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Posted Date: 13 March 2023

doi: 10.20944/preprints202212.0045.v7

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Preprints.org (www.preprints.org) | NOT PEER-REVIEWED | Posted: 13 March 2023 doi:10.20944/preprints202212.0045.v7

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The Imaginary Universe

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Abstract: Imaginary dimensions in physics require an imaginary set of base Planck units and some negative parameter c_n corresponding to the speed of light in vacuum c. Fresnel coefficients for the normal incidence of electromagnetic radiation on monolayer graphene introduce the second, negative fine-structure constant $\alpha_2^{-1} \approx -140.178$ as a fundamental constant of nature and this constant introduces these imaginary base Planck units along with this negative parameter $c_n \approx -3.06 \times 10^8$ [m/s]. Neutron stars and white dwarfs, considered as *objects* emitting perfect black-body radiation, are conjectured to possess energy exceeding their mass-energy equivalence ratios, wherein the imaginary parts of two complex energies, inaccessible for direct observation, make storing excess of these energies possible. With this assumption, black holes are fundamentally uncharged; charged micro neutron stars and white dwarfs with masses lower than 5.7275×10^{-10} [kg] cannot be observed; and the radii of white dwarfs' cores are limited to $R_{WD} < 6.7933\, GM_{WD}/c^2$, where M_{WD} is a white dwarf mass. This sets the bounds on charged blackbody objects' minimum masses and maximum radii.

Keywords: emergent dimensionality; Planck units; fine-structure constant; black holes; neutron stars; white dwarfs; holographic principle; mathematical physics

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1. Introduction

The universe began with the Big Bang, which is a current prevailing scientific opinion. But this Big Bang was not an explosion of 4-dimensional spacetime, which also is a current prevailing scientific opinion, but an explosion of dimensions. More precisely, in the -1-dimensional void, a 0-dimensional point appeared implying the appearance of countably infinitely other points indistinguishable from the first one. The breach made by the first operation of the *dimensional successor function* of the Peano axioms inevitably continued leading to the formation of the first two 1-dimensional, real and imaginary lines allowing for an ordering of points using multiples of real units (ones) or imaginary units ($a \in \mathbb{R} \Leftrightarrow a = 1b$[1]$, a \in \mathbb{I} \Leftrightarrow a = ib, b \in \mathbb{R}$). These two lines again implied the appearance of countably infinitely other ones, wherein an initial point (0) on each curve decomposed them into two half-lines or rays. Then out of two lines of each kind, crossing each other only at one initial point $(0,0)$, the dimensional successor function formed 2-dimensional $\mathbb{R}^2$, $\mathbb{I}^2$, and $\mathbb{R} \times \mathbb{I}$ Euclidean planes, with $\mathbb{I}^2$ plane being a mirror reflection of $\mathbb{R}^2$ plane. And so on, forming n-dimensional Euclidean spaces $\mathbb{R}^a \times \mathbb{I}^b$ with $a \in \mathbb{N}$ real and $b \in \mathbb{N}$ imaginary lines, $a + b = n$, and the scalar product defined by

$$\mathbf{X} \cdot \mathbf{Y} = (x_1, \ldots, x_a, ix'_1, \ldots, ix'_b)\,(y_1, \ldots, y_a, iy'_1, \ldots, iy'_b) := \\ := \sum_{k=1}^{a} x_k y_k + \sum_{l=1}^{b} x'_l \overline{y'_l}. \tag{1}$$

With the onset of the first 0-dimensional point, information began to evolve [1–6].

However, dimensional properties are not uniform. Concerning regular convex n-polytopes, for example, there are countably infinitely many regular convex polygons, five regular convex polyhedra (Platonic solids), six regular convex 4-polytopes, and only three regular convex 4-polytopes if $n > 3$,

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[1] This is, of course, a circular definition, but it is given for clarity.

$n \in \mathbb{N}$ [7]. In particular, 4-dimensional euclidean space is endowed with a peculiar property known as exotic $\mathbb{R}^4$ [8]. This property allowed for variation of phenotypic traits within populations of individuals [9] perceiving emergent Euclidean $\mathbb{R}^3 \times \mathbb{I}$ space of three real and one imaginary (time) dimension. The evolution of information extended into biological evolution.

Each dimension requires certain units of measure. In real dimensions, these *natural units of measure*, were introduced by Max Planck in 1899 as "independent of special bodies or substances, thereby necessarily retaining their meaning for all times and for all civilizations, including extraterrestrial and non-human ones" [10].

This study introduces the complementary set of Planck units applicable for imaginary dimensions, including the imaginary base units, and outlines certain prospects for their research. As the speed of electromagnetic radiation (EMR) is $c = \nu\lambda$, where ν is the frequency, and λ is the wavelength (which are real and at least non-negative), and both these quantities are imaginary in imaginary dimensions, some real but negative parameter $c_n = \nu_i \lambda_i$ corresponding to the speed of light in vacuum c (i.e., the Planck speed) is also necessary ($i^2 = -1$). It also turns out that the imaginary Planck energy $E_{\mathrm{P}i}$ and temperature are larger up to moduli than the Planck energy E_{P} and temperature T_{P}. Thus, the minimum energy principle sets more favorable conditions for biological evolution to prefer $\mathbb{R}^3 \times \mathbb{I}$ Euclidean space over $\mathbb{I}^3 \times \mathbb{R}$ Euclidean one.

The study shows that the energies of neutron stars and white dwarfs exceed their mass–energy equivalences. Therefore, the excess of this energy must be stored in imaginary dimensions and is inaccessible to direct observations.

The paper is structured as follows. Section 2 shows that Fresnel coefficients for the normal incidence of EMR on MLG introduce the second, negative fine-structure constant α_2 as a fundamental constant of nature. Section 3 shows that nature endows us with the imaginary base Planck units by this second fine-structure constant. Section 4 introduces the concept of a black-body *object* in thermodynamic equilibrium emitting black-body radiation and discusses its necessary properties. Section 5 introduces the concept of two complex energies and discusses their applications to black-body *objects*. Section 6 discusses the findings of this study.

2. The second Fine-Structure Constant

Numerous publications provide Fresnel coefficients for the normal incidence of EMR on monolayer graphene (MLG), which are remarkably defined only by π and the fine-structure constant α having the reciprocal

$$\alpha^{-1} = \left(\frac{q_{\mathrm{P}}}{e}\right)^2 = \frac{4\pi\epsilon_0\hbar c}{e^2} \approx 137.036. \tag{2}$$

Transmittance (T) of MLG

$$T = \frac{1}{\left(1 + \frac{\pi\alpha}{2}\right)^2} \approx 97.746\% \tag{3}$$

for normal EMR incidence was derived from the Fresnel equation in the thin-film limit [11] (Eq. 3), whereas spectrally flat absorptance (A) $A \approx \pi\alpha \approx 2.3\%$ was reported [12,13] for photon energies between about 0.5 and 2.5 [eV]. T was related to reflectance (R) [14] (Eq. 53) as $R = \pi^2\alpha^2 T/4$, i.e,

$$R = \frac{\frac{1}{4}\pi^2\alpha^2}{\left(1 + \frac{\pi\alpha}{2}\right)^2} \approx 0.013\%, \tag{4}$$

The above equations for T and R, as well as the equation for the absorptance

$$A = \frac{\pi\alpha}{\left(1 + \frac{\pi\alpha}{2}\right)^2} \approx 2.241\%, \tag{5}$$

were also derived [15] (Eqs. 29-31) based on the thin film model (setting $n_s = 1$ for substrate).

The sum of transmittance (3) and the reflectance (4) at normal EMR incidence on MLG was also derived [16] (Eq. 4a) as

$$\begin{aligned} T + R &= 1 - \frac{4\sigma\eta}{4 + 4\sigma\eta + \sigma^2\eta^2 + k^2\chi^2} \\ &= \frac{1 + \frac{1}{4}\pi^2\alpha^2}{\left(1 + \frac{\pi\alpha}{2}\right)^2} \approx 97.759\%, \end{aligned} \tag{6}$$

where η is the vacuum impedance

$$\eta = \frac{4\pi\alpha\hbar}{e^2} = \frac{1}{\epsilon_0 c} \approx 376.73 \;\; [\Omega], \tag{7}$$

where ϵ_0 is vacuum permittivity, $\sigma = e^2/(4\hbar) = \pi\alpha/\eta$ is the MLG conductivity [17], and $\chi = 0$ is the electric susceptibility of vacuum.

These coefficients are thus well-established theoretically and experimentally confirmed [11–13,16, 18,19].

As a consequence of the conservation of energy

$$(T + A) + R = 100\%. \tag{8}$$

In other words, the transmittance in the Fresnel equation describing the reflection and transmission of EMR at normal incidence on a boundary between different optical media is, in the case of the 2-dimensional (boundary) of MLG, modified to include its absorption.

The reflectance $R = 0.013\%$ (4) of MLG can be expressed as a quadratic equation with respect to α

$$\frac{1}{4}(R-1)\,\pi^2\alpha^2 + R\pi\alpha + R = 0, \tag{9}$$

having two roots with reciprocals

$$\alpha^{-1} = \frac{\pi - \pi\sqrt{R}}{2\sqrt{R}} \approx 137.036, \quad \text{and} \tag{10}$$

$$\alpha_2^{-1} = \frac{-\pi - \pi\sqrt{R}}{2\sqrt{R}} \approx -140.178. \tag{11}$$

Therefore, the quadratic equation (9) introduces the second, negative fine-structure constant α_2.

The sum of the reciprocals of these fine-structure constants (10) and (11)

$$\alpha^{-1} + \alpha_2^{-1} = \frac{\pi - \pi\sqrt{R} - \pi - \pi\sqrt{R}}{2\sqrt{R}} = -\pi, \tag{12}$$

is remarkably independent of the reflectance R (The same result can be obtained for T and A, as shown in Appendix A).

Furthermore, this result is intriguing in the context of a peculiar algebraic expression for the fine-structure constant [20]

$$\alpha^{-1} = 4\pi^3 + \pi^2 + \pi \approx 137.036303776 \tag{13}$$

that contains a *free* π term and is very close to the physical definition (2) of α^{-1}, which according to the CODATA 2018 value is 137.035999084. Notably, the value of the fine-structure constant is not *constant* but increases with time [21–25]. Thus, the algebraic value given by (13) can be interpreted as the asymptote of the α increase.

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Using Equations (12) and (13), we can express the negative reciprocal of the 2nd fine-structure constant α_2^{-1} that emerged in the quadratic equation (9) also as a function of π only

$$\alpha_2^{-1} = -\pi - \alpha_1^{-1} = -4\pi^3 - \pi^2 - 2\pi \approx -140.177896429, \tag{14}$$

and this value can also be interpreted as the asymptote of the α_2 decrease with the current value of $\alpha_2^{-1} \approx -140.177591737$, assuming the rate of change is the same for α and α_2.

Transmittance T (3), reflectance R (4), and absorptance A (5) of MLG for normal EMR incidence can be expressed just by π and introduce two π-like constants for two surfaces with positive and negative Gaussian curvatures (cf. Appendix B).

3. α_2-set of Planck units

Planck units can be derived from numerous starting points [5,26] (cf. Appendix C). The definition of the Planck charge $q_P = \sqrt{4\pi\epsilon_0\hbar c}$ can be solved for the speed of light yielding $c = q_P^2/(4\pi\epsilon_0\hbar)$. Furthermore, the definition of the fine-structure constant $\alpha = e^2/q_P^2$ (2) applied for the negative α_2, requires an introduction of some imaginary Planck charge q_{Pi} so that its square would yield a negative value of α_2

$$\frac{q_{Pi}^2}{e^2} = \alpha_2^{-1} \approx -140.178 < 0. \tag{15}$$

Using the value of the elementary charge e in (15) and the value of the α_2 (11) or (14), the imaginary Planck charge q_{Pi} can be thus derived as

$$q_{Pi} = \pm\sqrt{\frac{e^2}{\alpha_2}} = \pm\sqrt{4\pi\epsilon_0\hbar c_n}. \tag{16}$$

Almost all physical constants of the $\sqrt{4\pi\epsilon_0\hbar c_n}$ term are positive[2], whereas the charge e is squared (and is a real value by the relation (44)). Only the $c_n = \nu_i\lambda_i$ parameter must be negative, as it corresponds to the speed of light $c = \nu\lambda$, and both these quantities (frequency ν_i and wavelength λ_i) are imaginary in imaginary dimensions. Therefore, equation (16) can be solved for c_n yielding

$$c_n = q_{Pi}^2/(4\pi\epsilon_0\hbar) \approx -3.066653 \times 10^8 \text{ [m/s]}, \tag{17}$$

which is greater than the speed of light in vacuum c in modulus[3].

The negative parameter c_n (17) introduces the imaginary set of base Planck units q_{Pi}, ℓ_{Pi}, m_{Pi}, t_{Pi}, and T_{Pi} that redefined by square roots containing c_n raised to an odd (1, 3, 5) power become imaginary and bivalued

$$q_{Pi} = \pm\sqrt{4\pi\epsilon_0\hbar c_n} = \pm q_P\sqrt{\frac{\alpha}{\alpha_2}} \approx \pm i1.8969 \times 10^{-18} \text{ [C]} \quad (> q_P), \tag{18}$$

$$\ell_{Pi} = \pm\sqrt{\frac{\hbar G}{c_n^3}} = \pm\ell_P\sqrt{\frac{\alpha_2^3}{\alpha^3}} \approx \pm i1.5622 \times 10^{-35} \text{ [m]} \quad (< \ell_P), \tag{19}$$

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[2] Vacuum permittivity ϵ_0 is the value of the absolute dielectric permittivity of classical vacuum. Thus, ϵ_0 cannot be negative. The Planck constant h is the uncertainty principle parameter. Thus, it cannot be negative; negative probabilities do not seem to withstand Occam's razor.

[3] Their average $(c + c_n)/2 \approx -3.436417 \times 10^6$ [m/s] is in the range of the Fermi velocity.

$$m_{\mathrm{P}i} = \pm\sqrt{\frac{\hbar c_n}{G}} = \pm m_{\mathrm{P}}\sqrt{\frac{\alpha}{\alpha_2}} \approx \pm i2.2012 \times 10^{-8}\ [\mathrm{kg}] \quad (> m_{\mathrm{P}})\,, \tag{20}$$

$$t_{\mathrm{P}i} = \pm\sqrt{\frac{\hbar G}{c_n^5}} = \pm t_{\mathrm{P}}\sqrt{\frac{\alpha_2^5}{\alpha^5}} \approx \pm i5.0942 \times 10^{-44}\ [\mathrm{s}] \quad (< t_{\mathrm{P}})\,, \tag{21}$$

$$T_{\mathrm{P}i} = \pm\sqrt{\frac{\hbar c_n^5}{G k_{\mathrm{B}}^2}} = \pm T_{\mathrm{P}}\sqrt{\frac{\alpha^5}{\alpha_2^5}} \approx \pm i1.4994 \times 10^{32}\ [\mathrm{K}] \quad (> T_{\mathrm{P}})\,, \tag{22}$$

and can be expressed[4], using the relation (31), in terms of base Planck units ℓ_{P}, m_{P}, t_{P}, and T_{P}.

Planck units derived from the imaginary base units (19)-(22) are generally not imaginary. The α_2 Planck volume

$$\ell_{\mathrm{P}i}^3 = \pm\left(\frac{\hbar G}{c_n^3}\right)^{3/2} = \pm \ell_{\mathrm{P}}^3\sqrt{\frac{\alpha_2^9}{\alpha^9}} \approx \pm i3.8127 \times 10^{-105}\ [\mathrm{m}^3] \quad \left(< \ell_{\mathrm{P}}^3\right)\,, \tag{23}$$

the α_2 Planck momentum

$$p_{\mathrm{P}i} = \pm m_{\mathrm{P}i} c_n = \pm\sqrt{\frac{\hbar c_n^3}{G}} = \pm m_{\mathrm{P}} c\sqrt{\frac{\alpha^3}{\alpha_2^3}} \approx \pm i6.7504\ [\mathrm{kg\ m/s}] \quad (> m_{\mathrm{P}} c)\,, \tag{24}$$

the α_2 Planck energy

$$E_{\mathrm{P}i} = \pm m_{\mathrm{P}i} c_n^2 = \pm\sqrt{\frac{\hbar c_n^5}{G}} = \pm E_{\mathrm{P}}\sqrt{\frac{\alpha^5}{\alpha_2^5}} \approx \pm i2.0701 \times 10^{9}\ [\mathrm{J}] \quad (> E_{\mathrm{P}})\,, \tag{25}$$

and the α_2 Planck acceleration

$$a_{\mathrm{P}i} = \pm\frac{c_n}{t_{\mathrm{P}i}} = \pm\sqrt{\frac{c_n^7}{\hbar G}} = \pm a_{\mathrm{P}}\sqrt{\frac{\alpha^7}{\alpha_2^7}} \approx \pm i6.0198 \times 10^{51}\ [\mathrm{m/s^2}] \quad (> a_{\mathrm{P}})\,, \tag{26}$$

are imaginary and bivalued. However, the α_2 Planck force

$$F_{\mathrm{P}2} = \pm\frac{E_{\mathrm{P}i}}{\ell_{\mathrm{P}i}} = \pm\frac{c_n^4}{G} = \pm F_{\mathrm{P}}\frac{\alpha^4}{\alpha_2^4} \approx \pm 1.3251 \times 10^{44}\ [\mathrm{N}] \quad (> F_{\mathrm{P}})\,, \tag{27}$$

[4] The notation $\ell_{\mathrm{P}i}(< \ell_{\mathrm{P}})$, for example, means that the modulus of the imaginary Planck length $\ell_{\mathrm{P}i} = \sqrt{\hbar G/c_n^3}$ is lower than the modulus of the Planck length $\ell_{\mathrm{P}} = \sqrt{\hbar G/c^3}$.

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and the α_2 Planck density

$$\rho_{P2} = \pm\frac{m_{Pi}}{\ell_{Pi}^3} = \pm\frac{c_n^5}{\hbar G^2} = \pm\rho_P\frac{\alpha^5}{\alpha_2^5} \approx \pm 5.7735 \times 10^{96}\ [\mathrm{kg/m^3}] \quad (> \rho_P)\,, \tag{28}$$

are real and bivalued. On the other hand, the α_2 Planck area

$$\ell_{Pi}^2 = \frac{\hbar G}{c_n^3} = \ell_P^2\frac{\alpha_2^3}{\alpha^3} \approx -2.4406 \times 10^{-70}\ [\mathrm{m^2}] \quad \left(< \ell_P^2\right), \tag{29}$$

is strictly negative, while the Planck area ℓ_P^2 is strictly positive.

Both α_2 and c_n introduce the second negative vacuum impedance

$$\eta_2 = \frac{4\pi\alpha_2\hbar}{e^2} = \frac{1}{\epsilon_0 c_n} \approx -368.29\ [\Omega] \quad (< \eta)\,, \tag{30}$$

which is lower in modulus than η (7). Solving both impedances (7) and (30) for $4\pi\hbar\epsilon_0/e^2$ and comparing with each other yields the following important relation between the speed of light in vacuum c, negative parameter c_n, and the fine-structure constants α, α_2

$$c\alpha = c_n\alpha_2 \quad (= v_e)\,, \tag{31}$$

where, notably, v_e is the electron's velocity at the first circular orbit in the Bohr model of the hydrogen atom.

The relations between time (21) and temperature α_2 Planck units (22) are inverted, $\alpha^5 t_{Pi}^2 = \alpha_2^5 t_P^2$, $\alpha_2^5 T_{Pi}^2 = \alpha^5 T_P^2$, and saturate Heisenberg's (energy-time version) uncertainty principle using the equipartition theorem for one degree of freedom (one bit of information [5,27])

$$\frac{1}{2}k_B T_P t_P = \frac{1}{2}k_B T_{Pi} t_{Pi} = \frac{\hbar}{2}. \tag{32}$$

Furthermore, eliminating α and α_2 from (18)-(20) yield

$$\frac{q_P^2}{m_P^2} = \frac{q_{Pi}^2}{m_{Pi}^2} = 4\pi\epsilon_0 G, \tag{33}$$

and

$$\ell_P m_P^3 = \ell_{Pi} m_{Pi}^3 \quad \text{and} \quad \ell_P q_P^3 = \ell_{Pi} q_{Pi}^3. \tag{34}$$

Base Planck units themselves admit negative values as negative square roots. By choosing complex analysis, within the framework of emergent dimensionality [5,9,28–30], we enter into bivalence by the very nature of this analysis. All geometric objects admitting both positive and negative volumes and surfaces [30] equal in moduli. On the other hand, imaginary and negative physical quantities are the subject of research. In particular, the subject of scientific research is thermodynamics in the complex plane. Lee–Yang zeros, for example, have been experimentally observed [31,32].

We note here that the imaginary Planck Units are not imaginary due to being multiplied by the imaginary unit i. Contrary to mathematics, they are truly imaginary numbers $\mathbb{I}$ by their very nature; thus, they define imaginary physical quantities inaccessible to direct observations. The complementary Planck units do not apply only to the time dimension but to any imaginary dimension. However, in our four-dimensional Euclidean $\mathbb{R}^3 \times \mathbb{I}$ space-time, Planck units apply in general to the spatial dimensions, while the imaginary ones in general to the imaginary temporal dimension. All the complementary

Planck units have a physical meaning. However, some of them are elusive, like the negative area or imaginary volume, which require two or three orthogonal imaginary dimensions.

4. Black Body Objects

There seem to be only three observable *objects* in nature that emit perfect black-body radiation: unsupported black holes (BH, the densest), neutron stars (NS) supported by neutron degeneracy pressure, and white dwarfs (WD), supported by electron degeneracy pressure (the least dense). We shall collectively call them black-body *objects* (BBO). It has recently been experimentally confirmed that the so-called *accretion instability* is a fundamental physical process [33] common for all BBOs.

As black-body radiation is radiation emitted by a body in thermodynamic equilibrium, it is patternless (thermal noise) radiation and depends only on the temperature of this body. In the case of BHs, this is known as Hawking radiation, wherein the BH temperature $T_{BH} = T_{\text{P}}/(2\pi d_{BH})$, where T_{P} is the Planck temperature as a function of the BH diameter [5] $D_{BH} = d_{BH}\ell_{\text{P}}$, where $d_{BH} \in \mathbb{R}$ (in the following d is also called a diameter) and ℓ_{P} is the Planck length.

As Hawking radiation depends only on the diameter of a BH, it must be the same for a given BH, even though it is momentary as it fluctuates (cf. Appendix D). As the interiors of the BBOs are inaccessible to an exterior observer [34], BBOs do not have interiors and can only be defined by their diameters (cf. [5] Fig. 2(b)). The term *object* as a collection of *matter* is a misnomer in general, as it neglects quantum nonlocality. But it is a particularly staring misnomer if applied to BBOs. Thus we use emphasis for *particle* and *object* as these terms have no substantial meaning in emergent dimensionality. In particular, given the recent observation of *quasiparticles* in classical systems [35].

But not only BBOs are perfectly spherical. Also, the early epochs of their collisions must be perfectly spherical, as it has been recently, experimentally confirmed [36] for NSs based on the AT2017gfo kilonova data. One can hardly expect a collision of two perfectly spherical, patternless thermal noises to produce some aspherical pattern instead of another perfectly spherical patternless noise. Where would the information about this pattern come from at the moment of the collision? From the point of impact? No point of impact is distinct on a patternless surface.

As black-body radiation is patternless, the triangulated [5] of the BBOs, as well as their early epoch collisions, must contain a balanced number of Planck area triangles, each carrying binary potential $\delta\varphi_k = -\{0,1\}c^2$, as it has been shown for BHs [5], based on Bekenstein-Hawking entropy

$$S_{BH} = \frac{1}{4}k_{\text{B}}N_{BH}, \tag{35}$$

where $N_{BH} := 4\pi R_{BH}^2/\ell_{\text{P}}^2 = \pi d_{BH}^2$ is the BH information capacity (i.e., the number of the triangular Planck areas at the BH horizon, corresponding to bits of information [27,34,37] and the fractional part triangle $\{\pi d_{BH}^2\}$ to small to carry a single bit of information [*sic*]) and $R_{BH} = 2GM_{BH}/c^2$ is the BH (Schwarzschild) radius. The BH entropy (35) can be derived from the Bekenstein bound

$$S \leq \frac{2\pi k_{\text{B}}RE}{\hbar c}, \tag{36}$$

an upper limit on the thermodynamic entropy S that can be contained within a sphere of radius R having energy E, where k_{B} is the Boltzmann constant and $\hbar$ is the reduced Planck constant, after plugging into (36) the BH radius R_{BH} and energy $E_{BH} = M_{BH}c^2$ taken from mass-energy equivalence.

Since the patternless nature of the perfect black-body radiation was derived [5] by comparing BH entropy (35) with the binary entropy variation $\delta S = k_{\text{B}}N_1/2$ ([5] Eq. (55)), which is valid for any holographic sphere, where $N_1 \in \mathbb{N}$ denotes the number of active Planck areas with binary potential

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$\delta\varphi_k = -c^2$, the BH entropy (35) must be valid also for NSs and WDs. Thus, defining the generalized radius of a holographic sphere of mass M as a function of GM/c^2 multiplier k [5]

$$R := k\frac{GM}{c^2}, \tag{37}$$

and the generalized energy E of this sphere as a function of Mc^2 multiplier a

$$E := aMc^2, \tag{38}$$

with $k, a \in \mathbb{R} : k \geq 2$, the generalized Bekenstein bound (36) becomes

$$S \leq \frac{1}{2}k_{\mathrm{B}}\frac{a}{k}N, \tag{39}$$

where $N := 4\pi R^2/\ell_{\mathrm{P}}^2$ is the information capacity of this sphere, the surface of which contains $\lfloor N \rfloor$ Planck triangles, where "$\lfloor x \rfloor$" is the floor function that yields the greatest integer less than or equal to its argument x.

The generalized Bekenstein bound (39) equals the BH entropy (35) if $\frac{a}{2k} = \frac{1}{4} \Rightarrow a = \frac{k}{2}$. Thus, the energy of all BBOs having a radius (37) is

$$E_{BBO} = \frac{k}{2}M_{BBO}c^2, \tag{40}$$

with $k = 2$ in the case of BHs and $k > 2$ for NSs and WDs.

Schwarzschild BHs are fundamentally uncharged, contrary to NSs and WDs, since the entropy (35) of any BH is equal to that of the uncharged Schwarzschild BH with the same area by the Penrose process. It is accepted that in the case of NSs, electrons combine with protons to form neutrons but it is never the case that all electrons and all protons become neutrons; WDs are charged by definition as they are composed mostly of electron-degenerate matter.

As the entropy of independent systems is additive, a collision of two BBOs, BBO_1 and BBO_2, having entropies $S_{BBO_1} = \frac{1}{4}k_{\mathrm{B}}N_{BBO_1} = \frac{1}{4}k_{\mathrm{B}}\pi d_{BBO_1}^2$ and $S_{BBO_2} = \frac{1}{4}k_{\mathrm{B}}\pi d_{BBO_2}^2$, produces another BBO_C having entropy

$$S_{BBO_C} = S_{BBO_1} + S_{BBO_2} \quad \Rightarrow d_{BBO_C}^2 = d_{BBO_1}^2 + d_{BBO_1}^2. \tag{41}$$

This shows that a collision of two primordial BHs, each having the Planck length diameter, the reduced Planck temperature $\frac{T_{\mathrm{P}}}{2\pi}$ (which is the largest physically significant temperature [28]), and no tangential acceleration a_{LL} [5,28], produces a BH having $d_{BH} = \pm\sqrt{2}$ which represents the minimum BH diameter allowing for the notion of time [28], while a collision of the latter two BHs produces a BH having $d_{BH} = \pm 2$ having the triangulation defining only one precise diameter between its poles (cf. [5] Fig. 3(b)). Diameter $d_{BH} = \pm 2$ is also recovered [5] from Heisenberg's Uncertainty Principle (cf. Appendix C).

The hitherto considerations may be unsettling for the reader as the energy (40) of BBOs other than BHs exceeds mass-energy equivalence $E = Mc^2$ for $k > 2$, which is the maximum *real* energy. Thus, a part of the energy of NSs and WDs must be imaginary and thus unmeasurable. We shall consider this question in the subsequent section.

5. Complex Energies

A complex energy formula

$$E_R := E_{M_R} + iE_{Q_R} = (1 + i\beta_R)\, M_R c^2, \tag{42}$$

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where $E_{M_R} = M_R c^2$ represents real and iE_{Q_R} imaginary energy of an *object* having mass M_R and charge Q_R[5] of the *object*, and

$$\beta_R := \frac{E_{Q_R}}{E_{M_R}} = \frac{Q_R}{2M_R\sqrt{\pi\epsilon_0 G}}, \tag{43}$$

is the imaginary-real energy ratio[6], was proposed in [38] (Eqs. (1), (3), and (4)). Equations (42) and (43) consider real (physically measurable) masses M_R and charges Q_R.

Planck charge relations (2) and (16) imply that the elementary charge e is the same both in real and imaginary dimensions, since

$$e^2 = \alpha q_\mathrm{P}^2 = \alpha_2 q_{\mathrm{P}i}^2. \tag{44}$$

On the other hand, there is no physically meaningful *elementary mass* $m_e = \pm 3.4566 \times 10^{-18}$ [kg] that would satisfy the analogous relation (20)

$$m_e^2 = \alpha m_\mathrm{P}^2 = \alpha_2 m_{\mathrm{P}i}^2. \tag{45}$$

Thus, as to the modulus, charges are the same in both real and imaginary dimensions, while masses are different. We note that the relations (44) and (20) reflect the inverse-square proportionality of both Coulomb's law and Newton's law of gravity.

We shall discretize charges by the elementary charge, $Q := q_e e$, $q_e \in \mathbb{Z}$, and modify the equation (42) to the form involving imaginary masses M_i and charges $Q_i = iQ = iq_e e$ defining the following two complex energies, the complex energy of real mass M and imaginary charge Q_i

$$\begin{aligned} E_{MQ_i} &:= E_M + E_{Q_i} = \\ &= \left(1 + \beta_{Q_i}\right) Mc^2 = \left(M + iq_e\sqrt{\alpha}m_\mathrm{P}\right)c^2, \end{aligned} \tag{46}$$

and the complex energy of real charge Q and imaginary mass M_i

$$\begin{aligned} E_{QM_i} &:= E_Q + E_{M_i} = \\ &= \left(\beta_Q + 1\right) M_i c_n^2 = \left(q_e\sqrt{\alpha_2}m_{\mathrm{P}i} + M_i\right)c_n^2, \end{aligned} \tag{47}$$

where

$$\beta_{Q_i} := \frac{Q_i}{2M\sqrt{\pi\epsilon_0 G}} = \frac{iq_e\sqrt{\alpha}m_\mathrm{P}}{M} \in \mathbb{I}, \tag{48}$$

$$\beta_Q := \frac{Q}{2M_i\sqrt{\pi\epsilon_0 G}} = \frac{q_e\sqrt{\alpha_2}m_{\mathrm{P}i}}{M_i} \in \mathbb{I}. \tag{49}$$

Equations (46)-(49) yield two different quanta of the charge-dependent energies corresponding to the elementary charge, the imaginary quantum

$$E_{Q_i}(q_e = \pm 1) = \pm i\sqrt{\alpha}E_\mathrm{P} \approx \pm i1.67 \times 10^8 \text{ [J]}, \tag{50}$$

and the larger in modulus, real quantum

$$E_Q(q_e = \pm 1) = \pm\sqrt{\alpha_2}E_{\mathrm{P}i} \approx \pm 1.75 \times 10^8 \text{ [J]}, \tag{51}$$

and furthermore $\alpha^2 E_{Qi} = i\alpha_2^2 E_Q$.

---

5 Charges in the cited study are defined in CGS units; here we adopt SI.
6 In the cited study it is called α, so we shall call it β to avoid confusion with the fine-structure constant α.

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Complex energies (46), (47) are real-to-imaginary balanced ($|E_M| = |E_{Qi}|$ and $|E_Q| = |E_{Mi}|$) if

$$
\begin{aligned}
M &= q_e\sqrt{\alpha}m_{\mathrm{P}} \approx \pm 1.8592 \times 10^{-9}\ [\mathrm{kg}],\\
M_i &= q_e\sqrt{\alpha_2}m_{\mathrm{P}} \approx \pm i1.8383 \times 10^{-9}\ [\mathrm{kg}].
\end{aligned} \tag{52}
$$

Furthermore, complex energies (46), (47) unify Coulomb's law and Newton's law of gravity between two *particles* in four complex force equations (cf. Appendix E).

The squared moduli of the energies (42) and (46) can be expressed as

$$
|E_{MQ_i}|^2 = M^2c^4\left(1-\beta_{Qi}^2\right) = \left(M^2 + q_e^2\alpha m_{\mathrm{P}}^2\right)c^4, \tag{53}
$$

and (using relations (31) and (20))

$$
\begin{aligned}
|E_{QM_i}|^2 &= M_i^2c_n^4\left(\beta_{Mi}^2 - 1\right) = \left(q_e^2\alpha_2 m_{\mathrm{P}i}^2 - M_i^2\right)c_n^4 =\\
&= \frac{\alpha^4}{\alpha_2^4}\left(q_e^2\alpha m_{\mathrm{P}}^2 - M_i^2\right)c^4.
\end{aligned} \tag{54}
$$

We assume these moduli are equal, which yields the value of the imaginary mass M_i corresponding to the *particle* having mass M and charge Q

$$
\begin{aligned}
\alpha_2^4\left(M^2 + q_e^2\alpha m_{\mathrm{P}}^2\right) &= \alpha^4\left(q_e^2\alpha m_{\mathrm{P}}^2 - M_i^2\right),\\
M_i &= \pm\sqrt{q_e^2\alpha m_{\mathrm{P}}^2\left(1 - \frac{\alpha_2^4}{\alpha^4}\right) - \frac{\alpha_2^4}{\alpha^4}M^2},
\end{aligned} \tag{55}
$$

In particular for an uncharged mass M ($q_e = 0$) this yields

$$
M_i\alpha^2 = \pm iM\alpha_2^2 \quad \text{or} \quad M_i = \pm i\frac{\alpha_2^2}{\alpha^2}M \approx \pm 0.9557iM. \tag{56}
$$

Since mass M_i is imaginary by definition, the argument of the square root in the relation (55) must be negative. This leads to

$$
|M| > |q_e|m_{\mathrm{P}}\sqrt{\alpha\left(\frac{\alpha^4}{\alpha_2^4} - 1\right)} \approx |q_e|5.7275 \times 10^{-10}\ [\mathrm{kg}], \tag{57}
$$

which means that masses of uncharged micro *BHs* ($M_{BH} < 5.7275 \times 10^{-10}$ [kg]) can be arbitrary but micro *NS*s and micro *WD*s cannot be observed, as achieving a net charge $Q = 0$ is impossible in this case. Even a single elementary charge renders the mass $M = 5.7275 \times 10^{-10}$ [kg] comparable to the mass of a grain of sand.

We can interpret the modulus of the generalized energy of *BBO*s (40) as the modulus of the complex energy of real mass (53), taking the observable real energy $E_{BBO} = M_{BBO}c^2$ of the *BBO* as the real part of this energy. Thus

$$
\begin{aligned}
\left(\frac{k}{2}M_{BBO}c^2\right)^2 &= \left(M_{BBO}^2 + q_{eBBO}^2\alpha m_{\mathrm{P}}^2\right)c^4,\\
q_{eBBO} &= \pm\frac{M_{BBO}}{m_{\mathrm{P}}}\sqrt{\frac{1}{\alpha}\left(\frac{k^2}{4} - 1\right)},
\end{aligned} \tag{58}
$$

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which is real for $k \geq 2$ and for $k = 2$ confirms vanishing net charge of BHs. Similarly, we can interpret the modulus of the generalized energy of BBOs (40) as the modulus of the complex energy of real charge (54). Thus

$$\begin{aligned} \frac{k^2}{4} M_{BBO}^2 &= \frac{\alpha^4}{\alpha_2^4} \left(q_{eBBO}^2 \alpha m_{\mathrm{P}}^2 - M_{iBBO}^2 \right), \\ M_{iBBO}^2 &= q_{eBBO}^2 \alpha m_{\mathrm{P}}^2 - \frac{\alpha_2^4}{\alpha^4} \frac{k^2}{4} M_{BBO}^2. \end{aligned} \tag{59}$$

Substituting q_{eBBO}^2 from the relation (58) into the relation (59) yields

$$\begin{aligned} M_{iBBO}^2 &= \left[\frac{k^2}{4} \left(1 - \frac{\alpha_2^4}{\alpha^4} \right) - 1 \right] M_{BBO}^2, \\ M_{iBBO} &= \pm M_{BBO} \sqrt{ \frac{k^2}{4} \left(1 - \frac{\alpha_2^4}{\alpha^4} \right) - 1}, \end{aligned} \tag{60}$$

which for $k = 2$ also corresponds to the relation (56) between uncharged masses M and M_i, where no assumptions concerning the BBO energy have been made.

Furthermore, the argument of the square root in the relation (60) must be negative, as mass M_i is imaginary by definition. This leads to

$$|k_{max}| < \frac{2}{\sqrt{1 - \frac{\alpha_2^4}{\alpha^4}}} \approx 6.7933. \tag{61}$$

Relations (58) and (60) are shown in Fig 1.

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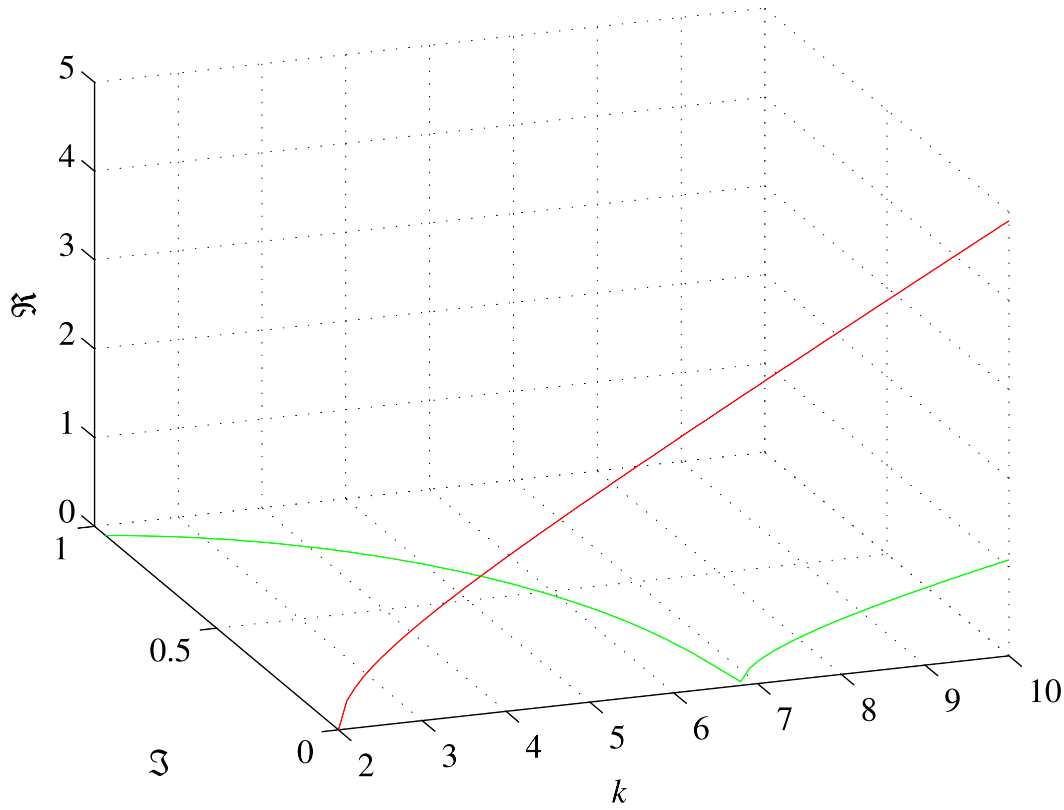

Figure 1. Imaginary mass M_{iBBO}/M_{BBO} (green) and charge $q_{BBO}m_{\mathrm{P}}\sqrt{\alpha}/M_{BBO}$ (red) of a BBO. Mass is imaginary up to $k < 6.79$.

The relation (61) sets the upper bound on the BBO radius and energy

$$R_{BBO} < \frac{R_{BH}}{\sqrt{1 - \frac{\alpha_2^4}{\alpha^4}}} \quad \text{and} \quad E_{BBO} < \frac{M_{BBO}c^2}{\sqrt{1 - \frac{\alpha_2^4}{\alpha^4}}} \tag{62}$$

where R_{BH} is the radius of BH having a mass of the BBO. As WDs are the largest BBOs, this bound defines the maximum radius of a WD core.

These results show that the radius of charged BBOs (i.e., BBOs other than BHs) is a continuous function of $k \in \mathbb{R} : 2 < k < k_{max}$. k_{max} is the largest k satisfying the BBO entropy relation (35), a necessary condition of perfect black body patternless radiation [5].

6. Discussion

We have shown that the reflectance of graphene under the normal incidence of electromagnetic radiation expressed as the quadratic equation for the fine-structure constant α must introduce the 2nd negative fine-structure constant α_2. The sum of the reciprocal of this 2nd fine-structure constant α_2 with the reciprocal of the fine-structure constant α (2) is independent of the reflectance value R and remarkably equals simply $-\pi$. Particular algebraic definition of the fine-structure constant $\alpha^{-1} = 4\pi^3 + \pi^2 + \pi$ (13), containing the free π term, can be interpreted as the asymptote of the CODATA value α^{-1}, the value of which increases with time. The negative fine-structure constant α_2 introduces the complementary set of Planck units applicable to imaginary dimensions, including five imaginary base Planck units (18)-(22). Real and imaginary mass and charge units (33), length and

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mass units (34) units, and temperature and time units (32) are directly related to each other. Also, the elementary charge e is common for real and imaginary dimensions (44).

It has been shown that the generalized energy (40) of all perfect black-body objects (black holes, neutron stars, and white dwarfs) having the generalized radius $R_{BBO} = kGM/c^2$ exceed mass-energy equivalence if $k > 2$. Applying the complementary Planck units to a complex energy formula [38] yields two complex energies (46), (47) allowing for storing the excess of this energy in their imaginary parts, inaccessible for direct observation. The results show that the perfect black-body objects other than black holes cannot have masses lower than 5.7275×10^{-10} [kg] and that the maximum radius of their cores is given by $k = 2/\sqrt{1 - \alpha_2^4/\alpha^4} \approx 6.7933$.

The findings of this study inquire further research in the context of information-theoretic approach [1–6] and emergent dimensionality [9,29,30].

Acknowledgments: I truly thank my wife for her support when this research [39,40] was conducted. I thank Wawrzyniec Bieniawski for inspiring discussions, constructive ideas concerning the layout and flow of this paper. I thank Tomasz Tomski for the definition of the scalar product for Euclidean spaces $\mathbb{R}^a \times \mathbb{I}^b$ (1).

Appendix A Other Quadratic Equations

The quadratic equation for the sum of transmittance (3) and absorptance (5), putting $C_{TA} := T + A$, is

$$\frac{1}{4}C_{TA}\pi^2\alpha^2 + (C_{TA} - 1)\,\pi\alpha + (C_{TA} - 1) = 0, \tag{A1}$$

and has two roots with reciprocals

$$\alpha^{-1} = \frac{C_{TA}\pi}{2\left(1 - C_{TA} + \sqrt{1 - C_{TA}}\right)} \approx 137.036, \tag{A2}$$

and

$$\alpha_2^{-1} = \frac{C_{TA}\pi}{2\left(1 - C_{TA} - \sqrt{1 - C_{TA}}\right)} \approx -140.178, \tag{A3}$$

whereas their sum $\alpha^{-1} + \alpha_2^{-1} = -\pi$ is, similarly as (12), also independent of T and A.

Other quadratic equations do not feature this property. For example, the sum of $T + R$ (6) expressed as the quadratic equation and putting $C_{TR} := T + R$, is

$$\frac{1}{4}\,(C_{TR} - 1)\,\pi^2\alpha^2 + C_{TR}\pi\alpha + (C_{TR} - 1) = 0, \tag{A4}$$

and has two roots with reciprocals

$$\alpha^{-1} = \frac{\pi(C_{TR} - 1)}{-2C_{TR} + 2\sqrt{2C_{TR} - 1}} \approx 137.036, \tag{A5}$$

and

$$\alpha_{TR}^{-1} = \frac{\pi(C_{TR} - 1)}{-2C_{TR} - 2\sqrt{2C_{TR} - 1}} \approx 0.0180, \tag{A6}$$

whereas their sum

$$\alpha_{TR_1}^{-1} + \alpha_{TR_2}^{-1} = \frac{-\pi C_{TR}}{C_{TR} - 1} \approx 137.054 \tag{A7}$$

is dependent on T and R.

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Appendix B Two π-Like Constants

With algebraic definitions of α (13) and α_2 (14), transmittance T (3), reflectance R (4) and absorptance A (5) of MLG for normal EMR incidence can be expressed just by π. For $\alpha^{-1} = 4\pi^3 + \pi^2 + \pi$ (13) they become

$$T(\alpha) = \frac{4\left(4\pi^2 + \pi + 1\right)^2}{\left(8\pi^2 + 2\pi + 3\right)^2} \approx 97.746\%, \tag{A8}$$

$$A(\alpha) = \frac{4\left(4\pi^2 + \pi + 1\right)}{\left(8\pi^2 + 2\pi + 3\right)^2} \approx 2.241\%, \tag{A9}$$

while for $\alpha_2^{-1} = -4\pi^3 - \pi^2 - 2\pi$ (14) they become

$$T(\alpha_2) = \frac{4\left(4\pi^2 + \pi + 2\right)^2}{\left(8\pi^2 + 2\pi + 3\right)^2} \approx 102.279\%, \tag{A10}$$

$$A(\alpha_2) = -\frac{4\left(4\pi^2 + \pi + 2\right)}{\left(8\pi^2 + 2\pi + 3\right)^2} \approx -2.292\%, \tag{A11}$$

with

$$R(\alpha) = R(\alpha_2) = \frac{1}{\left(8\pi^2 + 2\pi + 3\right)^2} \approx 0.013\%. \tag{A12}$$

Obviously $(T(\alpha) + A(\alpha)) + R(\alpha) = (T(\alpha_2) + A(\alpha_2)) + R(\alpha_2) = 1$ as required by the law of conservation of energy (8), whereas each conservation law is associated with a certain symmetry, as asserted by Noether's theorem. Nonetheless, physical interpretation of $T(\alpha_2) > 1$ and $A(\alpha_2) < 0$ invites further research. $A(\alpha) > 0$ implies a *sink*, whereas $A(\alpha_2) < 0$ implies a *source*, whereas the opposite holds true for the transmittance T, as illustrated schematically in Fig A1. Perhaps, the negative absorptance and transmittance exceeding 100% for α_2 (11) or (14) could be explained in terms of graphene spontaneous emission.

The quadratic equation (9) describing the reflectance R of MLG under normal incidence of EMR (or alternatively (A1)) can also be solved for π yielding two roots

$$\pi(R, \alpha_*)_1 = \frac{2\sqrt{R}}{\alpha_*(1 - \sqrt{R})}, \quad \text{and} \tag{A13}$$

$$\pi(R, \alpha_*)_2 = \frac{-2\sqrt{R}}{\alpha_*(1 + \sqrt{R})}, \tag{A14}$$

dependent on R and α_*, where α_* indicates α or α_2. This can be further evaluated using the MLG reflectance R (4) or (A12) (which is the same for both α and α_2), yielding four, yet only three distinct, possibilities

$$\pi_1 = \pi(\alpha)_1 = -\pi\frac{4\pi^2 + \pi + 1}{4\pi^2 + \pi + 2} = \pi\frac{\alpha_2}{\alpha} \approx -3.0712, \tag{A15}$$

$$\pi(\alpha)_2 = \pi(\alpha_2)_1 = \pi \approx 3.1416, \quad \text{and} \tag{A16}$$

$$\pi_2 = \pi(\alpha_2)_2 = -\pi\frac{4\pi^2 + \pi + 2}{4\pi^2 + \pi + 1} = \pi\frac{\alpha}{\alpha_2} \approx -3.2136. \tag{A17}$$

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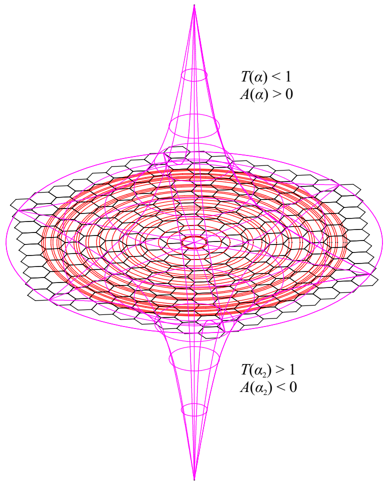

Figure A1. Illustration of the concepts of negative absorptance and excessive transmittance of EMR under normal incidence on MLG.

The modulus of π_1 (A15) corresponds to a convex surface having a positive Gaussian curvature, whereas the modulus of π_2 (A17) - to a negative Gaussian curvature. Their product $\pi_1\pi_2 = \pi^2$ is independent of α_*, and their quotient $\pi_1/\pi_2 = \alpha_2^2/\alpha^2$ is independent of π. It remains to be found, whether each of them describes the ratio of circumference of a circle drawn on the respective surface to its diameter (π_c) or the ratio of the area of this circle to the square of its radius (π_a). These definitions produce different results on curved surfaces, whereas $\pi_a > \pi_c$ on convex surfaces, while $\pi_a < \pi_c$ on saddle surfaces [41].

Appendix C Planck Units and HUP

Perhaps the simplest derivation of the squared Planck length is based on Heisenberg's uncertainty principle

$$\delta P_{\text{HUP}}\delta R_{\text{HUP}} \geq \frac{\hbar}{2} \quad \text{or} \quad \delta E_{\text{HUP}}\delta t_{\text{HUP}} \geq \frac{\hbar}{2}, \tag{A18}$$

where δP_{HUP}, δR_{HUP}, δE_{HUP}, and δt_{HUP} denote momentum, position, energy, and time uncertainties, by replacing energy uncertainty $\delta E_{\text{HUP}} = \delta M_{\text{HUP}}c^2$ with mass uncertainty and time uncertainty with position uncertainty, using mass-energy equivalence and $\delta t_{\text{HUP}} = \delta R/c_{\text{HUP}}$ [26], which yields

$$\delta M_{\text{HUP}}\delta R_{\text{HUP}} \geq \frac{\hbar}{2c}. \tag{A19}$$

Plugging $\delta M_{\text{HUP}} = \delta R_{\text{HUP}}c^2/(2G)$ for BH mass into (A19) we arrive at $\delta R_{\text{HUP}}^2 = \ell_{\text{P}}^2 \Rightarrow \delta D_{\text{HUP}} = \pm 2\ell_{\text{P}}$ and recover BH diameter $d_{BH} = \pm 2$.

However, using the same procedure but inserting the BH radius, instead of the BH mass, into the uncertainty principle (A19) leads to $\delta M_{\text{HUP}}^2 = \frac{1}{4}\hbar c/G = \frac{1}{4}m_{\text{P}}^2$. In general, using the generalized radius (37) in both procedures, one obtains

$$\delta M_{\text{HUP}}^2 = \frac{1}{2k}m_{\text{P}}^2 \quad \text{and} \quad \delta R_{\text{HUP}}^2 = \frac{k}{2}\ell_{\text{P}}^2. \tag{A20}$$

Thus, if k increases mass δM_{HUP} decreases, and δR_{HUP} increases and the factor is the same for $k = 1$ i.e., for *orbital speed radius* $\delta R = G\delta M/c^2$ or the *orbital speed mass* $\delta M = \delta Rc^2/G$.

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Appendix D Fluctuations of the Holographic Spheres

A simple relation describing the BH information capacity after absorption $(+16\pi^2 d_{BH}/l)$ or emission $(-16\pi^2 d_{BH}/l)$ of a *particle* having the wavelength λ multiplier $l = \lambda/\ell_P$ has been derived in [5] (Eq. (18)) as

$$N_{BH}^{A/E}(d_{BH}, l) = 64\pi^3 \frac{1}{l^2} \pm 16\pi^2 \frac{d_{BH}}{l} + \pi d_{BH}^2. \tag{A21}$$

Using the generalized radius (37), this equation can be generalized to all holographic spheres, including BBOs

$$N^{A/E}(d, l) = 16k^2\pi^3 \frac{1}{l^2} \pm 8k\pi^2 \frac{d}{l} + \pi d^2. \tag{A22}$$

Appendix E Complex Forces

Complex energies (46) and (47) of real and complex masses and charges (similarly to the complex energy of real masses and charges (42), [38] Eqs. (7), (8)) unify Coulomb's law and Newton's law of gravity between two *particles* in four complex force equations, the complex force between two real masses acting over a real distance $R := r\ell_P$, $r \in \mathbb{R}$

$$\begin{aligned} F_{MQ_i} &= \frac{G}{c^4 R^2} E_{1MQ_i} E_{2MQ_i} = \\ &= G\frac{M_1 M_2}{R^2}\left(1 + \beta_{1Q_i} + \beta_{2Q_i}\right) - \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{R^2}. \end{aligned} \tag{A23}$$

the complex force between two real masses acting over an imaginary distance $R_i := r_i \ell_{Pi}^2$, $r_i \in \mathbb{R}$

$$\begin{aligned} \tilde{F}_{MQ_i} &= \frac{G}{c_n^4 R_i^2} E_{1MQ_i} E_{2MQ_i} = \\ &= \frac{\alpha_2^4}{\alpha^4}\left[G\frac{M_1 M_2}{R_i^2}\left(1 + \beta_{1Q_i} + \beta_{2Q_i}\right) - \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{R_i^2}\right], \\ &= \frac{\alpha_2}{\alpha}\frac{r^2}{r_i^2}\left[G\frac{M_1 M_2}{R^2}\left(1 + \beta_{1Q_i} + \beta_{2Q_i}\right) - \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{R^2}\right] = \\ &= \frac{\alpha_2}{\alpha}\frac{r^2}{r_i^2} F_{MQ_i}, \end{aligned} \tag{A24}$$

the complex force between two imaginary masses acting over an imaginary distance R_i

$$\begin{aligned} \tilde{F}_{M_i Q} &= \frac{G}{c_n^4 R_i^2} E_{1M_i Q} E_{2M_i Q} = \\ &= G\frac{M_{1i} M_{2i}}{R_i^2}\left(1 + \beta_{1M_i} + \beta_{2M_i}\right) + \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{R_i^2}, \\ &= \frac{\alpha^3}{\alpha_2^3}\frac{r^2}{r_i^2}\left[G\frac{M_{1i} M_{2i}}{R^2}\left(1 + \beta_{1M_i} + \beta_{2M_i}\right) + \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{R^2}\right], \end{aligned} \tag{A25}$$

and the complex force between two imaginary masses acting over a real distance R

$$\begin{aligned} F_{M_i Q} &= \frac{G}{c^4 R^2} E_{1M_i Q} E_{2M_i Q} = \\ &= \frac{\alpha^4}{\alpha_2^4}\left[G\frac{M_{1i} M_{2i}}{R^2}\left(1 + \beta_{1M_i} + \beta_{2M_i}\right) + \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{R^2}\right] = \\ &= \frac{\alpha}{\alpha_2}\frac{r_i^2}{r^2}\tilde{F}_{M_i Q}, \end{aligned} \tag{A26}$$

We exclude mixed forces (of real and imaginary masses/charges), as real and imaginary dimensions are orthogonal. Thus

$$\alpha_2 r^2 F_{MQ_i} = \alpha r_i^2 \tilde{F}_{MQ_i} \quad \text{and} \quad \alpha_2 r^2 F_{M_iQ} = \alpha r_i^2 \tilde{F}_{M_iQ}. \tag{A27}$$

which, with a simplifying assumption of $r = r_i$, leads to

$$F_{MQ_i} \approx -1.0229 \tilde{F}_{MQ_i} \quad \text{and} \quad F_{M_iQ} \approx -1.0229 \tilde{F}_{M_iQ}, \tag{A28}$$

and

$$\begin{gathered} F_{M_iQ} + F_{MQ_i} = \frac{\alpha^4}{\alpha_2^4}\left[G\frac{M_{1i}M_{2i} + M_1M_2}{R^2} + \right. \\ \left. \frac{\sqrt{G}}{2\sqrt{\pi\epsilon_0}}\frac{Q_1(M_{2i} - M_2) + Q_2(M_{1i} - M_1)}{R^2}\right], \end{gathered} \tag{A29}$$

which, for uncharged masses, is

$$\begin{gathered} F_{M_iQ} + F_{MQ_i} = \frac{\alpha^4}{\alpha_2^4}\left[G\frac{M_{1i}M_{2i} + M_1M_2}{R^2}\right] \approx \\ \approx 1.0949\left[G\frac{M_{1i}M_{2i} + M_1M_2}{R^2}\right], \end{gathered} \tag{A30}$$

which using (56) becomes

$$F_{M_iQ} + F_{MQ_i} = G\frac{M_1M_2}{R^2}\left(1 + \frac{\alpha^4}{\alpha_2^4}\right) \approx 2.0949 G\frac{M_1M_2}{R^2}. \tag{A31}$$

Thus, forces acting over a real distance R are stronger and opposite to the corresponding forces acting over an imaginary distance R_i even though the Planck force is lower in modulus than the complementary (and, in this case, real) Planck force (27). This matter requires further research.

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