

Article

Not peer-reviewed version

On the Foundations of Quantum Mechanics

[Richard Oldani](#) *

Posted Date: 7 October 2025

doi: 10.20944/preprints202510.0504.v1

Keywords: matrix mechanics; time; quantum-classical divide; energy conservation; Hamilton's principle; action function; gravity



Preprints.org is a free multidisciplinary platform providing preprint service that is dedicated to making early versions of research outputs permanently available and citable. Preprints posted at Preprints.org appear in Web of Science, Crossref, Google Scholar, Scilit, Europe PMC.

Copyright: This open access article is published under a Creative Commons CC BY 4.0 license, which permit the free download, distribution, and reuse, provided that the author and preprint are cited in any reuse.

Article

On the Foundations of Quantum Mechanics

Richard Oldani

Independent Researcher, USA; oldani@juno.com

Abstract

We consider three long-standing conceptual difficulties of quantum mechanics; the time variable, the quantum-classical divide, and gravitational fields. We investigate possible solutions by extending matrix mechanics from single points in time to continuous time in order to describe the absorption energy of a quantum system. Due to the conservation of energy absorption is equal to emission, as described by wave functions; which is an illustration of the classical-quantum divide. Hamilton's principle of least action proves to be a natural fit for describing the resulting two-part quantum system and it specifies the true paths to be the ones with action equal to $\hbar$. Energy absorption followed by discrete, four-dimensional changes in structure and energy emission is shown to be a pattern that is repeated countless times throughout nature by phase transitions and other phenomena. Experimental evidence is cited to show that the resultant mathematical model varies consistently with changes in gravitational potential. A universal physical interpretation is proposed.

Keywords: matrix mechanics; time; quantum-classical divide; energy conservation; Hamilton's principle; action function; gravity

1. Introduction

Although matrix mechanics and wave mechanics are believed to be equivalent formulations of quantum mechanics, they are not equal participants. They are based on widely separated initial assumptions, distinct experiments, and different mathematical models. Their contributions to the foundations are also different. Wave mechanical models have predominated in analytical discussions while matrix mechanics plays a subordinate role. The reasons given for the imbalance is visualizability and ease of calculation, properties we attribute not to matter but to the theories themselves. Despite its many conceptual difficulties the wave function is the main focal point for physical interpretation whereas matrix theory has not advanced significantly beyond its initial assessment 100 years ago. Due to unfamiliarity a theory can seem ineffective, but it can also mean that there are possibilities that have yet to be explored. We will take the latter point of view since matrix mechanics does not suffer the paradoxes of wave mechanics. For example, the physical variables given by the diagonal matrix elements, frequency and line intensity, are measurable independently of probability distributions nor is there need for a collapse function when performing measurements. These specific physical characteristics of matrix mechanics are the premises for investigating its role in the foundations more thoroughly.

2. Matrix Mechanics Reinterpreted

Matrix mechanics, introduced by Heisenberg in 1925, uses matrices to describe how the properties of observables, the frequency and intensity of the spectral lines, are related to electron energy transitions. They describe the states of a system while the values within the matrices represent the physical quantities associated with those states. The matrix elements are infinite in number and together they describe an atom's energy state. Heisenberg's initial premise was that quantum mechanics should be "founded exclusively upon relationships between quantities which in principle are observable" [1]. The diagonal elements of the matrix specify the observed spectral lines while the non-diagonal elements are unobserved exchanges of energy that are assigned a value of zero. It is

common practice in the construction of mathematical models to make approximations. There is not a single theory of nature that pretends to describe nature exactly so no one questioned the validity of his methods.

Although it is true that the individual molecular impulses of heat do not contribute substantially to the energy of an isolated hydrogen atom, Nobel laureate Richard Feynman noted that there is a problem with treating atoms as particles in the manner of Heisenberg, by insisting that only observable electron transitions have physical meaning [2].

But suppose we look at the whole hydrogen atom as a “particle.” If we didn’t know that the hydrogen atom was made out of a proton and an electron, we might have started out and said: “Oh, I know what the base states are—they correspond to a particular momentum of the hydrogen atom.” No, because the hydrogen atom has internal parts. It may, therefore, have various states of different internal energy, and describing the real nature requires more detail.

Matrix mechanics describes the internal energy of a molecule by giving a detailed accounting of all possible energy transitions in terms of matrix elements, where each one of an infinite number of matrix elements represents a possible electron transition and energy state. The “states of internal energy” of the hydrogen atom are infinite in number and refer to all possible matrix configurations, also infinite in number. As Feynman noted, due to internal structure a hydrogen atom cannot be thought of as a proton whose electron occupies a single energy state. The electron is loosely bound and it can oscillate independently of the proton during a collision. Thus atoms have an elastic structure and absorb energy internally due to electronic structure and externally due to their motion through space in the form of kinetic energy. Each matrix element of an atom represents a possible electron transition, the possibility of an excited energy state, and an increased internal energy. The total internal energy of a hydrogen atom consists of contributions from all of the excitations described by matrix elements both far and near to the diagonal, nearly all of which Heisenberg chose to ignore because they are unobservable. Although off-diagonal elements represent small energy exchanges they are infinite in number and when integrated over the entire space of the atom they represent the influence of thermal energy which leads up to and produces the spectral lines. An atom whose internal energy is described by a matrix with non-diagonal elements equal to zero is an incomplete physical model.

A matrix is a detailed snapshot of an atom’s internal energy at a particular point in time. The matrix elements refer to electron transitions of definite energy, positive due to excitation and negative due to decay. Atoms are three-dimensional so there are an infinite number of electron transitions possible and no single transition determines internal energy. The electron has a probability of being anywhere in atomic space so internal energy is determined by the superposition of many infinitesimal transitions. To study the variation of the internal energy of an atom in time we apply heat incrementally and use matrices to describe change as a series of snapshots, initially by matrix elements far from the diagonal and progressing inwards towards the diagonal. We align the snapshots sequentially in time to describe the gradual increase of an atom’s internal energy in terms of a distribution of energy states in atomic space. Taken together we can use the frames to describe the internal energy of an atom in the form of a motion picture. As more and more energy is absorbed matrix elements closer and closer to the diagonal become excited until a spectral line is realized. The snapshots of energy form a continuous series in time that describe an atom’s absorption of energy. All atoms have electronic structure so the absorption of energy by a massive object can occur in the same way. Thus it is possible to use matrices to describe the classical absorption of heat energy by a distribution of unobservable energy states in atomic space. The classical-quantum divide occurs because atomic electrons exist in atomic space as probability distributions, not as localized particles, so that electron transitions superpose but *they do not superimpose*.

3. Energy Conservation

The purpose of the above analysis is to show that matrix and wave mechanics are equally important in the description of the emission and absorption of radiation. By using energy and time as physical variables to describe a radiating hydrogen atom we interpret the emission of a photon as a two-step process, first continuous absorption and then discrete emission described by a wave function. A continuous supply of heat energy raises the electron to higher and higher energy states and creates the initial conditions necessary for discrete electron decay and the emission of a photon in the visible spectrum. Continuous energy absorption by a material system followed by discrete changes in structure illustrates the classical-quantum divide and is a process that occurs throughout nature in many ways. Freezing, evaporation, condensation, sublimation, crystallization, and boiling are all examples of material systems that absorb energy continuously followed by discrete changes in structure and the emission of energy, and are referred to as phase transitions.

By formulating matrix mechanics with respect to energy and time we see the reason why it is equivalent in importance to wave mechanics. Due to the conservation of energy an isolated quantum system such as the hydrogen atom must absorb energy before it is able to emit in the form of a spectral line. Because energy can neither be created nor destroyed, it can only change by transforming from one type to another. The quantum system conserves energy for changes in electromagnetic potential in the same way that a classical system conserves energy for changes in gravitational potential. Energy input must be identical to energy output for an isolated material system such as the hydrogen atom. Because of energy conservation two quantum mechanical formulations are necessary instead of one.

To describe the emission and absorption of radiation as a transformation we use the energy-time conjugate variables in an “action function”. Electron excitations consist of a sum of infinitesimal increments over a complete period, as described in paragraph 2.0, followed by sudden decay. If we integrate a continuous function between two fixed endpoints and two specified times we obtain an action integral $\int L dt$, a relativistic invariant. The usual formulations of quantum mechanics; matrix mechanics, wave mechanics, and the path integral formulation; are non-relativistic. It means that they correctly predict the results of many experiments, but not those with gravitational fields. That is, the equations of motion derived quantum mechanically are not influenced by gravitational potentials even though experiments with atomic clocks have shown that they must be [3]. By integrating an action function over the time period of an electron transition we hope to formulate quantum mechanics relativistically.

4. Lagrangian Quantum Mechanics

A spectral line is determined by electrons “jumping” from the ground state to an excited state and subsequently returning to the ground state with the emission of a photon. The paths of excitation and decay occur between two specific endpoints, the atomic orbitals, and two specific times, the period τ ; where $\tau=(t_1 -t_2)$; which are the precise physical conditions that define Hamilton’s principle of least action. Thus the physical characteristics of a quantum system naturally define the correct equations that are used to describe it. The principle of least action simply means that of all the paths that an electron can take between two atomic orbitals, the ones actually taken are found by computing the action for each of these trajectories, and selecting the ones that have the least action. The paths selected are the ones that have an action equal to the reduced Planck’s constant $\hbar$.

The action is the time integral of the Lagrangian $\int L dt$, where $L=T-V$, T is the kinetic energy, and V is the potential energy. We compute the action of an electron transitioning from the ground state to an excited state by using generalized coordinates, three to describe its position on the electron shells $R_1 = (x_1,y_1,z_1)$ and $R_2 = (x_2,y_2,z_2)$, and three to describe its trajectory.

$$S[r(t)]=\int_{R_1}^{R_2}\int_{t_1}^{t_2}(T-V)dt \tag{1)}$$

The action, $S[r(t)]$, is a functional that describes the absorption process in four dimensions. It has as its argument an infinite number of functions, the possible electron paths $r(t)$. We know that it is experimentally correct because gravitational fields influence the action identically to the way that they influence clocks; that is, by causing time dilation.

Emission initiates from the excited state $R_2 = (x_2, y_2, z_2)$ at time t_2 and it finalizes at the ground state $R_1 = (x_1, y_1, z_1)$ at time t_1 . Each of the electron shells R_2 and R_1 determines a locus of points where the fields vanish [4]. The Lagrangian density over the region of space-time between the excited and ground states is "stationary for all small variations of the coordinates inside the region." Changes in action during emission are evaluated by integrating the Lagrangian density four-dimensionally thereby yielding a relativistic formulation.

$$S[\phi_i(t)] = \int_{R_2}^{R_1} \int_{t_2}^{t_1} L(\phi_i, \phi_{i,\mu}) d^3x dt = E\tau \quad 2)$$

The emission of a photon is described by the action integral of a Lagrangian density $\int \mathcal{L}(\phi_i, \phi_{i,\mu}) dt$ in the region of space-time between the excited and ground states given by the fields ϕ_i and its first derivatives $\phi_{i,\mu}$. This allows for a complete accounting of the energy interactions that occur during an electron transition within the volume $\int d^3x$ between the electron orbitals where ϕ_i is the current density described *radially* and $\phi_{i,\mu}$ is the electromagnetic field strength described *transversely* that results in the creation of a photon. The photon is described therefore as a four-dimensional localization of fields contained within the volume $\int d^3x$ and the time interval $t_2 - t_1 = \tau$ to obtain $E\tau = \hbar$ which is equivalent to the more familiar time-averaged relation $E = \hbar\nu$. The discrete and continuous properties of the emission process are described by a Lagrangian density over the time interval of a period which results in a photon being created. A more complete derivation is given elsewhere [5].

5. Conclusions

Measurements by their very nature are performed at single points in time, but the action is defined over a time interval. It cannot be evaluated in the usual way by a number. We need a different way to perform measurements of the action because it describes the path energy takes between two points in time. To evaluate action we integrate energy over a path and a time interval. The measurement of work, force times distance, can be interpreted as an action, for example, because force includes time continuously and distance defines a time period, the interval determined by a force.

This new formulation of quantum mechanics using energy-time conjugate variables has heuristic value in the search for a more complete understanding of nature and leads to further insights into the behavior of natural phenomena. The gradual increase in energy of a material system followed by discrete, four-dimensional changes in structure is a pattern that is repeated countless times throughout nature. We find it in phase transitions, period doubling phenomena in chaos theory [6], and the gravitational collapse of a star that results in the formation of a black hole. The same pattern is even found in a theory of evolutionary biology. Periods of uniformity in the fossil record are often followed by sudden structural change in a process referred to as "punctuated equilibrium" [7]. Thus we are able to appreciate that quantum mechanics is a truly universal theory of nature since it may be used to describe the microscopic world with position-momentum conjugate variables, and the macroscopic world with energy-time conjugate variables.

References

- [1] W. Heisenberg *Z Phys* 33 (1) (1925), in B.L. van der Waerden (ed.), *Sources of Quantum Mechanics* (Amsterdam, 1967). doi: <http://www.mat.unimi.it/users/galgani/arch/heis25ajp.pdf>
- [2] Feynman, Leighton, Sands *The Feynman Lectures on Physics, Vol III, Chap 8-3 "The Hamiltonian Matrix"* (1963).

3. [3] T. Bothwell *et al.* "Resolving the gravitational red shift within a millimeter atomic sample". *Nature* **602**, Issue 7897, 420 (2022) arXiv:2109.12238
4. [4] Dirac, P.A.M., "The Lagrangian in quantum mechanics" *Phys Zeit Sow* **3**, 1933, p.1. https://www.informationphilosopher.com/solutions/scientists/dirac/Lagrangian_1933.pdf
5. [5] Oldani, R. (2024) "Galactic symmetry" doi: preprints.org/manuscript/202009.0215/v8, p. 8.
6. [6] Feigenbaum, M. J. (1978). "Quantitative universality for a class of nonlinear transformations". *Journal of Statistical Physics*. **19** (1): 25–52. doi:10.1007/BF01020332.
7. [7] Eldredge, N.; Niles, S.J. (1972). "Punctuated equilibria: An alternative to phyletic gradualism" In Schopf, T.J.M. (ed.). *Models in Paleobiology*. San Francisco, CA: Freeman Cooper. p. 82.

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.