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Article

Quantum Tunneling and Bound States from Entropy Geometry: A TEQ-Based Derivation

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Abstract: We derive quantum tunneling probabilities and bound-state quantization directly from the entropy-weighted path integral and entropy curvature principles of the Total Entropic Quantity (TEQ) framework. Instead of invoking traditional quantum postulates such as wavefunctions, operator algebra, or boundary-condition quantization, we demonstrate that exponential suppression of entropy-unstable trajectories in high-curvature entropy geometries naturally produces canonical tunneling profiles and energy discretization. Standard quantum results—including the WKB tunneling formula and the Bohr–Sommerfeld quantization rule—are recovered explicitly as special limiting cases of a broader variational principle grounded in entropy geometry. These results provide structurally grounded, empirically falsifiable predictions distinct from conventional quantum mechanics, testable in engineered nanoscale and quantum optical systems.

Keywords: entropy geometry; quantum tunneling; energy quantization; entropy metric; entropy curvature; TEQ framework; path integral; resolution suppression; entropy-stable modes; empirical falsifiability; WKB approximation; quantum geometry

Meta-Abstract

This section provides a concise structural guide to the logic, assumptions, and derivational flow underpinning this work. It integrates both the derivation scope and clarification of which results are derived from first principles.

1. **Scope and Purpose:** This paper derives canonical quantum tunneling and quantized bound states from entropy geometry within the TEQ framework. It assumes prior derivation of the entropy-weighted action and entropy metric, as developed in [1,2], and focuses here on applying these principles to concrete quantum scenarios. No operator postulates, wavefunctions, or boundary-condition quantization are invoked. All results follow from entropy-stabilized path selection and the geometry of resolution.
2. **Axioms and Principles:** The derivation is based on two foundational postulates: (a) entropy geometry as the governing structure of distinguishability, and (b) the minimal principle of stable distinction. These are introduced and developed in [1,2] and underpin all results here.
3. **Derivation Pathway:** Section 2 reintroduces the entropy-weighted action and entropy metric $G_{ij}(\phi)$ following the structural derivation in [2], Sections 2 and 3 and [3], Appendix B. Sections 3 and 4 apply this framework to derive tunneling suppression and quantized modes, respectively, generalizing canonical results like the WKB approximation and Bohr–Sommerfeld rule without invoking wavefunctions or operator structure.
4. **Technical Justification:** The entropy flux functional $g(\phi, \dot{\phi}) = G_{ij}(\phi)\dot{\phi}^i\dot{\phi}^j$ arises from the Hessian of a structural entropy $S_{\text{struct}}(\phi)$ and is formally derived in [3], Equation (B.3). The suppression of unstable configurations is governed by the second variation $\delta^2 S_{\text{eff}}$, with spectral resolution justified in [2], Section 5.4.
5. **Assumptions and Limitations:** The derivation presumes smooth, well-defined entropy geometry and structural entropy fields. Deviations from conventional quantum predictions arise when

entropy curvature varies sharply or anisotropically. The approach does not rely on decoherence or stochastic collapse.

6. **Empirical Reach and Tests:** Sections 5–6 present empirical implications, including deviations from WKB and entropy-curvature-induced quantization anomalies. These results are testable in high-resolution tunneling and nanoscale systems, particularly in engineered entropy landscapes. Representative values of β across physical regimes are listed in Table 1, with derivation in Appendix A.
7. **Comparative Clarity:** Section 3.2 explicitly compares the TEQ tunneling result to the standard WKB formula. Section 4.2 derives the Bohr–Sommerfeld quantization rule as a special case of the entropy-metric length condition (Equation (15)), thereby structurally embedding canonical quantum conditions in TEQ entropy geometry. The selector-angle parameterization of β in [2], Section 3.4, further clarifies the transition between coherent and thermodynamic regimes. For a detailed contrast between the TEQ derivational pathway and standard operator- or wavefunction-based quantum mechanics, including a discussion of which results are or are not recovered by TEQ, see Appendix B.

This meta-abstract serves as both a derivational map and an interpretive guide, clarifying assumptions, result provenance, and empirical scope.

1. Introduction

Quantum tunneling and energy quantization are traditionally introduced as fundamental consequences of wave mechanics and operator algebra within the standard quantum formalism. Textbooks present these as postulates or as results of boundary condition constraints on wavefunctions, often without deeper structural derivation. While mathematically effective, such formulations obscure the underlying origin of quantization and tunneling behavior and offer limited insight into their geometric or thermodynamic meaning.

The Total Entropic Quantity (TEQ) framework replaces these postulates with a variational principle grounded in entropy geometry. Instead of assuming wave-like behavior or Hilbert space structure, TEQ treats the observable behavior of physical systems as emerging from the resolution geometry of distinguishability. The effective dynamics are determined by entropy-stabilized paths—those that minimize a complexified action functional balancing dynamical evolution and entropy cost. This approach yields an entropy-weighted path amplitude:

$$A[\phi(t)] \sim \exp\left(\frac{i}{\hbar}S[\phi] - \beta \int g(\phi, \dot{\phi})dt\right),$$

where $g(\phi, \dot{\phi})$ is the entropy flux functional defined by the local curvature of a structural entropy.

This paper applies the TEQ formalism to two classic quantum scenarios: tunneling through classically forbidden regions and energy quantization in bound systems. We show that both phenomena arise from entropy-suppressed dynamics governed by the entropy metric $G_{ij}(\phi)$. In doing so, we recover the exponential suppression associated with the WKB approximation and the Bohr–Sommerfeld quantization condition as special cases of a broader geometric framework.

All results in this paper follow from previously derived TEQ structures—specifically, the entropy-weighted action

$$S_{\text{eff}}[\phi] = \int dt(L(\phi, \dot{\phi}) - i\hbar\beta g(\phi, \dot{\phi})),$$

with $g(\phi, \dot{\phi}) = G_{ij}(\phi)\dot{\phi}^i\dot{\phi}^j$ and $G_{ij} = \partial_i\partial_j S_{\text{struct}}$, as derived in [2], Section 2.3 and [3], Appendix B.

Section Overview. The structure of this paper is as follows:

- Section 2 summarizes the entropy-weighted action and entropy metric formulation, recalling their derivation from structural entropy geometry.

- Section 3 derives tunneling suppression as an entropy-geometric phenomenon, showing how exponential decay arises in high-curvature regions (see also [2], Section 3.1 for entropy-driven suppression in constrained geometries).
- Section 4 applies the entropy curvature operator to bound-state systems, deriving quantization conditions from entropy-stabilized modes (derivations structurally mirror those in [2], Section 4–5, where eigenmodes are identified as entropy-stable configurations).
- Section 5 discusses concrete experimental scenarios and predictive deviations from the WKB limit, emphasizing entropy-engineered quantum systems.
- Section 6 outlines falsifiability conditions and error sources, specifying how and where TEQ predictions can be empirically tested or ruled out.
- Section 7 concludes with a summary of the derivational scope and theoretical implications for quantum structure and thermodynamic stability.

2. TEQ Framework and Entropy Geometry

Observable dynamics in the TEQ framework are governed by the entropy-weighted effective action:

$$S_{\text{eff}}[\phi] = \int dt (L(\phi, \dot{\phi}) - i\hbar\beta g(\phi, \dot{\phi})), \quad (1)$$

where $g(\phi, \dot{\phi}) = G_{ij}(\phi)\dot{\phi}^i\dot{\phi}^j$ encodes the entropy cost of resolving a trajectory $\phi(t)$ through configuration space. The entropy metric G_{ij} defines the resolution line element:

$$dR^2 = G_{ij}(\phi) d\phi^i d\phi^j, \quad (2)$$

which quantifies local distinguishability under coarse-grained measurement. The dimensionless parameter β acts as a structural Lagrange multiplier controlling the trade-off between action and entropy. It generalizes inverse temperature in statistical mechanics and is discussed in more detail in Section 3.2 and in [2], Section 3.1.

Stationary paths of the complex action S_{eff} dominate the physical ensemble. As shown in [2], Section 2.3, entropy-unstable paths are exponentially suppressed in the entropy-weighted path integral:

$$A[\phi(t)] \sim \exp\left(\frac{i}{\hbar} S[\phi] - \beta \int g(\phi, \dot{\phi}) dt\right), \quad (3)$$

leading to sharply peaked support on entropy-stabilized trajectories.

2.1. Derivation of the Entropy Metric and Entropy Curvature

We now outline the universal construction of the entropy metric as used throughout the TEQ framework, following the derivational pathway first introduced in [1] and made explicit in [3], Appendix B. Let \mathbf{x} denote coordinates on a smooth configuration manifold, and let $S_{\text{struct}}(\mathbf{x})$ be a structural entropy function encoding the locally accessible resolution structure.

1. **Entropy metric:** The entropy metric is defined as the second variation (Hessian) of the structural entropy:

$$G_{ij}(\mathbf{x}) = \frac{\partial^2 S_{\text{struct}}(\mathbf{x})}{\partial x^i \partial x^j}, \quad (4)$$

where S_{struct} characterizes the entropic cost of resolving infinitesimal displacements dx^i . This definition aligns with the resolution line element (2), and was first operationalized in [2], Equation (11).

2. **Entropy flux functional:** The entropy flux functional $g(\mathbf{x}, \dot{\mathbf{x}})$, which enters the entropy-weighted action (1), takes the quadratic form:

$$g(\mathbf{x}, \dot{\mathbf{x}}) = G_{ij}(\mathbf{x}) \dot{x}^i \dot{x}^j, \quad (5)$$

justified both from information geometry [4] and from TEQ's entropy-variational principle. In [3], Equation (B.3), this form arises as the lowest-order local expansion respecting reparametrization invariance and entropy positivity.

3. **Specialization to physical systems:** In concrete scenarios, S_{struct} may encode physical constraints such as energy thresholds, potential landscapes, or dynamical distinguishability. For instance, in a one-dimensional potential barrier problem, rapid growth of S_{struct} in the forbidden region yields

$$G(x) \propto V(x) - E,$$

as shown in [2], Section 3.2. However, the construction of G_{ij} and g is universal and does not depend on such identifications. This allows application to nonclassical, multidimensional, or anisotropic systems, as further discussed in [3], Section 4 and [2], Section 4.

The entropy metric formalism is thus algorithmic: once a structural entropy function S_{struct} is defined, the resolution geometry and resulting suppression dynamics follow directly. As emphasized in [10], this framework also governs the regularity of physical evolution near high-curvature or singular domains.

In the next section, we apply this structure to derive the suppression of tunneling amplitudes through high-entropy-curvature regions, and show how entropy geometry generalizes and subsumes the WKB approximation.

3. Entropy-Stabilized Tunneling

Quantum tunneling refers to the empirical phenomenon in which systems evolve across regions that are classically inaccessible—typically where a potential $V(x)$ exceeds the system's total energy E . In conventional quantum mechanics, this is explained by wavefunction continuity and exponential decay in classically forbidden regions. In contrast, the TEQ framework explains tunneling as a consequence of entropy geometry: configurations requiring high-resolution support in low-distinguishability regions are exponentially suppressed.

In this section, we derive the tunneling amplitude suppression directly from the entropy-weighted action (1), showing that exponential decay emerges from the entropy line integral over the minimal path, without reference to wave behavior or operator algebra.

3.1. Barrier Geometry and Entropy Suppression

Let $\mathbf{x}(t) \in \mathcal{C}$ denote a trajectory through configuration space crossing a potential barrier. The entropy suppression of such a path is governed by the entropy flux functional (5):

$$g(\mathbf{x}, \dot{\mathbf{x}}) = G_{ij}(\mathbf{x}) \dot{x}^i \dot{x}^j. \quad (6)$$

Here, $G_{ij}(\mathbf{x})$ is the entropy metric derived from the structural entropy S_{struct} via (4). In forbidden regions where resolution becomes costly—typically due to a steep entropy gradient—components of G_{ij} grow large along obstructed directions. This reflects the thermodynamic penalty of maintaining resolution through structurally unstable paths [2], Section 3.2.

The suppression of the entropy-weighted amplitude (3) is controlled by the total entropy-metric length of the path:

$$T_{\text{TEQ}} \sim \exp\left(-2\beta \int_{\gamma} \sqrt{G_{ij}(\mathbf{x}) dx^i dx^j}\right), \quad (7)$$

where γ is the entropy-geodesic trajectory connecting the classical turning points. This result generalizes Equation (B.4) in [3] and replaces the probabilistic interpretation of wavefunction tails with a structural suppression law over entropy curvature.

In one-dimensional or isotropic systems, Equation (7) simplifies to:

$$T_{\text{TEQ}} \sim \exp\left(-2\beta \int_{x_1}^{x_2} \sqrt{G(x)} dx\right), \quad (8)$$

where $G(x)$ is a scalar entropy metric and $[x_1, x_2]$ denotes the forbidden region. If $G(x) \propto V(x) - E$, this reproduces the standard WKB structure.

3.2. Comparison to WKB Result

In traditional quantum mechanics, the WKB tunneling probability through a barrier is given by:

$$T_{\text{WKB}} \sim \exp\left(-\frac{2}{\hbar} \int_{x_1}^{x_2} \sqrt{2m(V(x) - E)} dx\right). \quad (9)$$

This result assumes a semiclassical wave description and continuity constraints across turning points.

TEQ provides a deeper structural interpretation. In the TEQ framework, β is not an arbitrary parameter but a structural *Lagrange multiplier* enforcing entropy constraints in the path probability distribution. It governs the relative weighting between action and entropy in the entropy-weighted amplitude [1]:

$$A[\phi] = \exp\left(\frac{i}{\hbar} S[\phi] - \beta S_{\text{apparent}}[\phi]\right),$$

where S_{apparent} reflects the entropy flux functional $g(\phi, \dot{\phi})$. This expression arises from maximizing path entropy subject to average-action and entropy constraints.

The suppression factor derived from this framework generalizes the WKB expression. The entropy suppression factor

$$T_{\text{TEQ}} \sim \exp\left(-\alpha \frac{2}{\hbar} \int_{\gamma} \sqrt{G_{ij}(\mathbf{x})} dx^i dx^j\right), \quad (10)$$

emerges as a universal law for tunneling, where $\alpha = \beta\hbar$ is a dimensionless entropy-resolution parameter, and γ is the minimal-entropy-cost path across the barrier.

In one-dimensional systems where $G(x)$ is a scalar function and entropy geometry tracks potential curvature, this reduces to the standard WKB form:

$$G(x) = \frac{2m}{\hbar^2} (V(x) - E), \quad \beta = \frac{1}{\hbar}, \quad (11)$$

reproducing Equation (9) as a special case. This identification is not imposed but emerges in the limit where entropy curvature aligns with potential curvature and resolution follows dynamical constraints.

Thus, the TEQ suppression factor arises generically from entropy geometry, not from wave assumptions, with WKB behavior as a special limiting case.

3.2.1. Selector Angle Formulation

As shown in [2], β may be parametrized by a selector angle θ via:

$$\beta = \lambda \cos \theta,$$

where $\theta = 0$ corresponds to coherence-dominated (unitary) evolution and $\theta = \pi/2$ corresponds to purely thermal suppression. This formulation enables a smooth interpolation between quantum and classical regimes without invoking stochastic collapse or decoherence postulates. In this picture, the parameter $\alpha = \beta\hbar = \lambda\hbar \cos \theta$ governs the degree of entropy-induced suppression.

3.2.2. Representative Values Across Regimes

Feynman and Boltzmann weights emerge as distinct projections of the same entropy-weighted structure. The constants \hbar and k_B define the phase and entropy resolution scales, respectively. Table 1 summarizes representative values of β across regimes, clarifying when TEQ predictions reduce to classical or quantum approximations and when entropy-induced deviations become dominant.

Table 1. Representative values of β across physical regimes, illustrating the entropy–coherence spectrum. See Appendix A for derivation details.

| Regime | Representative β | Interpretation |
|---|--|---------------------|
| Room temperature ($T \approx 300$ K) | $\sim 2.4 \times 10^{20} \text{ J}^{-1}$ | Classical–thermal |
| Quantum limit (unitary) | $\beta = \frac{i}{\hbar} \sim i \times 10^{34} \text{ J}^{-1}$ | Coherence-dominated |
| CMB temperature ($T \approx 2.7$ K) | $\sim 2.7 \times 10^{22} \text{ J}^{-1}$ | Weak entropy flow |
| Planck temperature ($T_P \sim 1.4 \times 10^{32}$ K) | $\sim 10^{-9} \text{ J}^{-1}$ | Action-dominated |

In summary, the TEQ tunneling law (10) replaces the semiclassical assumption of wave continuity with a structural suppression principle rooted in entropy geometry. Deviations from the WKB approximation are predicted when the entropy metric G_{ij} deviates from potential curvature or when entropy flow is externally modulated. These empirical implications are explored in Sections 5–6.

4. Bound States and Mode Quantization

Quantized bound states arise in physical systems where particles are confined within finite regions of configuration space—such as electrons in atoms, vibrational modes in molecules, or confined particles in quantum dots. In standard quantum theory, quantization is typically explained via boundary conditions on standing wave solutions or as eigenvalues of Hermitian operators acting on Hilbert space. TEQ provides an alternative derivation: bound states correspond to stationary, entropy-stabilized paths selected by the entropy-weighted action (1).

In this section, we derive the quantization condition directly from the geometry of entropy resolution. The key idea is that stable eigenmodes of the entropy curvature operator correspond to resolvable, entropy-minimizing trajectories, generalizing the Bohr–Sommerfeld rule and reproducing canonical quantization as a special case.

4.1. Entropy-Stabilized Modes in Potential Wells

Within a finite entropy well, entropy-stationary paths extremize the entropy-weighted action (1) and define the structure of resolvable motion. The stability of such paths is governed by the second variation of the effective action:

$$\delta^2 S_{\text{eff}}[\phi] = H(\phi),$$

(12)

which defines the *entropy curvature operator* H . This operator encodes the local curvature of resolution geometry and determines the dynamics of fluctuations around entropy-stationary solutions [2], Section 4.2.

Its eigenmodes $\psi_n(\phi)$ satisfy:

$$H\psi_n = \lambda_n\psi_n, \quad E_n = \text{Re}(\lambda_n),$$

(13)

where $\lambda_n \in \mathbb{C}$ and the physically meaningful energy levels are given by the real parts E_n . This construction generalizes both the Schrödinger eigenvalue problem and the Feynman–Hibbs formulation. Crucially, no operator postulate or Hilbert space assumption is required: quantization follows from resolution geometry.

The entropy-stabilized eigenmodes ψ_n are sharply peaked in the entropy-weighted path ensemble (3), and all non-stationary configurations are exponentially suppressed. This mechanism parallels

the path localization results derived in [10], Section 3, where entropy geometry stabilizes finite-support solutions in high-curvature domains.

4.2. Quantization from Entropy Stability: Boundary Conditions and Uniqueness

A complementary derivation of quantization arises from requiring that bound-state trajectories form closed loops in entropy geometry. Let γ denote an entropy-stationary closed path. The entropy-metric length of this path,

$$\mathcal{R}(\gamma) = \int_{\gamma} \sqrt{G_{ij}(\mathbf{x})} dx^i dx^j,$$

must be an integer multiple of a fundamental entropy-metric unit S_0 . Thus, the quantization condition becomes:

$$\int_{\gamma} \sqrt{G_{ij}(\mathbf{x})} dx^i dx^j = nS_0, \quad (14)$$

where $n \in \mathbb{N}$, and $S_0 \sim \pi\hbar$ in simple systems. This is a geometric generalization of the Bohr–Sommerfeld quantization rule, justified in [2], Section 5 and [3], Section 3.

The quantity S_0 sets the fundamental entropy-metric unit of quantization. Its scale can be determined by matching to known results in simple systems. For example, in the one-dimensional potential well, the Bohr–Sommerfeld rule is:

$$\int p(x) dx = nh = 2\pi n\hbar,$$

which, when expressed in terms of entropy-metric length $\mathcal{R}(\gamma)$, implies that the minimal loop length corresponds to a half-wavelength condition $\mathcal{R}(\gamma) = n\pi\hbar$. Thus, identifying $S_0 \sim \pi\hbar$ ensures that TEQ reproduces standard quantization in this limit. In more general settings, S_0 may vary depending on boundary constraints and the topological properties of the entropy geometry, but $\pi\hbar$ provides a canonical scale fixed by comparison with semiclassical quantum structure.

Only closed paths satisfying (14) and minimizing the entropy-weighted action contribute to the observable spectrum. Boundary conditions further select physically realizable modes. These include:

- Vanishing amplitude at regions of divergent entropy curvature (e.g., infinite potential walls);
- Continuity of entropy flux $g(\phi, \dot{\phi})$ across classical-quantum boundaries;
- Compatibility with coarse-grained distinguishability constraints.

All other configurations are exponentially suppressed in amplitude due to entropy instability, as shown in the entropy-weighted integral (3). This aligns with results in [2], Section 4.3–4.4, where unstable modes are shown to vanish under entropy selection.

In one-dimensional systems, Equation (14) reduces to:

$$\int_{x_0}^{x_0+L} \sqrt{G(x)} dx = n\pi\hbar, \quad n \in \mathbb{N}, \quad (15)$$

recovering the classical Bohr–Sommerfeld rule as a special case of entropy-length quantization. This provides a structural derivation of quantization from entropy geometry, requiring no reference to waves, operators, or boundary condition postulates.

5. Empirical Implications and Representative Experimental Scenarios

Empirical deviations from standard quantum tunneling rates are among the most accessible predictions of the TEQ framework. Because the suppression of path amplitudes depends on the entropy metric $G_{ij}(\mathbf{x})$, rather than solely on potential energy differences, TEQ generically predicts measurable departures from the WKB formula when entropy curvature varies rapidly or is externally manipulated.

Such deviations are not expected in standard quantum mechanics unless one invokes environmental decoherence or phenomenological corrections [8]. In contrast, TEQ derives them structurally, as a consequence of the entropy-weighted amplitude (10).

5.1. Representative Experimental Setups and Predicted Effects

To clarify the empirical reach of the TEQ framework, Table 2 summarizes prototypical experimental systems where TEQ predicts observable shifts distinct from those expected in standard quantum mechanics or the WKB approximation. For each setup, the table indicates the predicted deviation and highlights the role of entropy geometry.

Table 2. Representative experimental scenarios where TEQ predicts measurable deviations from standard quantum mechanics, highlighting the role of entropy geometry.

| System/Setup | Observable Shift (TEQ vs. WKB/Standard QM) | Role of Entropy Geometry |
|--|---|---|
| Quantum dots with engineered nonuniform barriers | Tunneling rates deviate from WKB prediction in regions of sharp entropy-metric gradient; anomalous transmission when barrier curvature is manipulated | Entropy metric $G_{ij}(\mathbf{x})$ structurally amplifies or suppresses transmission independent of $V(x)$; geometric design directly tunes entropy suppression |
| Cold atom traps with tunable confinement | Quantization levels shift or split as entropy landscape is modulated via trap geometry; nonstandard level spacing in squeezed or anisotropic wells | Entropy curvature G_{ij} governs mode structure; confinement geometry controls quantization via entropy-stabilized eigenmodes |
| Scanning tunneling microscopy (STM) across entropy-shaped surfaces | Decay length of tunneling current departs from exponential WKB form in patterned substrates; signal anomalies when crossing high-curvature entropy domains | Local entropy metric determines decay profile; STM probes entropy-geometric rather than purely electronic structure |
| Photon interference in feedback-controlled DPIM setups | Suppression or enhancement of interference fringes beyond standard decoherence predictions; entropy-modulated transitions between wave-like and particle-like regimes | Entropy flow dynamically selects allowed interference; measurement feedback modulates G_{ij} in real time |
| Anisotropic quantum wells or Möbius strip geometries | Observation of half-integer or topologically shifted quantization spectra, not predicted by standard QM boundary conditions | Topologically nontrivial entropy-geodesic closure determines spectrum; quantization emerges from entropy geometry |

5.2. Predictive Calculation: Nonuniform Barrier

Suppose a nanoscale quantum dot system is engineered such that the potential landscape $V(\mathbf{x})$ includes a sharp cusp or step discontinuity. This leads to strong local gradients in the structural entropy S_{struct} , and thus to sharp features in the entropy metric $G_{ij}(\mathbf{x}) = \partial_i \partial_j S_{\text{struct}}(\mathbf{x})$. TEQ predicts that in such regions, tunneling suppression deviates from the standard WKB prediction (9).

The entropy-suppressed amplitude is given by:

$$T_{\text{TEQ}} \sim \exp\left(-\alpha \frac{2}{\hbar} \int_{\gamma} \sqrt{G_{ij}(\mathbf{x})} \, dx^i dx^j\right), \tag{16}$$

where γ is the entropy-geodesic across the barrier. If $G_{ij}(\mathbf{x})$ varies sharply—e.g., near a step discontinuity or designed asymmetry—then the suppression integral becomes highly sensitive to these features, resulting in a faster-than-WKB attenuation of the transmission probability.

Such modifications are not just theoretically distinct but experimentally testable. As shown in [3], Section 3.2 and [9], Section 4, measurable changes in current-voltage (I–V) characteristics can arise from entropy modulation alone.

5.3. Observable Effects and Interpretive Summary

The following effects serve as candidate observables for TEQ-induced deviations:

- **Entropy-dependent tunneling rates:** Under varying external measurement coupling or feedback (modifying entropy flow), the tunneling rate shifts due to changes in β or G_{ij} , even with unchanged $V(x)$.
- **Breakdown of Gaussian tails:** In regions of rapidly changing entropy curvature, TEQ predicts a departure from standard Gaussian suppression toward sharper exponential tails, consistent with Equation (16).
- **Anomalous quantization under entropy shaping:** If the entropy landscape is engineered (e.g., via anisotropic gating, squeezed confinement, or environmental feedback), quantization conditions shift even with unchanged mechanical potentials. This is especially clear in TEQ eigenmode structure [2], Section 5.1.

These effects can be tested in high-resolution cold atom traps, quantum dots, or scanning tunneling spectroscopy setups where both potential and environmental coupling are tunable. TEQ thereby offers a structurally grounded alternative to stochastic decoherence and opens new paths for falsifiability.

In one-dimensional or diagonal-metric cases, Equation (16) simplifies to:

$$T_{\text{TEQ}} \sim \exp\left(-\alpha \frac{2}{\hbar} \int_{x_1}^{x_2} \sqrt{G(x)} dx\right), \quad (17)$$

providing a direct generalization of the standard WKB result. Deviations are thus interpretable as entropic distortions of geometric length, rather than probabilistic collapse or phenomenological corrections.

6. Predicted Deviations, Error Sources, and Empirical Falsification

The TEQ framework yields specific, testable deviations from standard quantum predictions in regimes where entropy geometry departs from the assumptions underlying WKB and Hilbert-space quantization. In particular, when the entropy metric $G_{ij}(\mathbf{x})$ exhibits strong anisotropy, curvature discontinuities, or nonlocal structure, the standard quantum predictions fail to capture the full behavior of physical systems.

These deviations do not arise from stochastic processes or measurement postulates, but from entropy-stabilized resolution geometry. TEQ therefore allows both prediction and falsification based on structural diagnostics, as emphasized in [9], Section 4 and [3], Section 3.3.

6.1. Key Sources of Empirical Falsification

- **Tunneling in engineered nonuniform barriers:** If devices engineered with sharp entropy-metric gradients (e.g., discontinuous G_{ij}) fail to show measurable deviation from the WKB tunneling rate (9), this would contradict TEQ predictions, especially Equation (16).
- **Missing entropy-curvature shifts in high-confinement systems:** Systems with tight spatial confinement and steep entropy curvature (e.g., quantum dots or squeezed wells) should exhibit quantization anomalies per Equation (14). Their absence would suggest either redundancy or incorrectness of entropy-stabilized mode selection.
- **Failure in entropy-modulated interference:** Experiments such as those discussed in [7,9], where external entropy control is applied, must display entropy-sensitive suppression. Otherwise, the core principle of resolution-based dynamics would be challenged.
- **Absence of entropy-induced quantization anomalies:** Nontrivial $G_{ij}(\mathbf{x})$ structure—such as topologically nontrivial or anisotropic entropy curvature—should generically shift quantization

levels. Absence of such shifts in designed systems (see [2], Section 5.3) would undermine TEQ's generality.

- **Inconsistency in entropy—resolution calibration:** If the entropy-resolution parameter $\alpha = \beta\hbar$ cannot be determined consistently across comparable experiments, or if empirical tunneling or quantization data fail to fit a shared structural α , the universality of the framework would be in doubt.

6.2. TEQ-Predicted Corrections and Deviations

- **Entropy-sensitive tunneling modulation:** Tunneling amplitudes should shift measurably under external resolution changes—e.g., measurement-induced decoherence, entropy pumping, or coarse-graining biasing—modulating G_{ij} or β .
- **Energy level shifts in curvature-dominated systems:** Bound-state spectra in sharply curved entropy geometries should deviate from Schrödinger predictions, revealing entropy-stabilized corrections to Equation (13).
- **Topological entropy-induced quantization:** In systems with closed, nontrivial entropy-geodesics—such as toroidal fields or Möbius-entangled wells—the closure condition (14) may yield half-integer or anomalous spectral families.
- **Non-adiabatic transition suppression:** In systems undergoing rapid entropy metric changes (see [3], Section 4), TEQ predicts suppression of path bifurcation or non-classical transitions without invoking external observers or collapse.

6.3. Interpretive Summary

These deviations are not phenomenological add-ons, but structural consequences of entropy geometry—expressed in the entropy-weighted action (1), the suppression law (16), and the quantization condition (14). They represent falsifiable predictions rooted in curvature constraints on distinguishability, rather than in postulates of probabilistic evolution.

In this way, the TEQ framework broadens the empirical domain of quantum theory while minimizing assumptions. It provides a geometric resolution filter through which stability, quantization, and coherence emerge—not as imposed constraints, but as structural selections in the entropy landscape.

7. Conclusions

This paper has derived canonical quantum tunneling and bound-state quantization directly from the entropy geometry principles of the TEQ framework. Unlike standard quantum mechanics, which invokes wavefunction postulates or Hilbert space operators, TEQ treats observable behavior as emerging from a thermodynamic principle: stability under entropy-weighted path selection.

By varying the entropy-weighted action

$$S_{\text{eff}}[\phi] = \int dt (L(\phi, \dot{\phi}) - i\hbar\beta g(\phi, \dot{\phi})),$$

and constructing the entropy flux functional $g(\phi, \dot{\phi})$ from the Hessian of a structural entropy field S_{struct} , we showed that tunneling suppression and energy quantization arise from entropy-stabilized resolution geometry. In particular:

- The universal tunneling suppression law (16) reproduces the WKB result in one dimension as a special case, while predicting new corrections in high-curvature or anisotropic entropy geometries.
- Quantization conditions follow from closed entropy-geodesics in the configuration manifold, leading to the entropy-metric Bohr–Sommerfeld rule (14).
- The entropy curvature operator $H = \delta^2 S_{\text{eff}}$ selects stable eigenmodes that define the observable spectral structure without invoking operator postulates or boundary condition quantization.

These results reinforce the TEQ program's core claim: that quantum structure emerges from the geometry of distinguishability under entropy flow. As explored in companion works [2,10], this

same principle also underpins the emergence of spacetime structure, gravitational constraints, and quantum collapse.

Moreover, Sections 5–6 identified specific predictions and falsifiability criteria for TEQ-based tunneling and quantization behavior in engineered entropy environments. These include entropy-curvature-induced deviations from the WKB formula, entropy-modulated interference suppression, and anomalous quantization in non-Euclidean entropy metrics. Such effects are measurable in nanoscale systems and coherence-controlled quantum experiments.

In sum, entropy geometry provides not only a derivational foundation for quantum structure but also a predictive and testable generalization of it. It shifts the explanatory focus from assumed wave behavior to the thermodynamic constraints of resolution stability—replacing postulates with structure.

Acknowledgments: This work was carried out independently during a period of cognitive and physical rehabilitation following a brain hemorrhage. It reflects part of a personal recovery process rather than a formal research program. ChatGPT was used for language refinement and structural organization; all theoretical content is the author's own.

Appendix A. Order-of-Magnitude Estimate for β in TEQ

The parameter β arises in TEQ as a Lagrange multiplier enforcing entropy constraints during path selection. It determines the relative weight of entropy production versus classical action in the effective amplitude:

$$A[\phi] = \exp\left(\frac{i}{\hbar}S[\phi] - \beta S_{\text{apparent}}[\phi]\right),$$

and thus plays a central role in selecting entropy-stabilized trajectories. Its order of magnitude reflects physical conditions and the dominant entropic scale in the system.

Appendix A.1. Canonical Regimes

Following the classification in [1], Appendix C, we distinguish three principal regimes:

1. **Thermal Regime:** When thermodynamic entropy dominates and the system is in contact with a heat bath at temperature T , the canonical identification is:

$$\beta = \frac{1}{k_B T}.$$

At room temperature ($T \approx 300$ K), this yields:

$$\beta \approx 2.4 \times 10^{20} \text{ J}^{-1}.$$

2. **Quantum Regime:** For isolated, coherence-preserving systems, entropy is stationary and the amplitude becomes purely oscillatory. In this unitary limit, we recover:

$$\beta = \frac{i}{\hbar} \approx i \times 9.5 \times 10^{33} \text{ J}^{-1}.$$

3. **Entropy–Action Balance:** Defining the dimensionless parameter $\alpha := \beta\hbar$, we can assess the strength of entropy suppression relative to action. For example, at room temperature:

$$\alpha = \frac{\hbar}{k_B T} \approx 2.5 \times 10^{-14} \ll 1,$$

indicating that macroscopic coherence is strongly suppressed, consistent with classicality.

Appendix A.2. Gravitational and Cosmological Bounds

In gravitational thermodynamics and early-universe cosmology, entropy curvature becomes dominant and standard approximations break down. In the high-temperature limit $T \rightarrow T_P$ (Planck temperature), the effective $\beta \rightarrow 0$, signaling the breakdown of classical distinguishability. At the Planck temperature $T_P \sim 1.42 \times 10^{32}$ K, the minimal value of β is:

$$\beta_P = \frac{1}{k_B T_P} \sim 7.0 \times 10^{-10} \text{ J}^{-1},$$

representing the action-dominated boundary of entropy-resolved physics.

Appendix A.3. Unified Entropy-Resolution Geometry

The selector geometry formulation [2] represents β and \hbar^{-1} as orthogonal projections of a complex selector vector:

$$\beta = \lambda \cos \theta, \quad \frac{1}{\hbar} = \lambda \sin \theta,$$

with selector angle $\theta \in [0, \pi/2]$ and scale factor $\lambda \in \mathbb{R}^+$. This leads to:

$$\alpha := \beta \hbar = \cot \theta,$$

which interpolates between classical ($\theta \rightarrow 0$) and quantum-coherent ($\theta \rightarrow \pi/2$) regimes.

Appendix A.4. Summary Table

Table A1. Representative values of β across physical regimes.

| Regime | Typical β | Interpretation |
|-----------------------------------|--|-----------------------------|
| Room temperature | $\sim 2.4 \times 10^{20} \text{ J}^{-1}$ | Classical-thermal |
| Cosmic Microwave Background (CMB) | $\sim 2.7 \times 10^{22} \text{ J}^{-1}$ | Weak entropy flow |
| Quantum coherence | $\beta = \frac{i}{\hbar} \sim i \times 10^{34} \text{ J}^{-1}$ | Phase-stable limit |
| Planck temperature | $\sim 7.0 \times 10^{-10} \text{ J}^{-1}$ | Entropy curvature dominates |

These estimates support the view that β is a dynamically determined structural parameter, rather than a fixed input. Its value determines the dynamical regime, the strength of entropy suppression, and the transition between classical and quantum behavior.

Appendix B. Relation to Prior Work and Distinctive Features of TEQ

The TEQ framework reconstructs key quantum phenomena—such as tunneling, quantization, and spectral discreteness—from a variational principle based on entropy geometry, rather than from operator algebra, Hilbert space postulates, or wavefunction continuity. The differences are both conceptual and structural:

- **Origin of Quantization:**
 - *Standard QM:* Quantization arises from operator eigenvalue problems and boundary conditions on wavefunctions (e.g., Schrödinger equation, Hermitian operators).
 - *TEQ:* Quantization is a consequence of entropy-geometric closure and stability under the entropy-weighted action, with eigenmodes selected by entropy curvature—no operator postulate required.
- **Tunneling Phenomena:**
 - *Standard QM:* Tunneling is explained via wavefunction continuity and semiclassical (WKB) approximations, with transmission probability given by exponential decay in classically forbidden regions.

- *TEQ*: Tunneling suppression results from the entropy-metric line integral over the minimal path, generalizing and often correcting the WKB formula, with deviations governed by entropy curvature rather than potential energy alone.
- **Measurement and Collapse:**
 - *Standard QM*: The measurement postulate, Born rule, and wavefunction collapse are added as interpretive or axiomatic elements.
 - *TEQ*: All measurement effects, collapse, and emergence of classicality follow from the entropy geometry and stability conditions; no additional postulates or stochastic processes are introduced.
- **Structural Minimalism:**
 - *Standard QM*: Relies on a layered structure: Hilbert space, Hermitian operators, commutation relations, and measurement axioms.
 - *TEQ*: Derives all dynamics from a single structural principle: stability under entropy-weighted action, with explicit geometric (metric) formulation.

Appendix B.1. What Cannot (Yet) Be Recovered in TEQ

While TEQ generalizes and subsumes many standard results, some elements are not automatically recovered or are presently outside its scope:

- **Detailed multi-particle entanglement dynamics:** TEQ can describe entropic correlations and stability, but a fully developed many-body theory with all standard Bell-type predictions requires further generalization.
- **Field-theoretic renormalization:** The formalism handles path integrals and entropy geometry, but explicit handling of field divergences and renormalization group flows is not yet incorporated.
- **All possible boundary/initial conditions:** The structural entropy metric must be defined; pathological or discontinuous domains may require refined geometric tools.
- **Operational equivalence with all quantum predictions:** While TEQ predicts new effects (especially under entropy modulation), it does not guarantee exact equivalence with all quantum results in every regime; empirical validation is needed.

For a detailed structural comparison, see [2], Section 6 and Table 2. TEQ thus serves as both a unification and an extension of quantum mechanics, grounded in a thermodynamic-geometric principle with explicit pathways to empirical falsification and further theoretical development.

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