Article

Spin, Magnetic Moment, Structure, Shape and Sizes of an Electron, Muon, and Proton

Askar Abdukadyrov *

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Spin, Magnetic Moment, Structure, Shape and Sizes of an Electron, Muon, and Proton

Askar Abdukadyrov

Kyrgyz State Technical University, Bishkek, Kyrgyzstan; abdukadyrov64@mail.ru

Abstract: In particle physics, spin is an intrinsic form of angular momentum carried by the elementary particles. It is believed that spin is a solely quantum-mechanical phenomenon, which does not have a counterpart in classical mechanics. We have derived the simplest equations of spin and magnetic moment of a charged elementary particle. These formulas enable us to understand the physical nature of the spin and the structure of elementary particles. It is shown that the electron and muon are in the shape of a thin ring with a circular cross section (torus), and the proton is in the form of a ring with a non-circular cross section (toroid). The connection between the photon's spin of and the propagation of electromagnetic waves is considered.

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…you need not be a genius to make an important contribution to physics…

S.A. Goudsmit

"The discovery of the electron spin"

1. Introduction

Mechanical movement is common to all bodies of nature. If a body is moving progressively, to describe his movement is sufficient to describe the motion of the body's center of mass (*center of inertia*). In this case, the center of mass conventionally contains the whole mass of the body.

The rotational motion is a type of the mechanical motion. Rotation around a fixed axis is a special case of rotational motion. The angular momentum (moment of momentum, or rotational momentum) is the amount of rotation an object has, taking into account its mass and shape.

In classical mechanics, the own angular momentum S of a *rigid body* rotating around an axis passing through its center of mass is defined by the formula:

$$S = J\omega = mr_i^2\omega, \tag{1}$$

where $J = mr_i^2$ is the body's moment of inertia around an axis (axis moment of inertia), m is the mass of a body, r_i is its radius of inertia (radius of gyration), ω is angular velocity [1].

Equation (1) can be rewritten as:

$$S = mr_i v_i, \tag{2}$$

where $v_i = r_i\omega$ is the linear velocity of the radius of inertia (the *integral* linear velocity).

Should be recalled that the beginning of the radius of inertia is the body's center of mass. The end of the radius of inertia describes in the plane of rotation a *circumference of inertia* (circumference of mass), which conventionally contains the whole mass of a rotating body. Thus, the circumference of inertia plays the *same* role for a rotating body, as the center of inertia does for a progressively moving body.

In 1858, an German physicist G. Helmholtz published a memoir devoted to the vortex motion in a homogeneous incompressible fluid which has no viscosity (i.e. has no internal friction) [2]. He found

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that in such an "ideal" fluid the vortex motion must occur in the form of vortex filaments, ending at the boundaries of the fluid. In the case of an infinite "ideal" fluid, only *closed vortex rings* are possible, i.e. in this case, the vortex filaments always form *closed contours*. Consequently, the vortex ring in a limitless liquid that does not have internal friction, no physical effects cannot be broken. Therefore, according to Helmholtz, these vortex rings are *absolutely not destroyed*. They can only be deformed under the influence of certain conditions, changing their geometric dimensions: to be reduced to small sizes or to increase their contour, depending on the circumstances.

Using Helmholtz's research, an English scientist W. Thomson (Kelvin) published in 1868 a large work on vortex motion [3]. He hypothesized that the space is continuously filled with an incompressible fluid, which is not affected by any external forces, and that material phenomena of any kind depend solely on the vortex motion in this fluid.

Even earlier, in 1867, Kelvin suggested that the matter atoms, which seemed to be absolutely indestructible and indecomposable to any smaller particles of matter at that time, are vortex rings [4].

Later, in 1946, a Soviet physicist V. Mitkevich wrote: "In connection with the achievements of modern physics... the vortex atom seems to have receded into the realm of legends... [However,] W. Thomson's hypothesis about the vortex nature of extremely small particles of matter preserves very important significance and at the present time, if only instead of an atom we mean the elementary constituents of an atom (electrons, protons, etc.)" [5].

The electron was the first discovered elementary particle; it was discovered by an English physicist J. Thomson in 1897 [6]. The particle was presented to scientists in the form of a negatively charged sphere of radius $r = ke^2/m_ec^2$ (where k is the coefficient of proportionality, e is the elementary charge, m_e is electron mass, c is speed of light in vacuum).

In 1915, a British chemist and physicist A. Parson proposed a new model for the electron with a ring-shaped geometry [7]. According to the model, the electron has an extremely small section (thickness) of the ring and a relatively large radius of the ring. The model assumes that a unitary charge flows through the ring, generating an electric current and an associated magnetic field.

The ring electron model postulates that the rotational velocity of the electric charge υ_r will match the speed of light $c = 3\times10^8$ m/s and that the angular momentum L will match the reduced Planck constant $\hbar = 1.054\times10^{-34}$ J s: $\upsilon_r = c$, $L = m_er\upsilon_r = m_erc = \hbar$.

As a consequence of this equations, the radius of the ring r will match the reduced Compton wavelength of the electron $\lambda\!\!\!^{-}_e$: $r = \hbar/m_e\upsilon_r = \hbar/m_ec = \lambda\!\!\!^{-}_e$.

Inspired by this paper, several other physicists of the time, developed Parson's ideas, however, the ring electron model was not widely accepted.

In 1925, the Dutch scholars G. Uhlenbeck and S. Goudsmit suggested that an electron possesses an intrinsic angular momentum (became known as *spin*) equal to $\hbar/2$ and a magnetic dipole moment (μ) resulting from its spin. They interpreted the spin as a real rotation of the "ball"-electron around its axis [8,9].

However, this interpretation met strong objections of other scientists. They noticed that if an electron with mass ~10^{-30} kg would be a rotating ball having radius $r = ke^2/m_ec^2 \sim 10^{-15}$ m then proceeding from the spin formula $S = m_er\upsilon_r$ the linear velocity υ_r on the surface of such electron would exceed the speed of light [10].

Indeed, in this case

$$\upsilon_r = S/m_er = (\hbar/2)/(ke^2/c^2) = \hbar c^2/2ke^2 = c/2\alpha = c\cdot137/2 = c\cdot68.5, \qquad (3)$$

where $\alpha = ke^2/\hbar c \sim 1/137$ is the fine-structure constant.

Therefore, concluded that the spin is a solely quantum-mechanical phenomenon, which does not have a counterpart in classical mechanics.

In this paper, we will reveal a secret of the spin and and determine the structure, shape and sizes of the electron, muon and proton.

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2. Method

For current physics the own moments (spin and magnetic moment) of elementary particles are mysterious, but they can be easily explained if to compare with the orbital moments.

If a charged elementary particle is considered as a *material point* (a body, the sizes of which can be neglected in thise conditions) moving with a speed v in a circular orbit of radius r, it has the orbital moments – rotational (L) and magnetic (μ_L):

$$L = mvr, \qquad \mu_L = evr/2, \tag{4}$$

where $e = 1.602\times10^{-19}$ C is an elementary charge in the SI system.

The gyromagnetic ratio of the orbital moments is equal to:

$$\mu_L/L = e/2m. \tag{5}$$

The gyromagnetic ratio of the own moments of elementary particles, as shown first experiments already [11,12], is equal to:

$$\mu/S = ne/m, \tag{6}$$

where n is the corresponding dimensionless factor (e.g., for an electron n_e = 1.00115965, for a proton n_p = 2.792847) [13].

The difference of the formulas (5) and (6) it is obvious, but there is one similarity: *these equations do not include speed*.

This means that, similar to the equations for orbital moments (4), the expressions for the own moments of a charged elementary particle (for μ and S) must include *one* linear speed v.

Substituting v_i = v in equation (2), we obtain the simplest formula for spin of a charged elementary particle:

$$S = mvr_i. \tag{7}$$

Using equations (6) and (7), we obtain the formula of the spin magnetic moment:

$$\mu = S\cdot ne/m = mvr_i\cdot ne/m = evr_i\cdot n. \tag{8}$$

In this case, the sizes of particles *cannot be ignored*, so we have derived the simplest equation of the magnetic moment [14]:

$$\mu = evr, \tag{9}$$

where $r = r_i\cdot n$ is the radius of a particle.

As we see from equations (7) and (9), one linear velocity v is connected with the radius of a particle r and its radius of inertia r_i. This allows us to understand a difference between the rotational momentum of an usual (rigid) body and the spin of a charged elementary particle:

1) for a rigid body rotating with a constant angular velocity (ω = const) the linear velocity v_l at each point *is proportional* to the distance l from this point to the axis of rotation: $v_l = l\omega$;

2) a charged elementary particle *is not* a rigid body; the linear velocity v at each point of a particle *is the same* and *does not depend* on the distance l from this point to the axis of rotation: $v(l)$ = const.

This extremely interesting property means that a charged elementary particle is a *ring* that contains *the closed vortex filaments* of different radius. These vortex filaments are located in parallel planes; the centers of filaments lie on the particle's axis.

Vortex filaments are revolved around the axis of a particle with same linear speed, v = const, but with different angular velocities, $\omega(r) \neq$ const. Thus, let us clarify that a charged elementary particle is an *irrotational* vortex ring.

As a whole, a charged elementary particle has a single moment of inertia J and an integral angular velocity $\omega_i = v/r_i$, therefore generally its spin

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$$S = J\omega_i = mr_i^2\omega_i = mvr_i\,. \qquad (10)$$

3. Shape and sizes of particles

The resulting expression

$$r = r_i\, n, \qquad (11)$$

binds the radius of a particle and its radius of inertia. It allows us to understand the shape of charged elementary particles.

For example, for an electron $r = r_i{\cdot}1.00115965$, i.e. the radius of an electron is only *slightly* more than its radius of inertia. Consequently, an electron is a ring (torus) having a very small cross section, i.e. is approximately the closed vortex filament. Namely *for this reason* the researchers *cannot determine* the size of an electron in experiments and mistakenly consider it as a point [15]. But we can calculate its size.

If an electron to consider as a closed filament or a *material circumference* (a ring, the cross section of which in thise conditions can be neglected), then we can assume that the electron charge moves with velocity υ_r along the circular orbit of radius r and creates the orbital magnetic moment μ_e , according to equation (4):

$$\mu_e = er\upsilon_r/2. \qquad (12)$$

The orbital speed *cannot exceed* the speed of light, i.e. $\upsilon_r \le c$. So

$$\mu_e \le erc/2. \qquad (13)$$

On the other hand, according to equation (9), the spin magnetic moment of an electron

$$\mu_e = er\upsilon_e\,, \qquad (14)$$

where υ_e is the linear speed of electron rotation.

Comparing equations (13) and (14) we obtain:

$$er\upsilon_e \le erc/2. \qquad (15)$$

Hence, the linear speed of electron rotation *cannot exceed* half the speed of light, i.e. υ_e is less than or equal to $c/2$: $\upsilon_e \le c/2$.

Therefore, the electron's radius of inertia r_i *cannot be less* than the reduced Compton wavelength of an electron $ƛ_e$, i.e. r_i is greater than or equal to $ƛ_e$, $r_i \ge ƛ_e$, because

$$r_i = S/m_e\upsilon_e\,, \qquad (16)$$

$$ƛ_e = \hbar/m_e c = (\hbar/2)/(m_e c/2) = S/(m_e c/2), \qquad (17)$$

where $S = \hbar/2$ is spin of an electron.

The reduced Compton wavelength ƛ is natural representation for mass on the quantum scale, and as such, it appears in many of the fundamental equations of quantum mechanics.

Thus, we conclude that an electron ($m_e = 0.9109{\times}10^{-30}$ kg) has the linear velocity of rotation equal to half the speed of light, $\upsilon_e = c/2$, and its radius of inertia equal to the reduced Compton wavelength, $r_i = ƛ_e = 3.86159{\times}10^{-13}$ m [13].

Knowing the values of the radius of inertia and linear velocity, we find the integral time of rotation (revolution) of the electron t_i (the characteristic time of processes occurring due to electromagnetic forces):

$$t_i = 2\pi r_i/\upsilon = 4\pi ƛ_e/c = 1.618{\times}10^{-20}\ \text{s}. \qquad (18)$$

Since the electron has a circular cross-section, the circumference of inertia of an electron passes *exactly* through its center of section. Therefore, an electron has:

internal radius $r_{in} = r_i/n_e = r_i/1.00115965 = 3.8571{\times}10^{-13}$ m,

radius of section $r_s = r_i - r_{in} = 0.00449{\times}10^{-13}$ m $= 4.49{\times}10^{-16}$ m,

external radius $r = r_i n_e = r_i{\cdot}1.00115965 = r_i + r_s = 3.86606{\times}10^{-13}$ m.

Hence follows the relatively large magnetic moment of an electron (in the SI units):

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$$\mu_e = er\upsilon_e = en_e\lambda\!\!\!^{-}_e c/2 = 9.2847\times10^{-24}\ \text{A m}^2. \quad (19)$$

Here we will touch upon the issue of different systems of measurement.

It must be recalled that the International System of Units (SI) and the cm–g–s system of units (CGS) are variants of the metric system of physical units.

The SI system is a composite system: it includes, in particular, the MKS system for measuring mechanical quantities and the MKSA system for measuring electromagnetic quantities. The latter subsystem differs from the former primarily in that, along with the existing three basic units (meter, kilogram, second), it has a fourth basic unit – the ampere (A).

It should also be recalled that in addition to the CGS system for measuring mechanical quantities, there are various systems for measuring electromagnetic quantities that use centimeter, gram, second as basic units: absolute electrostatic system (CGSE), absolute electromagnetic system (CGSM), Gaussian system of units (type of unification of SGSE and CGSM).

In mechanics, the SI (MKS) system and the CGS system are built in an identical way, and the laws of mechanics do not depend on the choice of units of measurement. However, this is not the case in electrodynamics. This is because there is no one-to-one correspondence between the electromagnetic units in the SI (MKSA) and the same units in the CGSE, CGSM and Gaussian system, as is the case for mechanical units. Therefore, it is necessary to clearly *separate* the CGS system itself from the CGSE, CGSM and Gaussian systems.

In 2018, we showed that the electromagnetic units of the MKSA system (ampere, coulomb, ohm, volt, etc.) can be expressed using the base units of the MKS and CGS systems [16].

For example, in the MKS and CGS units, the ampere has the dimension of *force*, namely,

$$\text{A} = 10^{-6}\ \text{kg m s}^{-2}\ (\text{N}) = 0.1\ \text{g cm s}^{-2}\ (\text{dyn}). \quad (20)$$

Therefore, in units of the MKS and CGS systems the magnetic moment of an electron

$$\mu_e = 9.2847\times10^{-30}\ \text{kg m}^3\ \text{s}^{-2}\ (\text{N m}^2) = 9.2847\times10^{-21}\ \text{g cm}^3\ \text{s}^{-2}\ (\text{dyn cm}^2). \quad (21)$$

A stated argumentation concerning the electron is fully applicable to a muon (its mass m_μ = 1.8835×10^{-28} kg), because its spin $S = \hbar/2$, and the coefficient $n_\mu = 1.00116592$ [13]. Therefore, like an electron, a muon is a thin ring (torus), that has the following parameters:

linear speed of rotation $\upsilon_\mu = c/2$,

radius of inertia $r_i = \hbar/m_\mu c = \lambda\!\!\!^{-}_\mu = 1.867594\times10^{-15}$ m [13],

internal radius $r_{in} = r_i/n_\mu = r_i/1.0011659 = 1.8654\times10^{-15}$ m,

radius of section $r_s = r_i - r_{in} = 0.002194\times10^{-15}$ m $= 2.194\times10^{-18}$ m,

external radius $r = r_i n_\mu = r_i\cdot1.0011659 = r_i + r_s = 1.86977\times10^{-15}$ m.

Integral time of rotation (revolution) of the muon

$$t_i = 2\pi r_i/\upsilon_\mu = 4\pi\lambda\!\!\!^{-}_\mu/c = 7.822\times10^{-22}\ \text{s}. \quad (22)$$

The mean lifetime of muon $\tau = 2.2\times10^{-6}$ s [13], during this time it makes

$$\tau/t_i = 2.2\times10^{-6}\ \text{s}/7.822\times10^{-22}\ \text{s} = 2.81\times10^{15}\ \text{revolutions}. \quad (23)$$

The muon magnetic moment, in the MKSA units,

$$\mu_\mu = er\upsilon_\mu = en_\mu\lambda\!\!\!^{-}_\mu c/2 = 4.49\times10^{-26}\ \text{A m}^2 \quad (24)$$

or, in the MKS and CGS units,

$$\mu_\mu = 4.49\times10^{-32}\ \text{kg m}^3\ \text{s}^{-2}\ (\text{N m}^2) = 4.49\times10^{-23}\ \text{g cm}^3\ \text{s}^{-2}\ (\text{dyn cm}^2). \quad (25)$$

With regard to a proton, from the experience are known its mass $m_p = 1.6726\times10^{-27}$ kg, spin S = $\hbar/2$, and spin magnetic moment $\mu_p = 1.4106\times10^{-26}$ A m^2. However, the radius of the proton remains a puzzle to this day. Measurements show that the root mean square charge radius of a proton r is about 0.83÷0.87 fm (1 fm = 10^{-15} m) [13].

From equation (9) we find a linear speed of proton rotation υ_p:

$$\upsilon_p = \mu_p/er \approx 10^8\ \text{m/s} \approx c/3. \quad (26)$$

Thus, the linear speed of proton rotation is approximately about 3 times less then the speed of light. The proton radius of inertia

$$r_i = r/n_p = (0.83\div0.87)\ \text{fm}/2.792847 \approx 0.3\ \text{fm}. \tag{27}$$

Obviously that the proton's circumference of inertia is located *closer to the axis*, as $r_i < (r - r_i) = 0.53\div0.57$ fm. Therefore, unlike an electron and muon, *the cross-section of a proton is not a circle*, i.e. the proton has the shape of a toroid.

As we noted, modern theoretical physics considers the electron to be a point particle. This delusion leads to the problem of infinite values (*divergences*) obtained in the theory of elementary particles. A point-like electron at rest must have infinite self-energy and, consequently, infinite mass. The meaninglessness of this result clearly confirms that the electron is not a point particle; in fact, as we have shown, it has a well-defined size.

Scientists call the value $R_g = Gm/c^2$ or $R_g = G\rho V/c^2$ (where G is the gravitational constant, ρ is the density of the body, V is its volume) the "gravitational radius" of a physical body; this quantity is always many times smaller than the size R of the body, $R_g \ll R$.

For example:

mass of the Earth $m = 5.97\times10^{24}$ kg, its radius $R \approx 6370$ km, value $R_g = 4.43$ mm;

mass of the Moon $m = 7.34\times10^{22}$ kg, its radius $R \approx 1737$ km, value $R_g = 0.054$ mm.

In 1935, the Soviet physicist M. Bronstein rightly noted that in the quantum theory of the gravitational field «*the gravitational radius of the test body used for the measurement* ($G\rho V/c^2$) *cannot be larger than its actual linear dimensions* ($V^{1/3}$)*…*», however, he was mistaken in asserting that «*In electrodynamics there is no analogy to this fact…*» [17].

In fact, there is the value $r_0 = ke^2/mc^2$ which scientists call the "classical radius" of a charged elementary particle; this quantity is always much smaller than the radius r of the particle, $r_0 < r$.

For example, for an electron $r_0 = ke^2/m_ec^2 = 2.81\times10^{-15}$ м, $r = 3.866\times10^{-13}$ м;

for a muon $r_0 = ke^2/m_\mu c^2 = 1.36\times10^{-17}$ м, $r = 1.869\times10^{-15}$ м;

for a proton $r_0 = ke^2/m_pc^2 = 1.53\times10^{-18}$ м, $r = (0.83\div0.87)\times10^{-15}$ м.

Thus, the sizes of the electron and muon are determined correctly.

Modern physics cannot explain why the electron's magnetic moment is slightly larger than the *Bohr magneton* $\mu_B = e\hbar/2m_e$, and the proton's magnetic moment is much larger than the *nuclear magneton* $\mu_N = e\hbar/2m_p$:

$$\mu_e = \mu_B\, n_e, \qquad \mu_p = \mu_N\, n_p\,. \tag{28}$$

This discrepancy between the experimentally observed and assumed (according to the Dirac equation) values of the magnetic moment is considered an *anomaly*.

However, it must be so, because spin $S = mvr_i$, magnetic moment $\mu = evr = evr_i{\cdot}n$, and

$$\mu_B = e\hbar/2m_e = eS/m_e = ev_er_i\,, \tag{29}$$

$$\mu_N = e\hbar/2m_p = eS/m_p = ev_pr_i\,, \tag{30}$$

$$e\hbar/2m_\mu = eS/m_\mu = ev_\mu r_i\,. \tag{31}$$

Thus, in reality, there is no anomaly in the magnetic moment of the electron, muon, proton and other particles.

4. Spin and electromagnetic waves

With the rotational movement we encounter in study of light; it is known that the quantum of light (photon) has a spin.

Must be recalled that to explain the phenomenon of light, there are two different points of view, forming so-called *wave-particle duality*.

On one hand, it is believed that light is a *photon flux*, i.e. the energy transfer is a process associated with the transfer of matter.

On other hand, it is argued that light is a *wave process* due to fluctuations of electric and magnetic fields, i.e. the energy transfer without transfer of matter. Modern physics *cannot explain* this contradiction. We will try to explain it.

According to the Planck formula, the energy E of a photon is proportional to the frequency ν (or circular frequency ω):

$$E = h\upsilon = \hbar\omega. \tag{33}$$

This formula can be written as:

$$E = S_\gamma\omega, \tag{33}$$

where $S_\gamma = \hbar$ is the spin of a photon.

A photon, like other elementary particles, *is not* an ordinary (rigid) body; a photon has *one* linear velocity of rotation υ, so its spin

$$S_\gamma = m_\gamma \upsilon r_i\,, \tag{34}$$

where m_γ is the mass of a moving photon. Then,

$$S_\gamma\omega = m_\gamma \upsilon r_i\omega = m_\gamma \upsilon^2, \tag{35}$$

where $\upsilon = r_i\,\omega$.

As it is known, the light and, in general, the electromagnetic waves are propagated at the speed of light c. Therefore, the speed of light is the linear velocity of *the rotational motion* of a photon, and the photon energy E is its *energy of rotation*:

$$c = \upsilon = r_i\,\omega. \tag{36}$$

$$E = \hbar\omega = m_\gamma c^2. \tag{37}$$

The circular frequency of a photon is its angular velocity $\omega = 2\pi\nu$, where ν is the frequency of rotation of the photon, then the speed of light

$$c = r_i{\cdot}2\pi\nu = \lambda\!\!\!^{-}{\cdot}2\pi\nu = \lambda\nu, \tag{38}$$

where $\lambda = 2\pi\lambda\!\!\!^{-}$ is the *length* of the light wave, herewith the value $\lambda\!\!\!^{-} = r_i$ should be understood as the *radius* of the wave.

So, we have come to the known conclusion that the speed of light is the speed of the light wave, provided that the photons *rotate* at the speed of light.

At first glance, this condition seems paradoxical therefore we will remind some historical information.

In the XIX century it was assumed that light is propagated in a special light-bearing medium – *ether* that fills the entire world space.

In 1839, an Irish mathematician J. MacCullagh proposes an original theory of luminiferous ether. He believed that the ethereal medium has unusual properties, namely an elasticity only in relation to *rotation* and has no resistance to other types of deformation [18].

The theory of "rotational" ether was seemed absurd to other scientists; it was forgotten and remembered after the creation by J. Maxwell in 1864 the electromagnetic theory of light.

An Irish scientist G. FitzGerald saw the possibility of combining two theories. This was due to the fact that Maxwell's theory was analytically equivalent to MacCullagh's, i.e. in the part concerning to mathematical equations, both theories are exactly the same. In 1878, the scientist came to the conclusion that the magnetic intensity H is a *flow rate* in ether, and the electric induction D is determined by *rotation* of smallest parts (elements) of ether [19,20]:

$$\partial r/\partial t = \boldsymbol{H}, \qquad \mathrm{rot}\ r = \boldsymbol{D}, \tag{39}$$

where r is a displacement of the elements of ether in which the electromagnetic waves are propagated, $\boldsymbol{H}$ is a vector of magnetic intensity, $\boldsymbol{D}$ is a vector of electric induction.

It turns out that Fitzgerald's assumption is quite correct.

Using transformation of the electromagnetic units of the MKSA system into units of the MKS and CGS systems, we showed in [16], that

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$$\partial r/\partial t = \boldsymbol{H} = \partial \boldsymbol{G}/\partial t, \qquad \text{rot}\ r = \boldsymbol{D} = \text{rot}\ \boldsymbol{G}, \tag{40}$$

where r is the displacement of the elements of space in which the electromagnetic waves are propagated (since the concept of ether was rejected in the XX century), the $\boldsymbol{G}$ is a vector of conductance.

These formulas complement Maxwell's equations, which do not answer the question of how electromagnetic waves propagate.

From the formula $\boldsymbol{D} = \text{rot}\ \boldsymbol{G}$ follows that the conductance of space, i.e. *the transfer of energy* by electromagnetic waves from one element of space to another, is carried out by *rotation* of these elements.

Thus, three different methods – MacCullagh's theory (1839), unit conversion (2018) and the current analysis of the spin (2023) – lead to the same conclusion.

5. Conclusions

We have derived the simplest equations of spin and magnetic moment of a charged elementary particle. These formulas enable us to determine that a charged elementary particles – an electron, muon, proton – have a shape of the ring and is contained the closed vortex filaments, which rotating around the axis of the particle with same linear speed, but with different angular velocities (an *irrotational vortex*). Thus, the spin of elementary particles is due to the real rotation of their internal structure. Also have shown that the elements of space are *closed vortex filaments*, through the rotation of which the electromagnetic waves are propagated.

Competing interests: The author declares that he has no competing interests.

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