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Article

# Unifying Penrose Process and Blandford–Znajek Mechanism Using Negative Phase Velocity

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## Abstract

The extraction of rotational energy from a Kerr black hole admits several celebrated descriptions, including the Penrose process (Penrose (1969)), wave superradiance (Zel'dovich (1971), Misner (1972), Teukolsky, Press (1974); see also Thorne, Price, Wheeler (1986)), and the Blandford–Znajek mechanism (Blandford, Znajek (1977)). Traditionally treated as distinct, these processes are shown here to be unified through a single criterion: the existence of negative Killing energy flux across the horizon, locally manifested as *negative phase velocity* (NPV) in the ergosphere (Setiawan, Mackay, Lakhtakia (2005)). We demonstrate that (i) the Penrose process corresponds to particle trajectories with  $E_\chi < 0$ , (ii) superradiance corresponds to wave modes with  $\omega < m\Omega_H$ , and (iii) the Blandford–Znajek mechanism corresponds to electromagnetic field lines with  $\Omega_F < \Omega_H$ . In all cases, the horizon absorbs negative energy while positive energy escapes to infinity. By expressing these mechanisms through the NPV condition  $\mathbf{S} \cdot \mathbf{k} < 0$ , we present a unified theoretical framework for black hole energy extraction.

**Keywords:** negative phase velocity (NPV); penrose process; Blandford–Znajek mechanism; superradiance; Kerr black hole/ergosphere; negative killing energy; energy extraction from black holes; Poynting flux/electromagnetic jets; frame-dragging/horizon physics; GRMHD diagnostics/numerical simulations; wave amplification/mode scattering

## 1. Introduction

Rotating black holes provide one of the most remarkable natural laboratories for relativistic energy transfer. The ergosphere—the spacetime region outside the horizon where the timelike Killing vector becomes spacelike—permits negative-energy states relative to infinity. This peculiar feature allows rotational energy to be tapped, an idea pioneered by Penrose (1969).

Over the decades, three paradigms have emerged:

- **Penrose process:** a test particle entering the ergosphere can split into fragments, one of which falls into the black hole with negative Killing energy, allowing the other to escape with enhanced energy.
- **Superradiance:** classical or quantum fields scattered by a rotating black hole can emerge amplified if their frequency satisfies  $\omega < m\Omega_H$ .
- **Blandford–Znajek mechanism:** magnetic field lines threading the horizon, coupled to a force-free magnetosphere, can extract rotational energy and drive astrophysical jets.

Although derived in different contexts—particles, waves, and electromagnetic fields—these processes share the same root: *the absorption of negative energy by the horizon*. This work demonstrates that a local diagnostic, *negative phase velocity* (NPV), provides a unifying description. In all channels, energy extraction occurs precisely when the phase velocity points opposite to the energy flux, i.e.,  $\mathbf{S} \cdot \mathbf{k} < 0$ .

### 1.1. Relation to Previous Equivalence Results for the Penrose and Blandford–Znajek Processes

A substantial literature has established that the Blandford–Znajek (BZ) mechanism can be interpreted as an electromagnetic generalization of the Penrose process. Komissarov (2009) showed

that electromagnetic extraction may be formulated in terms of negative Killing-energy flux carried by fields into the horizon, directly paralleling the negative-energy particle states of the Penrose process. GRMHD simulations by Koide et al. (2008, 2014) demonstrated that regions of negative energy density form inside the ergosphere and are responsible for powering the outgoing jet, while Lasota et al. (2013) clarified the BZ mechanism as an “electromagnetic Penrose process.” Koide (2018) further analyzed the correspondence in relativistic magnetospheres. Recently, Noda et al. (2025) provided numerical evidence that the negative-energy inflow picture persists in turbulent, time-dependent magnetospheres.

These works establish a *global equivalence* at the level of horizon fluxes and energy budgets. However, they do not provide a *local, channel-independent diagnostic* that applies uniformly to particles, waves, and force-free electromagnetic fields.

The present work introduces such a diagnostic: *negative phase velocity* (NPV), defined by

$$\mathbf{S} \cdot \mathbf{k} < 0, \quad (1)$$

as the local manifestation of negative Killing-energy transport in the ergosphere. The novelty emphasized here is threefold: (i) a single *local* criterion valid in the particle, wave, and electromagnetic channels; (ii) an explicit bridge between energy extraction and curved-spacetime wave propagation (analogous to NPV in effective media); and (iii) a practical diagnostic implementable in numerical simulations via the sign of  $\mathbf{S} \cdot \mathbf{k}$  rather than global horizon integrals. Thus, the aim is not merely to re-state Penrose–BZ equivalence, but to place it within a unified spacetime–optical framework applicable across channels.

**Table 1.** Connection between Penrose process, superradiance, and Blandford–Znajek (BZ) mechanism, and their relation to negative phase velocity (NPV).

Phenomenon	Description	Relation to NPV
Penrose process	Particle in the ergosphere splits; one fragment with negative energy falls into the hole, the other escapes with greater-than-incoming energy.	Negative-energy state corresponds to energy flow opposing momentum (phase direction) — a mechanical analog of NPV.
Superradiance	Incident waves scattering off a rotating black hole are amplified when the superradiant condition holds.	Amplification arises because, within the ergosphere, local phase and energy flow oppose each other, creating an NPV region that extracts rotational energy.
Blandford–Znajek (BZ)	Magnetic field lines threading the rotating hole drive currents; energy is extracted as outward Poynting flux.	Magnetosphere behaves as a macroscopic NPV-like system: outward Poynting flux with co-rotating field structure; horizon acts like a rotating conductor with effectively negative impedance.

## 2. Theoretical Framework

### 2.1. Geometry and Conserved Currents

Consider a Kerr black hole with mass  $M$  and angular momentum  $J = aM$ . The horizon generator is

$$\chi^\mu = t^\mu + \Omega_H \phi^\mu, \quad \Omega_H = \frac{a}{2Mr_+}, \quad (2)$$

with  $r_+$  the horizon radius.

For a stress-energy tensor  $T_{\mu\nu}$ , the conserved flux of energy across the horizon is

$$\dot{E}_H = \int_{\mathcal{H}} T_{\mu\nu} \chi^\mu d\Sigma^\nu, \quad (3)$$

while the angular momentum flux is

$$\dot{J}_H = - \int_{\mathcal{H}} T_{\mu\nu} \phi^\mu d\Sigma^\nu. \quad (4)$$

Horizon regularity imposes the relation

$$\dot{E}_H = \Omega_H \dot{J}_H. \quad (5)$$

Thus, if  $\dot{J}_H < 0$ , the horizon absorbs negative energy: rotational energy is extracted.

## 2.2. Negative Phase Velocity Condition

In a locally non-rotating (ZAMO) frame, the Poynting vector  $\mathbf{S}$  gives the direction of energy flow, while the wavevector  $\mathbf{k}$  sets the direction of phase propagation. The *NPV condition* is

$$\mathbf{S} \cdot \mathbf{k} < 0, \quad (6)$$

which coincides with the condition that the horizon absorbs negative Killing energy.

In ordinary media, the phase velocity (direction of wavefront propagation) and the energy flow (given by the Poynting vector) point in the same direction. In negative-phase-velocity (NPV) or left-handed media, the phase velocity points opposite to the energy flow. Such media exhibit negative refraction, reverse Doppler, and reverse Cherenkov effects. This can also arise not just in meta-materials, but in curved spacetime near rotating black holes, where the spacetime metric itself acts as an effective medium with exotic constitutive parameters.

In Kerr geometry (i.e. rotating black holes), the frame-dragging of spacetime inside the ergosphere produces regions where the effective electromagnetic constitutive tensor allows NPV propagation. This means that Waves can have phase velocity directed inward while energy flow (Poynting vector) points outward, and the local observer sees “backward” wave propagation relative to the global energy flux. This is the electromagnetic analog of negative energy orbits in the Penrose process.

## 2.3 Negative Killing Energy and Negative Phase Velocity: Local Equivalence

The equivalence between negative Killing energy and negative phase velocity can be made explicit in the eikonal (geometric-optics) limit. For a wave with four-wavevector  $k_\mu = \nabla_\mu \Theta$  and stress-energy tensor  $T^{\mu\nu}$ , the energy current associated with the stationary Killing vector is

$$\mathcal{E}^\mu = -T^{\mu\nu} \chi_\nu. \quad (7)$$

In a locally non-rotating (ZAMO) frame, the spatial part of  $\mathcal{E}^\mu$  reduces to the Poynting vector  $\mathbf{S}$ , while the spatial projection of  $k^\mu$  defines the phase-propagation direction  $\mathbf{k}$ .

Using the standard 3 + 1 decomposition,

$$E_\chi = \int_{\Sigma} T^{\mu\nu} \chi_\mu n_\nu d\Sigma \propto \omega_{\text{loc}} - m\Omega_{\text{loc}}, \quad (8)$$

where  $\Omega_{\text{loc}}$  is the local frame-dragging angular velocity. Inside the ergosphere  $\Omega_{\text{loc}} > \omega/m$  may occur, yielding  $E_\chi < 0$ .

In the same region, the metric-induced constitutive relations imply

$$\mathbf{S} \cdot \mathbf{k} = \omega_{\text{loc}}(\omega_{\text{loc}} - m\Omega_{\text{loc}}) |\mathbf{A}|^2, \quad (9)$$

so that

$$E_\chi < 0 \iff \mathbf{S} \cdot \mathbf{k} < 0. \quad (10)$$

Hence negative Killing energy states are locally equivalent to negative phase velocity propagation, providing a formal justification for the identification used throughout this work.

Interpretation.

Equation (9) shows that NPV is not merely an analogy: in the geometric–optics (eikonal) limit it is a *local signature* of negative Killing-energy states. The ergosphere is therefore simultaneously (i) the region permitting  $E_\chi < 0$  relative to infinity and (ii) an effective spacetime medium supporting  $\mathbf{S} \cdot \mathbf{k} < 0$ . This provides the conceptual basis for the statement that “negative phase velocity is the local manifestation of negative Killing energy” used throughout the paper.

### 3. Results: Three Channels of Energy Extraction

#### 3.1. Penrose Process (Particle Channel)

A particle with 4-momentum  $p^\mu$  has Killing energy

$$E_\chi = -p_\mu \chi^\mu. \quad (11)$$

In the ergosphere,  $E_\chi$  can become negative. If a particle splits such that one fragment has  $E_\chi < 0$ , the escaping fragment must carry away energy in excess of the original. This is the particle realization of the NPV condition: momentum direction anti-aligned with energy flux.

NPV condition for the particle channel.

For a massive particle one may associate a mechanical wavevector  $\mathbf{k} \parallel \mathbf{p}$  and energy flux  $\mathbf{S} \parallel E\mathbf{v}$ . Inside the ergosphere,

$$E_\chi = -p_\mu \chi^\mu < 0 \Rightarrow \mathbf{p} \cdot \mathbf{v} < 0, \quad (12)$$

i.e. momentum (phase direction) is antiparallel to the energy flow. Therefore,

$$E_\chi < 0 \Rightarrow \mathbf{S} \cdot \mathbf{k} < 0, \quad (13)$$

establishing the NPV criterion for the Penrose process.

#### 3.2. Superradiance (Wave Channel)

For a scalar field mode

$$\Phi \sim e^{-i\omega t + im\phi}, \quad (14)$$

the horizon flux evaluates to

$$\dot{E}_H \propto (\omega - m\Omega_H) |\Phi|^2. \quad (15)$$

Amplification occurs when

$$\omega < m\Omega_H, \quad (16)$$

so that  $\dot{E}_H < 0$ . Locally, this corresponds to NPV: the phase velocity associated with  $\mathbf{k}$  is opposite to the outward energy flux  $\mathbf{S}$ .

#### 3.3. Blandford–Znajek Mechanism (Electromagnetic Channel)

In a force-free (or magnetically dominated) magnetosphere, magnetic field lines rotate with angular velocity  $\Omega_F$  and the system is often treated as stationary and axisymmetric. To connect this to a phase-velocity criterion, we emphasize that even a stationary, rigidly rotating field configuration possesses a well-defined *phase structure*: the electromagnetic pattern co-rotates at the *pattern speed*  $\Omega_F$ .

Concretely, one may decompose the field into azimuthal Fourier components whose phase is

$$\Theta = m\phi - \omega t, \quad \omega = m\Omega_F. \quad (17)$$

Constant-phase surfaces correspond to the rotating field pattern; thus  $\Omega_F$  is the *phase (pattern) velocity* in the azimuthal direction:

$$v_{\text{ph}} = \frac{\omega}{m} = \Omega_F. \quad (18)$$

The associated four-wavevector (a *pattern wavevector*, not a radiative mode) is

$$k_\mu = \nabla_\mu \Theta = (-\omega, 0, 0, m). \quad (19)$$

This addresses the stationary character of the BZ solution: the relevant “ $\mathbf{k}$ ” is the gradient of the rotating pattern phase, analogous to the wavevector of a rigidly rotating spiral mode rather than a freely propagating wavepacket.

The horizon energy flux in the BZ mechanism is negative when  $\Omega_F < \Omega_H$ , producing outward Poynting power and an astrophysical jet. The standard BZ power scaling is

$$P_{\text{BZ}} \sim \kappa \Omega_H^2 \Phi_B^2 \left( \frac{\Omega_F}{\Omega_H} \right) \left( 1 - \frac{\Omega_F}{\Omega_H} \right), \quad (20)$$

with  $\Phi_B$  the magnetic flux threading the horizon and  $\kappa$  a dimensionless constant.

NPV condition for Blandford–Znajek.

In the ZAMO frame, the electromagnetic energy transport direction is given by the Poynting vector  $\mathbf{S}$ . Using the pattern phase (17)–(19), one finds that near the horizon the local sign relevant for energy extraction obeys

$$\mathbf{S} \cdot \mathbf{k} \propto (\Omega_F - \Omega_H) \Omega_F B^2, \quad (21)$$

so that

$$\Omega_F < \Omega_H \quad \iff \quad \mathbf{S} \cdot \mathbf{k} < 0. \quad (22)$$

Hence, the standard BZ extraction condition is mathematically identical to the NPV criterion when the rotating field pattern is treated as a phase wave.

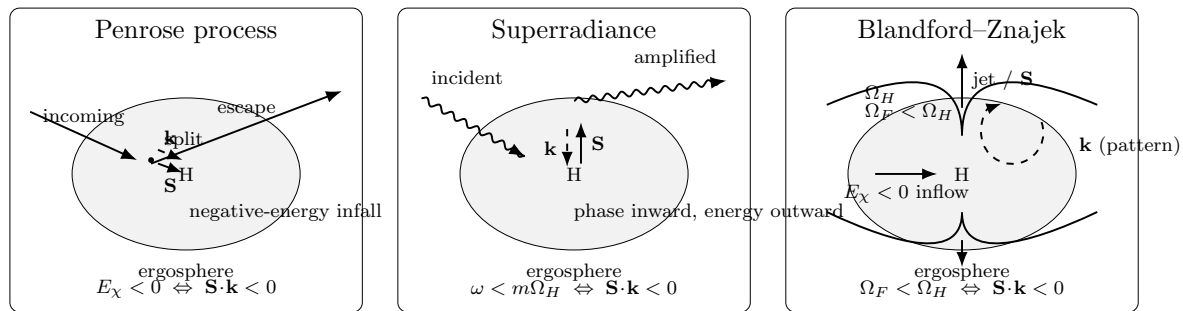
**Table 2.** Unified physical picture across particle, wave, and field/plasma regimes, highlighting how NPV manifests in each.

Process	Mechanism	NPV manifestation
Penrose (particle)	Negative-energy orbits in the ergosphere enable net energy extraction.	Momentum/phase direction vs. energy flow are misaligned (mechanical analog of NPV).
Superradiance (wave)	Wave amplification by a rotating horizon under the superradiant condition.	Phase velocity opposes energy (Poynting) flow within the ergosphere (wave-level NPV).
Blandford–Znajek (field/plasma)	Magnetically driven jet (outward Poynting flux) powered by black-hole spin.	Effective negative electromagnetic impedance; macroscopic NPV-like energy extraction.
NPV (EM/geometric)	Phase–energy opposition enabled by frame-dragging and constitutive effects.	Underlies all of the above energy-extraction phenomena.

## 4. Discussion

The above results demonstrate that all three energy-extraction processes from a Kerr black hole are governed by a single criterion: the absorption of negative Killing energy, equivalently expressed as negative phase velocity (NPV) propagation in the ergosphere. Figure 1 illustrates the geometry of energy and phase transport for each channel, emphasizing that negative-energy inflow occurs within the ergosphere while positive energy is carried outward.

- **Penrose process:** NPV arises in the momentum-energy misalignment of particles.
- **Superradiance:** NPV is realized when the reflected wave has group velocity outward but phase velocity inward.
- **BZ mechanism:** NPV manifests as field lines with pattern speed below the horizon angular velocity, leading to reversed energy flux across the horizon.



**Figure 1.** Schematic illustration of the three channels of black-hole energy extraction and their NPV interpretation. **Left:** Penrose process—particle splitting inside the ergosphere yields a fragment with negative Killing energy falling into the horizon. **Middle:** superradiance—phase fronts propagate inward while energy flux propagates outward. **Right:** Blandford–Znajek mechanism—a split-monopole-like jet configuration with field-line rotation  $\Omega_F < \Omega_H$ , producing negative electromagnetic energy inflow within the ergosphere and outward Poynting flux along the jet. Solid arrows denote energy flux  $\mathbf{S}$ ; dashed arrows denote phase/pattern direction  $\mathbf{k}$ .

Figure 1 schematically illustrates the three channels. The left panel shows particle splitting and negative-energy inflow in the ergosphere (Penrose process). The middle panel depicts inward phase propagation with outward energy flux for superradiant waves. The right panel represents the BZ configuration, where magnetic field lines co-rotate with angular velocity  $\Omega_F < \Omega_H$ , leading to negative electromagnetic energy inflow inside the ergosphere and outward Poynting flux along the jet.

This unification has several consequences:

1. It clarifies that BZ is not fundamentally different from superradiance but its *steady, force-free, large-scale limit*.
2. It provides a diagnostic for simulations: identifying regions with  $\mathbf{S} \cdot \mathbf{k} < 0$  can signal ongoing energy extraction in numerical GRMHD models.
3. It suggests that analogous processes may occur in other rotating systems with ergoregions, such as analogue gravity experiments or rotating compact stars.

### 4.1 Scope and novelty of the present unification

Previous studies established that negative Killing energy flux is the key ingredient common to Penrose-type and electromagnetic extraction mechanisms. The new element emphasized here is the identification of *negative phase velocity as the universal local signature* of this flux, valid for discrete particle trajectories, linear wave scattering, and nonlinear force-free electromagnetic fields.

This reformulation enables: (i) a geometrical-optics interpretation of energy extraction, (ii) direct comparison between analytical theory and numerical simulations through local field diagnostics, and (iii) extension to analogue-gravity systems where phase velocity may be directly measured.

## 5. Conclusions

We have demonstrated that the Penrose process, superradiant scattering, and the Blandford–Znajek mechanism are three manifestations of a unified energy-extraction phenomenon from rotating black holes. The ergosphere permits negative Killing energy states, whose local signature is *negative phase velocity*.

- In the particle channel, this condition allows the Penrose process.
- In the wave channel, it yields superradiant amplification.
- In the electromagnetic channel, it drives the Blandford–Znajek jet.

This unified perspective highlights the ergosphere as the essential stage and NPV as the diagnostic connecting microscopic, mesoscopic, and macroscopic energy extraction processes.

**Data Availability Statement:** No data were generated or analyzed in support of this research.

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