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Article

Minimum Cost Design for Rectangular Isolated Footings Taking into Account That the Column Is Located in Any Part of the Footing

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Abstract: This work presents a new model to obtain the minimum cost design for a rectangular isolated footing, taking into account that the column is located in any part of the footing. The methodology is developed by integration to obtain the moments, bending shear and punching shear according to the American Concrete Institute ACI 318-14. This document presents the simplified and precise equations of the four moments, four bending shears and one punching shear acting on the footing. Some designs have been developed by the trial and error method to determine the footing dimensions, and later the thickness and steel area of the footing are obtained. Some authors present the minimum cost design for a rectangular isolated footing taking into account that the column is located in the center of gravity of the footing, and other authors present very complex algorithm. Numerical examples are presented to obtain the minimum cost design of rectangular isolated footings under biaxial bending, and some results are compared with those of other authors considering the same conditions. The new model presents a smaller contact area with the soil and a lower design cost than those presented by other authors.

Keywords: minimum cost design; minimum area; moments; bending shear; punching shear

1. Introduction

Footing or foundation is the structural member (sub-structure) that transfers the loads of the superstructure to the soil. Footings are used in various types of construction in structural engineering, such as buildings and bridges.

The main object of geotechnical and structural engineers is to obtain the smallest area and the minimum cost of the footing to support the loads imposed by the superstructure.

Optimization techniques have been successfully used in various foundation problems such as: Hannan et al. [1] showed a strategy to obtain minimum volume for reinforced concrete footing that support a wind load on the superstructure. Basudhar et al. [2, 3] investigated the optimal cost design for rigid raft foundation and circular isolated footings taking into account the cost of concrete, cost of excavations, cost of backfilling works and cost of steel reinforcement based in the mathematical programming problem using Powell's conjugate direction search method. Al-Douri [4] proposed an optimal model for the design of trapezoidal combined footings taking into account the cost of concrete, cost of excavations, cost of backfilling works and cost of steel reinforcement. Wang and Kulhawy [5] proposed the minimum construction cost as objective function for a foundation considering the serviceability limit state (SLS), ultimate limit state (ULS), and economics. Rizwan et al. [6] developed an optimal design of reinforced concrete combined footings using computational procedure, and objective function includes cost of excavation, filling, concrete and reinforcement for the footings. Jelušič and Žlender [7-8] developed an optimal design for reinforced pad and strip foundations based on optimizations as are: multiparametric, mixed-integer, and nonlinear programming (MINLP). Velázquez-Santillán et al. [9] presented the optimal design for reinforced

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concrete rectangular combined to obtain the minimum cost design, the radius, the thickness, and the reinforced steel areas in both directions. López-Chavarría et al. [10] designed an optimal model for circular isolated footings to obtain the minimum cost design, the radius, the thickness, and the reinforced steel areas in both directions. Khajuria and Singh [11] proposed a metaheuristic optimization for the design of reinforced concrete footings supporting a column based of the gravitational search algorithm (GSA). Al-Ansari and Afzal [12] presented a simplified analysis for the design of irregularly shaped reinforced concrete footings with an eccentric load subject to biaxial bending that support a square column, the shapes of the footings studied are: square, triangular, circular, and trapezoidal. Kashani et al. [13] studied the optimal design of reinforced concrete combined footings according to the American Concrete Institute ACI 318-05, using five swarm intelligence algorithms such as: particle swarm optimization (PSO), accelerated particle swarm optimization (APSO), whale optimization algorithm (WOA), ant lion optimizer (ALO), and moth flame optimization (MFO). Nigdeli and Bekdas [14] investigated the orientation of the column mounted on the footings using the optimal design based harmony search algorithm. Kamal et al. [15] presented a study on the optimal design of footing systems applied in the prefabricated industrial buildings, and the concrete bracket system using as the connection type in the reinforced concrete frames.

The main contributions through optimization techniques in the design of rectangular isolated footings are: Camp and Assadollahi [16] presented the optimal design of reinforced concrete footings according to the American Concrete Institute ACI 318-11 subjected to vertical load using a hybrid Big Bang-Big Crunch (BB-BC) algorithm. Luévanos-Rojas et al. [17] developed an optimal model for the design of rectangular isolated footings using computation to solve various integrals according to the American Concrete Institute ACI 318-13. Galvis and Smith-Pardo [18] presented design aids and simplified closed form equations to obtain the coupled axial load and biaxial moment capacity of hollow and solid circular and rectangular shallow foundations. Rawat and Mittal, Rawat et al. [19, 20] developed a simplified approach for the design of reinforced concrete rectangular isolated footings with eccentric load according to the American Concrete Institute ACI 318-14. Solorzano and Plevris [21] studied genetic algorithms (GA) with a selection technique applied to the optimal design of reinforced concrete rectangular isolated footings according to the American Concrete Institute ACI 318-19. Chaudhuri and Maity [22] investigated the optimal design of reinforced concrete isolated footings according to the Indian Standard (IS) 4568:2000, using two swarm intelligence algorithms such as: genetic algorithms (GA) and unified particle swarm optimization (UPSO). Also, metaheuristic optimization has been enthusiastically applied in the design of reinforced concrete rectangular isolated footings as a practical tool in parametric research used different algorithms, such as Harmony Search (HS), Teaching Learning Based Optimization (TLBO) algorithm and Flower Pollination Algorithm (FPA) [23], and Evolutionary Algorithm (EA) and the Genetic Algorithm (GA) [24].

This study shows a new model to obtain the minimum cost design for a rectangular isolated footing subjected to biaxial bending by the application of the load and moments of the column and also the column is located in any part of the footing. The least cost design is carried out in two stages: First stage: the objective function is the minimum area of contact with the soil; the known or constant parameters (independent variables) are σ_{max}, P, e_x. e_y, M_x and M_y; the unknown or decision variables (dependent variables) are: A_{min}, h_x and h_y. Second stage: the objective function is the minimum cost; the known or constant parameters (independent variables) are σ_{max}, P_u, M_{ux} and M_{uy}, h_x and h_y; the unknown or decision variables (dependent variables) are: C_{min}, d, A_{sx} and A_{sy}. Numerical examples are shown to find the minimum cost design of rectangular isolated footings subjected to biaxial bending for four example: Example 1: When the column is located at the center of gravity of the footing. Example 2: When the column is located at the limit of the footing in the Y direction. Example 3: When the column is located at the limit of the footing in the X direction. Example 4: When the column is located at the limit of the footing in the X and Y directions or at a corner. Also, some results are compared with those of other authors considering the same conditions to observe the differences.

2. Formulation of the model

The loads and moments are obtained from a structural analysis, where the analysis of the structural framework is developed by any of the known methods (stiffness method, slope-deflection method and Hardy Cross method) that include dead, live, wind, and earthquake loads.

Figure 1 shows a rectangular isolated footing subjected to biaxial bending and the column located in any part of the footing on elastic soils, and assuming a linear distribution of soil pressure.

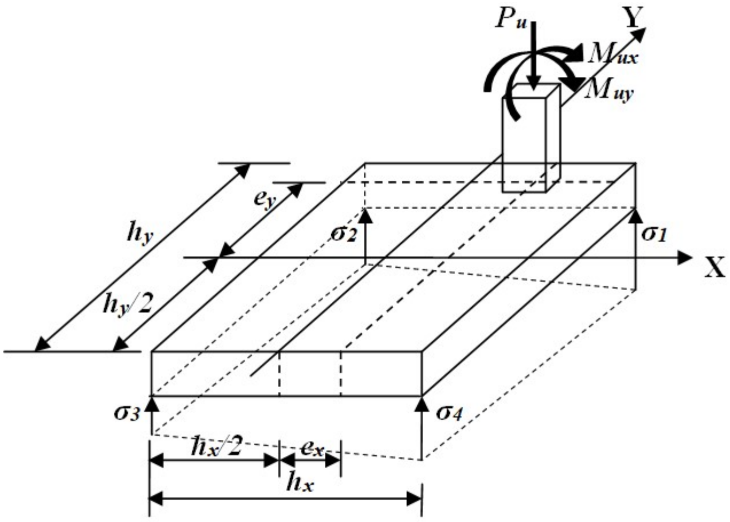

Figure 1. Rectangular isolated footing subjected to biaxial bending. Where: e_x and e_y are the location coordinates of the column.

General equation for any type of footing subjected to biaxial bending is:

$$\sigma = \frac{P}{A} + \frac{M_{xT} y}{I_x} + \frac{M_{yT} x}{I_y}, \quad (1)$$

where: σ is the pressure of the soil in any part of the footing, P is the axial load, M_{xT} and M_{yT} are the total moments on the X and Y axes, A is the area or surface of contact of the footing with the soil, I_x and I_y are the moments of inertia of the footing on the X and Y axes, x and y are the coordinates where it is desired to find the soil pressure on the footing.

Equation (1) assumes that the contact area of the footing with the soil works totally under compression.

Now, substituting $A = h_x h_y$, $I_x = h_x h_y^3/12$, $I_y = h_y h_x^3/12$, $M_{xT} = M_x + Pe_y$, $M_{yT} = M_y + Pe_x$, and the corresponding coordinates at each corner of the rectangular isolated footing, the pressures are obtained:

$$\sigma_1 = \frac{P}{h_x h_y} + \frac{6(M_x + Pe_y)}{h_x h_y^2} + \frac{6(M_y + Pe_x)}{h_x^2 h_y}, \quad (2)$$

$$\sigma_2 = \frac{P}{h_x h_y} + \frac{6(M_x + Pe_y)}{h_x h_y^2} - \frac{6(M_y + Pe_x)}{h_x^2 h_y}, \quad (3)$$

$$\sigma_3 = \frac{P}{h_x h_y} - \frac{6(M_x + Pe_y)}{h_x h_y^2} - \frac{6(M_y + Pe_x)}{h_x^2 h_y}, \quad (4)$$

$$\sigma_4 = \frac{P}{h_x h_y} - \frac{6(M_x + Pe_y)}{h_x h_y^2} + \frac{6(M_y + Pe_x)}{h_x^2 h_y}, \quad (5)$$

where: h_x and h_y are the sides of the footing in the directions X and Y.

2.1. Optimal area for rectangular isolated footing

The objective function for the minimum area "A_{min}" is obtained:

$$A_{min} = h_x h_y, \tag{6}$$

The constraint functions are: the equations (2) to (5), $0 \leq \sigma_1, \sigma_2, \sigma_3, \sigma_4 \leq \sigma_{max}$ (allowable bearing capacity of the soil).

Note: The base area of the footing will be determined from unfactored load P, the unfactored moments M_x and M_y transmitted by footing to the soil.

2.2. Optimal cost for rectangular isolated footing

Substituting $A = h_x h_y$, $I_x = h_x h_y^3/12$, $I_y = h_y h_x^3/12$, $M_{xT} = M_{ux} + P_u e_y$, $M_{yT} = M_{uy} + P_u e_x$ into Equation (1) to obtain the pressure in function of the coordinates for a rectangular isolated footing. The pressure equation is:

$$\sigma_u(x,y) = \frac{P_u}{h_x h_y} + \frac{12(M_{ux} + P_u e_y)y}{h_x {h_y}^3} + \frac{12(M_{uy} + P_u e_x)x}{h_y {h_x}^3}. \tag{7}$$

where: P_u is the factored load; M_{ux} and M_{uy} are the factored moments on the X and Y axes.

2.2.1. Equations for moments

Figure 2 shows the critical sections for moments of a rectangular isolated footing subjected to biaxial bending and the column located in any part of the footing. The critical sections for moments are presented on the faces of the columns.

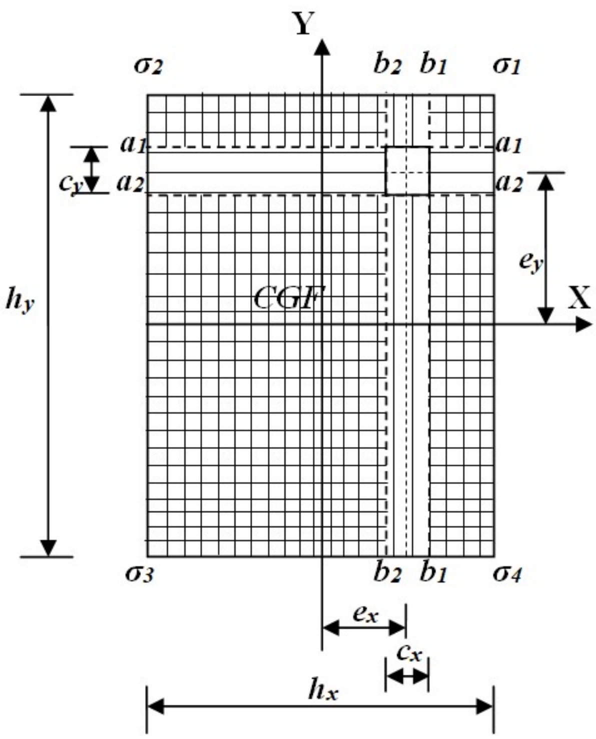

Figure 2. Critical sections for moments.

General equation for the moment on the a_1 axis "M_{ua1}" is obtained as follows:

$$M_{ua1} = -\int_{e_y+\frac{c_y}{2}}^{\frac{h_y}{2}} \int_{-\frac{h_x}{2}}^{\frac{h_x}{2}} \sigma_u(x,y)\left(y - e_y - \frac{c_y}{2}\right) dxdy. \tag{8}$$

Substituting the Equation (7) into Equation (8) M_{ua1} is obtained:

$$M_{ua1} = -\frac{\left\{P_u {h_y}^2 \left[\left(h_y - c_y\right)^2 - 4e_y\left(h_y - e_y - c_y\right)\right] + 2\left(P_u e_y + M_{ux}\right)\left[2{h_y}^3 - 3{h_y}^2\left(2e_y + c_y\right) + \left(2e_y + c_y\right)^3\right]\right\}}{8{h_y}^3}. \quad (9)$$

where: c_x and c_y are the sides of the column in the directions X and Y.

General equation for the moment on the a_2 axis "M_{ua2}" is obtained as follows:

$$M_{ua2} = \frac{P_u c_y}{2} + M_{ux} - \int_{e_y - \frac{c_y}{2}}^{\frac{h_y}{2}} \int_{-\frac{h_x}{2}}^{\frac{h_x}{2}} \sigma_u(x, y)\left(y - e_y + \frac{c_y}{2}\right) dxdy. \quad (10)$$

Substituting the Equation (7) into Equation (10) M_{ua2} is obtained:

$$M_{ua2} = -\frac{\left\{P_u {h_y}^2 \left[\left(h_y - c_y\right)^2 + 4e_y\left(h_y + e_y - c_y\right)\right] - 2\left(P_u e_y + M_{ux}\right)\left[2{h_y}^3 + 3{h_y}^2\left(2e_y - c_y\right) - \left(2e_y - c_y\right)^3\right]\right\}}{8{h_y}^3}. \quad (11)$$

General equation for the moment on the b_1 axis "M_{ub1}" is obtained as follows:

$$M_{ub1} = -\int_{-\frac{h_y}{2}}^{\frac{h_y}{2}} \int_{e_x + \frac{c_x}{2}}^{\frac{h_x}{2}} \sigma_u(x, y)\left(x - e_x - \frac{c_x}{2}\right) dxdy. \quad (12)$$

Substituting the Equation (7) into Equation (12) M_{ub1} is obtained:

$$M_{ub1} = -\frac{\left\{P_u {h_x}^2 [(h_x - c_x)^2 - 4e_x(h_x - e_x - c_x)] + 2\left(P_u e_x + M_{uy}\right)[2{h_x}^3 - 3{h_x}^2(2e_x + c_x) + (2e_x + c_x)^3]\right\}}{8{h_x}^3}. \quad (13)$$

General equation for the moment on the b_2 axis "M_{ub2}" is obtained as follows:

$$M_{ub2} = \frac{P_u c_x}{2} + M_{uy} - \int_{-\frac{h_y}{2}}^{\frac{h_y}{2}} \int_{e_x - \frac{c_x}{2}}^{\frac{h_x}{2}} \sigma_u(x, y)\left(x - e_x + \frac{c_x}{2}\right) dxdy. \quad (14)$$

Substituting the Equation (7) into Equation (14) M_{ub2} is obtained:

$$M_{ub1} = -\frac{\left\{P_u {h_x}^2 [(h_x - c_x)^2 - 4e_x(h_x - e_x - c_x)] + 2\left(P_u e_x + M_{uy}\right)[2{h_x}^3 - 3{h_x}^2(2e_x + c_x) + (2e_x + c_x)^3]\right\}}{8{h_x}^3}. \quad (15)$$

2.2.2. Equations for bending shear

Figure 3 shows the critical sections for bending shear of a rectangular isolated footing subjected to biaxial bending and the column located in any part of the footing. The critical sections for bending shear occur at a distance d (effective depth of footing) from the column faces.

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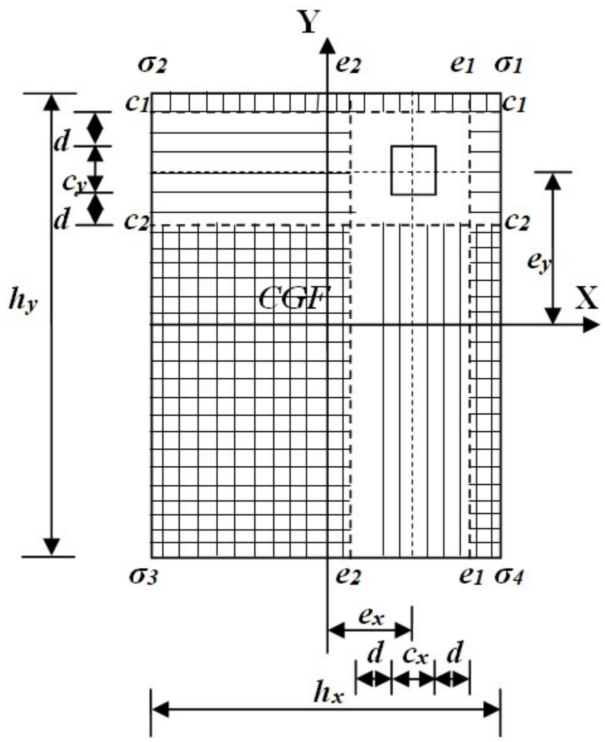

Figure 3. Critical sections for bending shear.

General equation for the bending shear on the c_1 axis "V_{ufc1}" is obtained as follows:

$$V_{ufc1} = -\int_{e_y+\frac{c_y}{2}+d}^{\frac{h_y}{2}} \int_{-\frac{h_x}{2}}^{\frac{h_x}{2}} \sigma_u(x,y)dxdy. \tag{16}$$

Substituting the Equation (7) into Equation (16) V_{fc1} is obtained:

$$V_{ufc1} = -\frac{\left\{P_u {h_y}^2\left[h_y - c_y - 2\left(e_y + d\right)\right] + 3\left(P_u e_y + M_{ux}\right)\left[{h_y}^2 - \left(2e_y + c_y + 2d\right)^2\right]\right\}}{2{h_y}^3}. \tag{17}$$

General equation for the bending shear on the c_2 axis "V_{ufc2}" is obtained as follows:

$$V_{ufc2} = P_u - \int_{e_y-\frac{c_y}{2}-d}^{\frac{h_y}{2}} \int_{-\frac{h_x}{2}}^{\frac{h_x}{2}} \sigma_u(x,y)dxdy. \tag{18}$$

Substituting the Equation (7) into Equation (18) V_{ufc2} is obtained:

$$V_{ufc2} = \frac{\left\{P_u {h_y}^2\left[h_y - c_y + 2\left(e_y - d\right)\right] - 3\left(P_u e_y + M_{ux}\right)\left[{h_y}^2 - \left(2e_y - c_y - 2d\right)^2\right]\right\}}{2{h_y}^3}. \tag{19}$$

General equation for the bending shear on the e_1 axis "V_{ufe1}" is obtained as follows:

$$V_{ufe1} = -\int_{-\frac{h_y}{2}}^{\frac{h_y}{2}} \int_{e_x+\frac{c_x}{2}+d}^{\frac{h_x}{2}} \sigma_u(x,y)dxdy. \tag{20}$$

Substituting the Equation (7) into Equation (20) V_{ufe1} is obtained:

$$V_{ufe1} = -\frac{\{P_u {h_x}^2[h_x - c_x - 2(e_x + d)] + 3\left(P_u e_x + M_{uy}\right)[{h_x}^2 - (2e_x + c_x + 2d)^2]\}}{2{h_x}^3}. \tag{21}$$

General equation for the bending shear on the e_2 axis "V_{ufe2}" is obtained as follows:

$$V_{ufe2} = P_u - \int_{-\frac{h_y}{2}}^{\frac{h_y}{2}} \int_{e_x - \frac{c_x}{2} - d}^{\frac{h_x}{2}} \sigma_u(x,y)dxdy. \quad (22)$$

Substituting the Equation (7) into Equation (22) V_{ufe2} is obtained:

$$V_{ufe2} = \frac{\{P_u {h_x}^2[h_x - c_x + 2(e_x - d)] - 3(P_u e_x + M_{uy})[{h_x}^2 - (2e_x - c_x - 2d)^2]\}}{2{h_x}^3}. \quad (23)$$

2.2.3. Equations for punching shear

Figure 4 shows the critical section for punching shear of a rectangular isolated footing subjected to biaxial bending and the column located in any part of the footing. The critical section for punching shear occurs at a perimeter formed at a distance d/2 from the column faces in two directions.

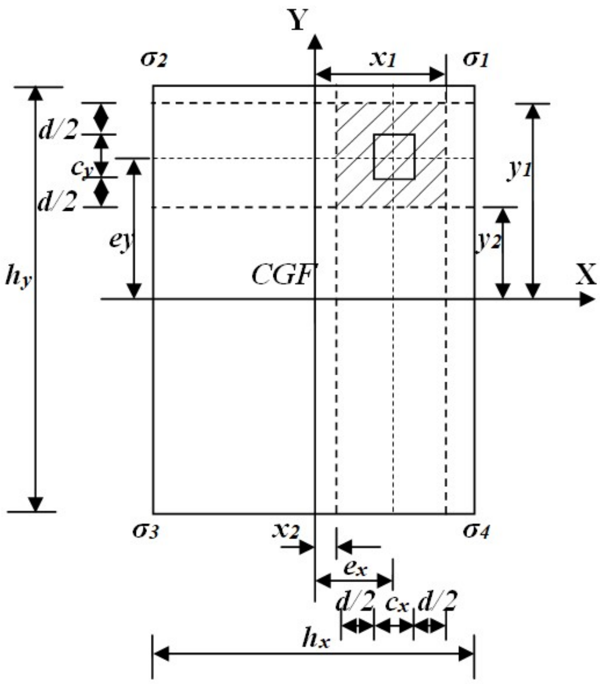

Figure 4. Critical sections for punching shear.

General equation for the punching shear "V_p" is obtained as follows:

$$V_{up} = P_u - \int_{y_2}^{y_1} \int_{x_2}^{x_1} \sigma_u(x,y)dxdy. \quad (24)$$

Substituting the Equation (7) into Equation (24) V_p is obtained:

$$V_{up} = \frac{P_u {h_x}^3 {h_y}^3 - (x_1 - x_2)(y_1 - y_2)\{P_u {h_x}^2 {h_y}^2 - 6[(P_u e_y + M_{ux}){h_x}^2(y_1 + y_2) + (P_u e_x + M_{uy}){h_y}^2(x_1 + x_2)]\}}{{h_x}^3 {h_y}^3}. \quad (25)$$

where: x_1, x_2, y_1 and y_2 are the coordinates of the corners of the critical perimeter in the directions X and Y.

2.2.4. Objective function to minimize the cost

The minimum cost "C_{min}" is equal to steel cost more the concrete cost (The cost includes materials and manpower). The minimum cost of a rectangular isolated footing is:

$$C_{min} = V_c C_c + V_s \gamma_s C_s, \quad (26)$$

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where: C_{min} = Minimum cost in Dollars, C_c = Ready mix concrete cost in Dollars/m³, C_s = Steel cost in Dollars/kN, V_c = Concrete volume in m³, V_s = Steel volume in m³, γ_s = Concrete density is 24 kN/m³, γ_s = Steel density is 78 kN/m³.

The volumes for a rectangular isolated footing are:

$$V_s = A_{sx}h_x + A_{sy}h_y, \tag{27}$$

$$V_c = h_x h_y (d + r), \tag{28}$$

where: r is concrete cover.

Substituting the $\gamma_s C_s = \alpha C_c$ (where $\alpha = \gamma_s C_s / C_c$), and Equations (27) and (28) into Equation (26) to find "C_{min}". Equation is obtained:

$$C_{min} = C_c\left[h_x h_y (d + r) + (\alpha - 1)\left(A_{sx}h_x + A_{sy}h_y\right)\right], \tag{29}$$

2.2.5. Constraint functions

Equations for the design of a rectangular isolated footing are [25]:

Equations for the moments are:

$$|M_{ua1}|, |M_{ua2}| \leq \emptyset_f f_y d A_{sy}\left(1 - \frac{A_{sy}f_y}{1.7h_x d f'_c}\right), \tag{30}$$

$$|M_{ub1}|, |M_{ub2}| \leq \emptyset_f f_y d A_{sx}\left(1 - \frac{A_{sx}f_y}{1.7h_y d f'_c}\right), \tag{31}$$

where: f_y = Specified yield strength of reinforcement of steel (MPa), f'_c = Specified compressive strength of the concrete at 28 days (MPa) A_{sy} = Steel area in the Y direction, A_{sx} = Steel area in the X direction, $\emptyset_f$ = Bending strength reduction factor is 0.90.

Equations for the bending shear are:

$$|V_{ufc1}|, |V_{ufc2}| \leq 0.17\emptyset_v \sqrt{f'_c}\, h_x d, \tag{32}$$

$$|V_{ufe1}|, |V_{ufe2}| \leq 0.17\emptyset_v \sqrt{f'_c}\, h_y d. \tag{33}$$

where: $\emptyset_v$ = Shear strength reduction factor is 0.85.

Equation for the punching shear is:

$$|V_{up}| \leq \begin{cases} 0.17\emptyset_v\left(1 + \dfrac{2}{\beta_c}\right)\sqrt{f'_c}\, b_0 d \\ 0.083\emptyset_v\left(\dfrac{\alpha_s d}{b_0} + 2\right)\sqrt{f'_c}\, b_0 d, \\ 0.33\emptyset_v \sqrt{f'_c}\, b_0 d \end{cases} \tag{34}$$

where: β_c = Relationship of the long side between the short side of the column; b_0 = Perimeter for the punching shear (m); α_s = 20 for corner columns, α_s = 30 for edge columns, and α_s = 40 for interior columns.

Equations for the percentage of reinforcing steel are:

$$\rho_x, \rho_y \leq 0.75\left[\frac{0.85\beta_1 f'_c}{f_y}\left(\frac{600}{600 + f_y}\right)\right], \tag{35}$$

$$\rho_x, \rho_y \geq \begin{cases} \dfrac{0.25\sqrt{f'_c}}{f_y} \\ \dfrac{1.4}{f_y} \end{cases}, \tag{36}$$

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$$0.65 \leq \beta_1 = \left(1.05 - \frac{f'_c}{140}\right) \leq 0.85, \tag{37}$$

where: ρ_x and ρ_y = Percentage of reinforcing steel in the X and Y directions, β_1 is the factor relating depth of equivalent rectangular compressive stress block to neutral axis depth.

Equations for the reinforcing steel are:

$$A_{sy} = \rho_y h_x d, \tag{38}$$

$$A_{sx} = \rho_x h_y d. \tag{39}$$

Figure 5 shows the flowchart of the algorithm for the optimal design procedure of reinforced concrete rectangular isolated footing.

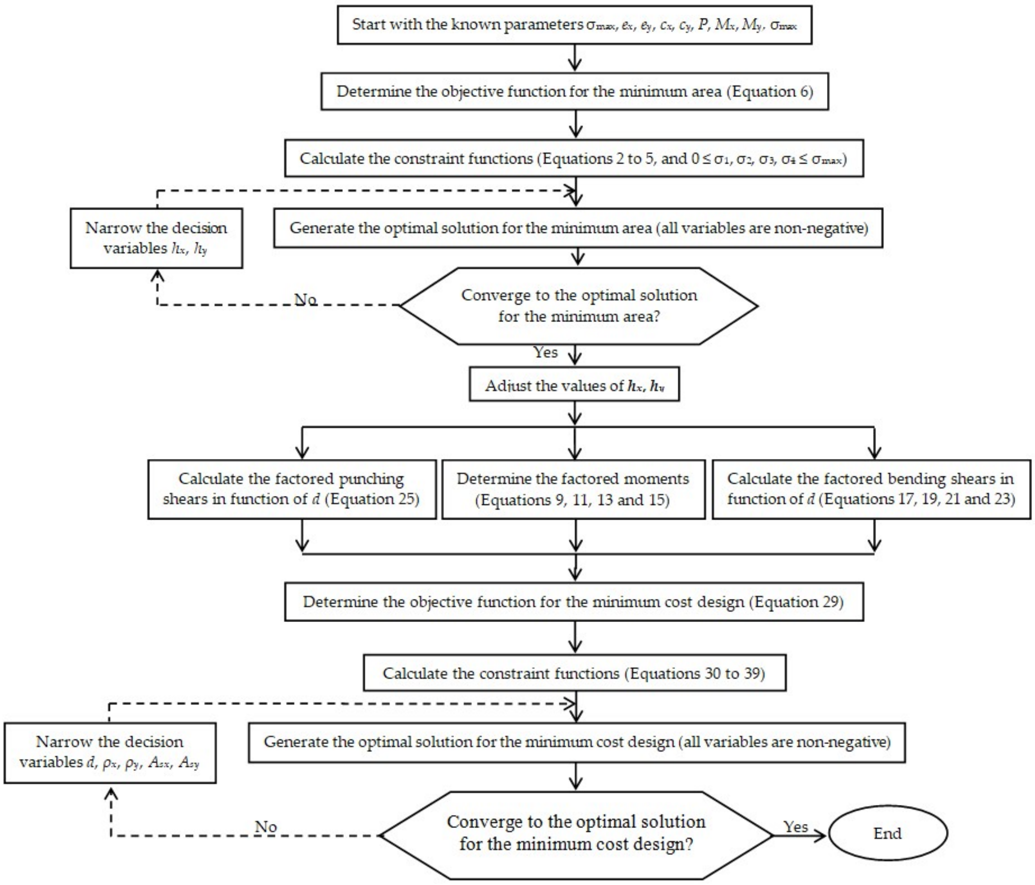

Figure 5. Flowchart for the procedure of rectangular isolated footing optimal design.

Figure 6 shows the flowchart for the use of Maple software for the reinforced concrete rectangular isolated footing optimal design.

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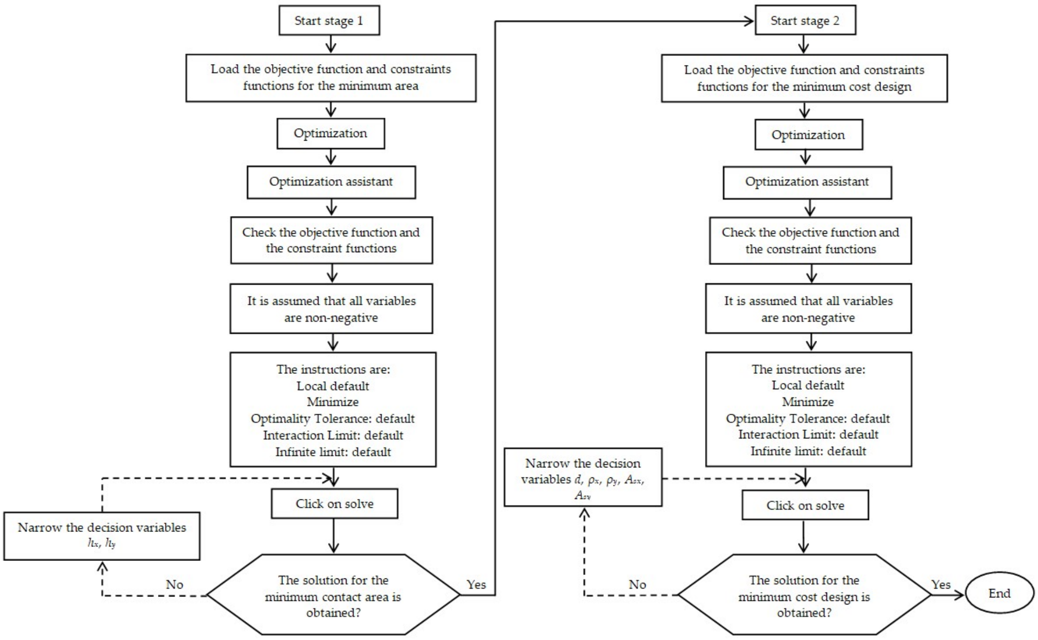

Figure 6. Flowchart for the use of Maple software for the rectangular isolated footing optimal design.

3. Numerical problems

Tables 1, 2, 3 and 4 present the four examples to obtain the rectangular isolated footing optimal design subjected to biaxial bending (An axial load, a moment on the X axis and a moment on the Y axis). Example 1: When the column is located at the center of gravity of the footing. Example 2: When the column is located at the limit of the footing in the Y direction. Example 3: When the column is located at the limit of the footing in the X direction. Example 4: When the column is located at the limit of the footing in the X and Y directions or at a corner. The general data for all footings are: $c_x = c_y = 0.40$ m, r = 0.08 m, $\sigma_{max} = 180$ kN/m², f′c = 21 Mpa, $f_y = 420$ Mpa and $\alpha = 90$.

Table 1. Optimal cost design of rectangular isolated footing ($e_x = 0$ and $e_y = 0$).

Example	h_x (m)	h_y (m)	σ_1 (kN/m²)	σ_2 (kN/m²)	σ_3 (kN/m²)	σ_4 (kN/m²)	A_{min} (m²)	P_u (kN)	M_{ux} (kN-m)	M_{uy} (kN-m)	A_{sx} (cm²)	A_{sy} (cm²)	d (m)	ρ_x	ρ_y	C_{min} ($)
1.1	2.55	3.80	176.29	103.44	30.11	102.96	9.69	1400	300	200	45.19	52.36	0.36	0.00333	0.00575	$7.03C_c$
1.2	2.45	3.65	177.49	95.33	12.61	94.77	8.94	1200	300	200	41.90	45.88	0.34	0.00333	0.00543	$6.20C_c$
1.3	2.40	3.60	173.61	86.81	0	86.81	8.64	1000	300	200	35.94	45.86	0.30	0.00333	0.00637	$5.52C_c$
1.4	3.00	4.50	88.89	44.44	0	44.44	13.50	800	300	200	35.51	62.77	0.24	0.00333	0.00883	$7.74C_c$

where: h_x and h_y are the fitted sides; σ_1, σ_2, σ_3 and σ_4 are the pressures at each corner of the footing due to the fitted sides; A_{min} is the minimum area; P_u, M_{ux} and M_{uy} are the factored load and moments; A_{sx} and A_{sy} are the steel areas in the X and Y directions; d is the effective depth of footing; ρ_x and ρ_y are the percentage of reinforcing steel in the X and Y directions; C_{min} is the minimum cost.

Table 2. Optimal cost design of rectangular isolated ($e_x = 0$ and $e_y = h_y/2 - c_y/2$).

Example	h_x (m)	h_y (m)	σ_1 (kN/m²)	σ_2 (kN/m²)	σ_3 (kN/m²)	σ_4 (kN/m²)	A_{min} (m²)	P_u (kN)	M_{ux} (kN-m)	M_{uy} (kN-m)	A_{sx} (cm²)	A_{sy} (cm²)	d (m)	ρ_x	ρ_y	C_{min} ($)
2.1	9.05	1.00	178.94	156.97	47.57	69.55	9.05	1400	-300	200	51.12	258.39	0.86	0.00596	0.00333	14.90C_c
2.2	6.95	1.00	178.92	141.66	74.32	111.58	6.95	1200	-300	200	39.66	167.88	0.73	0.00547	0.00333	9.54C_c
2.3	5.15	1.00	179.56	111.70	111.70	179.56	5.15	1000	-300	200	29.79	102.89	0.60	0.00496	0.00333	5.78C_c
2.4	4.00	1.15	179.35	81.52	81.52	179.35	4.60	800	-300	200	25.90	59.18	0.44	0.00507	0.00333	3.94C_c

Table 3. Optimal cost design of rectangular isolated ($e_x = h_x/2 - c_x/2$, $e_y = 0$).

Example	h_x (m)	h_y (m)	σ_1 (kN/m²)	σ_2 (kN/m²)	σ_3 (kN/m²)	σ_4 (kN/m²)	A_{min} (m²)	P_u (kN)	M_{ux} (kN-m)	M_{uy} (kN-m)	A_{sx} (cm²)	A_{sy} (cm²)	d (m)	ρ_x	ρ_y	C_{min} ($)
3.1	1.00	9.05	178.94	69.55	47.57	156.97	9.05	1400	200	-300	258.39	51.12	0.86	0.00333	0.00596	14.90C_c
3.2	1.00	6.95	178.92	111.58	74.32	141.66	6.95	1200	200	-300	167.88	39.66	0.73	0.00333	0.00547	9.54C_c
3.3	1.00	5.15	179.56	179.56	111.70	111.70	5.15	1000	200	-300	102.89	29.79	0.60	0.00333	0.00496	5.78C_c
3.4	1.15	4.00	179.35	179.35	81.52	81.52	4.60	800	200	-300	59.18	25.90	0.44	0.00333	0.00507	3.94C_c

Table 4. Optimal cost design of rectangular isolated ($e_x = h_x/2 - c_x/2$, $e_y = h_y/2 - c_y/2$).

Example	h_x (m)	h_y (m)	σ_1 (kN/m²)	σ_2 (kN/m²)	σ_3 (kN/m²)	σ_4 (kN/m²)	A_{min} (m²)	P_u (kN)	M_{ux} (kN-m)	M_{uy} (kN-m)	A_{sx} (cm²)	A_{sy} (cm²)	d (m)	ρ_x	ρ_y	C_{min} ($)
4.1	2.00	2.35	149.39	149.39	169.76	169.76	4.70	1000	-1000	-800	41.60	43.51	0.53	0.00333	0.00409	4.53C_c
4.2	2.15	2.60	38.09	108.60	178.37	107.85	5.59	800	-1000	-800	40.86	51.08	0.47	0.00333	0.00503	5.05C_c
4.3	2.65	3.20	1.50	51.56	104.63	54.57	8.48	600	-1000	-800	48.17	63.75	0.40	0.00379	0.00606	7.00C_c
4.4	3.80	4.65	0.72	16.80	33.24	17.15	17.67	400	-1000	-800	59.40	77.92	0.34	0.00378	0.00607	12.62C_c

Table 1 shows the results for P_D = 500 kN and P_L = 500 kN (Example 1.1), P_D = 400 kN and P_L = 450 kN (Example 1.2), P_D = 500 kN and P_L = 250 kN (Example 1.3), P_D = 400 kN and P_L = 200 kN (Example 1.4), M_{xD} = 150 kN-m, M_{xL} = 75 kN-m, M_{yD} = 100 kN-m, M_{yL} = 50 kN-m, e_x = 0, e_y = 0 (The column is located at the center of gravity of the footing). Therefore the unfactored loads and moments are: P = 1000 kN, M_x = 225 kN-m, M_y = 150 kN-m (Example 1.1); P = 850 kN, M_x = 225 kN-m, M_y = 150 kN-m (Example 1.2); P = 750 kN, M_x = 225 kN-m, M_y = 150 kN-m (Example 1.3); P = 600 kN, M_x = 225 kN-m, M_y = 150 kN-m (Example 1.4).

Table 2 shows the results for P_D = 600 kN and P_L = 425 kN (Example 2.1), P_D = 520 kN and P_L = 360 kN (Example 2.2), P_D = 500 kN and P_L = 250 kN (Example 2.3), P_D = 400 kN and P_L = 200 kN (Example 2.4), M_{xD} = -150 kN-m, M_{xL} = -75 kN-m, M_{yD} = 100 kN-m, M_{yL} = 50 kN-m, e_x = 0, $e_y = h_y/2 - c_y/2$ (The column is located at the limit of the footing in the Y direction). Therefore the unfactored loads and moments are: P = 1025 kN, M_x = -225 kN-m, M_y = 150 kN-m (Example 2.1); P = 880 kN, M_x = -225 kN-m, M_y = 150 kN-m (Example 2.2); P = 750 kN, M_x = -225 kN-m, M_y = 150 kN-m (Example 2.3); P = 600 kN, M_x = -225 kN-m, M_y = 150 kN-m (Example 2.4).

Table 3 shows the results for P_D = 600 kN and P_L = 425 kN (Example 3.1), P_D = 520 kN and P_L = 360 kN (Example 3.2), P_D = 500 kN and P_L = 250 kN (Example 3.3), P_D = 400 kN and P_L = 200 kN (Example 3.4), M_{xD} = 100 kN-m, M_{xL} = 50 kN-m, M_{yD} = -150 kN-m, M_{yL} = -75 kN-m, $e_x = h_x/2 - c_x/2$, e_y = 0 (The column is located at the limit of the footing in the X direction). Therefore the unfactored loads and moments are: P = 1025 kN, M_x = 150 kN-m, M_y = -225 kN-m (Example 3.1); P = 880 kN, M_x = 150 kN-m, M_y = -225 kN-m (Example 3.2); P = 750 kN, M_x = 150 kN-m, M_y = -225 kN-m (Example 3.3); P = 600 kN, M_x = 150 kN-m, M_y = -225 kN-m (Example 3.4).

Table 4 shows the results for P_D = 500 kN and P_L = 250 kN (Example 4.1), P_D = 420 kN and P_L = 185 kN (Example 4.2), P_D = 300 kN and P_L = 150 kN (Example 4.3), P_D = 200 kN and P_L = 100 kN (Example 4.4), M_{xD} = -500 kN-m, M_{xL} = -250 kN-m, M_{yD} = -400 kN-m, M_{yL} = -200 kN-m, $e_x = h_x/2 - c_x/2$, $e_y = h_y/2 - c_y/2$ (The column is located at the limit of the footing in the X and Y directions or at a corner). Therefore the unfactored loads and moments are: P = 750 kN, M_x = -750 kN-m, M_y = -600 kN-m (Example 4.1); P = 605 kN, M_x = -750 kN-m, M_y = -600 kN-m (Example 4.2); P = 450 kN, M_x = -750 kN-m, M_y = -600 kN-m (Example 4.3); P = 300 kN, M_x = -750 kN-m, M_y = -600 kN-m (Example 4.4).

4. Results

The way to verify the new model is as follows:

1.- For moments

a. When the column is located at the end of the footing in the positive Y direction and on the Y axis, then e_x = 0 and $e_y = h_y/2 - c_y/2$ are substituted into Equation (9) and $M_{ua1} = 0$ is obtained.

b. When the column is located at the end of the footing in the negative Y direction and on the Y axis, then e_x = 0 and $e_y = -h_y/2 + c_y/2$ are substituted into Equation (11) and $M_{ua2} = 0$ is obtained.

c. When the column is located in the center of the footing, then ex = 0 and ey = 0 are substituted into Equation (9) and $M_{ua1} = -\left[P_u {h_y}^2 + 2M_{ux}(2h_y + c_y)\right](h_y - c_y)^2/8{h_y}^2$ is obtained. When the column is located in the center of the footing, then e_x = 0 and e_y = 0 are substituted into Equation (11) and $M_{ua} = -\left[P_u {h_y}^2 - 2M_{ux}(2h_y - c_y)\right](h_y - c_y)^2/8{h_y}^2$ is obtained. Now, if c_y = 0 is substituted and $M_{ua1} - M_{ua2}$ is performed, the following result is obtained: $-M_{ux}$. This means that it is in equilibrium.

d. When the column is located at the end of the footing in the positive X direction and on the X axis, then $e_x = h_x/2 - c_x/2$ and e_y = 0 are substituted into Equation (13) and $M_{ub1} = 0$ is obtained.

e. When the column is located at the end of the footing in the negative X direction and on the X axis, then $e_x = -h_x/2 + c_x/2$ and e_y = 0 are substituted into Equation (15) and $M_{ub2} = 0$ is obtained.

f. When the column is located in the center of the footing, then e_x = 0 and e_y = 0 are substituted into Equation (13) and $M_{ub1} = -\left[P_u {h_x}^2 + 2M_{uy}(2h_x + c_x)\right](h_x - c_x)^2/8{h_x}^2$ is obtained. When the column is located in the center of the footing, then e_x = 0 and e_y = 0 are substituted into Equation (15) and $M_{ub2} = -\left[P_u {h_x}^2 - 2M_{uy}(2h_x - c_x)\right](h_x - c_x)^2/8{h_x}^2$ is obtained. Now, if c_x = 0 is substituted and $M_{ub1} - M_{ub2}$ is performed, the following result is obtained: $-M_{uy}$. This means that it is in equilibrium.

2.- For bending shear

a. When the column is located at a distance $h_y/2 - c_y/2 - d$ from the center of the footing in the positive Y direction and on the Y axis, then e_x = 0 and $e_y = h_y/2 - c_y/2 - d$ are substituted into Equation (17) and $V_{ufc1} = 0$ is obtained.

b. When the column is located at a distance $– h_y/2 + c_y/2 + d$ from the center of the footing in the negative Y direction and on the Y axis, then $e_x = 0$ and $e_y = – h_y/2 + c_y/2 + d$ are substituted into Equation (19) and $V_{ufc2} = 0$ is obtained.

c. When the column is located in the center of the footing, then $e_x = 0$ and $e_y = 0$ are substituted into Equation (15) and $V_{ufc} = -[P_u h_y{}^2 + 3M_{ux}(h_y + c_y + 2d)](h_y - c_y - 2d)/2h_y{}^3$ is obtained. When the column is located in the center of the footing, then $e_x = 0$ and $e_y = 0$ are substituted into Equation (11) and $V_{ufc2} = [P_u h_y{}^2 - 3M_{ux}(h_y + c_y + 2d)](h_y - c_y - 2d)/2h_y{}^2$ is obtained. Now, if $c_y = 0$ and $d = 0$ are substituted and $V_{ufc1} – V_{ufc2}$ is performed, the following result is obtained: – P. This means that it is in equilibrium.

d. When the column is located at a distance $h_x/2 – c_x/2 – d$ from the center of the footing in the positive X direction and on the X axis, then $e_x = h_x/2 – c_x/2 – d$ and $e_y = 0$ are substituted into Equation (21) and $V_{ufe1} = 0$ is obtained.

e. When the column is located at a distance $– h_x/2 + c_x/2 + d$ from the center of the footing in the negative X direction and on the X axis, then $e_x = – h_x/2 + c_x/2 + d$ and $e_y = 0$ are substituted into Equation (23) and $V_{ufe2} = 0$ is obtained.

f. When the column is located in the center of the footing, then $e_x = 0$ and $e_y = 0$ are substituted into Equation (21) and $V_{ufe1} = -[P_u h_x{}^2 + 3M_{uy}(h_x + c_x + 2d)](h_x - c_x - 2d)/2h_x{}^3$ is obtained. When the column is located in the center of the footing, then $e_x = 0$ and $e_y = 0$ are substituted into Equation (23) and $V_{ufe2} = [P_u h_x{}^2 - 3M_{uy}(h_x + c_x + 2d)](h_x - c_x - 2d)/2h_x{}^2$ is obtained. Now, if $c_x = 0$ and $d = 0$ are substituted and $V_{ufe1} – V_{ufe2}$ is performed, the following result is obtained: – P. This means that it is in equilibrium.

3.- For punching shear

a. When the column is located in the center of the footing, then $x_1 = c_x/2 + d/2$, $x_2 = – c_x/2 – d/2$, $y_1 = c_y/2 + d/2$, and $y_2 = – c_y/2 – d/2$ are substituted into Equation (25) and $V_{up} = [P_u h_x h_y - (c_x + d)(c_y + d)]/h_x h_y$.

Tables 1 to 4 show the minimum cost design for the rectangular isolated footings subjected to an axial load and two bending moments in X and Y directions.

Table 1 presents the following: when the axial load P decreases; d and A_{sx} decrease for all the examples; A_{min}, A_{sy}, C_{min} decrease until $P_u = 1000$ kN and then it increases; ϱ_x is the same for all the examples; ϱ_y decreases until $P_u = 1200$ kN and then it increases.

Table 2 shows the following: when the axial load P decreases; d, A_{sx}, A_{min}, A_{sy}, C_{min} decrease for all the examples; ϱ_x decreases until $P_u = 1000$ kN and then it increases; ϱ_y is the same for all the examples.

Table 3 presents the following: when the axial load P decreases; d, A_{sx}, A_{min}, A_{sy}, C_{min} decrease for all the examples; ϱ_x is the same for all the examples; ϱ_y decreases until $P_u = 1000$ kN and then it increases.

Table 4 shows the following: when the axial load P decreases; A_{min}, A_{sy}, ϱ_y, C_{min} increase for all the examples; d decreases; ϱ_x is the same until $P_u = 800$ kN; A_{sx} decreases until $P_u = 800$ kN and then it increases.

Also, a comparison is made with the results of Solorzano and Plevris (2022) to show the advantages of the new model according to the American Concrete Institute ACI 318-19 [26].

Table 5 shows the results of the example 1 of Solorzano and Plevris (2022) for $P_D = 1500$ kN, $P_L = 850$ kN, $e_x = 0.20$ m, $e_y = 0$ m, $c_x = c_y = 0.50$ m, $r = 0.05$ m, $\sigma_{max} = 400$ kN/m², $f'_c = 35$ Mpa, $f_y = 410$ Mpa and $C_s = 15C_c$. Therefore the unfactored loads and moments are: P = 2350 kN, $M_x = 0$ kN-m, $M_y = 470$ kN-m.

Table 5. Comparison of the model proposed by Solorzano and Plevris (Example 1) against the new model.

Model	h_x (m)	h_y (m)	σ_1 (kN/m²)	σ_2 (kN/m²)	σ_3 (kN/m²)	σ_4 (kN/m²)	A_{min}	P_u (kN)	M_{ux} (kN-m)	M_{uy} (kN-m)	A_{sx} (cm²)	A_{sy} (cm²)	d (m)	t (m)	ϱ_x	ϱ_y	C_{min} ($)

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Model	h_x (m)	h_y (m)	σ_1 (kN/m²)	σ_2 (kN/m²)	σ_3 (kN/m²)	σ_4 (kN/m²)	A_{min} (m²)	P_u (kN)	M_{ux} (kN-m)	M_{uy} (kN-m)	A_{sx} (cm²)	A_{sy} (cm²)	d (m)	t (m)	ρ_x	ρ_y	C_{min} ($)
Solorzano and Plevris	3.49	2.34	400.00	201.72	201.72	400.00	8.16	3160	0	632	153.20	122.90	0.48	0.53	0.01364	0.00734	5.55C_c
New model	1.00	7.12	400.00	288.97	288.97	400.00	7.12	3160	0	632	159.47	17.21	0.48	0.53	0.00470	0.00361	4.17C_c

where: t is the total thickness of the footing.

Table 6 shows the results of the example 2 of Solorzano and Plevris (2022) for P_D = 1600 kN, P_L = 900 kN, e_x = 0 m, e_y = 0 m, c_x = 0.50 m, c_y = 0.80 m, r = 0.05 m, σ_{max} = 400 kN/m², f'_c = 35 Mpa, f_y = 410 Mpa and $C_s = 15C_c$. Therefore the unfactored loads and moments are: P = 2500 kN, M_x = 0 kN-m, M_y = 0 kN-m.

Table 6. Comparison of the model proposed by Solorzano and Plevris (Example 2) against the new model.

Model	h_x (m)	h_y (m)	σ_1 (kN/m²)	σ_2 (kN/m²)	σ_3 (kN/m²)	σ_4 (kN/m²)	A_{min} (m²)	P_u (kN)	M_{ux} (kN-m)	M_{uy} (kN-m)	A_{sx} (cm²)	A_{sy} (cm²)	d (m)	t (m)	ρ_x	ρ_y	C_{min} ($)
Solorzano and Plevris	2.14	3.02	399.29	399.29	399.29	399.29	6.46	3360	0	632	106.10	103.40	0.44	0.49	0.00798	0.01098	3.98C_c
New model	2.54	2.54	399.93	399.93	399.93	399.93	6.45	3360	0	632	44.14	39.83	0.43	0.48	0.00400	0.00361	3.45C_c

Table 5 presents the following: A_{min}, ρ_x, ρ_y, A_{sy} and C_{min} are smaller, and A_{sx} is greater in the new model compared to the one presented by Solorzano and Plevris, and d is the same for both models. The new model shows a saving of 12.75% in contact area with soil and of 24.86% in cost with respect to the model proposed by Solorzano and Plevris.

Table 6 shows the following: d, ρ_x, ρ_y, A_{sx}, A_{sy} and C_{min} are smaller, and A_{min} is greater in the new model compared to the one presented by Solorzano and Plevris. The new model shows a saving of 0.15% in contact area with soil and of 13.32% in cost with respect to the model proposed by Solorzano and Plevris.

Note: In the new model, the footing dimensions in Tables 5 and 6 were not adjusted to observe the differences between the two models.

5. Conclusions

In this paper, a mathematical model is presented to determine the minimum cost design of a rectangular isolated footing subjected to an axial load and two moments due to a column located in any part of the footing on elastic soils, and assuming a linear distribution of soil pressure.

The new model is presented in two stages. The first stage is to obtain the minimum area, and the second is to find the minimum cost design once the dimensions of the footing are known. The first stage (minimum area), the known or constant parameters (independent variables) are: σ_{max}, P, M_x and M_y, and the unknown or decision variables (dependent variables) are: A_{min}, h_x, h_y. The second stage (minimum cost design), the known or constant parameters (independent variables) are: h_x, h_y, P_u, M_{ux} and M_{uy}, and the unknown or decision variables (dependent variables) are: d, ρ_x, ρ_y, A_{sx}, A_{sy} and C_{min}.

The main contributions showed in this paper are:

1.- Some engineers use the trial and error method to determine the dimensions for rectangular isolated footings subjected to biaxial bending, and later the design is obtained considering the maximum and uniform pressure along the underside of the footing.

2.- Other authors present the minimum cost design for rectangular isolated footings subjected to biaxial bending rested on elastic soils, but only consider the column located at the center of gravity of the footing.

3.- Some authors present very complex algorithm to obtain the minimum cost design for rectangular isolated footings subjected to biaxial bending rested on elastic soils.

4.- The new model presents a significant reduction in design costs for rectangular isolated footings (see Tables 5 and 6).

5.- Equations for moments, bending shear and punching shear are verified by equilibrium.

6.- The new model can be used for any other building code, taking into account the equations that resist the moments, the bending shear and the punching shear. Also, equation of the reinforcing steel areas proposed for any other building code.

The main advantage of this work over other works is that the moment, bending shear and punching shear equations are presented in detail, as well as the optimization algorithm and its equations.

Future works may be:

Minimum cost design of a rectangular isolated footing subjected to an axial load and two moments due to a column located in any part of the footing, assuming that the contact area of the footing with the soil works partially under compression.

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