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Quantum Tunneling through Structured Barrier Media with Quantized Modes: Transmission Probability and Tunneling Time

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Abstract

Quantum tunneling is conventionally described using the stationary Schrödinger equation with an externally imposed potential barrier. In this work, we present a dynamical formulation of quantum tunneling in which the barrier is modeled as a structured medium possessing quantized internal modes that interact coherently with an incident electron. Using the Heisenberg operator formalism and a second-quantized representation of the barrier medium, we derive coupled dynamical equations governing the electron–barrier interaction. Under a continuum-mode and mean-field approximation, the collective response of the barrier modes generates an effective potential that reproduces the conventional rectangular barrier model and the standard tunneling transmission probability obtained from the Schrödinger equation. Within this framework, a tunneling traversal time is naturally defined from the dynamical evolution of the electron and is shown to depend on the barrier width, barrier height, and incident electron energy. Numerical simulations illustrating the transmission probability and tunneling-time behavior are presented. The results provide a complementary microscopic interpretation of tunneling processes in structured quantum media and may be relevant to nanoscale transport, photonic barriers, and coherent quantum devices.:

Keywords: quantum tunneling; structured barrier media; quantized barrier modes; coherent particle–medium interaction; transmission probability; tunneling time; Heisenberg formalism; quantum transport

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1. Introduction

Quantum tunneling is one of the most fundamental phenomena in quantum mechanics and plays a central role in a wide range of physical systems and applications. Originally introduced in the context of alpha decay by Gamow [1] and independently by Gurney and Condon [2], tunneling describes the penetration of a quantum particle through a classically forbidden potential barrier. Since then, tunneling phenomena have become essential in numerous areas of physics, including semiconductor heterostructures and tunnel diodes [3,4], resonant tunneling devices [5], molecular and chemical reaction dynamics [6], superconducting junctions [7], and scanning tunneling microscopy [8].

In conventional quantum mechanics, tunneling is typically described using the stationary Schrödinger equation with an externally imposed static potential barrier [9–11]. For simple barrier geometries, analytical expressions for the transmission probability can be obtained and exhibit the well-known exponential dependence on barrier width and barrier height. Semiclassical approaches such as the Wentzel–Kramers–Brillouin (WKB) approximation provide additional physical insight into the exponential suppression of wave amplitudes in classically forbidden regions [12]. These methods have been remarkably successful in describing a broad range of tunneling phenomena.

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Despite this success, the physical interpretation of quantum tunneling continues to attract significant interest, particularly regarding the microscopic origin of effective barrier potentials and the meaning of tunneling time. Various tunneling-time concepts have been proposed over the years, including phase time, dwell time, Wigner delay time, traversal time, and Larmor-clock approaches [13–16]. Recent advances in weak-measurement techniques and attosecond experiments have enabled increasingly direct investigations of tunneling-time phenomena [17–19]. Nevertheless, different operational definitions often lead to different interpretations, and the physical picture underlying quantum traversal through classically forbidden regions remains an active topic of research.

From a microscopic perspective, a physical barrier in condensed-matter, molecular, or photonic systems is not merely an abstract static potential but rather a structured medium composed of atoms, lattice vibrations, electromagnetic modes, or other collective excitations. It is therefore natural to consider the possibility that a tunneling barrier possesses internal dynamical degrees of freedom capable of interacting with an incident particle. Related ideas have previously appeared in studies of particle–environment coupling, dissipative tunneling, localized interaction models, and open quantum systems [20–24]. In particular, the pioneering work of Büttiker and Landauer demonstrated how localized interactions can provide insight into tunneling-time measurements and traversal dynamics [14].

Motivated by these considerations, we investigate a dynamical description of tunneling in which the barrier is modeled as a structured medium containing quantized internal modes. Rather than introducing the barrier as an externally prescribed potential, the present formulation treats the barrier as an effective collective response arising from coherent interactions between the electron and the internal modes of the medium. A schematic illustration of the proposed structured barrier-medium framework is shown in Figure 1.

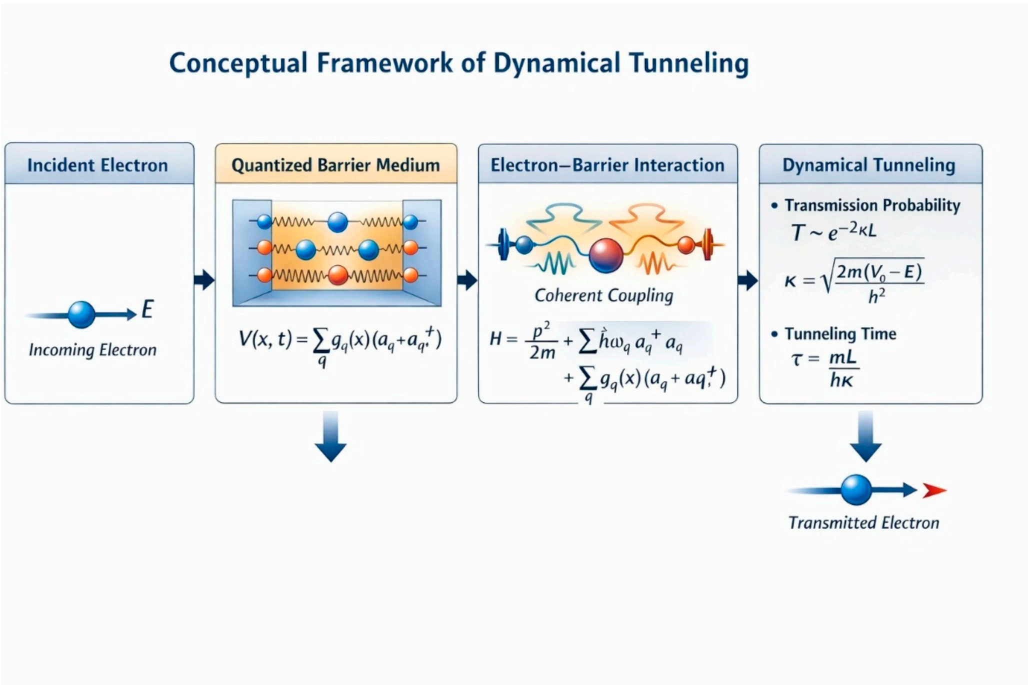

Figure 1. Conceptual framework of dynamical quantum tunneling. An incident electron interacts coherently with the quantized modes of the barrier medium while traversing the barrier region. The barrier potential is represented as a superposition of quantized field modes $V(x,t) = \sum_q g_q(x)(a_q + a_q^\dagger)$. The coupled electron–barrier system is described by a Hamiltonian containing the electron kinetic energy, the barrier-mode energy, and the interaction between the electron and the barrier modes. This dynamical interaction leads to the tunneling process, from which the transmission probability $T \sim e^{-2\kappa L}$ and the tunneling time $\tau = mL/(\hbar\kappa)$ can be obtained.

Using the Heisenberg operator formalism and a second-quantized representation of the barrier medium, we derive coupled equations of motion describing the electron–barrier interaction. Under a continuum-mode and mean-field approximation, the collective response of the barrier modes generates an effective potential that reproduces the conventional rectangular barrier model and the standard tunneling transmission probability obtained from the stationary Schrödinger equation. In this sense, the familiar static tunneling barrier may be viewed as a continuum-limit description of an underlying dynamical particle–medium interaction.

Within the same framework, a tunneling traversal time emerges naturally from the dynamical evolution of the electron across the barrier region. The resulting expression depends explicitly on the barrier width, barrier height, and incident electron energy. While the present formulation is not intended as a universal theory of tunneling, it provides a complementary microscopic interpretation of tunneling dynamics in structured environments and establishes a unified framework for discussing both transmission probability and tunneling traversal time.

The remainder of this paper is organized as follows. Section 2 introduces the Hamiltonian model and the quantized barrier-medium description. Section 3 derives the coupled dynamical equations using the Heisenberg formalism. Section 4 discusses the continuum-mode approximation and the recovery of the conventional Schrödinger tunneling model. Sections 5 and 6 present the transmission-probability and tunneling-time analyses, respectively. Section 7 discusses the physical implications, limitations, and possible experimental relevance of the model. Finally, Section 8 summarizes the main conclusions.

2. Dynamical Model and Quantized Barrier Medium

In conventional quantum tunneling theory, the barrier is represented by an externally prescribed static potential $V(x)$appearing in the Schrödinger equation. While this description has been highly successful, it does not explicitly address the microscopic origin of the barrier or its possible internal dynamical structure. In many physical systems, including condensed-matter, molecular, and photonic environments, barriers are formed by structured media possessing internal degrees of freedom such as lattice vibrations, electromagnetic modes, molecular excitations, or other collective states [20–24].

Motivated by this observation, we model the barrier as a structured medium containing quantized internal modes that interact coherently with an incident electron. The resulting framework allows the effective barrier potential to emerge from the collective response of the medium rather than being imposed externally.

The total Hamiltonian is written as

$$\hat{H} = \hat{H}_e + \hat{H}_b + \hat{H}_{\text{int}}, \qquad (1)$$

where

$$\hat{H}_e = \frac{\hat{p}^2}{2m} \qquad (2)$$

describes the kinetic energy of the electron and

$$\hat{H}_b = \sum_q \hbar\,\omega_q \hat{a}_q^\dagger \hat{a}_q \qquad (3)$$

represents the quantized internal modes of the barrier medium.

Here,

$$[\hat{a}_q, \hat{a}_{q'}^\dagger] = \delta_{qq'} \qquad (4)$$

and $\omega_q$denotes the frequency of the q-th barrier mode.

The interaction Hamiltonian is taken as

$$\hat{H}_{\text{int}} = \sum_q g_q\,(\hat{x})\big(\hat{a}_q + \hat{a}_q^\dagger\big), \qquad (5)$$

where $g_q(\hat{x})$is a position-dependent coupling function localized primarily within the barrier region.

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Combining Eqs. (1)–(5), the total Hamiltonian becomes

$$\hat{H} = \frac{\hat{p}^2}{2m} + \sum_q \hbar\, \omega_q \hat{a}_q^\dagger \hat{a}_q + \sum_q g_q\, (\hat{x})(\hat{a}_q + \hat{a}_q^\dagger). \qquad (6)$$

The physical picture underlying the model is illustrated schematically in Figure 1. An incident electron interacts coherently with quantized internal modes localized within the barrier region. These internal modes may represent phonons, optical excitations, electromagnetic modes, cavity modes, or other collective degrees of freedom associated with the structured barrier medium. The collective response of these modes generates an effective barrier potential governing the tunneling dynamics.

2.1. Effective Barrier Potential

Within a mean-field description, the collective response of the barrier medium is characterized by the expectation values of the barrier-mode operators. The effective potential acting on the electron is defined as

$$V_{\text{eff}}(x) = \sum_q g_q\, (x) < \hat{a}_q + \hat{a}_q^\dagger >. \qquad (7)$$

The expectation value is evaluated with respect to a stationary coherent or displaced state of the barrier medium.

Importantly, if the barrier modes occupy their vacuum state,

$$< \hat{a}_q + \hat{a}_q^\dagger > = 0,$$

and no effective barrier is generated. Therefore, the present model assumes that the barrier medium possesses a nonzero collective background response.

This effective-potential approximation corresponds to a mean-field treatment in which fluctuations of the barrier modes are neglected. The resulting effective potential captures the collective coherent response of the structured medium while leaving fluctuation and correlation effects for future studies.

2.2. Continuum Representation

For a dense spectrum of barrier modes, the discrete summation can be replaced by a continuum description,

$$V_{\text{eff}}(x) = \int d\omega\, D(\omega)\, g_\omega(x) < \hat{a}_\omega + \hat{a}_\omega^\dagger >, \qquad (8)$$

where $D(\omega)$ is the mode density and $g_\omega(x)$ is the corresponding coupling function.

The continuum representation provides a bridge between the microscopic barrier-medium description and the effective single-particle tunneling picture developed in later sections.

2.3. Assumptions Underlying the Effective Barrier

The emergence of the effective rectangular barrier used in the subsequent analysis requires several simplifying assumptions.

First, the coupling functions $g_\omega(x)$ are assumed to be localized within a finite spatial region $0 < x < L$, corresponding to the physical barrier region.

Second, the collective response of the barrier medium is assumed to be approximately uniform inside the barrier. Under this assumption,

$$D(\omega)\, g_\omega(x) < \hat{a}_\omega + \hat{a}_\omega^\dagger >$$

has only weak spatial variation for $0 < x < L$.

Third, the collective response is assumed to be negligible outside the barrier region.

Fourth, the barrier modes are assumed to occupy stationary coherent or displaced background states, producing nonzero expectation values and therefore a nonvanishing effective potential.

Finally, the present treatment employs a mean-field approximation in which fluctuations and higher-order correlations of the barrier modes are neglected.

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Under these assumptions, the effective potential reduces approximately to

$$V_{\text{eff}}(x) = \begin{cases} 0, & x < 0, \\ V_0, & 0 < x < L, \\ 0, & x > L, \end{cases} \tag{9}$$

which corresponds to the conventional rectangular tunneling barrier widely used in quantum mechanics [9–12].

The present model should therefore be viewed as a dynamical structured barrier-medium framework whose continuum-limit behavior reproduces the familiar static barrier description.

3. Dynamical Equations of Motion

The dynamics of the coupled electron–barrier system are governed by the Heisenberg equations of motion. In the Heisenberg picture, the time evolution of an operator $\hat{O}$ is determined by

$$\frac{d\hat{O}}{dt} = \frac{i}{\hbar}[\hat{H}, \hat{O}], \tag{10}$$

where $\hat{H}$ is the total Hamiltonian given by Eq. (6). The Heisenberg formalism provides a natural framework for describing the dynamical interaction between the electron and the quantized barrier medium [25–27].

Applying Eq. (10) to the electron position operator $\hat{x}$ yields

$$\frac{d\hat{x}}{dt} = \frac{i}{\hbar}[\hat{H}, \hat{x}] = \frac{\hat{p}}{m}, \tag{11}$$

which is the standard velocity relation for a nonrelativistic particle.

Similarly, the momentum operator evolves according to

$$\frac{d\hat{p}}{dt} = \frac{i}{\hbar}[\hat{H}, \hat{p}]. \tag{12}$$

Substituting Eq. (6) into Eq. (12) gives

$$\frac{d\hat{p}}{dt} = -\sum_q \frac{\partial g_q(\hat{x})}{\partial x}(\hat{a}_q + \hat{a}_q^\dagger). \tag{13}$$

Equation (13) shows that the electron experiences a force arising from the spatial dependence of the electron–mode coupling. Physically, this force originates from the collective response of the structured barrier medium.

The dynamics of the barrier-mode operators are obtained in the same manner. For the annihilation operator $\hat{a}_q$,

$$\frac{d\hat{a}_q}{dt} = \frac{i}{\hbar}[\hat{H}, \hat{a}_q], \tag{14}$$

which gives

$$\frac{d\hat{a}_q}{dt} = -i\omega_q \hat{a}_q - \frac{i}{\hbar} g_q(\hat{x}). \tag{15}$$

Equation (15) indicates that the barrier modes evolve under the combined influence of their intrinsic oscillation and the driving induced by the electron. The coupled equations therefore describe a dynamical feedback process between the electron and the internal degrees of freedom of the barrier medium.

The present formulation is related to bilinear particle–environment coupling models commonly encountered in open quantum systems and quantum Langevin dynamics [20–24]. However, unlike conventional dissipative approaches, which focus on decoherence and energy relaxation, the present work emphasizes coherent collective interactions and their role in generating an effective tunneling barrier.

3.1. Emergence of the Effective Potential

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When the barrier modes respond collectively to the presence of the electron, their expectation values generate an effective potential acting on the electron. This potential reflects the coherent response of the barrier medium to the traversing particle and provides the connection between the microscopic interaction picture and the effective single-particle tunneling description.

Within the mean-field approximation introduced in Section 2, the effective potential is

$$V_{\text{eff}}(x) = \sum_{q} g_q\,(x) < \hat{a}_q + \hat{a}_q^{\dagger} >. \qquad (16)$$

Taking the expectation value of Eq. (13) therefore yields

$$< \frac{d\hat{p}}{dt} > = -\frac{\partial V_{\text{eff}}(x)}{\partial x}, \qquad (17)$$

which has the same form as Newton's equation of motion in an effective potential.

In the continuum representation introduced in Eq. (8), the effective potential becomes

$$V_{\text{eff}}(x) = \int d\omega\, D(\omega)\, g_{\omega}(x) < \hat{a}_{\omega} + \hat{a}_{\omega}^{\dagger} >. \qquad (18)$$

Under the assumptions summarized in Section 2.3, the effective potential reduces approximately to the rectangular barrier form of Eq. (9). Consequently, the conventional tunneling barrier may be interpreted as an emergent collective response of the structured medium in the continuum-mode limit.

The coupled equations derived above provide the dynamical foundation for the tunneling analysis developed in the following sections. In particular, they establish the connection between the microscopic electron–barrier interaction and the effective barrier potential responsible for quantum tunneling.

4. Continuum-Mode Limit and Recovery of the Schrödinger Tunneling Model

The coupled dynamical equations derived in the previous section describe the coherent interaction between the electron and the quantized modes of the structured barrier medium. In general, the full system constitutes a many-body dynamical problem involving both particle and environmental degrees of freedom. However, under appropriate conditions, the collective response of the barrier modes may be represented by an effective stationary potential acting on the electron.

In the present model, this reduction occurs in the continuum-mode limit, where the barrier medium possesses a dense spectrum of internal modes whose collective response varies slowly on the timescale of the electron motion. Under these conditions, the microscopic barrier dynamics become effectively encoded in an emergent potential experienced by the electron.

4.1. Conditions for the Continuum-Mode Limit

The continuum-mode approximation employed in the present work relies on several assumptions.

First, the barrier medium is assumed to possess a sufficiently dense spectrum of internal modes so that the discrete summation over modes may be replaced by a continuum representation,

$$\sum_{q} \rightarrow \int d\omega\, D(\omega), \qquad (19)$$

where $D(\omega)$ is the density of states of the barrier medium.

Second, the expectation values of the barrier-mode operators are assumed to vary slowly compared with the characteristic electron traversal time. Under this condition, the collective response of the barrier medium can be regarded as approximately stationary during the tunneling process.

Third, the barrier modes are assumed to occupy coherent or displaced background states that generate nonzero collective expectation values. These expectation values are responsible for producing the effective barrier potential introduced in Section 2.

Fourth, fluctuations and higher-order correlations of the barrier modes are neglected within the mean-field approximation. The present analysis therefore focuses on the collective coherent response of the barrier medium rather than fluctuation-induced corrections.

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Under these assumptions, the barrier medium behaves effectively as a stationary structured background, and the electron dynamics reduce to an effective single-particle problem.

4.2. Effective Single-Particle Description

The resulting effective Hamiltonian may be written as

$$\hat{H}_{\text{eff}} = \frac{\hat{p}^2}{2m} + V_{\text{eff}}(\hat{x}), \qquad (20)$$

where $V_{\text{eff}}(x)$is given by Eq. (16).

For the rectangular barrier configuration introduced in Eq. (9), the effective Hamiltonian becomes identical to the conventional one-dimensional tunneling Hamiltonian widely used in quantum mechanics [9–12].

The corresponding stationary Schrödinger equation is

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V_{\text{eff}}(x)\right]\psi(x) = E\psi(x), \qquad (21)$$

where $E$is the incident electron energy.

Inside the barrier region,

$$0 < x < L,$$

where

$$V_{\text{eff}}(x) = V_0,$$

and for energies satisfying

$$E < V_0,$$

the wavefunction obeys

$$\frac{d^2\psi}{dx^2} = \kappa^2\psi, \qquad (22)$$

with

$$\kappa = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}. \qquad (23)$$

The resulting solution inside the barrier takes the familiar evanescent form

$$\psi(x) \propto e^{-\kappa x}, \qquad (24)$$

which describes the exponential attenuation characteristic of quantum tunneling.

Consequently, the transmission probability for an opaque barrier becomes

$$T \sim e^{-2\kappa L}, \qquad (25)$$

which is identical to the standard result obtained from the conventional Schrödinger and WKB formulations [11,12,28–30].

4.3. Relation to Conventional Tunneling Theory

The agreement between Eq. (25) and the conventional tunneling expression is expected because the Schrödinger and Heisenberg formulations of quantum mechanics are formally equivalent [26,27]. The present framework therefore does not seek to replace the standard theory of quantum tunneling. Rather, it provides a complementary microscopic interpretation in which the effective barrier emerges from coherent interactions between the particle and a structured medium possessing quantized internal modes.

From this perspective, the static potential barrier appearing in the Schrödinger equation may be viewed as an effective continuum-limit description of underlying particle–medium interactions. The conventional tunneling problem is therefore recovered naturally as the continuum-mode limit of the present structured barrier-medium framework.

The continuum-limit analysis developed here forms the basis for the transmission-probability and tunneling-time calculations presented in the following sections.

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5. Transmission Probability

In the continuum-mode limit developed in Section 4, the collective response of the quantized barrier medium generates an effective stationary potential barrier. The electron tunneling problem therefore reduces to an effective single-particle description governed by the Hamiltonian of Eq. (20). The transmission probability may then be evaluated using the corresponding effective wave dynamics.

For an incident electron with energy $E < V_0$, the wavefunction inside the barrier region exhibits exponential attenuation characterized by

$$\kappa = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}, \qquad (26)$$

where $V_0$is the effective barrier height and $L$is the barrier width. The quantity κ^{-1}represents the characteristic decay length of the evanescent wave inside the classically forbidden region.

Within the semiclassical WKB approximation [11,12,28,29], the tunneling probability is determined by the action accumulated across the barrier,

$$S = \int_0^L p(x)\, dx. \qquad (27)$$

Inside the barrier region, where $E < V_0$, the effective momentum formally becomes

$$p(x) = i\hbar\kappa. \qquad (28)$$

The appearance of an imaginary momentum does not imply a physical velocity inside the barrier. Rather, it reflects the exponential attenuation of the evanescent wavefunction in the classically forbidden region.

Substituting Eq. (28) into Eq. (27) yields

$$S = i\hbar\kappa L. \qquad (29)$$

The corresponding transmission probability is therefore

$$T \sim e^{-2\kappa L}, \qquad (30)$$

which is identical to the conventional tunneling result obtained from the stationary Schrödinger equation and from the WKB approximation [11,12,28,29].

This agreement is expected because the present structured barrier-medium framework reduces to the conventional tunneling model in the continuum-mode limit. The novelty of the present work therefore does not lie in modifying the exponential transmission law itself. Rather, it lies in providing a microscopic dynamical interpretation of how the effective tunneling barrier may emerge from coherent interactions between the electron and the internal modes of a structured medium.

In the present picture, the barrier is not introduced as an externally imposed static potential. Instead, the effective barrier arises from the collective response of quantized environmental modes to the presence of the electron. The conventional tunneling barrier therefore appears as an emergent continuum-limit description of the underlying particle–medium interaction.

Numerical Results

To illustrate the transmission behavior predicted by the model, numerical simulations were performed using Eq. (30). The calculations employed the free-electron mass

$$m = m_e.$$

Unless otherwise specified, the following baseline parameters were used:

$$V_0 = 1.0 \text{ eV},$$
$$L = 1.0 \text{ nm},$$
$$E = 0.5 \text{ eV}.$$

Figure 2 shows the dependence of the transmission probability on:

1. barrier width L,
2. barrier height V_0,
3. incident electron energy E.

In each case, a single parameter is varied while the remaining parameters are held fixed.

The results exhibit the expected tunneling behavior. The transmission probability decreases exponentially with increasing barrier width and barrier height, reflecting the increased attenuation of the evanescent wave inside the barrier region. Conversely, the transmission probability increases rapidly as the incident electron energy approaches the barrier height, where the attenuation constant κbecomes smaller.

These trends are fully consistent with conventional tunneling theory and confirm that the present dynamical barrier-medium model correctly reproduces the expected continuum-limit behavior. At the same time, the model provides a complementary microscopic interpretation by relating the effective tunneling barrier to coherent interactions between the electron and the quantized modes of the structured medium.

To illustrate these dependencies quantitatively, numerical simulations were performed based on the analytical expressions derived above. The transmission probability was evaluated as a function of the barrier width L, barrier height V_0, and incident electron energy E. The results are summarized in Figure 2.

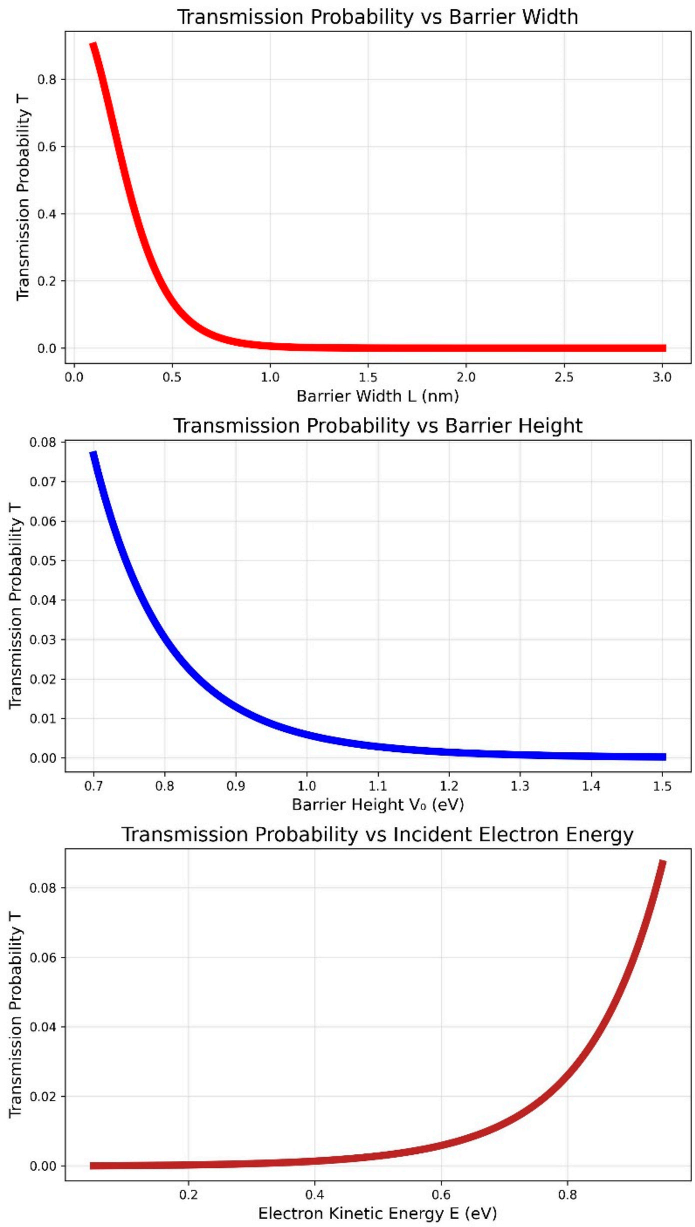

Figure 2. Dependence of transmission probability on barrier parameters. Composite plots showing the transmission probability Tas a function of (a) barrier width L, (b) barrier height V_0, and (c) incident electron

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energy E. The simulations are based on Eq. (30), $T \sim e^{-2\kappa L}$, where $\kappa = \sqrt{\frac{2m(V_0-E)}{\hbar^2}}$. Unless otherwise specified, the parameters used are $m = m_e$, $V_0 = 1.0$ eV, $L = 1.0$ nm, and $E = 0.5$ eV. In each subplot, one parameter is varied while the remaining parameters are held fixed. The results reproduce the conventional exponential tunneling behavior expected in the continuum-mode limit while providing a dynamical interpretation in terms of coherent interactions between the electron and the quantized barrier medium.

In each subplot, one parameter is varied while the remaining parameters are held fixed. The results reproduce the conventional tunneling behavior expected in the continuum-mode limit while providing a dynamical interpretation in terms of coherent interactions between the electron and the quantized modes of the structured barrier medium.

6. Dynamical Tunneling Time

One of the longstanding questions in quantum mechanics concerns the definition and interpretation of tunneling time. Various concepts have been proposed over the years, including phase time, dwell time, Wigner delay time, Büttiker–Landauer traversal time, and Larmor-clock time [13–16,30,31]. Although these approaches often agree in certain limits, they generally correspond to different operational definitions and physical interpretations.

Within the present structured barrier-medium framework, tunneling is described as a dynamical process arising from the interaction between the electron and the collective modes of the barrier medium. The Heisenberg formalism therefore provides a natural basis for defining a characteristic traversal time associated with the propagation of the electron through the effective barrier region.

The equation of motion for the electron position operator is

$$\frac{d\hat{x}}{dt} = \frac{\hat{p}}{m}, \qquad (31)$$

as obtained previously from Eq. (11).

Taking the expectation value yields

$$\frac{d\langle\hat{x}\rangle}{dt} = \frac{\langle\hat{p}\rangle}{m}. \qquad (32)$$

Equation (32) describes the average propagation of the electron through the barrier region.

In the classically forbidden region, the wavefunction exhibits exponential attenuation characterized by

$$\kappa = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}. \qquad (33)$$

The corresponding effective momentum takes the form

$$p = i\hbar\kappa. \qquad (34)$$

The appearance of an imaginary momentum should not be interpreted as a physical velocity inside the barrier. Instead, it represents the exponential attenuation scale associated with the evanescent wavefunction. The quantity κ^{-1} therefore characterizes the spatial decay length of the tunneling state rather than the trajectory of a classical particle.

Within the present dynamical framework, a characteristic traversal time may be associated with propagation across the barrier width L. Integrating Eq. (32) across the barrier region yields

$$\tau = \frac{mL}{\hbar\kappa}. \qquad (35)$$

This expression provides a characteristic timescale associated with traversal of the effective barrier.

6.1. Action-Based Interpretation

The same result may be obtained from the tunneling action.

Using Eq. (29),

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$$S = i\hbar\kappa L, \qquad (36)$$

the characteristic tunneling time can be written as

$$\tau = \left| \frac{\partial S}{\partial E} \right|, \qquad (37)$$

which gives

$$\tau = \frac{mL}{\hbar\kappa}. \qquad (38)$$

Equations (35) and (38) therefore provide equivalent interpretations of the tunneling traversal time within the present model.

6.2. Relation to Existing Tunneling-Time Definitions

The tunneling-time expression derived here should be interpreted as a traversal-time scale associated with the structured barrier-medium framework. It is not intended to represent a universally unique tunneling time applicable to all tunneling systems.

The present result is closely related to the traversal-time concept introduced by Büttiker and Landauer [14], where a characteristic time is associated with propagation through the barrier region. It is also related to semiclassical action-based descriptions of tunneling dynamics.

By contrast, Wigner delay time [15] is defined through the energy derivative of the scattering phase shift, while Smith time delay [16] is formulated in terms of the scattering lifetime matrix. Larmor-clock approaches employ spin precession in weak magnetic fields to infer traversal times [30,31].

The present formulation differs from these approaches in that the tunneling time emerges directly from the dynamical evolution of the electron interacting with a structured barrier medium. Thus, the traversal time is obtained within the same framework that generates the effective tunneling barrier itself.

Numerical Results

Figure 3 shows the tunneling traversal time calculated from Eq. (35) for different barrier widths, barrier heights, and incident electron energies. Unless otherwise specified, the simulations employ

$$V_0 = 1.0\ \text{eV}, L = 1.0\ \text{nm}, E = 0.5\ \text{eV}. \qquad (39)$$

The tunneling time increases approximately linearly with increasing barrier width L. Physically, a wider barrier requires the electron to traverse a larger evanescent region, resulting in a larger accumulated action and a correspondingly longer traversal time.

The tunneling time also increases with increasing barrier height V_0. As the barrier height increases, the attenuation constant κincreases, leading to stronger suppression of the tunneling wave and a larger traversal timescale.

Conversely, the tunneling time decreases as the incident electron energy approaches the barrier height. In this regime, the attenuation constant becomes smaller, reducing the effective barrier strength and allowing more rapid traversal through the barrier region.

These trends are consistent with the analytical expression of Eq. (35) and provide a physically transparent interpretation of tunneling dynamics within the structured barrier-medium framework.

For sufficiently wide barriers, additional effects beyond the present mean-field approximation may become important. Nevertheless, within the range of parameters considered here, the approximately linear dependence on barrier width is a natural consequence of the traversal-time expression derived above.

Figure 3 shows that the tunneling time increases approximately linearly with increasing barrier width. Physically, a wider barrier requires the electron to traverse a larger evanescent region, leading to greater accumulated tunneling and a correspondingly longer traversal time.

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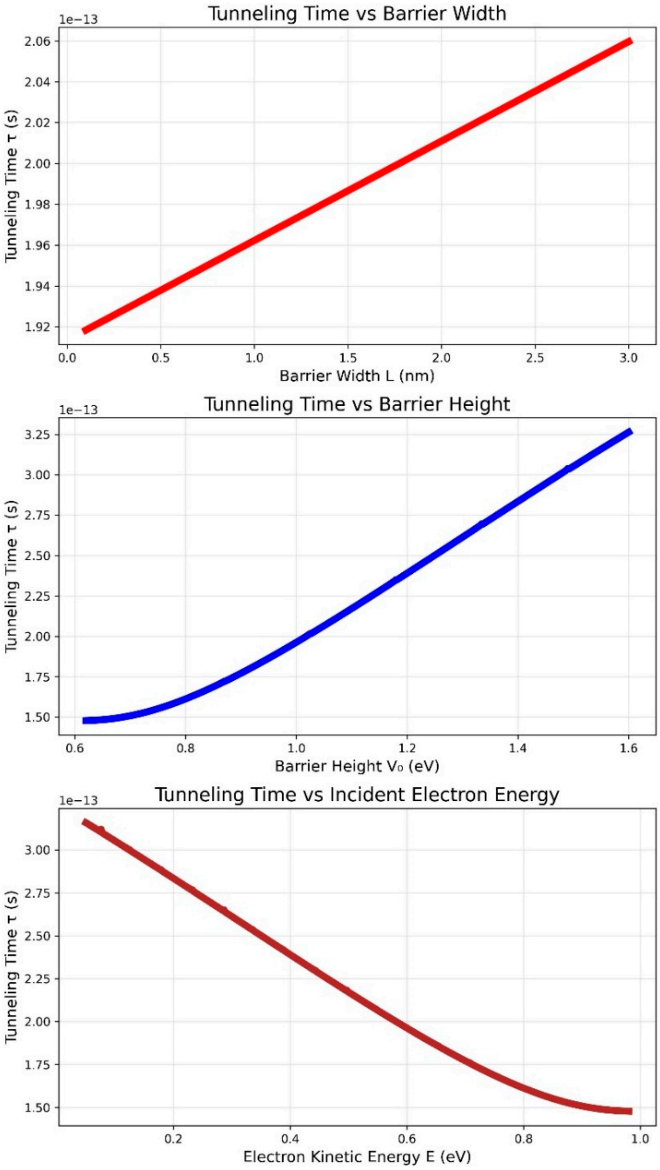

Figure 3. Dependence of tunneling time on barrier parameters. Composite plots showing the tunneling traversal time τas a function of (a) barrier width L, (b) barrier height V_0, and (c) incident electron energy E. The tunneling time is calculated using Eq. (35), $\tau = \frac{mL}{\hbar\kappa}$, where $\kappa = \sqrt{\frac{2m(V_0-E)}{\hbar^2}}$. Unless otherwise specified, the parameters used are $m = m_e$, $V_0 = 1.0\ \text{eV}$, $L = 1.0\ \text{nm}$, and $E = 0.5\ \text{eV}$. In each subplot, one parameter is varied while the remaining parameters are held fixed. The results show that the tunneling time increases with barrier width and barrier height, while decreasing as the incident electron energy approaches the barrier height.

The results show that the tunneling time increases with barrier width and barrier height while decreasing as the incident electron energy approaches the barrier height. The figure illustrates the traversal-time interpretation arising from the dynamical structured barrier-medium framework.

7. Discussion

The present work introduces a dynamical formulation of quantum tunneling in which the barrier is modeled as a structured medium possessing quantized internal modes that interact coherently with an incident electron. Under the continuum-mode and mean-field approximations, the collective response of the barrier medium generates an effective potential barrier that reproduces

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the conventional stationary tunneling problem. The resulting framework therefore remains fully consistent with standard quantum mechanics while providing a complementary microscopic interpretation of the tunneling process.

A central feature of the present model is the interpretation of the barrier as an emergent effective structure arising from coherent particle–medium interactions rather than as an externally imposed static potential. By explicitly incorporating internal degrees of freedom of the barrier medium, the model establishes a direct connection between tunneling phenomena and particle–environment interactions commonly encountered in condensed-matter systems, molecular junctions, photonic structures, and open quantum systems [20–24].

The present formulation therefore provides a microscopic interpretation of the barrier potential as arising from coherent particle–medium interactions rather than solely from an externally prescribed potential profile. In this sense, the static barrier appearing in the Schrödinger equation may be viewed as an effective continuum-limit description of underlying interactions within a structured medium.

An important result of the present analysis is the recovery of the conventional transmission probability, $T \sim e^{-2\kappa L}$, in the continuum-mode limit. This agreement is expected because the Schrödinger and Heisenberg formulations of quantum mechanics are formally equivalent [26,27]. Consequently, the novelty of the present work does not lie in modifying the standard exponential tunneling law. Rather, the novelty lies in providing a dynamical microscopic interpretation for the emergence of the barrier and the associated tunneling process.

Table 1. Comparison Between Conventional Tunneling Theory and the Present Structured Barrier-Medium Framework.

Aspect	**Conventional Schrödinger Tunneling**	**Present Structured Barrier-Medium Framework**
Barrier description	Externally prescribed static potential	Emergent effective potential generated by coherent particle–medium interactions
Mathematical formulation	Stationary Schrödinger equation	Heisenberg operator dynamics with quantized barrier modes
Barrier degrees of freedom	Not explicitly included	Internal quantized modes explicitly included
Microscopic origin of barrier	Assumed phenomenologically	Arises from collective response of the barrier medium
Transmission probability	Obtained from wave scattering	Recovered in the continuum-mode limit
Tunneling-time interpretation	Multiple operational definitions	Traversal-time interpretation within the dynamical framework

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Aspect	**Conventional Schrödinger Tunneling**	**Present Structured Barrier-Medium Framework**
Environmental interaction	Usually neglected	Explicit coherent particle–medium coupling
Relation to conventional theory	Standard formulation	Reduces conventional tunneling in the continuum-mode limit
Main emphasis	Wave propagation through a static barrier	Dynamical tunneling in structured media

7.1. Relation to Existing Tunneling-Time Studies

The tunneling-time problem has been discussed extensively over many decades and remains one of the most actively debated aspects of quantum tunneling [13–16]. Various approaches have been developed, including Wigner delay time, dwell time, Büttiker–Landauer traversal time, and Larmor-clock methods [14–16,30,31].

More recently, advances in weak-measurement techniques and attosecond spectroscopy have enabled increasingly direct investigations of tunneling-time phenomena [17–19]. Experiments performed by Steinberg and co-workers, as well as more recent attosecond measurements, have provided valuable insight into traversal-time concepts and quantum transport through classically forbidden regions.

The present work should not be viewed as a replacement for these established approaches. Instead, it provides a complementary framework in which the effective barrier and the traversal time emerge from the same underlying dynamical interaction between the electron and the structured barrier medium. In this sense, the model offers a microscopic interpretation of tunneling dynamics while remaining consistent with conventional tunneling theory in the continuum-mode limit.

7.2. Experimental Relevance

The present framework may be relevant to a variety of physical systems in which the tunneling barrier possesses internal collective degrees of freedom. Possible examples include semiconductor heterostructures [4,5], superconducting tunnel junctions [7], molecular junctions [32], photonic lattices and optical barriers [33], and engineered quantum systems with controllable environmental coupling.

In particular, photonic barriers and optical lattice systems may provide attractive platforms for exploring the concepts developed here because the internal mode structure of such systems can often be manipulated experimentally. Similarly, semiconductor nanostructures and molecular junctions offer opportunities to investigate the influence of collective environmental modes on tunneling transport.

One possible experimental test of the present model would be to examine whether modifications of the barrier-mode spectrum lead to measurable changes in tunneling-time behavior while preserving the overall transmission characteristics. Such effects would be difficult to describe within a purely static-potential framework but arise naturally within the structured barrier-medium interpretation developed here.

7.3. Limitations of the Present Model

Several limitations of the present formulation should be emphasized.

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First, the present analysis employs a mean-field approximation in which fluctuations and higher-order correlations of the barrier modes are neglected. The effective barrier is therefore generated by the collective average response of the medium rather than by its full quantum dynamics.

Second, the barrier modes are assumed to occupy coherent or displaced stationary background states. A more complete treatment would include time-dependent barrier states, nonequilibrium effects, and dynamical back-action between the electron and the medium.

Third, the continuum-mode approximation assumes a sufficiently dense spectrum of barrier modes and slowly varying collective response during the tunneling process. Outside this regime, additional corrections may become important.

Consequently, the present formulation should be regarded as an effective dynamical model intended to illustrate how coherent particle–medium interactions can generate tunneling-like behavior in structured environments. It is not intended to replace the general theory of quantum tunneling, but rather to provide a complementary microscopic perspective relevant to systems possessing internal environmental degrees of freedom.

8. Conclusion

In this work, we have presented a dynamical formulation of quantum tunneling in which the barrier is modeled as a structured medium possessing quantized internal modes that interact coherently with an incident electron. Using the Heisenberg operator formalism and a second-quantized description of the barrier medium, we derived coupled dynamical equations governing the electron–barrier interaction and investigated the resulting tunneling behavior.

Under the continuum-mode and mean-field approximations, the collective response of the barrier modes generates an effective potential barrier that reproduces the conventional rectangular tunneling model and the standard transmission probability obtained from the stationary Schrödinger equation. In this sense, the present formulation provides a microscopic interpretation of the effective tunneling barrier as an emergent consequence of coherent particle–medium interactions within a structured environment.

The analysis further shows that a characteristic tunneling traversal time arises naturally from the dynamical evolution of the electron through the barrier region. The resulting tunneling-time expression depends explicitly on the barrier width, barrier height, and incident electron energy, and exhibits the expected qualitative behavior associated with tunneling through an effective potential barrier. Numerical simulations illustrating both the transmission probability and tunneling-time dependence on barrier parameters were presented and discussed.

An important feature of the present framework is that it recovers the conventional tunneling transmission probability in the continuum-mode limit while simultaneously providing a microscopic interpretation of the origin of the barrier. The novelty of the approach therefore lies not in modifying the established exponential tunneling law, but in offering a dynamical barrier-medium perspective in which the effective potential emerges from coherent interactions between the electron and the internal modes of a structured medium.

The present formulation should be regarded as a complementary dynamical framework rather than a replacement for conventional quantum tunneling theory. Its primary value lies in providing an alternative microscopic viewpoint for understanding tunneling processes in systems where the barrier possesses internal environmental degrees of freedom. Possible areas of relevance include semiconductor heterostructures, molecular junctions, superconducting tunnel devices, photonic barriers, optical lattices, and engineered quantum systems with controllable collective modes.

The theory also suggests a possible experimental direction for future investigation. In conventional tunneling models, the barrier is treated as a fixed external potential whose internal structure plays no explicit role. In contrast, the present framework predicts that modifications of the internal mode structure of the barrier medium may influence tunneling dynamics through changes in the collective response of the medium. Experimental platforms that permit controlled

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manipulation of phonon spectra, optical modes, cavity fields, or engineered environmental couplings may therefore provide opportunities to test the structured barrier-medium interpretation proposed here.

Several limitations of the current model should also be acknowledged. The present treatment employs a mean-field approximation in which fluctuations and higher-order correlations of the barrier modes are neglected, and the barrier medium is assumed to remain in a stationary coherent background state during the tunneling process. Future work may extend the framework to include fluctuation effects, nonequilibrium dynamics, dissipation, multimode correlations, and fully time-dependent barrier responses.

In summary, the present study suggests that quantum tunneling in structured environments may be viewed not only as wave propagation through an externally imposed static potential but also as an emergent dynamical process arising from coherent interactions between quantum particles and collective modes of the surrounding medium. Further theoretical and experimental investigations may help clarify the role of such interactions in quantum transport, nanoscale devices, and photonic tunneling systems.

Author Contributions: J. T. initiated the project and conceived the model. J.T., L. Z., and J-H P. discussed the work and wrote the manuscript.

Data Availability Statement: This work contains theoretical derivations with no experiments. The data is available upon reasonable request.

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Appendix A. Derivation of the Heisenberg Equations of Motion

This appendix provides additional details regarding the derivation of the coupled dynamical equations used in the main text.

Starting from the total Hamiltonian

$$\hat{H} = \frac{\hat{p}^2}{2m} + \sum_q \hbar\,\omega_q \hat{a}_q^\dagger \hat{a}_q + \sum_q g_q\,(\hat{x})(\hat{a}_q + \hat{a}_q^\dagger), \qquad \text{(A1)}$$

the Heisenberg equation of motion for an arbitrary operator $\hat{O}$ is

$$\frac{d\hat{O}}{dt} = \frac{i}{\hbar}[\hat{H}, \hat{O}]. \qquad \text{(A2)}$$

Applying Eq. (A2) to the position operator $\hat{x}$ gives

$$\frac{d\hat{x}}{dt} = \frac{i}{\hbar}\left[\frac{\hat{p}^2}{2m}, \hat{x}\right]. \qquad \text{(A3)}$$

Using the canonical commutation relation

$$[\hat{x}, \hat{p}] = i\hbar, \qquad \text{(A4)}$$

one obtains

$$\frac{d\hat{x}}{dt} = \frac{\hat{p}}{m}, \qquad \text{(A5)}$$

which corresponds to Eq. (11) in the main text.

For the momentum operator,

$$\frac{d\hat{p}}{dt} = \frac{i}{\hbar}[\hat{H}, \hat{p}]. \qquad \text{(A6)}$$

Since the kinetic-energy term commutes with $\hat{p}$, only the interaction Hamiltonian contributes,

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$$\frac{d\hat{p}}{dt} = -\sum_q \frac{\partial g_q(\hat{x})}{\partial x}(\hat{a}_q + \hat{a}_q^\dagger). \qquad \text{(A7)}$$

This expression corresponds to Eq. (13) in the main text and shows that the electron experiences an effective force generated by the spatial variation of the particle–medium coupling.

For the barrier-mode annihilation operator,

$$\frac{d\hat{a}_q}{dt} = \frac{i}{\hbar}[\hat{H}, \hat{a}_q], \qquad \text{(A8)}$$

which yields

$$\frac{d\hat{a}_q}{dt} = -i\omega_q \hat{a}_q - \frac{i}{\hbar} g_q(\hat{x}). \qquad \text{(A9)}$$

The coupled equations (A5), (A7), and (A9) constitute the microscopic dynamical basis of the structured barrier-medium model developed in this work.

Appendix B. Numerical Evaluation of Transmission Probability and Tunneling Time

This appendix summarizes the numerical procedures used to generate Figures 2 and 3.

The transmission probability was calculated using

$$T = \exp(-2\kappa L), \qquad \text{(B1)}$$

where

$$\kappa = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}. \qquad \text{(B2)}$$

The tunneling traversal time was evaluated from

$$\tau = \frac{mL}{\hbar\kappa}. \qquad \text{(B3)}$$

The calculations employed the free-electron mass

$$m = 9.109 \times 10^{-31}\ \text{kg}, \qquad \text{(B4)}$$

and Planck's reduced constant

$$\hbar = 1.055 \times 10^{-34}\ \text{J s}. \qquad \text{(B5)}$$

Unless otherwise specified, the baseline simulation parameters were

$$V_0 = 1.0\ \text{eV}, \qquad \text{(B6)}$$
$$L = 1.0\ \text{nm}, \qquad \text{(B7)}$$

and

$$E = 0.5\ \text{eV}. \qquad \text{(B8)}$$

For Figure 2, one parameter was varied while the remaining parameters were held fixed:

- Barrier width:

$$0.2 \le L \le 3.0\ \text{nm}$$

- Barrier height:

$$0.6 \le V_0 \le 2.0\ \text{eV}$$

- Incident electron energy:

$$0.1 \le E \le 0.95V_0$$

Similarly, Figure 3 was generated using Eq. (B3) over the same parameter ranges.

The numerical results are intended primarily to illustrate the qualitative dependence of transmission probability and tunneling traversal time on barrier parameters within the continuum-

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mode approximation. They are not intended to represent a detailed simulation of any specific experimental system.

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