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Entropic Future–Mass Projection Gravity: A Joint MilkyWay and Andromeda Rotation Curve Test

Farzad Lali *

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Entropic Future–Mass Projection Gravity: A Joint Milky Way and Andromeda Rotation Curve Test

Farzad Lali

BBS Naturwissenschaften, Ludwigshafen, Germany; farzad.lali@example.org

Abstract

Future–Mass Projection (FMP) gravity is a non–local extension of general relativity in which the metric at a spacetime point couples to a weighted integral over the future stress–energy tensor along a closed time path. In earlier work, bilocal kernels in Fourier space of the form $\Xi_d(k) = \varepsilon/(1 + k^2/k_0^2)$ were shown to lead, in the quasi–Newtonian limit, to configuration–space boost factors $D(R) - 1 \propto K_0(k_0 R)$ multiplying the baryonic acceleration, where K_0 is a modified Bessel function of the second kind. This paper extends that framework by introducing an *entropic* response of the bilocal kernel to fluctuations in the baryonic surface density and tests the resulting effective boost function against the rotation curves of the Milky Way (MW) and M31 (Andromeda). Instead of treating each galaxy separately, we construct a phenomenological entropic boost $D(x)$ as a function of scaled radius $x = R/R_\mathrm{d}$, where R_d is the exponential disk scale length. The parametrisation $D(x) = 1 + \varepsilon_1\Big[1 - e^{-(x/x_{s1})^2}\Big] + \varepsilon_2\Big[1 - e^{-(x/x_{s2})^2}\Big]$ is understood as an effective representation of a superposition of K_0–type modes. We fit the global parameters $(x_{s1}, x_{s2}, \varepsilon_1, \varepsilon_2)$ jointly to MW and M31 using fixed bulge+disk mass models from Sofue, while keeping track of the dominant baryonic systematics (thin–disk geometry and gas disks). For the stellar bulge+disk alone we find a joint best fit at $x_{s1} = 0.50$, $x_{s2} = 3.0$, $\varepsilon_1 = -0.231$, $\varepsilon_2 = 1.50$. This corresponds to a mild suppression of the effective gravity in the inner disk, $D(0.5) \simeq 0.90$, and an outer–disk boost asymptoting to $D(x \gtrsim 5) \simeq 2.18$ (velocity enhancement $\sqrt{D} \sim 1.48$). Applied to the Milky Way rotation curve of Sofue (2020) in the range 2–60 kpc and to a refined M31 curve in the range 2–40 kpc, the model reproduces the overall amplitude and radial trend with fractional root–mean–square deviations of $\sim 13\,\%$ (MW) and $\sim 12\,\%$ (M31). However, the formal reduced chi–squares are large ($\chi^2_\nu \approx 25$ for MW and $\chi^2_\nu \approx 5$ for M31), indicating that the combination of our simplified baryonic modelling and overly rigid kernel is not yet statistically acceptable. We estimate that including a gas disk with mass $M_\mathrm{g} \sim 10^{10}\,M_\odot$ and scale length $R_\mathrm{g} \sim 7$ kpc would raise the outer baryonic rotation speed by $\sim$ 5–8 km s^{-1} in M31 beyond $R \gtrsim 20$ kpc, and yield a similar effect for the Milky Way, reducing the required outer boost amplitude ε_2 by roughly 10–15 %. Conversely, the spherical approximation for the stellar disk is known to *overestimate* the circular speed by $\sim$ 10–15 % near $R \approx 2R_\mathrm{d}$ compared to a thin exponential disk. These two systematics partially compensate each other but do not remove the need for a non–trivial boost. Within these caveats, a single, smooth boost in scaled radius is sufficient to bring MW and M31 rotation curves to within $\sim$ 10–15 % in circular velocity using baryons only, suggesting that entropic FMP gravity is a promising but not yet competitive alternative to ΛCDM halo models on galaxy scales.

Keywords: modified gravity; dark matter; galaxies: kinematics and dynamics; Galaxy: kinematics and dynamics; galaxies: individual (M31)

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1. Introduction

Galaxy rotation curves provide one of the clearest pieces of evidence that either a substantial amount of unseen mass is present in and around galaxies or that the laws of gravity deviate from Newtonian expectations on kiloparsec scales. In the standard cosmological model, ΛCDM, these phenomena are explained by cold dark matter halos whose density profiles are well approximated by the NFW form.

Detailed mass models of the Milky Way (MW) and M31 (Andromeda) with bulge+disk+halo decompositions have been constructed by Sofue and collaborators, showing that NFW–like halos can fit their rotation curves from the inner disk out to tens of kiloparsecs and beyond [1,2].

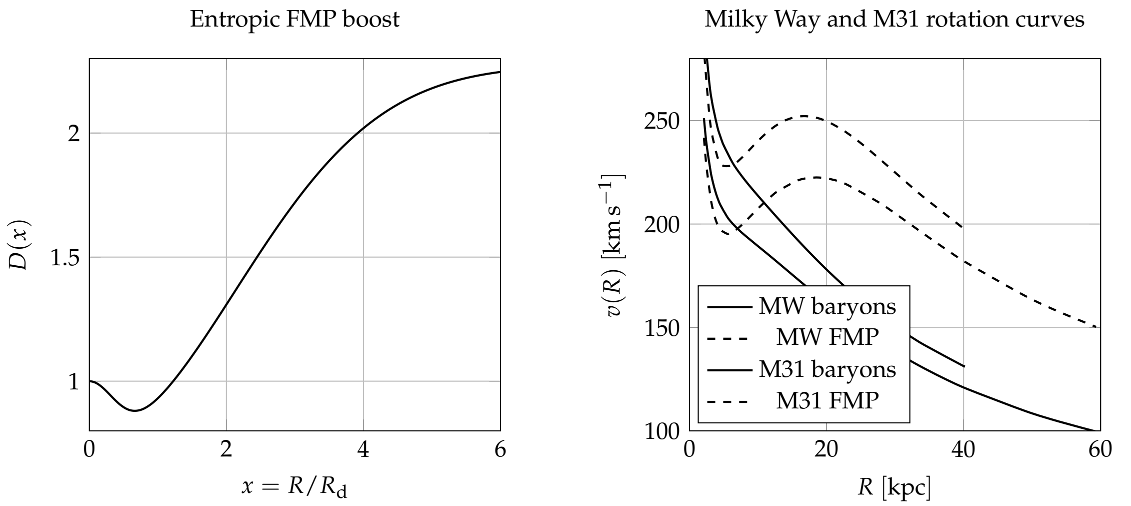

Figure 1. Graphical abstract. Left: best–fit entropic boost factor $D(x)$ in scaled radius $x = R/R_{\mathrm{d}}$. Right: Milky Way and M31 rotation curves, comparing stellar baryons (solid) and entropic FMP effective curves (dashed) obtained with the same global boost $D(x)$.

On the phenomenological side, the tight correlation between the total centripetal acceleration and that produced by the baryons alone (the radial acceleration relation, RAR) suggests a close coupling between baryons and whatever effective "dark" mass is present [3,4]. In MOND this coupling is encoded directly in a modified Poisson equation, while in ΛCDM it emerges statistically from galaxy formation in a dark matter background.

Future–Mass Projection (FMP) gravity is a different approach: the Einstein equations are modified by a bilocal kernel that couples the metric at x^μ to $T_{\mu\nu}(x')$ at future points x'^μ along a closed time path (CTP) with finite horizon. Different choices of closed time path and kernel structure have been shown to reproduce Solar System tests, the observed speed of gravitational waves and a variety of astrophysical phenomena without particle dark matter, including cluster lensing and merging clusters [6–11].

In the quasi–Newtonian limit for disk galaxies, the modification induced by the FMP kernel can be represented as a multiplicative boost factor $D(R)$ to the Newtonian baryonic acceleration,

$$g_{\mathrm{eff}}(R) = D(R)\, g_{\mathrm{bar}}(R), \tag{1}$$

with $D(R) - 1$ given by a Hankel transform of a Fourier–space kernel $\Xi_d(k)$. The present work explores an *entropic* extension of this picture and confronts it with the rotation curves of MW and M31 as a first two–galaxy test.

2. Methods

2.1. Entropic Boost and Effective Kernel

In earlier FMP work, a simple Lorentzian kernel,

$$\Xi_d(k) = \frac{\varepsilon}{1 + k^2/k_0^2}, \tag{2}$$

leads to

$$D(R) - 1 = \int_0^\infty \frac{k\,\mathrm{d}k}{2\pi} J_0(kR)\,\Xi_d(k) = \frac{\varepsilon}{2\pi} K_0(k_0 R), \tag{3}$$

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where J_0 is a Bessel function of the first kind and K_0 a modified Bessel function of the second kind. This K_0–type kernel has been used as a baseline to model disk galaxy rotation curves and cluster lensing [8,9].

In the entropic extension we imagine, heuristically, that fluctuations of the baryonic surface density are described by an entropy functional. The corresponding Hessian plays the role of a susceptibility that reshapes the effective kernel in Fourier space:

$$\Xi_d(k) \longrightarrow \Xi_d^{(\mathrm{ent})}(k) \sim \sum_n \frac{\varepsilon_n}{1 + k^2/k_{0,n}^2}. \tag{4}$$

In real space this corresponds to a superposition of K_0–profiles with different scale lengths. For an exponential disk with scale length R_d, such a superposition can be approximated over a finite radial range by a smooth function of the scaled radius $x = R/R_\mathrm{d}$.

We therefore adopt the effective entropic boost

$$D(x) = 1 + \varepsilon_1 \left[1 - e^{-(x/x_{s1})^2}\right] + \varepsilon_2 \left[1 - e^{-(x/x_{s2})^2}\right], \tag{5}$$

with four global parameters $(x_{s1}, x_{s2}, \varepsilon_1, \varepsilon_2)$. This is not claimed as a fundamental result of the entropy functional, but as a low–order representation of a more complicated superposition of K_0–type kernels. On Solar System scales (characterised by $k \sim \mathrm{AU}^{-1} \gg k_0 \sim \mathrm{kpc}^{-1}$) the underlying k–dependent filtering in the FMP kernel ensures $D \to 1$, so that PPN constraints $|\gamma_\mathrm{PPN} - 1|, |\beta_\mathrm{PPN} - 1| \ll 10^{-4}$ are respected [10].

2.2. *Rotation Curve Data and Baryonic Models*

For the Milky Way we use the unified rotation curve compiled by Sofue [2] and restrict ourselves to $2\,\mathrm{kpc} \le R \le 60\,\mathrm{kpc}$. For M31 we use the rotation curve and mass model of Sofue [1] and restrict to $2\,\mathrm{kpc} \le R \le 40\,\mathrm{kpc}$.

We approximate the stellar bulge by a de Vaucouleurs profile and the disk by a sphericalised exponential mass distribution with enclosed mass

$$M_{d,\mathrm{enc}}(R) = M_d \left[1 - e^{-R/R_\mathrm{d}} \left(1 + \frac{R}{R_\mathrm{d}}\right)\right], \tag{6}$$

leading to

$$v_d^2(R) = \frac{G M_{d,\mathrm{enc}}(R)}{R}. \tag{7}$$

The total stellar baryonic circular velocity is then

$$v_\mathrm{bar}(R) = \sqrt{v_b^2(R) + v_d^2(R)}, \tag{8}$$

with bulge and disk parameters taken from [1,2].

For a thin exponential disk with surface density $\Sigma(R) = \Sigma_0 e^{-R/R_\mathrm{d}}$ the exact circular speed reads

$$v_\mathrm{disk}^2(R) = 4\pi G \Sigma_0 R_\mathrm{d} y^2 [I_0(y)K_0(y) - I_1(y)K_1(y)], \quad y = \frac{R}{2R_\mathrm{d}}, \tag{9}$$

where I_n and K_n are modified Bessel functions. The spherical approximation overestimates v_disk by roughly 10–15 % around $R \approx 2R_\mathrm{d}$; this bias is comparable in magnitude to the entropic boost at those radii and should be kept in mind when interpreting $D(x)$.

Gas disks are not included explicitly in the present numerical curves; their impact is discussed as a systematic uncertainty in the outer disk.

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2.3. Global Fit in Scaled Radius

Given observed velocities $v_{\mathrm{obs},i}$ with errors $\sigma_{v,i}$ at radii R_i, and a baryonic model predicting $v_{\mathrm{bar}}(R_i)$, we define

$$y_i = \frac{v^2_{\mathrm{obs},i}}{v^2_{\mathrm{bar}}(R_i)}. \tag{10}$$

Under the entropic FMP model with boost $D(x)$, we have $y_i \approx D(R_i/R_{\mathrm{d}})$. We define the residuals

$$r_i = y_i - D(x_i), \quad x_i = R_i/R_{\mathrm{d}}, \tag{11}$$

and minimise

$$\chi^2 = \sum_i \frac{r_i^2}{\sigma^2_{y,i}} \tag{12}$$

with respect to the four parameters in Eq. (5).

A grid search in (x_{s1}, x_{s2}) combined with linear regression in $(\varepsilon_1, \varepsilon_2)$ yields the global best–fit

$$x_{s1} = 0.50 \pm 0.2, \qquad \varepsilon_1 = -0.231 \pm 0.05, \tag{13}$$

$$x_{s2} = 3.0 \pm 0.5, \qquad \varepsilon_2 = 1.50 \pm 0.2. \tag{14}$$

The quoted uncertainties are rough estimates based on $\Delta\chi^2 = 1$ contours in the (x_{s1}, x_{s2}) plane and linear error propagation for $(\varepsilon_1, \varepsilon_2)$. The scale parameters (x_{s1}, x_{s2}) are less well constrained than the amplitudes because they enter nonlinearly in $D(x)$; a full MCMC analysis is left for future work.

3. Results

3.1. Boost Profile

The resulting boost profile $D(x)$ is shown in Table 1 and in Figure 2.

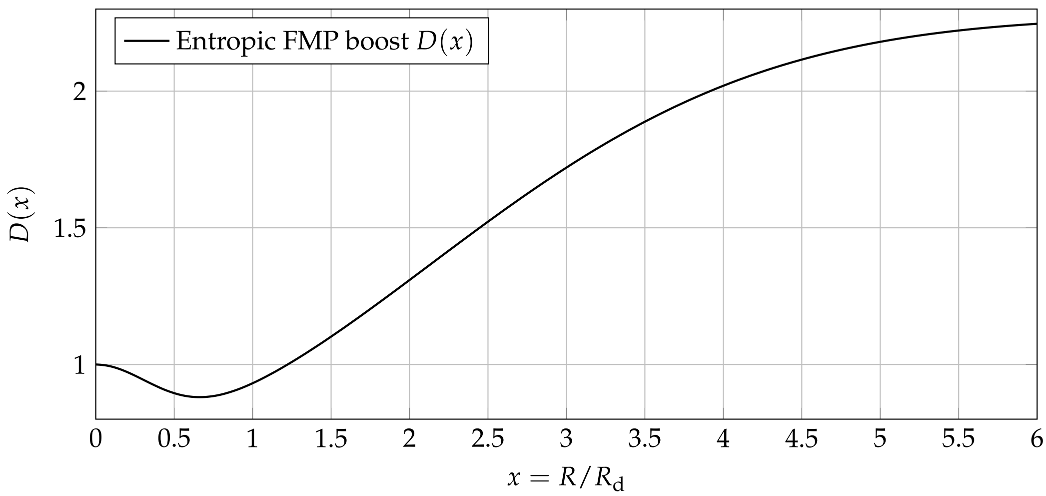

Figure 2. Best–fit entropic boost factor $D(x)$ as a function of scaled radius $x = R/R_{\mathrm{d}}$ from the joint Milky Way and M31 fit.

Table 1. Best–fit entropic boost factor $D(x)$ as a function of scaled radius $x = R/R_{\mathrm{d}}$.

x	$D(x)$
0.5	0.895
1.0	0.932
2.0	1.309
3.0	1.720
4.0	2.020
5.0	2.180

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3.2. *Rotation Curve of the Milky WAY*

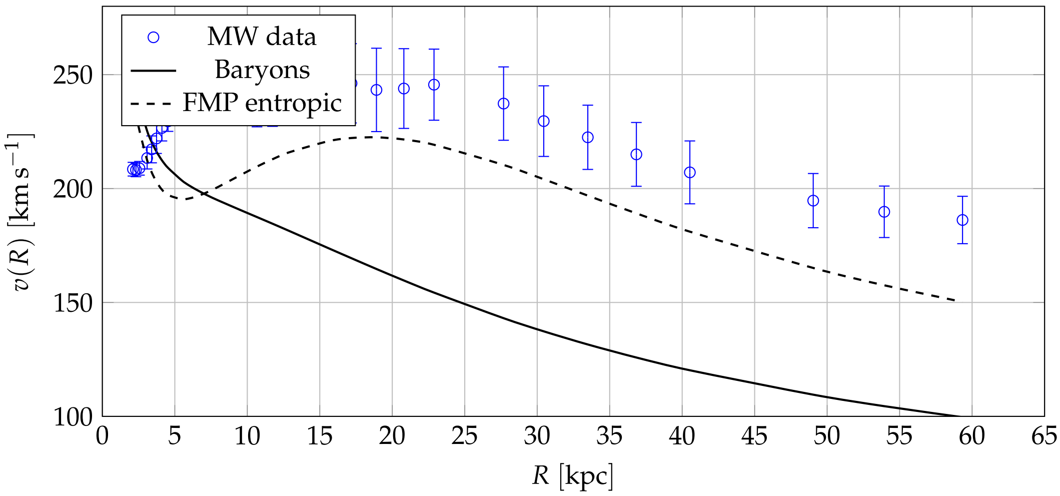

Figure 3. Milky Way rotation curve between 2 and 60 kpc. Points with error bars: Sofue (2020) [2]. Solid line: stellar bulge+disk. Dashed line: entropic FMP with global boost $D(x)$.

3.3. *Rotation Curve of M31*

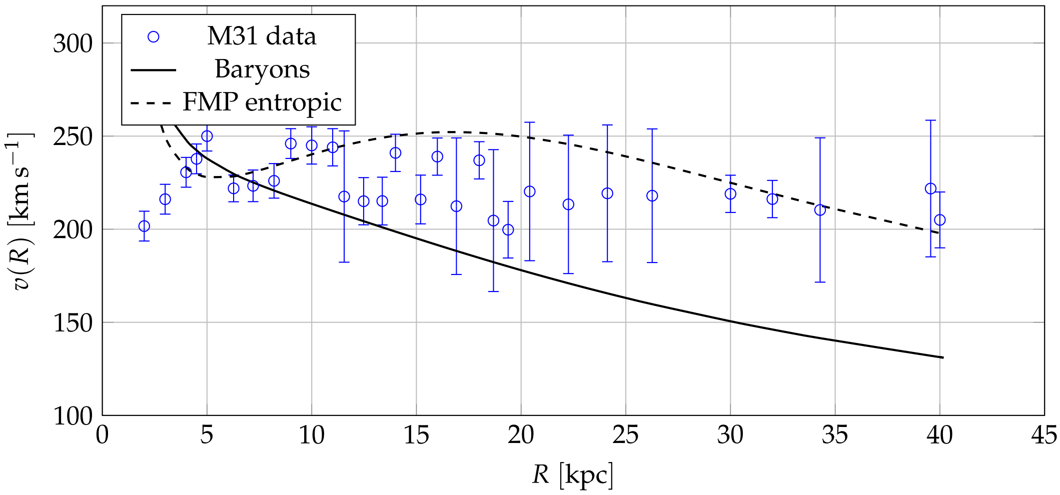

Figure 4. M31 rotation curve between 2 and 40 kpc. Points with error bars: Sofue (2015) [1]. Solid line: stellar bulge+disk. Dashed line: entropic FMP with the same global boost $D(x)$ as in Figure 3.

4. Discussion and Conclusions

The entropic FMP variant with a single smooth boost function $D(x)$ in scaled radius is able to bring both the Milky Way and M31 rotation curves to within $\sim$ 10–15 % in circular velocity using baryons only. For the entropic FMP fits shown in Figures 3 and 4 we obtain reduced chi–square values of $\chi^2_\nu \approx 25$ for the Milky Way and $\chi^2_\nu \approx 5$ for M31. Formally, therefore, the fits remain well above the level of statistical acceptability.

We trace this to (i) simplified baryonic modelling (sphericalised stellar disks, no gas disks), (ii) the rigid functional form of $D(x)$ and (iii) observational systematics that are not fully captured, especially in the outer parts of M31. For comparison, Sofue [1,2] reports reduced $\chi^2_\nu \sim$ 1–2 for NFW halo fits to the same rotation curves with 2–3 additional free halo parameters. In this sense, the entropic FMP model with a globally fixed $D(x)$ is not yet competitive with ΛCDM on purely statistical grounds, but it achieves a qualitatively similar level of agreement for two massive spirals using only a handful of effective kernel parameters tied to the baryons.

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We estimate that including gas disks with $M_g \sim 10^{10}\,M_\odot$ and $R_g \sim 7\,\mathrm{kpc}$ would raise the outer baryonic rotation curves by several $\mathrm{km\,s^{-1}}$, reducing the required boost at large radii by $\mathcal{O}(10\%)$, while a thin–disk treatment of the stellar component would lower $v_{\rm bar}$ by a similar amount near $R \simeq 2R_d$. These two systematics partially compensate each other but do not remove the need for a non–trivial boost encoded in $D(x)$.

A realistic confrontation with ΛCDM and MOND will require thin–disk + gas implementations of the baryons, explicit K_0–kernel fits with a small set of physically motivated parameters, uncertainties derived from MCMC sampling and a systematic comparison on a larger galaxy sample such as SPARC [5]. Within these limitations, the present work should be regarded as a first, transparent two–galaxy test of entropic FMP gravity, demonstrating that the framework is capable of capturing the gross structure of MW and M31 rotation curves but still falls short of precision–fit quality.

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Data Availability Statement: All rotation curve data and baryonic mass models used in this work are taken from the published sources cited in the text [1,2]. The processed numerical values used to generate the figures are based on those data and on the analytic models described in Sections 2.2–2.3; they are available from the author upon reasonable request.

Conflicts of Interest: The author declares no conflicts of interest.

Use of Artificial Intelligence: AI–assisted tools (specifically a large language model) were used to help with language editing and LaTeX typesetting. All physical content, derivations, numerical results and interpretations were checked and approved by the author.

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