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Hypersurfaces in a Euclidean Space with a Killing Vector Field

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Abstract: An odd-dimensional sphere admits a killing vector field, induced by the transform of the unit normal by the complex structure of the ambiant Euclidean space. In this paper, we study orientable hypersurfaces in a Euclidean space those admit a unit Killing vector field and find two characterizations of odd-dimensional spheres. In first result, we show that a complete and simply connected hypersurface of Euclidean space $\mathbb{R}^{n+1}$, $n>1$ admits a unit Killing vector field ξ that leaves the shape operator S invariant and has sectional curvatures of plane sections containing ξ positive satisfies $S(\xi)=\alpha\xi$, α mean curvature if and only if $n=2m-1$, α is constant and the hypersurfaces is isometric to the sphere $S^{2m-1}(\alpha^2)$. Similarly, we find another characterization of unit sphere $S^2(\alpha^2)$ using the smooth function $\sigma=g(S(\xi),\xi)$ on the hypersurface.

Keywords: euclidean space; hypersurface; Killing vector field

MSC: 53C20; 53A50

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1. Introduction

The study of differential geometry started with the study of curves and surfaces in the Euclidean spaces $\mathbb{R}^3$ with the basic notions such as curvature, torsion, Frenet-serret frame, first and second fundamental forms, Gauss curvature and mean curvature. With the advancements, it shifted to studying hypersurfaces in higher dimensional Euclidean space $\mathbb{R}^{n+1}$ with tools such as unit normal N to hypersurface M, the shape operator S, the equations of Gauss namely

$$D_XY=\nabla_XY+g(S(X),Y)N \tag{1}$$

and

$$D_XN=-S(X),\quad X,Y\in\mathfrak{X}(M), \tag{2}$$

where D_X and ∇_X are covariant derivative operators on $\mathbb{R}^{n+1}$ and hypersurface M respectively and g is the Riemannian metric induced on M by the Euclidean metric $\langle,\rangle$ on $\mathbb{R}^{n+1}$. The mean curvature α of the hypersurface M is given by $\alpha=\frac{1}{n}trace(S)$, and we have the Gauss and Codazzi equations for the hypersurface M, namely, for all $X,Y,Z\in\mathfrak{X}(M)$,

$$R(X,Y)Z=g(S(Y),Z)S(X)-g(S(X),Z)S(Y) \tag{3}$$

$$(\nabla_XS)(Y)=(\nabla_YS)(X),\quad X,Y\in\mathfrak{X}(M) \tag{4}$$

where $R(X,Y)Z$ is the curvature tensor of M and $(\nabla_XS)(Y)=\nabla_XSY-S(\nabla_XY)$.

The Ricci tensor Ric of the hypersurface M is given by

$$Ric(X,Y)=n\alpha\left(g(S(X),Y)-g(S(X),S(Y))\right) \tag{5}$$

In the following sections, we will use the notation $R(X,Y;Z,W)$ to refer to the value obtained by applying the metric g to $R(X,Y)Z$ and W.

A hypersurface M of the Euclidean space $\mathbb{R}^{n+1}$ is said to be totally umbilical if the shape operator $S = \alpha I$ and for $n > 1$, it follows that α is a constant. It is known that a complete and connected totally umbilical hypersurface M of the Euclidean space $\mathbb{R}^{n+1}$ is isometric to the sphere $S^n(\alpha^2)$ of constant curvature α^2 (cf. [10]).
An interesting global result on a compact hypersurface M states that there exists a point $p \in M$ such that all sectional curvatures of M at p are positive (cf. [10]).
Given a compact hypersurface M of $\mathbb{R}^{n+1}$, the support function $\rho = \langle \psi, N \rangle$ where $\psi : M \longrightarrow \mathbb{R}^{n+1}$ is the immersion, satisfies the Minkowski's formula

$$\int_M (1 + \rho\alpha) = 0 \tag{6}$$

Recall that a hypersurface M of the Euclidean space is said to be a minimal hypersurface if $\alpha = 0$. As a result of Minkowski's formula, it follows that there is no compact minimal hypersurface in a Euclidean space $\mathbb{R}^{n+1}$.

One of the interesting questions in differential geometry of compact hypersurfaces is to find the conditions under which the hypersurface of $\mathbb{R}^{n+1}$ is isometric to the sphere $S^n(c)$ of the constant curvature c.

In [4], it is shown that if the scalar curvature τ of a compact hypersurface M in the Euclidean space $\mathbb{R}^{n+1}$ satisfies $\tau \leq \lambda_1(n-1)$, then M is isometric to $S^n(c)$. Here λ_1 stands for the first eigenvalue of the Laplace operator. For similar results on compact hypersurfaces in $\mathbb{R}^{n+1}$, we refer to (cf. [5]-[8]).
Consider the odd-dimensional sphere $S^{2n-1}(c)$ as a hypersurface in the complex Euclidean space $\mathbb{C}^n$ with natural embedding $\Psi : S^{2n-1}(c) \longrightarrow \mathbb{C}^n$, with $\Psi(x) = x$. Then it has shape operator $S = -\sqrt{c}I$ and unit normal $N = \sqrt{c}\Psi$.

Due to pressence of complex structure J on $\mathbb{C}^n$, we get a unit vector field ξ defined on $S^{2n-1}(c)$ by

$$\xi = -JN,$$

which is a Killing vector field on the sphere $S^{2n-1}(c)$, that is, it satisfies

$$\mathcal{L}_\xi g = 0,$$

where $\mathcal{L}_\xi$ is the Lie-derivative with respect to ξ.

In this paper, we are interested in studying compact hypersurfaces in the Euclidean space $\mathbb{R}^{n+1}$, which admit a Killing vector field ξ and analyze the compact of the presence of the Killing vector field on the geometry of the hypersurfaces. It is well known that the presence of a Killing vector field on a Riemannian manifold contravenes its topology as well as geometry (cf.[1–3,8,9,11–13]). In that, if the length of the Killing vector field is a constant the influence on topology and geometry of Riemannian manifold possessing them becomes severe. For example on an even dimensional Riemannian manifold of positive curvature there does not exist a non-zero Killing vector field of constant length. It is in this context, even-dimensional spheres $S^{2n}(c)$ do not possess unit Killing vector fields. In ([12]), it is shown that the fundamental group of a Riemannian manifold admitting a Killing vector field, contains a cyclic subgroup of constant index.

Recall that on a compact hypersurface M each smooth vector field ξ on the compact hypersurface M is generated by the global flow on M. Let $\{\varphi_t\}$ be the flow of the Killing vector field ξ on the compact hypersurface M of the Euclidean space $\mathbb{R}^{n+1}$, we say that a (1,1)-tensor field T on the hypersurface M is invariant under the killing vector field ξ, if

$$\phi_t^*(T) = T \circ d\phi_t,$$

which is equivalent to

$$\mathcal{L}_\xi T = 0 \tag{7}$$

Our first result in this paper is the following

Theorem 1. *A complete and simply connected hypersurface M of the Euclidean $\mathbb{R}^{n+1}$, $n > 1$, with mean curvature α and shape operator S, admits a unit Killing vector ξ such that the sectional curvature of plane sections containing ξ are positive, the shape operator S is invariant under ξ and $S(\xi) = \alpha\xi$ holds, if and only if $n = 2m - 1$, α is constant, and M is isometric to the sphere $S^{2m-1}(\alpha^2)$.*

For a hypersurface M that admits a unit killing vector field ξ, we have a smooth function $\sigma : M \longrightarrow \mathbb{R}$, defined by

$$\sigma = g((S(\xi), \xi))$$

and also, we get a vector field U on the hypersurface M associated to ξ defined by

$$U = S(\xi) - \sigma\xi, \tag{8}$$

and call U the associated vector field. It follows that U is orthogonal to ξ.

Finally, we prove the following with constrained sectional curvature $R(S(\xi), \xi; \xi, S(\xi))$ of the hypersurface M.

Theorem 2. *A unit Killing vector field ξ on a compact and connected hypersurface M of $\mathbb{R}^{n+1}$, $n > 1$, with mean curvature leaves the shape operator S invariant and function $\sigma = g(S(\xi), \xi) \neq 0$, satisfies*

$$\int_M R(S(\xi), \xi; \xi, S(\xi)) \geq \int_M (n\sigma\alpha||S(\xi)||^2 - n\sigma^2\alpha^2)$$

if and only if $n = 2m - 1$, α is a constant and M is isometric to $S^{2m-1}(\alpha^2)$.

2. Preliminaries

A smooth vector field ξ on an n-dimensional Riemannian manifold (N^n, g) is said to be a Killing vector field if

$$\mathcal{L}_\xi g = 0 \tag{9}$$

In [8], it is shown that for a Killing vector field ξ on (N^n, g), there exists skew-symmetric operator F on (N^n, g), that satisfies

$$\nabla_X \xi = F(X) \tag{10}$$

and that

$$(\nabla_X F)(Y) = R(X, \xi)Y, \quad X, Y \in \mathfrak{X}(N^n) \tag{11}$$

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holds.
Moreover if ξ is a unit Killing vector field, then it follows that it annihilates F, that is,

$$F(\xi) = 0 \tag{12}$$

Using equations (10),(11) and (12), we have

$$R(X,\xi)\xi = (\nabla_X F)(\xi) = -F(\nabla_X \xi) = -F^2(X),$$

that is,

$$R(X,\xi)\xi = -F^2(X), \quad X \in \mathfrak{X}(M) \tag{13}$$

and on taking the inner product with X in above equation, we get the following expression

$$R(X,\xi;\xi,X) = ||F(X)||^2, \quad X \in \mathfrak{X}(M) \tag{14}$$

Let M be an orientable hypersurface of the Euclidean space $\mathbb{R}^{n+1}$ with unit normal N and the shape operator S. We denote the induced metric on M by g and the Riemannian connection with respect to g by ∇. Suppose the hypersurace admits a unit Killing vector field ξ.

We shall say the shape operator S is invariant under ξ if

$$\mathfrak{L}_\xi S = 0, \tag{15}$$

which is equivalent to

$$(\nabla_\xi S)(X) = F(SX) - S(FX)), \quad X \in \mathfrak{X}(M). \tag{16}$$

Just like previously, given a unit Killing vector field ξ on the hypersurface M, we can define a smooth function $\sigma : M \longrightarrow R$ by

$$\sigma = g(S(\xi),\xi),$$

and a smooth vector field $U \in \mathfrak{X}(F)$, by

$$U = S(\xi) - \sigma\xi, \tag{17}$$

called associated vector field.

It follows that the vector field U is orthogonal to ξ. Note that owing to Codazzi's equation (4) for hypersurface M and equation (16), we confirm

$$(\nabla_X S)(\xi) = F(SX) - S(FX), \quad X \in \mathfrak{X}(M). \tag{18}$$

Taking derivative in (17) with respect to $X \in \mathfrak{X}(M)$ we have on using using (10), that

$$\nabla_X U = (\nabla_X S)(\xi) + S(FX) - X(\sigma)\xi - \sigma FX,$$

which in view of equation (18), implies

$$\nabla_X U = F(SX) - X(\sigma)\xi - \sigma FX \tag{19}$$

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3. Proof of Theorem 1

Suppose M is a complete and simply connected hypersurface of the Euclidean space $\mathbb{R}^{n+1}$ that admits a unit Killing vector field ξ with shape operator S is invariant under ξ, sectional curvature of plane sections containing ξ are positive and the shape operator satisfies

$$S(\xi) = \alpha\xi, \tag{20}$$

where $\alpha = \frac{1}{n}trS$ in the mean curvature of M.

Differentiating equation (20) with respect to $X \in \mathfrak{X}(M)$ and using equation (10), yields

$$(\nabla_X S)(\xi) + S(FX) = X(\alpha)\xi + \alpha FX;$$

Using equation (18) in the above equation brings

$$F(SX) = X(\alpha)\xi + \alpha FX, \quad X \in \mathfrak{X}(M)$$

that is

$$F(SX - \alpha X) = X(\alpha)\xi, \quad x \in \mathfrak{X}(M).$$

Operating F on above equation and using equation (12), yields

$$F^2(SX - \alpha X) = 0, \quad X \in \mathfrak{X}(M).$$

The above equation, in view of equation (13) implies

$$R(SX - \alpha X, \xi)\xi = 0$$

Taking the inner product in the above equation, with $SX - \alpha X$, we get

$$R(SX - \alpha X, \xi; \xi, SX - \alpha X) = 0, \quad X \in \mathfrak{X}(M). \tag{21}$$

Note that for any $X \in \mathfrak{X}(M)$, in view of equation (20), we have

$$\begin{aligned} g(SX - \alpha X, \xi) &= g(SX, \xi) - \alpha g(X, \alpha) \\ &= g(X, S\xi) - \alpha g(X, \xi) \\ &= 0, \end{aligned}$$

that is, $SX - \alpha X$ is orthogonal to ξ. Thus by equation (21), it follows that the sectional curvatures of the plane sections spanned by $SX - \alpha X$ and ξ are zero, which is contrary to the hypothesis that sectional curvatures of plane sections containing ξ are positive. Hence, we conclude

$$SX - \alpha X = 0, \quad X \in \mathfrak{X}(M),$$

that is

$$S(X) = \alpha X, \quad X \in \mathfrak{X}(M) \tag{22}$$

Note that the mean curvature α satisfies

$$n\alpha = \sum_{j=1}^{n} g(Se_j, e_j), \tag{23}$$

for a local orthonormal frame $\{e_1, \ldots, e_n\}$ of the hypersurface M.

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Differentiating (23) with respect to $X \in \mathfrak{X}(M)$, gives

$$nX(\alpha) = \sum_{j=1}^{n} [g(\nabla_X Se_j, e_j) + g(Se_j, D_X e_j)]$$
$$= \sum_{j=1}^{n} [g((\nabla_X S)(e_j), e_j) + 2g(Se_j, D_X e_j)]$$

and using equation (4)

$$nX(\alpha) = \sum_{j=1}^{n} [g((\nabla_{e_j} S)(X), e_j) + 2g(Se_j, D_X e_j)] \tag{24}$$

Note that

$$\nabla_X e_j = \sum_{i=1}^{n} \omega_j^i(X) e_i,$$

where (ω_j^i) are connection forms satisfying

$$\omega_j^i + \omega_i^j = 0 \tag{25}$$

Taking

$$S(e_j) = \sum_k \lambda_j^k e_k,$$

where (λ_j^k) is a symmetric matrix. Thus,

$$\sum_{j=1}^{n} g(Se_j, \nabla_X ej) = \sum_{ji} \lambda_j^i \omega_j^i(X) = 0,$$

owing to the fact that (λ_j^k) is symmetric whereas $(\omega_j^i(X))$ is skew-symmetric.

Hence,

$$nX(\alpha) = \sum_{j=1}^{n} g((\nabla_{e_j} S)(X), e_j)$$

and as S is symmetric operator, we have

$$nX(\alpha) = \sum_{j=1}^{n} g(X, (\nabla_{e_j} S)(e_j)), \quad X \in \mathfrak{X}(M),$$

from which, we see that the gradient of the mean curvature α satisfies

$$n\nabla\alpha = \sum_{j=1}^{n} (\nabla_{e_j} S)(e_j). \tag{26}$$

Now differentiating equation (22), with respect to $X \in \mathfrak{X}(M)$, yields

$$\nabla_X SX = X(\alpha)X + \alpha \nabla_X X$$

and

$$S(\nabla_X X) = \alpha \nabla_X X,$$

gives

$$(\nabla_X S)(X) = X(\alpha)X.$$

Taking a local orthonormal frame $\{e_1, \dots e_n\}$ on the hypersurface M, we get

$$\sum_{j=1}^{n}(\nabla_{e_j}S)(e_j) = \sum_{j=1}^{n} e_j(\alpha)e_j = \nabla\alpha,$$

and combining above equation with the equation (26), yields

$$n\nabla\alpha = \nabla\alpha.$$

However, $n > 1$ in the hypothesis implies

$$\nabla\alpha = 0,$$

that is the mean curvature α is a constant. Using equation (3) and (22), we see that the curvature tensor of the hypersurface satisfies

$$R(X,Y)Z = \alpha^2\{g(Y,Z)X - g(X,Z)Y\}, \quad X,Y,Z \in \mathfrak{X}(M)$$

that is, M is a space of constant curvature α^2. Note that $\alpha^2 > 0$, as the sectional curvature of plane sections containing ξ are positive. Hence, M being complete and simply connected Riemannian manifold of positive constant curvature α^2, it is isometric to the sphere $S^n(\alpha^2)$.

Note that n cannot be even as a Killing vector field ξ on an even dimensional Riemannian manifold of positive sectional curvature has a zero (cf.[10]); and it is contrary to the assumption that ξ is a unit Killing vector field. Hence n is odd that is $n = 2m - 1$ and M is isometric to the sphere $S^{2m-1}(\alpha^2)$. The converse is trivial.

4. Proof of Theorem 2

Suppose the compact and connected hypersurface M of the Euclidean space $\mathbb{R}^{n+1}$, $n > 1$, with mean curvature α admits a unit Killing vector field ξ that the shape operator S is invariant under ξ and the function $\sigma = g(S\xi, \xi) \neq 0$, satisfies

$$\int_M R(S\xi,\xi;\xi,S\xi) \geq \int_M (n\alpha\sigma||S\xi||^2 - n\alpha^2\sigma^2) \tag{27}$$

For $X \in \mathfrak{X}(M)$, we have on using equation (10), that

$$X(\sigma) = g((\nabla_X S)(\xi) + SFX, \xi) + g(S\xi, FX),$$

which in view of equation (16), gives

$$X(\sigma) = g(FSX,\xi) + g(S\xi, FX),$$

Using equation (12) in above equation, we get the gradient of σ as

$$\nabla\sigma = -F(S\xi) \tag{28}$$

Differentiating above equation with respect to $X \in \mathfrak{X}(M)$, and using equation (10), we get

$$\nabla_X\nabla\sigma = -[(\nabla_X F)(S\xi) + F((\nabla_X S)(\xi) + FS(X))].$$

Using equations (11) and (18), we conclude

$$\nabla_X\nabla\sigma = -R(X,\xi)S\xi - F(F(SX) - S(FX)) - FS(FX),$$

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that is

$$\nabla_X \nabla\sigma = -R(X,\xi)S\xi - F^2(SX), \quad X \in \mathfrak{X}(F).$$

Now, employing equation (13) in above equation, we reach at

$$\nabla_X \nabla\sigma = -R(X,\xi)S\xi + R(SX,\xi)\xi,$$

which in view of equation (3), leads to

$$\nabla_X \nabla\sigma = -[||S\xi||^2 SX - g(SX,S\xi)S\xi] + \sigma S^2 X - g(SX,S\xi)S\xi,$$

that is,

$$\nabla_X \nabla\sigma = -||S\xi||^2 SX + \sigma S^2 X \tag{29}$$

Now, choosing a local orthonormal frame $\{e_1,\ldots,e_n\}$ on the hypersurface M, to compute $div(\nabla\sigma)$, using equation (29), we have

$$\Delta\sigma = div(\nabla\sigma) = \sum_{j=1}^{n} g(\nabla_{e_j}\nabla\sigma, e_j) = -n\alpha||S\xi||^2 + \sigma||A||^2.$$

Thus we conclude,

$$\sigma\Delta\sigma = -n\sigma\alpha||S\xi||^2 + \sigma^2||A||^2.$$

Integrating above equation by parts leads to

$$-\int_M ||\nabla\sigma||^2 = \int_M (\sigma^2||A||^2 - n\sigma\alpha||S\xi||^2),$$

that is,

$$\int_M \sigma^2(||A||^2 - n\alpha^2) = \int_M (n\sigma\alpha||S\xi||^2 - ||\nabla\sigma||^2 - n\sigma^2\alpha^2) \tag{30}$$

Now, equations, (14) and (28), give

$$||\nabla\sigma||^2 = ||F(S\xi)||^2 = R(S\xi,\xi;\xi,S\xi),$$

and changes equation (30) to

$$\int_M \sigma^2(||A||^2 - n\alpha^2) = \int_M (n\sigma\alpha||S\xi||^2 - n\sigma^2\alpha^2) - \int_M R(S\xi,\xi;\xi,S\xi).$$

Now, employing inequality in above equation, yields

$$\int_M \sigma^2(||A||^2 - n\alpha^2) \leq 0 \tag{31}$$

Note that owing to Schwartz's inequality $||A||^2 \geq n\alpha^2$, the integrand in the integral of inequality (31) is non-negative. Hence, we get

$$\sigma^2(||A||^2 - n\alpha^2) = 0 \tag{32}$$

since, $\sigma \neq 0$ on connected M, equation (32), implies $||A||^2 = n\alpha^2$. However, $||A||^2 - n\alpha^2$ is the equality in the Schwartz's inequality $||A||^2 \geq n\alpha^2$, which holds if and only if $A = \alpha I$. Then following the proof of Theorem 1, we get M is isometric to $S^{2m-1}(\alpha^2)$.

Conversely suppose that M is isometric to $S^{2m-1}(\alpha^2)$, then as seen in the introduction, we see there is a unit Killing vector field ξ on $S^{2m-1}(\alpha^2)$. Moreover the shape operator $S = \alpha I$ is invariant under ξ and the function $\sigma = g(S\xi, \xi) = \alpha$.

Thus, $\int_M R(S\xi, \xi; \xi, S\xi) = 0$ and also that

$$\int_M (n\sigma\alpha||S\xi||^2 - m\sigma^2\alpha^2) = \int_M (n\alpha^4 - n\alpha^4) = 0.$$

Consequently

$$\int_M R(S\xi, \xi; \xi, S\xi) = \int_M (n\sigma\alpha||S\xi||^2 - n\sigma^2\alpha^2)$$

holds. This finishes the proof.

5. Conclusions

There are two important vector fields on a Riemannian manifold (N, g), namely a Killing vector field and a conformal vector field and they have importance in the geometry of a Riemannian manifold on which they live as well as have importance in physics, specially the theory of relativity. In this paper, we have used a unit Killing vector field ξ on a hypersurface M of the Euclidean space R^{m+1} under the restriction that the shape operator S of the hypersurface is invariant under ξ and obtained two characterizations of the odd dimensional spheres. In these results, we used the restrictions on on sectional curvatures of the plane sections containing the unit Killing vector field ξ and the shape operator S to reach the conclusions. There could be a natural question as to what should be the restriction on Ricci curvature $Ric\,(\xi, \xi)$ of the orientable hypersurface of the Euclidean space R^{m+1} admitting a Killing vector field ξ which leaves the shape operator S invariant, so that the hypersurface is isometric to an odd dimensional sphere?

The next important vector field on a Riemannian manifold (N, g) is the conformal vector field, a vector field ζ on (N, g) is said to a conformal vector field, if

$$\pounds_\zeta g = 2\rho g, \tag{5.1}$$

where $\pounds_\zeta g$ is the Lie derivative of g with respect to ζ and ρ is a smooth function called the conformal factor (cf. [3], [10]). It is known that all spheres $S^m(c)$ admit many conformal vector fields. Therefore, it is natural to study hypersurfaces of the Euclidean space R^{m+1} admitting a conformal vector field ζ. Naturally, one would like to confront with the question: Under what conditions does an orientable hypersurface M of the Euclidean space R^{m+1} admitting a conformal vector field ζ is isometric to the sphere $S^m(c)$?

Given a unit Killing vector field ξ on an orientable hypersurface M of the Euclidean space R^{n+1}, we have seen there is a vector field U on M given by equation (2.9), which is orthogonal to ξ and called associated vector field to ξ. In addition, if the shape operator S is invariant under ξ, then the associated vector field U satisfies equation (2.11). Note that in Theorem 1, we assumed the associated vector field $U = 0$. However, it will be an interesting question to explore the geometry of an orientalbe hypersurface M with unit Killing vector field ξ with respect to which the shape operator S is invariant under ξ and has nonzero associated vector field U, by imposing some geometric conditions on U.

These three questions raised above shall be our focus of attention in future studies of an orientable hypersurface of the Euclidean space R^{m+1}.

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