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Self-Interacting Extended Scalar Field and Emergent Scale Dynamics in Flat Spacetime

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Abstract

This work explores a self-interacting scalar field in flat spacetime, where the dynamics of an extended configuration is governed by coupling to its ground state profile. The Klein-Gordon equation yields analytical solutions under constraints, which organize accessible states and induce a mass relaxation process that stabilizes at late times. The resulting residual value aligns with the mass scale of light neutrinos and is interpreted as a remnant of field stabilization. A key contribution is the conceptual shift from the energy level to quantization of geometry, establishing direct relations between scale parameters and energy density consistent with standard estimates. Quantum transitions between neighboring arrangements, driven by vacuum fluctuations, lead to an effective expansion rate compatible with observed cosmic dynamics. The model provides a unified description of the dark sector arising from field self-organization, highlighting the role of geometric emergence within this framework.

Keywords: self-interacting extended field; emergent geometry; quantum constraints; vacuum energy density; mass relaxation mechanism; dark sector unification

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1. Introduction

The relationship between spacetime structure and the dynamics of quantum fields remains one of the fundamental problems in theoretical physics [1]. Following the development of general relativity and quantum processes, numerous frameworks have explored how the internal properties of fields can influence the geometry of the universe [2,3]. In this context, models based on the Klein-Gordon equation occupy a central role, especially with self-interaction terms that generate nontrivial dynamics beyond the free field regime [4–6].

The present work studies a class of extended configurations that exhibit intrinsic scalar properties from which geometric features emerge. This approach aligns with extended formulations used in theoretical paradigms and quantum cosmology [7], whose structure allows the introduction of scalable geometric parameters in a flat spacetime. Alternatively, several recent proposals examine the possible emergence of geometric properties from extended fields. One line considers quantum gravity compositeness, viewing spacetime as a graviton condensate where saturation levels control the vacuum energy [8]. Another explores scenarios that feature dark energy as a dynamical quantity with a scale reduced by the vacuum structure [9]. Both perspectives suggest that certain geometric variables may arise from a more fundamental field substrate, establishing relations between the energy density and the horizon scale to motivate the present analysis.

The central hypothesis is that constrained field configurations determine the emergent geometric scale, rather than individual energy eigenstates. This necessitates a description of the system within an auxiliary ξ-space, implemented here via a rescaled Wick rotation. By mapping physical coordinates onto this dimensionless domain, the metric scale is decoupled from any prescribed background, allowing the extended field to dictate the scaling properties of the system. The expansion of the physical volume is not a free geometric evolution, but an energetically driven process arising from the transitions between these structural configurations.

To develop this hypothesis, we start from a Lagrangian with a self-interaction term of the form $\lambda\chi^4$ treated in a specific regime. The choice $\lambda > 0$ is not arbitrary: in this case [10,11], the associated potential is stable and tends to favor expansive behavior and an increase in the effective mass parameter [12,13]. In contrast, for $\lambda < 0$ the potential becomes unstable and leads to contractive behavior incompatible with consistent physical scenarios [14,15]. From the Lagrangian with $\lambda > 0$, the Klein-Gordon equation is derived and the assumptions required to obtain an analytical solution are discussed. An ansatz χ_0' for the ground state is proposed, allowing the self-interacting dynamics to be approximated by a term of the form $\lambda\chi_0'^2\chi^2$.

The model bridges a hierarchy between the Planck scale and light neutrino masses, offering a way to estimate the vacuum energy density [16]. This connection is realized through an analytical mapping that reveals how the internal structure of the field and its interactions can manifest at macroscopic levels [17,18]. The results suggest that quantum dynamics may play a meaningful role even at cosmological distances [19]. Consequently, the transition across scales is described by a relaxation process leading to a stable mass value, a mechanism examined in detail in Section 3.2.

Furthermore, the regime $\lambda > 0$ corresponds to an expansive behavior consistent with the observed cosmic acceleration. Generally attributed to dark energy [20,21], this provides an effective mechanism to describe large-scale evolution. In our analysis, the self-interaction of the extended field drives an expansion rate determined by transitions between neighboring configurations, yielding values consistent with classical estimates [22]. In this context, quantum constraints impose discrete values on the product of the characteristic scale and effective mass, allowing certain geometric features to be interpreted as manifestations of the field internal coupling [23].

This formalism employs the positive self-interaction parameter to ensure stability, providing natural mechanisms for spacetime expansion and mass spectrum generation. Our results suggest that both the dark sector and the geometric scale emerge as the field transitions toward a stable macroscopic state. Although the analysis is performed in flat spacetime, it establishes a robust basis for interpreting large-scale cosmic evolution as an emergent property of the internal field dynamics.

2. Self-Interaction Framework and Analytic Construction

An extended scalar field χ is considered, with local fluctuations around a ground state profile χ_0 driving the system through nonlinear dynamics. The analysis is restricted to a neighborhood of size δ around a reference point x'^μ, defining the increment as $\Delta x^\mu = x^\mu - x'^\mu$. Such a localization allows the configuration χ_0 to effectively capture the leading variations of the field. This approach enables an effective description without requiring an exact global solution for the underlying profile. The approximation reflects the fact that dominant couplings are controlled by local variations rather than global behavior. Consequently, χ_0 is represented by a simplified ansatz, sufficiently accurate within the δ-neighborhood to facilitate the analytical treatment [6]. Mathematically, this takes the form

$$(\Delta x^\mu)^2 < \delta^2 \Rightarrow \chi_0(\Delta x^\mu) \approx \chi_0'(\Delta x^\mu), \tag{1}$$

where χ_0' denotes an approximation used to derive closed form solutions. By introducing appropriate scaling parameters, this work aims to identify the quantum constraints imposed on those scales. These conditions define the physical states of the system and reveal how geometric features emanate from the field, effectively establishing a link with the resultant spacetime geometry.

Furthermore, such an ansatz simplifies the resolution of the Klein-Gordon equation. From a physical perspective, χ_0' incorporates the self-interaction effects that generate the emergent geometry. This local representation remains robust for vacuum energy estimates and defines the regime in which the field and spacetime geometry arise as a single physical entity.

2.1. Lagrangian Formulation and Choice of Interaction

The starting point is typically a scalar potential with both a mass term and a nonlinear coupling, given by $V(\chi) = \frac{1}{2}m^2\chi^2 + \frac{1}{4!}\lambda\chi^4$. In the present case, the interaction acts as a quartic term in the

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vicinity of the ground state. However, for higher states, the field is considered to converge toward a semiclassical background defined by χ_0. This leads to a reformulation of the potential that encodes the coupling between the fundamental state and the excited modes of χ, resulting in

$$V(\chi) = \frac{1}{2}m^2\chi^*\chi + \frac{1}{2}\lambda\chi_0^2(\chi^*\chi). \tag{2}$$

Although the interaction term is written as $\chi_0^2(\chi^*\chi)$, only χ requires complex conjugation to ensure the reality of the Lagrangian. The profile χ_0 acts as a reference state and remains strictly real at all stages, including within the transformed space presented below. Accordingly, the Lagrangian reads

$$\mathcal{L} = \frac{1}{2}\partial_\mu\chi^*\partial^\mu\chi - \frac{1}{2}m^2\chi^*\chi - \frac{1}{2}\lambda\chi_0^2(\chi^*\chi). \tag{3}$$

It provides a consistent way to track the emergence of geometry, linking the effective potential to the intrinsic field structure; namely, a self-organizing system in line with Refs. [24,25]. The analysis demonstrates how this yields the self-interacting Klein-Gordon equation, establishing the quantum constraints that define the allowed states.

The introduction of a specific coupling $\chi_0^2(\chi^*\chi)$, rather than the standard quartic potential χ^4, models the emergence of geometry from an ordered medium. Unlike a canonical term χ^4 describing local interactions, the proposed coupling establishes χ_0 as a macroscopic phase regulator. The ground state profile governs the dynamics of its own excitations, constraining their degrees of freedom into a synchronized collective mode. This intrinsic organization allows the system to transcend the free field regime and manifest emergent metric features. Consequently, expansion arises from an ordered evolution of the field profile driven by this fundamental state rather than external forces. Although entropic gravity or thermodynamic approaches in Refs. [26,27] point toward a non-fundamental geometry, our model explores this emergence via a mechanism rooted in the non-linear dynamics of χ.

2.2. Ground State Ansatz and Quantum Constraints

Starting from Lagrangian (3), the Klein-Gordon equation for a self-interacting potential is derived, in which the field χ depends on the spacetime coordinates x^μ. To facilitate an analytical solution, χ_0 is replaced by a suitable ansatz χ_0', allowing the resulting field equation to be expressed as

$$\partial_\mu\partial^\mu\chi + m^2\chi + \lambda\chi_0'^2\chi = 0. \tag{4}$$

Defining the temporal and spatial increments as $\Delta x^0 = x^0 - x'^0$ and $\Delta x^i = x^i - x'^i$, respectively, the proposed form of the approximation function is given by

$$\chi_0' = v_0\Big[1 - \frac{1}{2}(1/\delta_{\mathrm{t}}^2)(\Delta x^0)^2 - \frac{1}{2}(1/\delta_{\mathrm{s}}^2)(\Delta x^i)^2\Big]. \tag{5}$$

This expression depends on the spacetime x-coordinates, with the distances from an arbitrary fixed point x' scaled by the parameters δ_{t} and δ_{s}. The distinction between temporal and spatial scaling introduces a specific factor for each coordinate type. To preserve homogeneity, the same factor δ_{s} is applied in all spatial components x^i. In addition, a parameter v_0 is introduced to represent the amplitude of the ground state. However, negative signs within the quadratic terms of the ansatz are required to ensure asymptotic stability of the field, providing a consistent basis for the coordinate transformation. Based on these considerations, a transition is made from physical spacetime to the dimensionless auxiliary ξ-space, defined as

$$\xi^0 = (1/\delta_{\mathrm{t}})\Delta x^0, \quad \xi^i = (1/\delta_{\mathrm{s}})\Delta x^i, \tag{6}$$

given this transformation, the field is written as $\chi(x) = \tilde{\chi}(\xi)$. This mapping effectively deconstructs the physical spacetime interval, delegating the recovery of measurable lengths to the scale parameters, consistent with the constraint relations. Since $|\xi|^2 = (\xi^0)^2 + \xi^i\xi^i$, the ansatz acquires the form

$$\tilde{\chi}'_0 = v_0\Big(1 - \frac{1}{2}|\xi|^2\Big). \tag{7}$$

In accordance with condition (1), the approximation $|\xi|^2 < 1$ is applied to the ansatz, ensuring that its quadratic form remains valid within the local region where the auxiliary ξ-coordinates are defined, so that $\tilde{\chi}'^2_0 \approx v_0^2(1 - |\xi|^2)$ can be used directly in the Klein-Gordon equation

$$\frac{1}{\delta_t^2}\partial^2_{\xi^0}\tilde{\chi} - \frac{1}{\delta_s^2}\nabla^2_{\xi}\tilde{\chi} + m^2\tilde{\chi} + \lambda v_0^2(1 - |\xi|^2)\tilde{\chi} = 0. \tag{8}$$

A normalizable solution for the ground state is given by $\tilde{\chi}_0 = v_0 e^{-\frac{1}{2}|\xi|^2}$; indeed, once the ansatz has been proposed in the indicated form, a decaying exponential with amplitude v_0 is expected for the ground state. This implies that within the approximation level adopted in (1), a particular solution is obtained consistent with that requirement. For verification, the decaying exponential can be approximated to first order in a polynomial expansion to reproduce the ansatz, ensuring that $\tilde{\chi}_0 \to \tilde{\chi}'_0$ when $|\xi|^2 < 1$. Moreover, this function is well-behaved and bounded, vanishing outside the limits where the approximation holds. The constraints required to satisfy the governing equation are determined by substituting the partial derivatives into the original expression. The temporal component is given by $\partial^2_{\xi^0}\tilde{\chi}_0 = -\tilde{\chi}_0 + (\xi^0)^2\tilde{\chi}_0$ and the homogeneous spatial components $\nabla^2_{\xi}\tilde{\chi}_0 = -3\tilde{\chi}_0 + \xi^i\xi^i\tilde{\chi}_0$. Substituting both differential expressions yields

$$\frac{1}{\delta_t^2}(-\tilde{\chi}_0 + (\xi^0)^2\tilde{\chi}_0) - \frac{1}{\delta_s^2}(-3\tilde{\chi}_0 + \xi^i\xi^i\tilde{\chi}_0) + m^2\tilde{\chi}_0 + \lambda v_0^2\tilde{\chi}_0 - \lambda v_0^2|\xi|^2\tilde{\chi}_0 = 0.$$

Matching the quadratic terms in ξ, the following conditions must be satisfied

$$\frac{1}{\delta_t^2} = \lambda v_0^2, \quad \text{and} \quad -\frac{1}{\delta_s^2} = \lambda v_0^2, \tag{9}$$

the remaining terms cancel as

$$-(1/\delta_t^2) + (3/\delta_s^2) + m^2 + \lambda v_0^2 = 0. \tag{10}$$

Here, the coefficients -1 and $+3$ stem from the second time derivative and the diagonal terms of the Laplacian evaluated over the ground state solution. Moreover, as a consequence of these conditions, the temporal and spatial scale parameters are uniquely fixed to the values

$$\delta_t = \delta, \quad \text{and} \quad \delta_s = i\delta. \tag{11}$$

The appearance of the imaginary unit in the spatial scale parameter $\delta_s = i\delta$ is not a consequence of the sign convention of the metric itself, but of the Wick-type rotation implicitly performed when transforming Minkowski space into an effectively Euclidean ξ-space. Depending on whether the rotation is applied to the temporal or spatial sector, the factor i would appear in δ_t or δ_s, respectively. The choice made here is purely operational and does not affect the physical content of the field. Consequently, this reduces both relations that appear in (9) to become a single one $\lambda\delta^2 v_0^2 = 1$, and then (10) simplifies in this particular case to $\delta^2 m^2 = 3$.

Up to this point, the particular solution has been analyzed. The governing equation and its ground state solution are mathematically very similar to the standard approach used to solve the quantum harmonic oscillator [28]. We propose a general solution consisting of the Hermite polynomial $H_n(\xi) = \prod_{\mu=1}^{4} H_n(\xi^\mu)$ multiplied by the decaying exponential of the particular solution; this is $\tilde{\chi}_n = \sum C_n H_n(\xi) e^{-1/2|\xi|^2}$, consequently, the amplitude for the ground state, $n = 0$, must be $C_0 = v_0$. The field states depend on the excitation level, where $n = n_0 = n_1 = n_2 = n_3$ is a non-negative integer ($n \in \{0, 1, 2, \dots\}$) assumed for simplicity to ensure homogeneity in the spacetime coordinates. This

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solution has the usual form associated with the separation of variables. A parameter α^2 could have been introduced to redefine the bare mass m^2, namely $m'^2 = m^2 + \alpha^2$; however, this has no significant effect in our case. Due to the structure based on Hermite polynomials, the field $\chi(x)$ may acquire complex components after the inverse transformation $\xi = \xi(x)$, since this mapping introduces an imaginary factor. This behavior does not affect the analytical treatment in transformed space, with $\tilde{\chi}(\xi)$ consistently treated as a real field. The product $\chi^*\chi$ in the physical Lagrangian ensures hermiticity without imposing restrictions on the quantum number n or on the parity of the excitation levels, preserving the coherence of the formalism. A consistency relation similar to that of the particular case is found $\lambda\delta^2 v_0^2 = 1$. By induction, the general case produces $\delta^2 m^2 = 2(4n + 3/2)$, which depends on n. These algebraic relations are referred to as quantum constraints because they arise from the discrete structure of the Hermite solutions.

Thus, the quadratic ansatz $\tilde{\chi}_0' = v_0(1 - \frac{1}{2}|\xi|^2)$ is not merely an approximation for mathematical convenience; it is the leading order Taylor expansion of the Gaussian solution for $|\xi|^2 < 1$.This quadratic structure ensures that the field equations are satisfied locally, representing the ground state profile of the field within the expansion domain of validity.

Applying the transformation (6) and the scaling conditions in (11) yields a direct mapping between the physical spacetime distances and their counterparts in the auxiliary ξ-space

$$s^2 = \delta^2|\xi|^2, \tag{12}$$

the left-hand side of this expression is purely geometric. In fact, it identifies with the relativistic interval $s^2 = \Delta x_\mu \Delta x^\mu$; therefore, the transformation has a geometric character, relating the Lorentz invariant interval s^2 to the dimensionless Euclidean distance $|\xi|^2$. This relation is governed by the parameter δ, which rescales the spacetime interval through a transformation analogous to a Wick rotation (11). In this context, the geometric identity in (12) is physically more robust than the individual mappings in (6). Although the latter appear as simple scaling rules, the coupling with the field constraints integrates this relation into an invariant structure in which the interval scaling depends on the quantum state $\tilde{\chi}_n$. Consequently, the auxiliary ξ-space acts as the primary domain to derive the physical geometry. As shown in Section 3.3, this approach employs δ to project results from the internal ξ-space onto the x-space, suggesting that the field configuration provides a physical basis for the description of macroscopic spacetime.

2.3. Time Independent Regime and Extended Configuration

In this particular case, and to isolate the purely spatial behavior of the system, we work directly in the transformed ξ-space, assuming from the outset a field $\tilde{\chi}(\xi)$ with an explicit dependence on the ξ-coordinates. This implies starting from a transformed Lagrangian, which serves as the basis for the subsequent development

$$\tilde{\mathcal{L}}(\xi) = \frac{1}{2\delta_t^2}(\partial_{\xi^0}\tilde{\chi})^2 - \frac{1}{2\delta_s^2}(\nabla_\xi\tilde{\chi})^2 - \frac{1}{2}m^2\tilde{\chi}^2 - \frac{1}{2}\lambda\tilde{\chi}_0^2\tilde{\chi}^2, \tag{13}$$

it is the Lagrangian in (3) but rewritten using the scaling rules in (6). Furthermore, the action $S(x)$ transforms as follows $\tilde{S}(\xi) = \int \delta_t\delta_s^3 d^4\xi\tilde{\mathcal{L}}(\xi)$. Now, we assume that the scaling parameters satisfy the condition $\delta_t \gg \delta_s$. As a consequence, the transformed time coordinate satisfies $\xi^0 \ll \xi^i$. This condition effectively suppresses the time dependence of the field in the transformed description, making $\tilde{\chi}$ independent of ξ^0 in a controlled approximation, so that $\tilde{\chi}(\xi^0, \xi^1, \xi^2, \xi^3) \to \tilde{\chi}(\xi^1, \xi^2, \xi^3) = \tilde{\chi}(\xi^i)$. From a physical perspective, this amounts to assuming an implicit dimensionless time τ that is fully extended and evolves much more slowly than t, with the relation $\tau = t/\delta_t$; this auxiliary time does not play a dynamical role, but merely accounts for a slow background evolution. Since the field depends only on spatial components, the interval s^2 is spatial and equal to the proper distance L^2. The temporal coordinate τ is effectively decoupled, acting as a global evolution parameter independent of the local reference frame. In this way, any undefined or ambiguous behavior is avoided when determining the time evolution of the system parameters. These conditions lead to

$$\tilde{\mathcal{L}}(\xi) = -\frac{1}{2\delta_{\mathrm{s}}^2}(\nabla_\xi \tilde{\chi})^2 - \frac{1}{2}m^2\tilde{\chi}^2 - \frac{1}{2}\lambda\tilde{\chi}_0^2\tilde{\chi}^2, \tag{14}$$

and, consequently, the action transforms as $\tilde{S}(\xi) = \int \delta_{\mathrm{s}}^3 d^3\xi \tilde{\mathcal{L}}(\xi)$. Thereafter, the procedure to derive the Klein-Gordon equation and obtain a general solution follows that of Section 2.2, with only minor differences. For example, in this case $|\xi|^2 = \xi^i\xi^i$, the notation $\xi^i\xi^i$ is used instead of $|\xi|^2$ to emphasize that only spatial components are involved. Similarly, the ansatz $\tilde{\chi}_0'$ is employed to represent the background configuration within its Taylor validity range, which is given by $\tilde{\chi}_0'^2 \approx v_0^2(1-\xi^i\xi^i)$. Substituting this approximation into the Klein-Gordon equation derived from the proposed Lagrangian

$$-\frac{1}{\delta_{\mathrm{s}}^2}\nabla_\xi^2\tilde{\chi} + m^2\tilde{\chi} + \lambda v_0^2(1-\xi^i\xi^i)\tilde{\chi} = 0. \tag{15}$$

The ground state solution to this equation is given by a decaying exponential with amplitude v_0, of the form

$$\tilde{\chi}_0 = v_0 e^{-\frac{1}{2}\xi^i\xi^i}, \tag{16}$$

with $\delta_{\mathrm{s}} = i\delta$, the same quantum constraint equation is thus recovered

$$\lambda\delta^2 v_0^2 = 1, \tag{17}$$

as well as a mass constraint that depends on the excitation level

$$\delta^2 m^2 = 2(3n+1), \tag{18}$$

where $n = n_1 = n_2 = n_3$ corresponds to the three spatial dimensions in the homogeneous case. A general solution for the field can also be written as

$$\tilde{\chi}_n = \sum C_n H_n(\xi) e^{-\frac{1}{2}\xi^i\xi^i}, \tag{19}$$

with $H_n(\xi) = \prod_{i=1}^3 H_n(\xi^i)$, where $H_n(\xi^i)$ denotes the Hermite polynomials for each individual coordinate component. The amplitude C_0 of the ground state must coincide with the amplitude v_0 defined for the ansatz. To clarify, the appearance of the factor i in δ_{s} reflects the Wick-type structure of the transformation and does not imply any complex nature for $\tilde{\chi}(\xi)$, which remains strictly real in this space.

Considering the definition of the relativistic interval s, together with the fact that $\xi^0 \ll \xi^i$, and recalling that the sign convention $(+,-,-,-)$ is being used, a physical distance $L = (x^i - x'^i)(x^i - x'^i)$ can be related to $L^2 = -s^2_{(\xi^0 \ll \xi^i)}$. Under this convention, the spatial interval is negative; hence, a minus sign is introduced in its definition to ensure positivity, yielding

$$L^2 = \delta^2\xi^i\xi^i. \tag{20}$$

This identity aligns with the invariant form in (12). Because only the spatial part is evaluated under the assumption of an extended time, the equivalence between physical distance and spatial interval allows L to be considered a geometric invariant.

Although the general solution (19) is presented in Cartesian coordinates to simplify the algebra, the postulated homogeneity and isotropy of the field imply an underlying spherical symmetry. This consistency is easily verified by transforming the field equation into spherical coordinates. The solutions for the radial excitation levels are given by associated Laguerre polynomials $\tilde{\chi}_n(\xi) \propto Lg_n^{1/2}(\xi^i\xi^i)e^{-1/2\xi^i\xi^i}$ for $n = n_1 = n_2 = n_3$, depending exclusively on the spatial invariant $\xi^i\xi^i$. Consequently, any excited level $\tilde{\chi}_n$ preserves its functional structure under rotational transformations, ensuring that the physical

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consistency of the model is maintained for any arbitrary orientation and reinforcing the geometric nature of the relation (20).

The general solution (19) and the methodology developed around it is similar to that used in the quantum harmonic oscillator, so its properties can also be applied here. This allows us to define differential operators $\hat{a}(\xi)$ and $\hat{a}^\dagger(\xi)$ that induce excitations, namely

$$\hat{a}(\xi) = \frac{1}{\sqrt{2}}(\xi + \partial_\xi), \quad \hat{a}^\dagger(\xi) = \frac{1}{\sqrt{2}}(\xi - \partial_\xi). \tag{21}$$

These operators retain the same algebraic structure as in the standard harmonic oscillator formulation, since the differential representation in ξ-space is fully real. Furthermore, they are not occupation operators in the sense of creating or annihilating particles in a given mode; in an extended field of the type considered here, this cannot be defined in such a direct and simple manner. Rather, $\hat{a}(\xi)$ acts as an operator that removes excitations, while $\hat{a}^\dagger(\xi)$ acts as an operator that generates excitations. With this in mind, the corresponding relations are given by

$$\hat{a}(\xi)|n\rangle = \sqrt{n}|n-1\rangle, \quad \hat{a}^\dagger(\xi)|n\rangle = \sqrt{n+1}|n+1\rangle. \tag{22}$$

In this setting, the number operator $\hat{N}(\xi)$ is defined to characterize the discrete configurations of the field, expressed as

$$\hat{N}(\xi) = \hat{a}^\dagger(\xi)\hat{a}(\xi), \tag{23}$$

while these operators mirror those of the usual creation-annihilation algebra, their role here is purely to classify spatial excitations of $\tilde{\chi}$. Then, $\hat{N}(\xi)$ satisfies

$$\hat{N}(\xi)|n\rangle = n|n\rangle. \tag{24}$$

Applied to a given state, the operator yields the number of excitations; this property will be used to determine, in the limit of large n, the evolution of the field through its emergent parameters in Section 3.4.

Although the mathematical structure resembles that of the quantum harmonic oscillator, this similarity is only apparent; specifically, the system Hamiltonian cannot be explicitly expressed in terms of the operators $\hat{a}(\xi)$ and $\hat{a}^\dagger(\xi)$. This is because the Lagrangian does not incorporate a standard harmonic potential, but rather an ansatz that describes the ground state profile through a quadratic approximation. Despite it may appear similar to such a potential, it possesses a crucial distinguishing feature: the sign of the interaction term is opposite to that of a restorative potential. Physically, the systems also differ significantly. Whereas the proposed Lagrangian describes self-interactions $\lambda\tilde{\chi}_0^2\tilde{\chi}^2$ with a positive λ coefficient, corresponding to an expanding system, a strict harmonic potential would describe bound and confined states. Accordingly, we argue that the system defines discrete configurations of the field geometry rather than energy levels. This stems from the inherent requirement for discrete states linked to the scale δ.

3. Results and Discussion

In this section, we evaluate the physical implications of the constraint relations derived in Section 2.3, focusing on the effective mass, emergent spectrum, vacuum energy density, and dynamical behavior of the scale parameter.

3.1. Effective Mass and Level Dependence

To compute a physically measurable effective mass, the Lagrangian (14) can be rewritten and grouped as follows

$$\tilde{\mathcal{L}}(\xi) = \frac{1}{2\delta^2}(\nabla_\xi\tilde{\chi})^2 - \frac{1}{2}(m^2 + \lambda\tilde{\chi}_0^2)\tilde{\chi}^2, \tag{25}$$

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it is important to stress that the effective mass defined here does not arise from radiative corrections in the usual quantum field theoretic sense, but from the coupling between $\tilde{\chi}$ and its background configuration $\tilde{\chi}_0$ defined in (16). This allows us to identify the following term m^2_{eff}, which is commonly referred to as the effective mass in this context, then

$$m^2_{\text{eff}} = m^2 + \lambda\langle\tilde{\chi}^2_0\rangle. \tag{26}$$

The expectation value of the extended field $\tilde{\chi}$ is defined as the identity $\langle 0|\hat{\tilde{\chi}}|0\rangle / \langle 0|0\rangle = \tilde{\chi}_0$ under a mean-field approximation. This approach, standard in Bose-Einstein condensates and topological defects, determines the physical observables from the order parameter profile rather than individual particle fluctuations. Consequently, $\langle\tilde{\chi}^2_0\rangle$ is computed as the spatial average over the ground state extent, namely $\langle\tilde{\chi}^2_0\rangle = (\int d^3\xi \tilde{\chi}^2_0 e^{-1/2\xi^i\xi^i}) / (\int d^3\xi e^{-1/2\xi^i\xi^i})$. With this definition, the following calculation can be carried out

$$\langle\tilde{\chi}^2_0\rangle = \frac{1}{(2\pi)^{3/2}} \int_{-\infty}^{\infty} d^3\xi \tilde{\chi}^2_0 e^{-\frac{1}{2}\xi^i\xi^i} = \frac{\sqrt{3}}{9} v^2_0, \tag{27}$$

by substituting this value into the expression for the effective mass and applying the constraint relations $\lambda\delta^2 v^2_0 = 1$ and $\delta^2 m^2 = 2(3n+1)$ from Section 2.3, the following result is obtained

$$m^2_{\text{eff}} = \left[1 + \frac{\sqrt{3}}{18(3n+1)}\right] m^2. \tag{28}$$

The effective mass, m_{eff}, is determined by the bare mass, m, and modified by a factor depending on n. As n increases from 0 to ∞, m_{eff} varies only marginally from $\approx 1.05m$ to m. This mild dependence results from the large hierarchy induced by the constraint $\lambda\delta^2 v^2_0 = 1$, keeping $m_{\text{eff}} \approx m$ throughout the range.

3.2. Hierarchy of Scales and Mass Stabilization

Assuming that the effective mass is equal to the bare value, m_{eff1} can be estimated for case $n = 0$ by setting the Planck length as the lower bound for the ground state scale parameter, $\delta = \ell_{\text{p}}$. In this way, using (18) directly, it can be written

$$m_{\text{eff1}} = m = \frac{\sqrt{2}}{\delta} = \frac{\sqrt{2}}{\ell_{\text{p}}} = \sqrt{2} m_{\text{p}}, \tag{29}$$

this yields a value of the order of the Planck mass m_{p}, which can be regarded as the expected value for a supposedly primordial field state.

Another interesting case is to assume that the mass evolves from this initial primordial state to values with $n \gg 0$. We can represent this evolution by increasing both terms in relation (18). On the left-hand side, small and continuous increments are assumed, ensuring differentiability; this is justified because the variables involved are expected to approach a continuous spectrum for very large values of n. The term on the right-hand side, which depends only on n, increases by one unit, leading to $\Delta(\delta^2 m^2) = 2[(3(n+1)+1) - (3n+1)]$. A straightforward calculation yields

$$\Delta(\delta^2 m^2) = 2\delta m^2 \Delta\delta + 2\delta^2 m \Delta m = 6. \tag{30}$$

The identity $\Delta(\delta^2 m^2) = 6$ encapsulates the entire relaxation mechanism: it is the algebraic bridge between the discrete quantum structure of the field and the emergent large-scale dynamics. Imposing the stabilization threshold $\Delta m = 0$ fixes the residual mass purely through geometric information. This condition defines the point at which the internal dynamics no longer modify m. Consequently, a transition occurs wherein the scale parameter δ becomes dependent only on n; and under these assumptions, the mass is

$$m(\delta) = \sqrt{3}(\delta\Delta\delta)^{-1/2}. \tag{31}$$

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This relation reveals that the stabilized mass is determined solely by the ratio between the global scale δ and the minimal step $\Delta\delta$, linking microscopic and macroscopic scales within a single algebraic identity. Thus, δ can be set to a value equal to the radius of the observable universe, $\delta = R = 4.4 \times 10^{26}$ m. To maintain consistency with the first case, the variation of δ is taken to be the Planck length, $\Delta\delta = \ell_{\mathrm{p}}$. Substituting these limits into the previous expression yields the following result

$$m_{\mathrm{eff2}} = m = \sqrt{3}(R\ell_{\mathrm{p}})^{-1/2} = 4.04 \times 10^{-3}\ \mathrm{eV}. \tag{32}$$

The value obtained for m_{eff2} is of the same order as the lightest neutrinos, the smallest particle mass observed. This numerical coincidence follows directly from the internal structure of the model, justifying the proposal in Ref. [16]. Relaxation initiated from a primordial state [29,30] concludes once the underlying geometry is established at $\delta = R$. As a result, the mass evolves from $m_{\mathrm{eff1}} = \sqrt{2}m_{\mathrm{p}}$ at $n = 0$ toward a stabilized spectrum, yielding $m_{\mathrm{eff2}} = 4.04 \times 10^{-3}$ eV for $n \gg 0$.

Now, substituting the mass value obtained in (31) into the constraint relation (18), the following can be written

$$\delta = 2\ell_{\mathrm{p}} n, \tag{33}$$

here, δ acts as a discrete distance tied to the Planck length. Its dependence on the quantum number arises from the stabilization condition $\Delta m = 0$, once the mass relaxation process is complete. In this manner, the field state $\tilde{\chi}_n$ effectively determines a spatial length element through the relation (20). At the same time, both n and δ appear as effective quantum variables subject to imposed constraints, suggesting a self-consistency between the field and the background geometry. According to (33), the field excitations described by n can assign length elements as a statistical property; the dynamical concept is further explored in Section 3.4. Despite the quantum constraints on the underlying variables, the derived physical lengths can remain continuous. This distinction ensures that the geometric background preserves its smooth structure, even as it emerges from a discrete state. Consequently, the value of n can be determined from which stabilization occurs

$$n = \frac{1}{2}\frac{R}{\ell_{\mathrm{p}}} \approx 10^{61}. \tag{34}$$

This order of magnitude coincides with that provided by other authors in Ref. [28], from which the era dominated by dark energy is reached, but using different arguments than those presented here. Such large values of n justify treating the spectrum as effectively continuous in the dynamical analysis of Section 3.4. Likewise, under the same conditions, the ground state amplitude is computed as

$$v_0 = \frac{1}{\lambda^{1/2} 2\ell_{\mathrm{p}} n}, \tag{35}$$

the constraint relation (17) has been used together with the dependence of δ on n for this purpose. In the stabilization regime, the ground state reconfigures its amplitude in response to excited states as a consequence of the Lagrangian self-interaction terms.

3.3. Vacuum Energy Density and Scaling Relations

In general, the quantum vacuum is difficult to define precisely, and this becomes even more problematic for extended fields. This occurs because, in such cases, the canonical Fock space formalism cannot be rigorously applied. Furthermore, it is often impossible to unambiguously define particle creation and annihilation operators, and thus cannot assign well-defined occupation numbers to the field modes. In fact, identifying those modes with precision is itself ambiguous, and even the notion of localized individual particles becomes diffuse and ill defined.

The $n = 0$ state stands out as the only configuration whose functional form is strictly invariant and independent of the geometry used to describe the field. Unlike the excited levels, this solution remains identical in both Cartesian and spherical coordinate representations. Such a mathematical

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property grants it the character of a unique primordial vacuum state, as it is the only level that does not rely on the choice of coordinate system to preserve its structure [31].

The ground state amplitude v_0 establishes a permanent energetic substrate that persists alongside any further scaling dynamics, since this configuration must coexist indefinitely with any field state. Because this state acts as a primordial dynamical vacuum and a transient local equilibrium, the field evolves toward $n \gg 0$ in search of global stability. During this transition, the system moves along the potential gradient, geometrically redistributing the original energy, a result to be verified at the end of this section.

With this in mind, we propose a general relation for computing $\rho_{\rm vac}$, consisting of two terms: a kinetic contribution $T(\tilde{\chi}_0)$ and a potential contribution $V(\tilde{\chi}_0)$. This relationship is based on the Hamiltonian derived from the Lagrangian (25), given by $\tilde{\mathcal{H}} = \dot{\tilde{\chi}}\tilde{\pi}_\chi - \tilde{\mathcal{L}}$. Here, the conjugate momentum vanishes because the field does not explicitly depend on time $\tilde{\pi}_\chi = 0$, and the resulting Hamiltonian then becomes $\tilde{\mathcal{H}} = -\tilde{\mathcal{L}} = -\tilde{T} + \tilde{V}$.

The vacuum energy density $\rho_{\rm vac}$ is defined as the expectation value of the Hamiltonian density operator $\hat{\mathcal{H}}$. For the extended field $\tilde{\chi}$, this leads to the identity $\langle 0|\hat{\mathcal{H}}|0\rangle/\langle 0|0\rangle = \mathcal{H}(\tilde{\chi}_0)$ under the same considerations as in Section 3.1. Consequently, $\rho_{\rm vac}$ is computed as the spatial average over the ground state extent, namely $\rho_{\rm vac} = (\int d^3\xi \mathcal{H}(\tilde{\chi}_0)e^{-1/2\xi^i\xi^i})/(\int d^3\xi e^{-1/2\xi^i\xi^i})$. Thus, it can be written as

$$\rho_{\rm vac} = \langle \mathcal{H}(\tilde{\chi}_0)\rangle = -\langle T(\tilde{\chi}_0)\rangle + \langle V(\tilde{\chi}_0)\rangle, \tag{36}$$

computing the kinetic energy term derived from the Lagrangian, with $\delta = \ell_{\rm p}$, as

$$T(\tilde{\chi}_0) = \frac{1}{2\ell_{\rm p}^2}(\nabla_\xi\tilde{\chi}_0)^2, \tag{37}$$

given the homogeneity of the spatial coordinate terms, $(\nabla_\xi\tilde{\chi}_0)^2 = 3(\partial_{\xi^i}\tilde{\chi}_0)^2$, differentiating the ground state, $\partial_{\xi^i}\tilde{\chi}_0 = -\xi^i\tilde{\chi}_0$, and computing the average kinetic energy contribution, the following is obtained

$$\langle T(\tilde{\chi}_0)\rangle = \frac{3}{2\ell_{\rm p}^2}\langle(-\xi^i\tilde{\chi}_0)^2\rangle = \frac{3}{2\ell_{\rm p}^2}\frac{1}{(2\pi)^{3/2}}\int_{-\infty}^{\infty} d^3\xi(\xi^i)^2\tilde{\chi}_0^2 e^{-\frac{1}{2}\xi^i\xi^i}.$$

This kinetic contribution is computed entirely in the ξ-space, where $\tilde{\chi}_0$ is real and the Euclidean structure ensures normalizability, the result is

$$\langle T(\tilde{\chi}_0)\rangle = \frac{\sqrt{3}}{18}\frac{v_0^2}{\ell_{\rm p}^2}. \tag{38}$$

In exactly the same way, the potential term can be evaluated

$$V(\tilde{\chi}_0) = \frac{1}{2}m^2\tilde{\chi}_0^2 + \frac{1}{2}\lambda\tilde{\chi}_0^4, \tag{39}$$

computing the average of the potential and considering that the first term has already been calculated in (27)

$$\langle V(\tilde{\chi}_0)\rangle = \frac{1}{2}m^2\langle\tilde{\chi}_0^2\rangle + \frac{1}{2}\lambda\langle\tilde{\chi}_0^4\rangle = \frac{1}{2}m^2\left(\frac{\sqrt{3}}{9}v_0^2\right) + \frac{1}{2}\frac{1}{(2\pi)^{3/2}}\lambda\int_{-\infty}^{\infty} d^3\xi\tilde{\chi}_0^4 e^{-\frac{1}{2}\xi^i\xi^i}.$$

The potential term must be evaluated consistently with the constraint relations derived in Section 2.3, that is, $m_{n=0}^2 = 2/\ell_{\rm p}^2$ and $\lambda_{\delta=\ell_{\rm p}} = 1/(\ell_{\rm p}^2 v_0^2)$. Simplifying and substituting terms

$$\langle V(\tilde{\chi}_0)\rangle = \left(\frac{50\sqrt{3}+9\sqrt{5}}{450}\right)\frac{v_0^2}{\ell_{\rm p}^2}. \tag{40}$$

Inserting the computed values of $\langle T(\tilde{\chi}_0)\rangle$ and $\langle V(\tilde{\chi}_0)\rangle$ in (36), and denoting the numerical factor as $\alpha = (25\sqrt{3}+9\sqrt{5})/450 \approx 0.141$, the vacuum energy density yields the following result

$$\rho_{\text{vac}} = \alpha \frac{v_0^2}{\ell_p^2}. \tag{41}$$

Finally, a value for ρ_{vac} is obtained that depends exclusively on the amplitude v_0 of the ground state. This dependence reflects the essential role of v_0 in regulating the field configuration. However, it must be interpreted carefully because it is dynamical in nature; as suggested in Refs. [32–34]. The reason is that v_0 depends directly on n for $n \gg 0$, and therefore on the quantum state of the field and, consequently, on its evolution, as will be shown in Section 3.4. The order of magnitude of ρ_{vac} is primarily governed by the amplitude, but its sign is determined by the functional form of the ground state and the associated quantum constraints of the field. This plays a decisive role in ensuring a positive energy density.

Given the constraint relation $\lambda\delta^2 v_0^2 = 1$, the vacuum energy density (41) can also be written as

$$\rho_{\text{vac}}(\delta) = \alpha \frac{1}{\lambda \delta^2 \ell_p^2}. \tag{42}$$

This is a relation in which $\rho_{\text{vac}}(\delta)$ is expressed in terms of its fundamental parameter, implying that different densities can be obtained for different states of the scale. Then, given the magnitude of $\delta = \ell_p$, the vacuum energy density of the contracted state ρ_{vacc} represents the maximum achievable energy density and is defined as

$$\rho_{\text{vacc}} = \alpha \frac{1}{\lambda \ell_p^4}. \tag{43}$$

At the other end of the scale, corresponding to $\delta = R$, the vacuum energy density of the extended state ρ_{vace} is defined as

$$\rho_{\text{vace}} = \alpha \frac{1}{\lambda R^2 \ell_p^2}. \tag{44}$$

Considering these two densities, which essentially reflect a dynamic transition between two different scales, we can propose a ratio that matches the well-known hierarchy between the naive QFT vacuum density and the observed vacuum energy scale, leading to the following

$$\frac{\rho_{\text{vacc}}}{\rho_{\text{vace}}} = \frac{R^2}{\ell_p^2} \sim 10^{122}, \tag{45}$$

which spans 120 orders of magnitude [35]. This value reflects the well-known enormous gap between the energy density assigned to a primordial contracted state and that assigned to a fully expanded state; furthermore, within our framework, it points to a possible transition of the field $\tilde{\chi}$ from the $n = 0$ state to states with $n \gg 0$. Alternatively, ρ_{vacc} is expressed solely as a function of the mass using (29)

$$\rho_{\text{vacc}} = \alpha \frac{m_{\text{eff1}}^4}{4\lambda} = \alpha \frac{m_p^4}{\lambda}. \tag{46}$$

The same can be done for ρ_{vace}, but in this case using (32) it is obtained

$$\rho_{\text{vace}} = \alpha \frac{m_{\text{eff2}}^4}{9\lambda}. \tag{47}$$

Moreover, if ρ_{vace} is identified with the energy density ρ_Λ associated with the cosmological constant Λ, then $\rho_{\text{vace}} = \rho_\Lambda$, yielding $\alpha\hbar c/(\lambda R^2 \ell_p^2) = \Lambda c^4/(8\pi G)$. Expressing all terms in the same units, the result is given by

$$\lambda = \frac{8\alpha\pi}{\Lambda R^2} \approx 0.17. \tag{48}$$

The fact that $\lambda = 0.17$ is of order unity provides a compelling argument for the naturalness of the model, as discussed in Ref. [16]. Unlike standard cosmological constant fine-tuning, this result aligns with the expectation that fundamental dimensionless parameters should not deviate by many orders of magnitude from unity. Using this adjusted self-interaction constant and returning to (47), the identity establishes a numerical relation between density ρ_{vace} and mass m_{eff2}^4. This expression is often described in the specialized literature as a coincidence when m_{eff2} is assumed to represent a neutrino mass [16]. The numerical equivalence follows directly from the fundamental relation (42) and, in particular, the stabilization condition $\Delta m = 0$. Consequently, this yields a residual mass value that, as previously shown in Section 3.2, is remarkably close to that of light neutrinos.

Additionally, assuming a surface $S = 4\pi\delta^2$ that encloses the density $\rho_{\text{vac}}(\delta)$ as a bounding surface, and using the expression (42), with the Planck length expressed in terms of the gravitational constant G, yields

$$S\rho_{\text{vac}} = \frac{4\alpha\pi c^4}{\lambda G}. \tag{49}$$

It is noteworthy that this geometric relation emerges without invoking gravitational dynamics, as the model remains strictly in flat spacetime. Furthermore, by expressing (49) in SI units, it is intrinsically independent of $\hbar$ at macroscopic scales. This suggests that ρ_{vac} should not be interpreted as a standard stochastic quantum fluctuation; instead, it emerges as a deterministic geometric manifestation of the field configuration, as supported by (42). Our result aligns with the proposals of Refs. [36–39] concerning the link between energy density and a physical scale of the system; yet it offers a divergent theoretical origin. Whereas these authors often rely on holographic quantum counting, we interpret the constancy of the $S\rho_{\text{vac}}$ product as a structural coupling mediated by the self-interaction term $\tilde{\chi}_0^2\tilde{\chi}^2$. If ρ_{vac} is associated with the ground state $\tilde{\chi}_0$ via v_0, then the surface S represents the dynamical boundary of the field $\tilde{\chi}$, such that any transition between field states is necessarily reflected in the evolution of S.

Next, a vacuum energy E_0 is defined within a volume V, where the density ρ_{vac} is expressed as $\rho_{\text{vac}} = \Delta E_0/\Delta V$. If this volume element is represented by $\Delta V = S\Delta\ell$, the relation (49) can be written

$$\Delta E_0 = \frac{4\alpha\pi c^4}{\lambda G}\Delta\ell. \tag{50}$$

The resulting equation implies a necessary linear correspondence between a portion of the vacuum energy ΔE_0 and the geometric length element $\Delta\ell$. This relation integrates the Einstein tension c^4/G together with the coupling parameter λ. The energy surplus derived from the mass relaxation process does not dissipate; it constitutes the fundamental basis of the field. This dynamic identity ensures that the system is energetically self-consistent across all scales, enabling an equilibrated closed model. It is relevant to note that the numerical factor, given by $4\alpha\pi/\lambda \approx 10.4$, indicates that the emergent geometry requires approximately ten units of Planck energy for every unit of Planck length to maintain its macroscopic scale. This factor of order ten is not arbitrary; it arises from the spherical symmetry and intensity of the self-interaction λ. The structure of this linear relation between energy and length naturally explains the independence of $\hbar$ in ρ_{vac}. Since ΔE_0 is proportional to $\Delta\ell$, the dependencies on $\sqrt{\hbar}$ in both the Planck mass and the Planck length cancel exactly across all scales, unlike the scale dependent behavior observed in (42).

The term $\tilde{\chi}_0^2\tilde{\chi}^2$ from (14) establishes the state $\tilde{\chi}_0$ as the mandatory substrate for any field configuration. Furthermore, unlike the amplitudes C_n of the excited modes, v_0 is strictly fixed by the internal constraints. Thus, $\tilde{\chi}_0$ persists alongside any transient excitations, ordering the self-interaction processes between the different field states. An evolution occurs from ground state energy dominance toward the geometrization of higher order field excitations.

3.4. Dynamical Scale Evolution and Emergent Expansion

The geometric transformation (20) shares some similarities with the flat FLRW metric. As discussed in Section 3.2, the evolution of the field $\tilde{\chi}$ requires a dynamical law for the scale parameter δ to govern the transitions between its states. Consider the spatial distance between two coordinate points (x_1, y_1, z_1) and (x_2, y_2, z_2). Assume that the measurement is performed simultaneously at both points, then $t_1 = t_2$. If the flat FLRW metric can be written as $ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2)$, the resulting physical length L follows $L^2 = a^2(t)L_c^2$. This means that L expands in time through a scale factor $a(t)$ multiplying a fixed comoving length L_c. A similar calculation can be performed by applying (20) to points 1 and 2, which yields an observable physical distance L equivalent to the previous case. This analogy does not imply that δ replaces the cosmological scale factor, but rather that it induces an emergent dilation compatible with FLRW type expansion. Thus, it is possible to express the physical distance as $L^2 = (x_2^i - x_1^i)(x_2^i - x_1^i) = \delta^2 \xi^i \xi^i$, where the distance can be dilated by a parameter δ that rescales the transformed dimensionless Euclidean distance $\xi^i \xi^i$.

Accordingly, for a physical length L evaluated in both frameworks, $a(t)$ and δ play equivalent roles in the expansion and dilation processes. These factors arise from distinct mathematical formalisms. However, despite the methodological differences in their origin, both interpretations of the physical length L coincide. A time dependent parameter $\delta(t)$ transforms static scaling into a dynamical evolution, effectively describing the dilation of observable physical space [40]. This mechanism provides a field theoretic basis for the expansion observed in the FLRW description. To this end, let us evolve both sides of equality (33) in dimensionless time τ, resulting in

$$\frac{\Delta\delta}{\Delta\tau} = 2\ell_\mathrm{P}\frac{\Delta n}{\Delta\tau} = 2\ell_\mathrm{P}\frac{1}{\Delta\tau}. \tag{51}$$

Describing the dynamical evolution in the $n \gg 0$ regime requires assuming that transitions between states $\tilde{\chi}_n$ occur sequentially and in an orderly manner. Consequently, each quantum jump n to $n+1$ is necessarily associated with a physical time increment Δt of the order of Planck time t_P. This assignment is not arbitrary but mandatory to preserve the consistency of the model. Since the system evolves from the minimum scale $\delta \approx \ell_\mathrm{P}$ to the current scale $\delta \approx R$ through $n \approx 10^{61}$ levels, the accumulated physical time $t = \sum \delta_\mathrm{t} \Delta\tau$ for this number of steps accurately recovers the observed cosmological chronology. The dimensionless interval $\Delta\tau = \Delta t/\delta_\mathrm{t}$ acts as a differential counting element within the global framework $\delta_\mathrm{t} \sim R/c$. Thus, the inverse of the interval $1/\Delta\tau$ represents the fundamental frequency of jumps per unit of temporal scale, providing a natural justification for the proportionality between the macroscopic expansion rate and the quantum transition probability, namely

$$\frac{1}{\Delta\tau} \propto P(n \to n+1) = |\langle n+1|\hat{N}(\varepsilon)|n\rangle|^2. \tag{52}$$

Here, the transition operator used is $\hat{N}(\varepsilon)$, this is the number operator $\hat{N}$ defined in (23), but with the aim of adding a perturbation term ε to destabilize the system and induce transitions between states. Since the Hamiltonian is not easily expressed through the ladder pair $\hat{a}$ and $\hat{a}^\dagger$, we adopt the number operator as the fundamental observable that governs the field configuration. In this scheme, the transition between states is treated as a discrete jump in the excitation level, which drives the bonding dynamics, rescaling the emergent metric with each step. Therefore, it can be written as

$$\hat{N}(\varepsilon) = (\hat{a}^\dagger + \varepsilon)(\hat{a} + \varepsilon), \tag{53}$$

how the operator $\hat{N}$, by itself, cannot induce transitions between different field states; thus, a perturbation is introduced to the creation and annihilation operators, whose nature and origin will be clarified later. The calculation proceeds via the orthonormality of states $|n\rangle$ and the relations in (22)

$$\langle n+1|\hat{N}(\varepsilon)|n\rangle = \langle n+1|(\hat{a}^\dagger + \varepsilon)(\hat{a} + \varepsilon)|n\rangle = \langle n+1|\hat{N} + \varepsilon(\hat{a}^\dagger + \hat{a}) + \varepsilon^2|n\rangle,$$

where all terms vanish except for $\langle n+1|\hat{a}^\dagger|n\rangle = \sqrt{n+1}$. In the limit $n \gg 1$, yields $\langle n+1|\hat{N}(\varepsilon)|n\rangle = \varepsilon\sqrt{n}$. Finally, $1/\Delta\tau$ can be written by introducing a proportionality constant $\mathcal{A}$ as

$$\frac{1}{\Delta\tau} \propto \varepsilon^2 n = \mathcal{A}\varepsilon^2 n. \tag{54}$$

Returning to (51), substituting the value obtained for $1/\Delta\tau$, by introducing the variation of δ with respect to time t as $\dot{\delta} = \Delta\delta/\Delta t = (\Delta\delta/\Delta\tau)(c/R)$, and using (33) again to eliminate n, results in the following

$$\frac{\dot{\delta}}{\delta} = \frac{c}{R}\mathcal{A}\varepsilon^2. \tag{55}$$

The constants $\mathcal{A}$ and ε encapsulate the effective transition rate between neighboring configurations, and their order of magnitude can be determined as will be shown below. Ultimately, we find that the dynamics of the parameter δ is consistent with the Hubble law, similar to $a(t)$, and thus it is compatible with the dark energy processes involved in the standard model.

The nature of the perturbation ε follows from the definition of the operator $\hat{a}(\varepsilon) = \hat{a} + \varepsilon$, which, expressed as $1/\sqrt{2}((\xi + \sqrt{2}\varepsilon) + \partial_\xi)$, identifies the relation $\xi(\varepsilon) = \xi + \sqrt{2}\varepsilon$. Consequently, ε appears as a dispersion σ_ε of the variable ξ, or equivalently, ε manifests itself as an intrinsic fluctuation of the system. Defining the perturbation limit as the dispersion of the ground state profile, according to (16), yields

$$\xi(\varepsilon) = \xi + \sigma_\varepsilon \Rightarrow \sqrt{2}\varepsilon = \sigma_\varepsilon \approx 1 \Rightarrow \varepsilon \approx \frac{1}{\sqrt{2}}. \tag{56}$$

The dynamical expansion results from the propagation of these fluctuations through the field, triggering transitions to higher states. This process is supported by a mathematical structure in which transition probabilities are proportional to the excitation number, driving an ordered extension of the field and the subsequent dilation of the surrounding space.

Concerning the value of $\mathcal{A}$ according to (55), the factor c/R acts as the fundamental frequency that scales the quantum probability to the macroscopic domain. It is physically consistent to assume that the proportionality constant $\mathcal{A}$ is of the order of unity $\mathcal{A} \approx 1$. This choice is justified by normalization of the system in the transformed ξ-space, where the interaction scales are reduced to unity. Furthermore, it is supported by the highly ordered character of the transitions in the $n \gg 0$ regime, in which the system evolves almost deterministically level by level. With these values, and considering that both $\mathcal{A}$ and ε^2 are of the order of unity, we can rewrite (55) as follows

$$H_0 = \frac{\dot{\delta}}{\delta} \sim \frac{c}{R}. \tag{57}$$

This relation is consistent with the expected order of magnitude of the Hubble parameter H_0 at late times.

In this context, $\Delta m = 0$, in (30), define the moment when the mass m has relaxed and stabilized, we interpret it as a regime change where the field enters an expansionary stage described by (57); a phase that in the standard model is associated with the dark energy domain [41]. In fact, the order of magnitude of this mass given in (32) coincides with that proposed by some authors, such as Ref. [42]. Imposing a mass stabilization condition on the field quantum constraints yields an evolutionary mechanism consistent with the FLRW metric. Two perfectly synchronized phases emerge from this process without forced dynamics [43]. A closer inspection of these implications reveals how $\Delta m = 0$ leads to a density evolving inversely to volume, $\rho \propto 1/\delta^3$, accompanied by a vanishing pressure $p = 0$. This identifies a state where the density evolves freely and without exerting pressure on its surroundings, a notable feature of dark matter once it enters the expansion regime driven by dark energy. Thus, a qualitative link is suggested between the stabilization of m and the onset of the expansion driven by δ, pointing to a unified description of dark matter and dark energy within the same field framework. Moreover, since the phase transition entails a significant energy redistribution, the overall activity of the system is reduced following the global equilibrium. The resulting mechanism aligns with the proposals of Mass Varying Neutrinos [44,45]; however, unlike those models, our approach does not require an external neutrino field, as the dynamics arise instead from intrinsic ground state fluctuations. By considering the extended field as the driver and regulator of the dynamics, we find that the energy relaxation during the phase transition facilitates conditions for high quantum occupation and coherent configurations of the vacuum state. Accordingly, the residual mass m_{eff2} is expected to undergo gravitational condensation into bosonic halos; these structures represent stable and localized remnants of field self-organization and populate the dark matter sector. This process is consistent with the "nugget formation" proposed in Ref. [46], albeit under a different interpretation. Thus, the emergence of cold dark matter and the expansion driven by dark energy are intrinsically linked through $\Delta m = 0$. This requirement, internal to the model, provides a consistent and unified description of the dark sector.

4. Conclusions

This work establishes a framework where the physical field $\tilde{\chi}$ is not a mere passive background, but an extended entity with intrinsic scaling dynamics. It is demonstrated that the self-organization mediated by the coupling term $\tilde{\chi}_0^2\tilde{\chi}^2$ generates fundamental quantum constraints. Rather than mere mathematical requirements, these conditions define the allowed configurations of the system, ensuring that a coherent geometry emerges as a direct consequence of the internal dynamics.

A central aspect of this study is the mass relaxation mechanism within this substrate. It has been demonstrated how $\tilde{\chi}$ evolves from the Planck scale to a stable residual magnitude. Although comparable to light neutrino masses, such a result is interpreted as a remnant of the stabilization process rather than a particle mass. This approach provides a robust bridge between microscopic and macroscopic scales, arising from the self-consistent field properties.

The energy density is treated as a persistent expectation value of the vacuum state. The present analysis demonstrates that ρ_{vac} stems from a geometric manifestation of the field configuration, strictly governed by the underlying constraints. A geometric-energetic correspondence has been established that ensures the exact cancelation of $\hbar$ across all scales, arising as a structural property of the system. Although limited to flat spacetime, these results link the derived relations to observational estimates of the cosmological vacuum density.

Furthermore, the scale parameters are not static and exhibit a dynamical behavior governed by transitions between neighboring configurations. By treating these transitions as the result of intrinsic ground state fluctuations, an effective expansion rate compatible with a late-time accelerated regime is obtained. This provides an interpretation of how large-scale evolution emerges from quantum processes without requiring external cosmological drivers.

Finally, our results suggest that $\tilde{\chi}$ constitutes an extended entity dominated by its ground state profile $\tilde{\chi}_0$. Once the mass stabilization condition $\Delta m = 0$ is reached, the remnants of this process constitute the fundamental properties of the emergent spacetime. Under this unified perspective, the dark sector is not composed of independent substances; rather, dark matter and dark energy arise as distinct manifestations of the field transition toward a stable macroscopic state.

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References

1. Peskin, M.E.; Schroeder, D.V. *An Introduction to Quantum Field Theory*; Addison-Wesley, 1995.
2. Ziaeepour, H. Comparing quantum gravity models: String theory, loop Quantum Gravity, and entanglement gravity versus SU(∞)-QGR. *Symmetry* **2022**, *14*, 58, [2109.05757].

3. Vaid, D. Connecting loop quantum gravity and string theory via quantum geometry. In Proceedings of the Springer Proc. Phys. (DAE-BRNS High Energy Physics Symposium 2018), 2021, pp. 391–399, [1711.05693].
4. Kleinert, H.; Schulte-Frohlinde, V. *Critical Properties of ϕ^4-Theories*; World Scientific, 2001.
5. Glimm, J.; Jaffe, A. A quantum field theory without cutoffs. I. *Phys. Rev.* **1968**, *176*, 1945–1971.
6. Denton, P.B. Techniques for solving static Klein-Gordon equation with self-interaction $\lambda\phi^4$ and arbitrary spherical source terms. *Phys. Lett. B* **2024**, *855*, 138860, [2301.11106].
7. Bojowald, M. Quantum cosmology: A review. *Rep. Prog. Phys.* **2015**, *78*, 023901, [1501.04899].
8. Arraut, I.; Mehta, A.K. Non-thermodynamic black hole evaporation. *arXiv e-prints* **2025**, [arXiv:hep-th/2505.06859].
9. Wetterich, C. Dark energy evolution from quantum gravity. *Phys. Rev. D* **2024**, *110*, 043532, [2407.03465].
10. Asenjo, F.A.; Hojman, S.A. Light-like propagation of self-interacting Klein–Gordon fields in cosmology. *Eur. Phys. J. Plus* **2022**, *137*, 20, [2106.08236].
11. Cespedes, S.; Qin, Z.; Wang, D.G. $\lambda\phi^4$ as an effective theory in de Sitter. *arXiv e-prints* **2025**, [2510.25826].
12. Felder, G.N.; Navros, O. Inflaton Fragmentation after $\lambda\phi^4$ inflation. *JCAP* **2007**, *02*, 014, [hep-ph/0701128].
13. Panotopoulos, G.; Videla, N. Warm $\lambda\phi^4$ inflationary universe model in light of Planck 2015 results. *Eur. Phys. J. C* **2015**, *75*, 564, [1510.06981].
14. Starobinsky, A.A.; Yokoyama, J. Equilibrium state of self-interacting scalar field in the Sitter background. *Phys. Rev. D* **1994**, *50*, 6357–6368, [astro-ph/9407016].
15. Gleiser, M.; Ramos, R.O. Microphysical approach to nonequilibrium dynamics of quantum fields. *Phys. Rev. D* **1994**, *50*, 2441–2455, [hep-ph/9311278].
16. LeClair, A. Comment on the cosmological constant for $\lambda\phi^4$ theory in d spacetime dimensions. *arXiv e-prints* **2023**, [2304.13075].
17. Parker, L. Quantized fields and particle creation in expanding universes. I. *Phys. Rev.* **1969**, *183*, 1057.
18. Fiscaletti, D.; Licata, I. Scales and hierarchies: Planckian signature in standard model. *Particles* **2024**, *7*, 25.
19. Lombriser, L. Cosmology in Minkowski space. *Class. Quantum Grav.* **2023**, *40*, 155005, [2306.16868].
20. Arraut, I. Solving the dark energy problem via symmetry constraints. *Eur. Phys. J. C* **2025**, *85*, 475, [2412.13219].
21. Santos, E. Effects of the quantum vacuum at a cosmic scale and of dark energy. *Entropy* **2024**, *26*, 1042, [2412.06876].
22. Weinberg, S. The Cosmological constant problems. *Rev. Mod. Phys.* **1989**, *61*, 1–23.
23. Snyder, H.S. Quantized space-time. *Phys. Rev.* **1947**, *71*, 38–41.
24. Reuter, M.; Saueressig, F. Quantum Einstein gravity. *New J. Phys.* **2012**, *14*, 055022, [1202.2274].
25. Bonanno, A.; et al. Critical reflections on asymptotically safe gravity. *Front. in Phys.* **2020**, *8*, 269, [2004.06810].
26. Verlinde, E.P. On the Origin of gravity and the laws of Newton. *JHEP* **2011**, *2011*, 029, [1001.0785].
27. Padmanabhan, T. Thermodynamical aspects of gravity: New insights. *Rep. Prog. Phys.* **2010**, *73*, 046901, [0911.5004].
28. Sanchez, N.G. Quantum discrete levels of the universe from the early trans-planckian vacuum to the late dark energy. *Phys. Rev. D* **2021**, *104*, 123517, [2004.09257].
29. Bauer, F.; Schrempp, L. Relaxing neutrino mass bounds by a running cosmological constant. *JCAP* **2008**, *04*, 006, [0711.0744].
30. Chitov, G.Y.; August, T.; Natarajan, A.; Kahniashvili, T. Mass varying neutrinos, quintessence, and the accelerating expansion of the universe. *Phys. Rev. D* **2011**, *83*, 045033, [0911.1728].
31. Akhmedov, E.K. Vacuum energy and relativistic invariance. *Phys. Lett. B* **2002**, *536*, 276, [hep-th/0204048].
32. Klinkhamer, F.R.; Volovik, G.E. Dynamics of the quantum vacuum: Cosmology as relaxation to the equilibrium state. *Phys. Rev. D* **2011**, *84*, 047701, [1102.3152].
33. Shapiro, I.L.; Sola, J. Scaling behavior of the cosmological constant: Interface between quantum field theory and cosmology. *JHEP* **2002**, *02*, 006, [hep-th/0012227].
34. Bauer, F. Scaling laws for the cosmological constant. *Class. Quantum Grav.* **2005**, *22*, 3533, [gr-qc/0512007].
35. Zeldovich, Y.B. The cosmological constant and the theory of elementary particles. *Sov. Phys. Usp.* **1968**, *11*, 381. [Reprinted in Gen. Rel. Grav. 40, 1557 (2008)].
36. Cohen, A.G.; Kaplan, D.B.; Nelson, A.E. Effective field theory, black holes, and the cosmological constant. *Phys. Rev. Lett.* **1999**, *82*, 4971, [hep-th/9803132].
37. Li, M. A model of holographic dark energy. *Phys. Lett. B* **2004**, *603*, 1, [hep-th/0403127].
38. Jalalzadeh, S.; Moradpour, H.; Tebyanian, H. Holographic vacuum energy regularization and corrected entropy of de Sitter space. *Eur. Phys. J. C* **2023**, *83*, 114, [2405.12314].

 doi:10.20944/preprints202602.1978.v2

39. Brevik, I.; et al. Rips and regular future scenario with holographic dark energy: A comprehensive look. *Phys. Dark Univ.* **2024**, *46*, 101625, [2411.10984].
40. Frusciante, N.; Perenon, L. Effective field theory of dark energy: A review. *Phys. Rep.* **2020**, *857*, 1–63, [1907.03150].
41. Bozorgnia, N.; et al. Dark matter candidates and searches. *Can. J. Phys.* **2025**, *103*, 671–703, [2410.23454].
42. de Vega, H.J.; Sanchez, N.G. Dark energy is the cosmological quantum vacuum energy of light particles: the axion and the lightest neutrino. *arXiv e-prints* **2007**, [astro-ph/0701212].
43. Zlatev, I.; Wang, L.; Steinhardt, P.J. Quintessence, cosmic coincidence, and the cosmological constant. *Phys. Rev. Lett.* **1999**, *82*, 896–899, [astro-ph/9807002].
44. Fardon, R.; Nelson, A.E.; Weiner, N. Dark energy from Mass Varying Neutrinos. *JCAP* **2004**, *10*, 005, [astro-ph/0309800].
45. Simpson, F.; Jimenez, R.; Pena-Garay, C.; Verde, L. Dark energy from the motions of neutrinos. *JCAP* **2018**, *06*, 017, [1607.02515].
46. Gogoi, A.; Phukan, R.; Das, S. Early mass varying neutrino dark energy: Nugget formation and Hubble anomaly. *JCAP* **2021**, *01*, 033, [2005.11889].