gravitational dipole repulsion (or, push).

Table 6 suggests possible explanations for some phenomena regarding the evolution of the universe.

Table 6. Possible explanations for some phenomena regarding the evolution of the universe. We suggest the possibilities that this table lists for before inflation. (Some popular modeling suggests notions of a so-called Big Crunch.) We suggest the possibilities that this table lists for the start of inflation. (Popular modeling suggests an inflationary epoch.) Observations suggest the two multibillion-year eras. We (and possibly some observations) suggest a possible period after the later multibillion-year era. ROE abbreviates the six-word phrase rate of expansion of the universe. The words push and pull refer to gravitational effects. NNCGC abbreviates the four-element term neighboring non-colliding galaxy clusters.

Phenomena	**Explanation**
Before inflation	Hexadecapole pull
Start of inflation	Pauli-exclusion bounce
Early inflationary epoch ROE	Octupole push (between proto NNCGC)
Start of the earlier multibillion-year era (decreasing ROE)	Quadrupole pull (between NNCGC)
Start of the later multibillion-year era (increasing ROE)	Dipole push (between NNCGC)
After the later multibillion-year era (decreasing ROE)	Monopole pull (between NNCGC)

There may be no data that directly pertain to before or during the would-be inflationary epoch [67–71]. Our notions regarding before inflation, early in the inflationary epoch, and after the later multibillion-year era might comport with some popular modeling notions of bounce cosmology [72,73].

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Regarding dominance by hexadecapole gravitational attraction, one might need to suggest a non-gravitational mechanism that associates with a transition from a Big Crunch [74] to a Big Bang. Without a notion such as the notion of a bounce based on fermion particles and the Pauli-exclusion principle, hexadecapole pull might lead toward effects that would associate with 32-pole aspects. The word dotriacontapole and the word triacontadipole associate with 32-pole. Assuming that hexadecapole gravitational attraction pertained, we suggest that the reach per instance for each one of hexadecapole gravitational attraction and octupole gravitational repulsion might be six isomers. Consequences of those reaches being six isomers might associate with notions that, throughout the known evolution of the universe, large-scale densities of stuff tend to be, to a first approximation, spatially uniform [75].

3.6. Recap of How Our Methods Seem to Explain Otherwise Unexplained Cosmic Data

This unit recaps phenomena that our work seeks to and seems to help explain. This unit points to some aspects of our work that contribute to the seeming explanations.

The following items recap phenomena that our work seeks to and seems to help explain. Each item names phenomena, notes an approximate characterization of the explanation that we suggest, and points to one or two relevant bases for the explanation. DM:OM denotes ratios of dark-matter effects to ordinary-matter effects. CMB denotes cosmic microwave background radiation. Isomers refers to the notion that nature includes six similar, but not necessarily identical, instances of each one of most elementary particles. Extended Newtonian gravity refers to notions that lead to and include Table 3 and Table 4.

- Dark-matter elementary particles: Quantitative (Isomers).
- Dark-matter stuff: Quantitative (Isomers).
- DM:OM regarding some depletion of CMB: Quantitative (Isomers).
- Galaxy evolution and DM:OM regarding some galaxies: Quantitative (Isomers, Extended Newtonian gravity).
- DM:OM regarding some galaxy clusters: Quantitative (Isomers).
- DM:OM densities of the universe: Quantitative (Isomers).
- Eras in the rate of expansion of the universe: Qualitative (Isomers, Extended Newtonian gravity).
- Gravitational phenomena, including so-called dark-energy: Qualitative (Isomers, Extended Newtonian gravity).

4. Discussion

This unit discusses relationships between our work and popular modeling. This unit suggests opportunities for observational work and for enhancing popular modeling.

4.1. Extended Newtonian Gravity, Isomers, and General Relativity

This unit suggests similarities and differences between aspects of our work and aspects of general relativity.

Our explanations for data do not use general relativity.

Our work above regarding extended Newtonian gravity has bases in properties of individual objects. Popular modeling includes two properties, energy and momentum, of individual objects that our work above somewhat scarcely mentions. General relativity has bases in a stress-energy tensor that has bases in properties (such as energy density and momentum density) that associate with regions.

We suggest that popular modeling might want to consider the extent to which each one of the following four statements pertains regarding gravitation and general relativity. Mass (as in gravitational $n_r = n_{3s} = 1$) associates with general relativistic notions of energy density. Mass current (as in gravitational $n_r = 1$ and $n_{3s} = 2$) associates with general relativistic notions of energy flux and of momentum density. Mass whirl-or-jitter (as in gravitational $n_r = n_{3s} = 2$) associates with general relativistic notions of pressure. Mass-whirl-or-jitter current (as in gravitational $n_r = 2$ and $n_{3s} = 3$) associates with general relativistic notions of shear stress and momentum flux.

More generally, popular modeling might want to consider the extent to which the following four statements pertain regarding gravitation and general relativity. Regarding two-body gravitational properties, the pair $n_{3s} \leftrightarrow odd$ and $n_{3s} = n_r$ associates with general relativistic notions of energy density. Regarding two-body gravitational properties, the pair $n_{3s} \leftrightarrow even$ and $n_{3s} = n_r + 1$ associates with general relativistic notions of energy flux and of momentum density. Regarding two-body gravitational properties, the pair $n_{3s} \leftrightarrow even$ and $n_{3s} = n_r$ associates with general relativistic notions of pressure. Regarding two-body gravitational properties, the pair $n_{3s} \leftrightarrow odd$ and $n_{3s} = n_r + 1$ associates with general relativistic notions of shear stress and momentum flux.

To the extent that the possible associations between extended Newtonian gravity and the general-relativity stress energy tensor pertain, the following statements might pertain. Our work suggests that general relativity might not be adequately accurate regarding circumstances for which at least one of the following sentences pertains. Non-gravitational effects, such as electromagnetic effects or chromodynamics effects, are significant. The isomeric composition of stuff varies significantly between regions. Significant (or dominant) effective reaches per instance vary with time.

Our suggested cautions about possible lacks of adequate accuracy regarding general relativity might not pertain significantly regarding circumstances for which general relativity has passed so-called precision tests [76]. For example, popular modeling regarding the perihelion precession of the orbit of the planet Mercury associates with just one isomer, namely isomer-0. Popular modeling of the deflection of light by the Sun also associates with the ordinary-matter isomer and with no other isomers.

4.2. Some Possible So-Called Cosmic Tensions

This unit discusses examples, such as the so-called Hubble tension, of possible mismatches between cosmic data and popular modeling. This unit discusses possibilities that, assuming further data does not resolve the tensions, our work points qualitatively to sources of the tensions.

Popular modeling uses the word tension to describe some possible mismatches between popular modeling and cosmic data. Some popular modeling suggests that early-multibillion-year-era values for the Hubble constant are not compatible with later-multibillion-year-era values for the Hubble constant [77]. Popular modeling associates the two-word term Hubble tension with this possible mismatch. Some popular modeling suggests that popular modeling overestimates large-scale lumpiness of stuff [78–85]. Popular modeling sometimes associates the two-element phrase S8 tension with some such possible overestimates.

Recent observations suggest that some such gaps between data and popular modeling might be smaller than previously thought or might be essentially nonexistent. Some recent observations push back against the notion of a Hubble tension [86]. Some recent observations push back against the notion of an S8 tension [87].

Our work suggests qualitative explanations for such possible mismatches. We suggest that the explanations might point toward total or partial closings, but not toward wider openings, of the possible gaps between modeling and data. Lacking means to quantify the would-be explanations, we do not try to suggest the usefulness of the explanations and we do not try to estimate the extents to which might-be-tension-related data might tend to confirm or refute our work.

4.2.1. The Hubble Tension

This unit suggests that our work might provide a candidate qualitative explanation for possible mismatches, known as the Hubble tension, between data and popular modeling regarding some large-scale phenomena.

Popular modeling estimates for a Hubble constant that would associate with the early universe might suggest a Hubble constant that is significantly less than estimates for a Hubble constant that would associate with the recent universe. Estimates tend to associate [88] with equation (24). c denotes the speed of light. ρ denotes energy density and is nonnegative. P denotes pressure and is nonpositive.

$$-(c\rho + 3P) \tag{24}$$

Popular modeling suggests that an increasing rate of expansion associates with equation (24) evaluating to a positive number. Popular modeling suggests that a decreasing rate of expansion associates with equation (24) evaluating to a negative number.

Table 6 suggests that interactions between galaxy clusters provide a basis for the rate of expansion. Regarding around the beginning of the multibillion-year era of decreasing rate, attraction (between neighboring non-colliding galaxy clusters) that associates with $n_r = n_{3s} = 3$ two-body gravitational pull dominates regarding changes in the rate of expansion. Regarding around the beginning of the multibillion-year era of increasing rate, repulsion (between neighboring galaxy clusters) that associates with $n_r = n_{3s} = 2$ two-body gravitational push dominates regarding changes in the rate of expansion.

Per discussion above, our notions of two-body-interaction reaches might apply to stress-energy-tensor components such as pressure. Here, our discussion assumes that notions of reaches pertain regarding pressure.

Table 7 suggests dominant contributions to pressure, relevant to popular modeling rate-of-expansion calculations, at selected times in the evolution of the universe. In Table 7, each one of the two numerically specified pairs of one n_{in} and one $R_{/in}$ comports with Table 5 and with data about galaxy evolution.

Table 7. Dominant contributions to pressure, relevant to popular modeling rate-of-expansion calculations, at selected times in the evolution of the universe. The information about dominant property, $n_r = n_{3s}$, gravitational instances n_{in}, and reaches per instance $R_{/in}$ pertains for interactions with the mass of a second object. TBD denotes to be determined. We do not know of enough data to determine the two TBD integers. PNR denotes possibly not relevant. For the possible future era, we suggest that the property of pressure loses significance compared to the significance of energy density.

Approximately at the start of ...	**Dominant property**	$n_r = n_{3s}$	n_{in}	$R_{/in}$
Inflation	Moments-of-inertia rotation	4	TBD	TBD
The multibillion-year era of decreasing rate	Moments of inertia	3	6	1
The multibillion-year era of increasing rate	Internal angular momentum	2	3	2
A possible future era	(None)	PNR	PNR	PNR

We suggest that a reach per instance of one for pressure might associate with the start of the multibillion-year era of decreasing rate of expansion of the universe. We suggest that a reach per instance of two for pressure might associate with the start of the multibillion-year era of increasing rate of expansion of the universe. Based on such an increasing-with-time-from-one-to-two effective reach, we suggest that extrapolating from so-called early-universe pressures might lead to underestimations of so-called later-universe pressures. Underestimations of later-universe pressures might associate with possible underestimations of later-universe values of the Hubble constant.

4.2.2. Lumpiness Tensions

This unit suggests that our work might provide a candidate qualitative explanation for some possible mismatches, including a so-called S8 tension, between data and popular modeling regarding some large-scale phenomena.

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We suggest that large-scale lumpiness associates, at least in part, with an $n_r = n_{3s} = 4$-through-$n_r = n_{3s} = 2$ sequence that we suggest above regarding galaxy evolution and regarding the rate of expansion of the universe. We suggest that popular modeling might underestimate repulsion that we associate with $n_r = n_{3s} = 2$. We suggest that the underestimate might associate with possible overestimates, by popular modeling, of large-scale lumpiness.

4.3. Relationships Among Our Work, Data, and Popular Modeling

This unit summarizes, in a context of present data and popular modeling, aspects of our work.

Compared to state-of-the-art popular modeling, our work seems to offer the following strengths. Our work explains some otherwise unexplained data. Our work offers additional insight into galaxy formation. Our work offers a better-defined candidate specification for dark matter. Our work offers a perhaps more-promising basic description of two-body gravity. Our work might better identify possible bases for possible tensions between data and popular modeling.

Compared to state-of-the-art popular modeling, our work might seem to exhibit the following weaknesses. People have yet to develop means to perform calculations and simulations based on our work. Our work does not yet say much about contributions, to two-body gravitational interactions, for which neither property is mass. Our work does not yet include a many-body-physics analog to general relativity.

Our work seems to offer the following opportunities. Advance scientific understanding regarding dark matter, gravitation, and galaxy formation. Develop quantum gravitation notions that parallel quantum electrodynamics notions. Advance techniques regarding modeling and simulations.

4.4. Suggestions Regarding Cataloging Types of Cosmic Data That Physics Collects

This unit suggests means for characterizing and cataloging types of cosmic data that physics collects.

Tables 2 and 3 use n_r and n_{3s} to catalog some two-body electromagnetic properties of objects and some two-body gravitational properties of objects.

We suggest that case-e3 and case-g3 point to possible advantages for using notions from characterization mathematics [89,90], along with or in place of using some popular modeling vocabulary, when discussing properties of objects. For example, the characterization gravitational property plus $n_r = 2$ plus $n_{3s} = 2$ might be more useful for some circumstances than the phrase object-internal angular momentum.

We suggest that popular modeling might want to consider using types of interactions, values of n_r, values of n_{3s}, and values of $R_{/in}$ to characterize observations.

Cosmological redshifts provide an example. Popular modeling suggests that the motion of a source of light relative to an observer of that light can pertain. One might characterize such so-called Doppler redshift effects as electromagnetic, with $n_{3s} = n_r + 1$ (or, velocity-related), with n_r not necessarily having relevance, and with $R_{/in} = 1$. Popular modeling suggests that interactions between light and gravitational fields can pertain. One might characterize such so-called gravitational redshift effects as gravitational, with $n_{3s} = n_r$ (or, not-necessarily velocity-related), with n_r having relevance, and with $R_{/in}$ possibly having relevance.

One might speculate regarding the possible relevance of object-properties for which $n_{3s} \geq n_r + 2$. $n_{3s} = n_r$ associates with relative spatial position. $n_{3s} = n_r + 1$ associates with relative spatial position and with relative velocity. By extrapolation, $n_{3s} = n_r + 2$ might associate with relative spatial position, with relative velocity, and with relative acceleration. Based on (for example) time-lapse observations or multi-messenger observations or multi-object simulations, an object-P might be able to infer an object-A retarded-time charge acceleration $a_{oA.rel.to.oP}$.

4.5. Suggestions for Observational Work

This unit suggests some opportunities, to which our work points, for verifying or refuting aspects of our work and for pinpointing opportunities for observational or experimental work.

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We suggest considering possible synergies between galaxy-evolution studies and our work. To what extent does our work comport with early-universe galaxy-formation and galaxy-evolution? Did nature form at least as many 1:0+ (or, dark-matter) galaxies as 0+:1 (or, ordinary-matter) galaxies? To what extent does it seem reasonable that some of today's 1:0+ galaxies and 0+:1 galaxies maintained those ratios from early in the evolution of the universe? To what extent do our suggested scenarios for the formation of ∼4:1 galaxies comport with nature?

We suggest considering observational work, regarding the aftermath of galaxy-cluster collisions, that could help verify or refute our notions of significantly-electromagnetically-active dark matter. To what extent does isomer-3 intergalactic medium exist? To what extent does isomer-3 intergalactic medium lag other dark-matter stuff? More specifically, our specification for dark matter might not necessarily be incompatible with data about collisions, such as the Bullet Cluster collision, of two galaxy clusters [91,92]. Popular modeling suggests two types of trajectories for stuff. Most dark matter, from either one of the clusters, exits the collision with trajectories that are consistent with having interacted just gravitationally with the other cluster. Ordinary-matter stars, from either cluster, exit the collision with trajectories that are consistent with having interacted just gravitationally with the other cluster. However, ordinary-matter intergalactic medium, from either cluster, lags the cluster's ordinary-matter stars and dark matter. Popular modeling suggests that the ordinary-matter intergalactic medium interacted electromagnetically with the other cluster's ordinary-matter intergalactic medium, as well as gravitationally with the other cluster. We suggest that the stuff that associates with four isomers, namely isomer-1, isomer-2, isomer-4, and isomer-5, does not form much electromagnetically active intergalactic medium. We suggest that further analysis of data might help determine the validity of our notion that the stuff that associates with isomer-3 forms electromagnetically interactive intergalactic medium.

We suggest considering observational (and perhaps even experimental) work that could help verify, refute, or extend a table such as Table 5. Which reach pertains for hyperfine phenomena? Which reach or reaches pertain for atomic transitions that are not hyperfine transitions? To what extent might data suggest a reach for magnetic moment? To what extent do the reaches for $n_{3s}+1$ current properties comport with the reaches for the counterpart n_{3s} properties?

We suggest considering experimental work that might detect dark matter of the types that we suggest.

We suggest considering work that would test general relativity for circumstances in which our work would suggest that dominant isomeric compositions change spatially. To what extent do gravitational-wave signatures from collisions differ between collisions of pairs of same-isomer-based neutron stars and collisions of pairs of different-isomer-based neutron stars?

We suggest considering work that would test general relativity for circumstances in which our work would suggest that dominant reaches change temporally or spatially. To what extent do gravitation-wave signatures from collisions differ among collisions of pairs of same-isomer-based neutron stars, collisions of pairs of different-isomer-based neutron stars, and collisions of pairs of supermassive black holes (for which each black hole might include roughly equal presences stuff that associates with each one of the six isomers)?

4.6. Suggestions for Enhancing Popular Modeling

This unit suggests some opportunities, to which our work points, for enhancing popular modeling.

We suggest considering how to evolve popular modeling to embrace notions that, while much dark matter might qualify as collisionless dark matter, significant amounts of dark matter might not qualify as collisionless dark matter.

We suggest considering how to incorporate notions regarding instances and reaches per instance into popular modeling and into numerical simulations.

We suggest considering the extent to which popular modeling might benefit by using the series gravitational scalar potential, gravitational vector potential, gravitational two-tensor potential, and so forth.

We suggest considering details regarding how to implement multipole expansions that feature potentials that associate with multiple properties, such as charge and magnetic moment or such as mass and object-internal angular momentum. We note, as an aside, that equation (25) might point [7,8] to some relationships between energies that associate with properties of the three known nonzero-mass boson elementary particles. S denotes the spin (or, object-internal angular momentum) of a particle divided by the spin of the Z boson or by the spin of the W boson. M' denotes the rest mass of a particle divided by one-third the rest mass of the Z boson. Q denotes the charge of a particle divided by the magnitude of the charge of a W boson. Data [93] do not rule out equation (25). We note, as a second-level aside, that, in some sense, the right-hand side of equation (25) might suggest that spin detracts from mass and that charge detracts from mass.

$$(4-S)^2+1=(M')^2+S^2+Q^2 \tag{25}$$

We suggest considering how to integrate into popular modeling (for electromagnetism and for gravitation) methods that we use to develop and apply extended Newtonian gravity. We suggest considering how to extend such methods to apply to torques (as well as to forces).

We suggest considering the notion that transitions from gravitational-octupole push to gravitational-quadrupole pull to gravitational-dipole push to gravitational-monopole pull associate with or might extend popular modeling notions of phase transitions. We suggest considering the notion that such transitions seem to scale from interactions between neighboring non-colliding solar-system sized objects to interactions between neighboring non-colliding larger objects.

5. Conclusion

This unit summarizes key aspects of this study.

We suggest details about dark matter and gravity.

The details associate with suggested reuses of and suggested extensions to familiar popular modeling. For example, regarding dark matter, we reuse, with variations regarding details, a list of known elementary particles. Also, regarding gravity, we reuse, with variations, notions of multipole expansions.

Our work quantitatively explains dark-matter phenomena that seemingly no other work explains. Our work suggests insight about galaxy formation and about the large-scale evolution of the universe.

The data that we explain pertain to galaxies, galaxy clusters, cosmic microwave background radiation, the expansion of the universe, and densities of the universe. Based in part on the breadth of the data that we explain, people might find our explanations credible and compelling.

We suggest opportunities for new uses of extant data, for new observational work, and for upgrades to popular modeling.

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References

1. Georges Lemaitre. Un Univers homogene de masse constante et de rayon croissant rendant compte de la vitesse radiale des nebuleuses extra-galactiques. *Annales de la Societe Scientifique de Bruxelles*, 47:49–56, April 1927. Link: https://archives.uclouvain.be/ark
2. Edwin Hubble. A relation between distance and radial velocity among extra-galactic nebulae. *Proceedings of the National Academy of Sciences*, 15(3):168–173, March 1929. DOI: 10.1073/pnas.15.3.168.
3. Fritz Zwicky. The Redshift of extragalactic Nebulae. *Helvetica Physica Acta*, (6):110–127, 1933. Link: https://ned.ipac.caltech.edu/level5/March17/Zwicky/translation.pdf.
4. F. Zwicky. On the Masses of Nebulae and of Clusters of Nebulae. *The Astrophysical Journal*, 86(86):217, October 1937. DOI: 10.1086/143864.
5. Katherine Garrett and Gintaras Duda. Dark Matter: A Primer. *Advances in Astronomy*, 2011:1–22, 2011. DOI: 10.1155/2011/968283.
6. A. Arbey and F. Mahmoudi. Dark matter and the early Universe: A review. *Progress in Particle and Nuclear Physics*, 119:103865, July 2021. DOI: 10.1016/j.ppnp.2021.103865.

7. Thomas J. Buckholtz. *Models for Physics of the Very Small and Very Large*, volume 14 of *Atlantis Studies in Mathematics for Engineering and Science*. Springer, 2016. Series editor: Charles K. Chui. DOI: 10.2991/978-94-6239-166-6.
8. Thomas J. Buckholtz. Predict particles beyond the standard model; then, narrow gaps between physics theory and data. In *Proceedings of the 9th Conference on Nuclear and Particle Physics (19-23 Oct. 2015 Luxor-Aswan, Egypt)*, May 2016. Link: http://www.afaqscientific.com/nuppac15/npc1509.pdf.
9. Judd D. Bowman, Alan E. E. Rogers, Raul A. Monsalve, Thomas J. Mozdzen, and Nivedita Mahesh. An absorption profile centred at 78 megahertz in the sky-averaged spectrum. *Nature*, 555(7694):67–70, March 2018. DOI: 10.1038/nature25792.
10. Rennan Barkana. Possible interaction between baryons and dark-matter particles revealed by the first stars. *Nature*, 555(7694):71–74, March 2018. DOI: 10.1038/nature25791.
11. Paolo Panci. 21-cm line Anomaly: A brief Status. In *33rd Rencontres de Physique de La Vallee d'Aoste*, July 2019. URL: https://cds.cern.ch/record/2688533.
12. Joshua D. Simon and Marla Geha. Illuminating the darkest galaxies. *Physics Today*, 74(11):30–36, November 2021. DOI: 10.1063/pt.3.4879.
13. Charles Day. A primordial merger of galactic building blocks. *Physics Today*, 2021(1):0614a, June 2021. DOI: 10.1063/PT.6.1.20210614a.
14. Yuta Tarumi, Naoki Yoshida, and Anna Frebel. Formation of an Extended Stellar Halo around an Ultra-faint Dwarf Galaxy Following One of the Earliest Mergers from Galactic Building Blocks. *The Astrophysical Journal Letters*, 914(1):L10, June 2021. DOI: 10.3847/2041-8213/ac024e.
15. Elena Asencio, Indranil Banik, Steffen Mieske, Aku Venhola, Pavel Kroupa, and Hongsheng Zhao. The distribution and morphologies of Fornax Cluster dwarf galaxies suggest they lack dark matter. *Mon Not R Astron Soc*, June 2022. DOI: 10.1093/mnras/stac1765.
16. Massimo Meneghetti, Guido Davoli, Pietro Bergamini, Piero Rosati, Priyamvada Natarajan, Carlo Giocoli, Gabriel B. Caminha, R. Benton Metcalf, Elena Rasia, Stefano Borgani, et al. An excess of small-scale gravitational lenses observed in galaxy clusters. *Science*, 369(6509):1347–1351, September 2020. DOI: 10.1126/science.aax5164.
17. Joshua D. Simon and Marla Geha. The Kinematics of the Ultra-faint Milky Way Satellites: Solving the Missing Satellite Problem. *Astrophys. J.*, 670(1):313–331, November 2007. DOI: 10.1086/521816.
18. Shannon Hall. Ghost galaxy is 99.99 per cent dark matter with almost no stars. *New Scientist*, August 2016. URL: https://www.newscientist.com/article/2102584-ghost-galaxy-is-99-99-per-cent-dark-matter-with-almost-no-stars/.
19. Pieter van Dokkum, Roberto Abraham, Jean Brodie, Charlie Conroy, Shany Danieli, Allison Merritt, Lamiya Mowla, Aaron Romanowsky, and Jielai Zhang. A High Stellar Velocity Dispersion and ˜100 Globular Clusters for the Ultra-diffuse Galaxy Dragonfly 44. *Astrophysical Journal*, 828(1):L6, August 2016. DOI: 10.3847/2041-8205/828/1/l6.
20. Kristi A Webb, Alexa Villaume, Seppo Laine, Aaron J Romanowsky, Michael Balogh, Pieter van Dokkum, Duncan A Forbes, Jean Brodie, et al. Still at odds with conventional galaxy evolution: the star formation history of ultradiffuse galaxy Dragonfly 44. *Monthly Notices of the Royal Astronomical Society*, 516(3):3318–3341, August 2022. DOI: 10.1093/mnras/stac2417.
21. Cosmology Calculators, March 2013. Link: https://ned.ipac.caltech.edu/help/cosmology[us]calc.html with [us] denoting the underscore symbol.
22. Peter Behroozi, Risa Wechsler, Andrew Hearin, and Charlie Conroy. UniverseMachine: The correlation between galaxy growth and dark matter halo assembly from z = 0-10. *Monthly Notices of The Royal Astronomical Society*, 488(3):3143–3194, May 2019. DOI: 10.1093/mnras/stz1182.
23. R. Genzel, N. M. Forster Schreiber, H. Ubler, P. Lang, T. Naab, R. Bender, L. J. Tacconi, E. Wisnioski, S. Wuyts, T. Alexander, et al. Strongly baryon-dominated disk galaxies at the peak of galaxy formation ten billion years ago. *Nature*, 543(7645):397–401, March 2017. DOI: 10.1038/nature21685.
24. R. Herrera-Camus, N. M. Forster Schreiber, S. H. Price, H. Ubler, A. D. Bolatto, R. L. Davies, D. Fisher, R. Genzel, D. Lutz, T. Naab, et al. Kiloparsec view of a typical star-forming galaxy when the Universe was $\sim$1 Gyr old. *Astronomy and Astrophysics*, 665:L8, September 2022. DOI: 10.1051/0004-6361/202142562.
25. Pavel E. Mancera Pina, Filippo Fraternali, Elizabeth A. K. Adams, Antonino Marasco, Tom Oosterloo, Kyle A. Oman, Lukas Leisman, Enrico M. di Teodoro, Lorenzo Posti, Michael Battipaglia, et al. Off the Baryonic Tully-Fisher Relation: A Population of Baryon-dominated Ultra-diffuse Galaxies. *Astrophysical Journal*, 883(2):L33, September 2019. DOI: 10.3847/2041-8213/ab40c7.

26. Pavel E. Mancera Pina, Filippo Fraternali, Tom Oosterloo, Elizabeth A. K. Adams, Kyle A. Oman, and Lukas Leisman. No need for dark matter: resolved kinematics of the ultra-diffuse galaxy AGC 114905. *Mon. Not. R. Astron Soc.*, December 2021. DOI: 10.1093/mnras/stab3491.
27. Qi Guo, Huijie Hu, Zheng Zheng, Shihong Liao, Wei Du, Shude Mao, Linhua Jiang, Jing Wang, Yingjie Peng, Liang Gao, et al. Further evidence for a population of dark-matter-deficient dwarf galaxies. *Nature Astronomy*, 4(3):246–251, November 2019. DOI: 10.1038/s41550-019-0930-9.
28. Pieter van Dokkum, Shany Danieli, Roberto Abraham, Charlie Conroy, and Aaron J. Romanowsky. A Second Galaxy Missing Dark Matter in the NGC 1052 Group. *Astrophysical Journal*, 874(1):L5, March 2019. DOI: 10.3847/2041-8213/ab0d92.
29. Sebastien Comeron, Ignacio Trujillo, Michele Cappellari, Fernando Buitrago, Luis E. Garduno, Javier Zaragoza-Cardiel, Igor A. Zinchenko, Maritza A. Lara-Lopez, Anna Ferre-Mateu, and Sami Dib. The massive relic galaxy NGC 1277 is dark matter deficient. From dynamical models of integral-field stellar kinematics out to five effective radii, March 2023. DOI: 10.48550/ARXIV.2303.11360.
30. Pieter van Dokkum, Zili Shen, Michael A. Keim, Sebastian Trujillo-Gomez, Shany Danieli, Dhruba Dutta Chowdhury, Roberto Abraham, Charlie Conroy, J. M. Diederik Kruijssen, et al. A trail of dark-matter-free galaxies from a bullet-dwarf collision. *Nature*, 605(7910):435–439, May 2022. DOI: 10.1038/s41586-022-04665-6.
31. Aaron J. Romanowsky, Enrique Cabrera, and Steven R. Janssens. A Candidate Dark Matter Deficient Dwarf Galaxy in the Fornax Cluster Identified through Overluminous Star Clusters. *Research Notes of the AAS*, 8(8):202, August 2024. DOI: 10.3847/2515-5172/ad7112.
32. Maria Luisa Buzzo, Duncan A. Forbes, Aaron J. Romanowsky, Lydia Haacke, Jonah S. Gannon, et al. A new class of dark matter-free dwarf galaxies? I. Clues from FCC 224, NGC 1052-DF2 and NGC 1052-DF4. Preprint, 2025. DOI: 10.48550/ARXIV.2502.05405.
33. J. Jimenez-Vicente, E. Mediavilla, C. S. Kochanek, and J. A. Munoz. Dark Matter Mass Fraction in Lens Galaxies: New Estimates from Microlensing. *Astrophysical Journal*, 799(2):149, January 2015. DOI: 10.1088/0004-637x/799/2/149.
34. J. Jimenez-Vicente, E. Mediavilla, J. A. Munoz, and C. S. Kochanek. A Robust Determination of the Size of Quasar Accretion Disks Using Gravitational Microlensing. *Astrophysical Journal*, 751(2):106, May 2012. DOI: 10.1088/0004-637x/751/2/106.
35. Man Ho Chan. Two mysterious universal dark matter–baryon relations in galaxies and galaxy clusters. *Physics of the Dark Universe*, 38:101142, December 2022. DOI: 10.1016/j.dark.2022.101142.
36. Ewa L. Lokas and Gary A. Mamon. Dark matter distribution in the Coma cluster from galaxy kinematics: breaking the mass-anisotropy degeneracy. *Monthly Notices of The Royal Astronomical Society*, 343(2):401–412, August 2003. DOI: 10.1046/j.1365-8711.2003.06684.x.
37. Elena Rasia, Giuseppe Tormen, and Lauro Moscardini. A dynamical model for the distribution of dark matter and gas in galaxy clusters. *Monthly Notices of The Royal Astronomical Society*, 351(1):237–252, June 2004. DOI: 10.1111/j.1365-2966.2004.07775.x.
38. Lawrence Rudnick. The Stormy Life of Galaxy Clusters: astro version. Preprint, January 2019. DOI: 10.48550/arXiv.1901.09448.
39. Lawrence Rudnick. The stormy life of galaxy clusters. Physics Today, January 2019. DOI: 10.1063/pt.3.4112.
40. R. L. Workman and Others. Review of Particle Physics. *PTEP*, 2022:083C01, 2022. DOI: 10.1093/ptep/ptac097.
41. S. Bilenky. Neutrino oscillations: from an historical perspective to the present status. *Journal of Physics: Conference Series*, 718:062005, May 2016. DOI: 10.1088/1742-6596/718/6/062005.
42. Electroweak measurements in electron-positron collisions at W-boson-pair energies at LEP. *Physics Reports*, 532(4):119–244, 2013. DOI: 10.1016/j.physrep.2013.07.004.
43. Sudhakantha Girmohanta and Robert Shrock. Fitting a self-interacting dark matter model to data ranging from satellite galaxies to galaxy clusters. *Physical Review D*, 107(6):063006, March 2023. DOI: 10.1103/PhysRevD.107.063006.
44. Xingyu Zhang, Hai-Bo Yu, Daneng Yang, and Haipeng An. Self-interacting Dark Matter Interpretation of Crater II. *The Astrophysical Journal Letters*, 968(1):L13, June 2024. DOI: 10.3847/2041-8213/ad50cd.
45. D. Cross, G. Thoron, T. E. Jeltema, A. Swart, D. L. Hollowood, et al. Examining the self-interaction of dark matter through central cluster galaxy offsets. *Monthly Notices of the Royal Astronomical Society*, 529(1):52–58, February 2024. DOI: 10.1093/mnras/stae442.
46. David N. Spergel and Paul J. Steinhardt. Observational Evidence for Self-Interacting Cold Dark Matter. *Physical Review Letters*, 84(17):3760–3763, April 2000. DOI: 10.1103/PhysRevLett.84.3760.

47. Daneng Yang, Ethan O. Nadler, and Hai-Bo Yu. Testing the parametric model for self-interacting dark matter using matched halos in cosmological simulations. *Physics of the Dark Universe*, 47:101807, February 2025. DOI: 10.1016/j.dark.2025.101807.
48. Gonzalo Alonso-Alvarez, James M. Cline, and Caitlyn Dewar. Self-Interacting Dark Matter Solves the Final Parsec Problem of Supermassive Black Hole Mergers. *Physical Review Letters*, 133(2):021401, July 2024. DOI: 10.1103/PhysRevLett.133.021401.
49. Xingyu Zhang, Hai-Bo Yu, Daneng Yang, and Ethan O. Nadler. The GD-1 Stellar Stream Perturber as a Core-collapsed Self-interacting Dark Matter Halo. *The Astrophysical Journal Letters*, 978(2):L23, January 2025. DOI: 10.3847/2041-8213/ada02b.
50. Manuel A. Buen-Abad, Zackaria Chacko, Ina Flood, Can Kilic, et al. Atomic Dark Matter, Interacting Dark Radiation, and the Hubble Tension. 2024. DOI: 10.48550/arXiv.2411.08097.
51. Sofiane M. Boucenna and Stefano Morisi. Theories relating baryon asymmetry and dark matter. *Frontiers in Physics*, 1, 2014. DOI: 10.3389/fphy.2013.00033.
52. Isaac Newton. Philosophiae Naturalis Principia Mathematica. 1687. DOI: 10.3931/E-RARA-440.
53. Charles-Augustin de Coulomb. First dissertation on electricity and magnetism. *History of the Royal Academy of Sciences*, pages 569–577, 1785. Link: https://library.si.edu/digital-library/book/mmoiressurllectr00coul.
54. Hendrik Lorentz. Simplified Theory of Electrical and Optical Phenomena in Moving Systems. *Proceedings of the Royal Netherlands Academy of Arts and Sciences*, 1:427–442, 1899. Indirect link: https://ui.adsabs.harvard.edu/abs/1898KNAB....1..427L/abstract.
55. O Heaviside. A gravitational and electromagnetic analogy the electrician, 31 281-2 (1893). *Reproduced in (1.) O. Heaviside, Electromagnetic Theory*, 1:455–465. Link: https://sergf.ru/Heavisid.htm.
56. O. Heaviside. *Electromagnetic Theory*. Number v. 1 in AMS Chelsea Publishing Series. American Mathematical Society, 2003. ISBN: 9780821835579.
57. Jairzinho Ramos Medina. *Gravitoelectromagnetism (GEM): A Group Theoretical Approach*. PhD thesis, Drexel University, August 2006. Link: https://core.ac.uk/download/pdf/190333514.pdf.
58. Giorgio Papini. Some Classical and Quantum Aspects of Gravitoelectromagnetism. *Entropy*, 22(10):1089, September 2020. DOI: 10.3390/e22101089.
59. John David Jackson. *Classical Electrodynamics*. WILEY, third edition, August 1998. Link: https://www.wiley.com/en-us/Classical Electrodynamics, 3rd Edition-p-9780471309321.
60. Alexander L. Kholmetskii, Oleg V. Missevitch, and Tolga Yarman. RELATIVISTIC TRANSFORMATION OF MAGNETIC DIPOLE MOMENT. *Progress In Electromagnetics Research B*, 47:263–278, 2013. DOI: 10.2528/pierb12110903.
61. N. G. Busca, T. Delubac, J. Rich, S. Bailey, A. Font-Ribera, D. Kirkby, J.-M. Le Goff, M. M. Pieri, A. Slosar, E. Aubourg, et al. Baryon acoustic oscillations in the Lyα forest of BOSS quasars. *Astronomy and Astrophysics*, 552(A96), April 2013. DOI: 10.1051/0004-6361/201220724.
62. S. Perlmutter, G. Aldering, G. Goldhaber, R. A. Knop, P. Nugent, P. G. Castro, S. Deustua, S. Fabbro, A. Goobar, Groom, et al. Measurements of Ω and Λ from 42 high-redshift supernovae Ω. *Astrophysical Journal*, 517(2):565–586, June 1999. DOI: 10.1086/307221.
63. Adam G. Riess, Alexei V. Filippenko, Peter Challis, Alejandro Clocchiatti, Alan Diercks, Peter M. Garnavich, Ron L. Gilliland, Craig J. Hogan, Saurabh Jha, Robert P. Kirshner, et al. Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant. *Astronomical Journal*, 116(3):1009–1038, September 1998. DOI: 10.1086/300499.
64. Adam G. Riess, Louis-Gregory Strolger, John Tonry, Stefano Casertano, Henry C. Ferguson, Bahram Mobasher, Peter Challis, Alexei V. Filippenko, Saurabh Jha, Weidong Li, et al. Type Ia Supernova Discoveries at $z > 1$ from the Hubble Space Telescope: Evidence for Past Deceleration and Constraints on Dark Energy Evolution. *Astrophysical Journal*, 607(2):665–687, June 2004. DOI: 10.1086/383612.
65. DESI Collaboration, K. Lodha, R. Calderon, W. L. Matthewson, A. Shafieloo, M. Ishak, et al. Extended Dark Energy analysis using DESI DR2 BAO measurements, 2025. DOI: 10.48550/ARXIV.2503.14743.
66. DES Collaboration, T. M. C. Abbott, M. Acevedo, M. Adamow, M. Aguena, et al. Dark Energy Survey: implications for cosmological expansion models from the final DES Baryon Acoustic Oscillation and Supernova data, 2025. DOI: 10.48550/ARXIV.2503.06712.
67. Alan H. Guth. Inflationary universe: A possible solution to the horizon and flatness problems. *Physical Review D*, 23(2):347–356, January 1981. DOI: 10.1103/PhysRevD.23.347.
68. Martin Bucher, Alfred S. Goldhaber, and Neil Turok. Open universe from inflation. *Phys. Rev. D*, 52:3314–3337, September 1995. DOI: 10.1103/PhysRevD.52.3314.

69. Tao Zhu, Anzhong Wang, Gerald Cleaver, Klaus Kirsten, and Qin Sheng. Pre-inflationary universe in loop quantum cosmology. *Phys. Rev. D*, 96:083520, October 2017. DOI: 10.1103/PhysRevD.96.083520.
70. Brian Green. *Until the End of Time: Mind, Matter, and Our Search for Meaning in an Evolving Universe*. Alfred A. Knopf, February 2020. Link: https://www.penguinrandomhouse.com/books/549600/until-the-end-of-time-by-brian-greene/.
71. Mark P. Hertzberg. Structure Formation in the Very Early Universe. *Physics Magazine*, 13(26), February 2020. DOI: 10.1103/physics.13.16.
72. Robert Brandenberger and Patrick Peter. Bouncing Cosmologies: Progress and Problems. *Foundations of Physics*, 47(6):797–850, February 2017. DOI: 10.1007/s10701-016-0057-0.
73. Georgios Minas, Emmanuel Saridakis, Panayiotis Stavrinos, and Alkiviadis Triantafyllopoulos. Bounce Cosmology in Generalized Modified Gravities. *Universe*, 5(3):74, March 2019. DOI: 10.3390/universe5030074.
74. Bartjan van Tent, Paola Delgado, and Ruth Durrer. Constraining the Bispectrum from Bouncing Cosmologies with Planck. *Physical Review Letters*, 130(19):191002, May 2023. DOI: 10.1103/physrevlett.130.191002.
75. Kelvin K. S. Wu, Ofer Lahav, and Martin J. Rees. The large-scale smoothness of the Universe. *Nature*, 397(6716):225–230, January 1999. DOI: 10.1038/16637.
76. Clifford M. Will. The confrontation between general relativity and experiment. *Living Reviews in Relativity*, 17(1), June 2014. DOI: 10.12942/lrr-2014-4.
77. Eleonora Di Valentino, Olga Mena, Supriya Pan, Luca Visinelli, Weiqiang Yang, et al. In the realm of the Hubble tension - a review of solutions. *Classical and Quantum Gravity*, 38(15):153001, July 2021. DOI: 10.1088/1361-6382/ac086d.
78. Christopher Wanjek. Dark Matter Appears to be a Smooth Operator. *Mercury*, 49(3):10–11, October 2020. URL: https://astrosociety.org/news-publications/mercury-online/mercury-online.html/article/2020/12/10/dark-matter-appears-to-be-a-smooth-operator.
79. Charlie Wood. A New Cosmic Tension: The Universe Might Be Too Thin. *Quanta Magazine*, September 2020. URL: https://www.quantamagazine.org/a-new-cosmic-tension-the-universe-might-be-too-thin-20200908/.
80. Maria Temming. Dark matter clumps in galaxy clusters bend light surprisingly well. *Science News*, September 2020. URL: https://www.sciencenews.org/article/dark-matter-clumps-galaxy-clusters-bend-light-surprisingly-well.
81. Khaled Said, Matthew Colless, Christina Magoulas, John R. Lucey, and Michael J. Hudson. Joint analysis of 6dFGS and SDSS peculiar velocities for the growth rate of cosmic structure and tests of gravity. *Monthly Notices of The Royal Astronomical Society*, 497(1):1275–1293, July 2020. DOI: 10.1093/mnras/staa2032.
82. Supranta S. Boruah, Michael J. Hudson, and Guilhem Lavaux. Cosmic flows in the nearby Universe: new peculiar velocities from SNe and cosmological constraints. *Monthly Notices of The Royal Astronomical Society*, August 2020. DOI: 10.1093/mnras/staa2485.
83. Kyu-Hyun Chae, Federico Lelli, Harry Desmond, Stacy S. McGaugh, Pengfei Li, and James M. Schombert. Testing the Strong Equivalence Principle: Detection of the External Field Effect in Rotationally Supported Galaxies. *The Astrophysical Journal*, 904(1):51, November 2020. DOI: 10.3847/1538-4357/abbb96.
84. Eleonora Di Valentino, Luis A. Anchordoqui, Ozgur Akarsu, Yacine Ali-Haimoud, Luca Amendola, et al. Cosmology intertwined III: fσ8 and S8. *Astroparticle Physics*, 131:102604, September 2021. DOI: 10.1016/j.astropartphys.2021.102604.
85. Ryo Terasawa, Xiangchong Li, Masahiro Takada, Takahiro Nishimichi, Satoshi Tanaka, et al. Exploring the baryonic effect signature in the Hyper Suprime-Cam Year 3 cosmic shear two-point correlations on small scales: The S8 tension remains present. *Physical Review D*, 111(6):063509, March 2025. DOI: 10.1103/physrevd.111.063509.
86. Wendy L. Freedman, Barry F. Madore, Taylor J. Hoyt, In Sung Jang, Abigail J. Lee, and Kayla A. Owens. Status Report on the Chicago-Carnegie Hubble Program (CCHP): Measurement of the Hubble Constant Using the Hubble and James Webb Space Telescopes. *The Astrophysical Journal*, 985(2):203, May 2025. DOI: 10.3847/1538-4357/adce78.
87. Angus H. Wright, Benjamin Stolzner, Marika Asgari, Maciej Bilicki, Benjamin Giblin, et al. KiDS-Legacy: Cosmological constraints from cosmic shear with the complete Kilo-Degree Survey, March 2025. DOI: 10.48550/ARXIV.2503.19441.
88. J. R. L. Santos, P. H. R. S. Moraes, D. A. Ferreira, and D. C. Vilar Neta. Building analytical three-field cosmological models. *The European Physical Journal C*, 78(2), February 2018. DOI: 10.1140/epjc/s10052-018-5614-6.

89. Anonymous. The Definitive Glossary of Higher Mathematical Jargon. Math Vault. URL: https://mathvault.ca/math-glossary.
90. Margherita Barile. Characterization. Wolfram Mathworld. URL: https://mathworld.wolfram.com/Characterization.html.
91. M. Markevitch, A. H. Gonzalez, D. Clowe, A. Vikhlinin, W. Forman, C. Jones, S. Murray, and W. Tucker. Direct Constraints on the Dark Matter Self-Interaction Cross Section from the Merging Galaxy Cluster 1E 0657-56. *Astrophysical Journal*, 606(2):819–824, May 2004. DOI: 10.1086/383178.
92. Emily M. Silich, Elena Bellomi, Jack Sayers, John ZuHone, Urmila Chadayammuri, et al. ICM-SHOX. I. Methodology Overview and Discovery of a Gas-Dark Matter Velocity Decoupling in the MACS J0018.5+1626 Merger. *The Astrophysical Journal*, 968(2):74, June 2024. DOI: 10.3847/1538-4357/ad3fb5.
93. S. Navas et al. Review of particle physics. *Phys. Rev. D*, 110(3):030001, 2024. DOI: 10.1103/PhysRevD.110.030001.