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An investigation of experimental reports on the relativistic relation for Doppler shift.

James McKelvie

Department of Mechanical and Aeronautical Engineering

University of Strathclyde.

Abstract. An exhaustive list of thirteen instances of reports confirming experimentally the relativistic Doppler relation, are examined. For those involving longitudinal Doppler, the non-relativistic relation is seen to be confirmed, within the reported experimental accuracies, to the same degree as the standard relativistic relation. Higher values of the speed of the emitter would be required to examine further the claimed confirmations. For those reports involving saturation spectroscopy, there is much confusion over the appropriate Doppler relation to be used, together with some serious analytical flaws. For the two cases that involve transverse Doppler, there are seen to be either serious faults in the theoretical part, or intrusion from the first order effect.

Therefore, the reported conclusions - that the results for the experiments confirm the relativistic SR relation - cannot be justified by any of the experimental works.

Key words: Doppler; relativistic; non-relativistic.

1. Background

There have been seven experimental reports where the results of experiments involving longitudinal Doppler effect have been reported as confirming an aspect of SR - either because they are highly consistent with the SR-modified relation for the frequency of the light emitted by a moving source, or, in the early days, by simply obtaining the general nature of the relation. There have also been four reports involving saturation spectroscopy and two on transverse Doppler. The work on longitudinal Doppler considers the matter in detail without involving relativity. Reports involving saturation spectroscopy are also examined, as are two which investigate transverse Doppler.

1.1 Relativistic Doppler effect

With the Doppler effect, without considering relativity, the frequency observed in the observer's frame from a moving source of proper frequency ν_o , is expressed as

$$\nu = \nu_o / (1 \pm \beta) \tag{1}$$

where the + (-) refers to the source and observer receding (approaching). The classical relativistic relation for the frequency ν, again observed in the observer's frame, travelling at $\mathbf{u}_s$ w.r.t the observer is, Møller [1]

$$\nu_o = \nu \frac{1-(\mathbf{u}_s.\mathbf{e})/c}{(1-u_s^2/c^2)^{1/2}} \quad \text{or} \quad \nu = \nu_o \frac{(1-u_s^2/c^2)^{1/2}}{1-(\mathbf{u}_s.\mathbf{e})/c} \tag{2}$$

The quantity $\mathbf{u}_s.\mathbf{e} = u_s\cos\alpha$ where α is the angle between the direction of $\mathbf{u}_s$ and the direction of interest in the observer's field. The u's are the magnitudes of the **u**'s. There are two cases of Doppler effect that have been investigated:

a) when the direction of $\mathbf{u}_s$ is α = 0 or π then we have $\mathbf{u}_s.\mathbf{e} = \pm u_s$, and for this case - the "longitudinal" effect - the frequency relation is

$$\nu = \nu_o / \gamma(1 \pm \beta) \quad (3)$$

where $\gamma = (1-\beta^2)^{-1/2}$, $\beta = u_s/c$, and the + (–) again indicates the source and observer receding (approaching);

b) when $\theta_S = \pm\pi/2$ we have $\mathbf{u_s.e} = 0$, and the "transverse" effect – with for example a source emitting when situated instantaneously on the y-axis and moving at u in the ±x direction, with the observer at the origin. This gives

$$\nu = \nu_o/\gamma \quad (4)$$

c) when the angle between the source and observer is a general angle θ, there are two classical expressions for the frequency observed

i) when the angle is as seen in the observer's frame

$$\nu = \nu_o / \gamma(1 + \beta\cos\theta) \quad (5)$$

ii) when the angle is as seen in the source's frame

$$\nu = \nu_o \gamma(1 - \beta\cos\theta) \quad (6)$$

that is because the angle θ is different as viewed in each frame. It is seen that for $\theta = 0$ or π, there is consistency between (5) and (3). Equation (6) is consistent with Einstein [2]. There appears to be some confusion regarding (5) and (6) in the literature, as will be seen below.

1.2 Longitudinal Doppler effect

There have been numerous reports of work that conclude that relativistic longitudinal Doppler has been demonstrated. These include Pound and Rebka [3], Mandelberg and Witten [4], Mandelberg [5], Otting [6], Ives and Stilwell [7], [8], and Olin *et al.* [9]. Ref. [4] is based on the work of [5]. In the Pound and Rebka [3] experiment, a frequency shift due to thermal excitation of the atoms was observed in Mossbauer emission, - due effectively due to fore-and-aft molecular velocities. However, that work of Pound and Rebka involved acceleration of the source and therefore is outside the terms of Einstein's statement of the conditions for SR, - [in his "popular" treatment [10], Einstein, referring to reference systems K and K′ "in which 'the Galilean law holds' (i.e. inertial systems) says as follows: "But in addition to K, all bodies of reference K′ should be given preference in this sense and they should be exactly equivalent to K for the formulation of natural laws, provided that they are in a state of *uniform rectilinear and non-rotary motion* [his italics] with respect to K….The principle of relativity was assumed only for these reference bodies but not for others (e.g. those possessing motion of a different kind). In this sense we speak of the *special* principle of relativity, or special theory of relativity"]. Since ref. [3] involves acceleration, it is outside the application of SR and would have to be considered under the General Theory [GR].

The cases [4], [5], represent an experiment that has been repeated in principle by other workers in [9]. They were concerned with confirming the relation predicted by SR for the frequency of the radiation emitted by electrons in atoms having constant velocities, whereas [6], [7], [8], are more concerned simply with the general nature of the relevant relation. They all use light emitted in the transition of electrons from a higher to a lower energy level.

1.3 Saturation spectroscopy.

There have been a number of experiments that report verification of relativistic Doppler using saturation spectroscopy. One method was proposed by Juncar *et al.* [11] and was followed up in [12], [13], [14]. Doppler shift is traditionally taken to be the change of frequency of a wave emitted, as observed by an observer in motion relative to the source. However, saturation spectroscopy involves the absorption of a photon rather than the emission, and the frequency of a photon absorbed by a moving atom cannot be regarded as truly representing Doppler effect. But, nevertheless, the relevant references will be considered here.

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1.4 Transverse Doppler effect

Two studies on transverse Doppler are MacArthur *et al.* [15] and Hasselkamp, Mondry and Scharmann [16]. The issues involved in the transverse Doppler reports are different from those in the longitudinal case, and will be dealt with separately.

2. Longitudinal Doppler: Considering references [4], [5], [6], [7],[8], [9].

2.1 The values of β and of the accuracy in the longitudinal experiments

It will be important to know the magnitude of relevant values of β in the various experiments referred to and considered here (apart from [3] which is not of interest): the highest value, quoted in [9], is 0.05 whereas the highest in others of interest is 0.009, (i.e. in [4] and [5]). For that reason [9] will be considered separately. A second factor is that, in the experiments referred to, the claimed accuracy is important, - because it is against that figure that the validity of the conclusions will be judged.

2.2 A comparison of the two equations (relativistic vs non-relativistic)

We now make a comparison between the answers obtained by using (3) and (1). We find that whatever the value of $\nu_o/(1\pm\beta)$, the ratio of the two relations is γ. With a γ of 1.00004 (the highest of those that are of immediate interest), the ratio is equal to 1 to less than 4 parts in 10^5. This is beyond the capability of the discrimination quoted in the experiments and their conclusions, that (3) is verified (as opposed to (1)), are therefore not valid.

2.3 The work of Olin *et al.* [9]

Reference [9] uses a β = 0.05, which is an order of magnitude larger than is typical of the others and it is interesting in that it uses a capture reaction $^{16}O + {}^{4}He \rightarrow {}^{20}Ne^{*}$ where the Ne is "excited" and then resorts back to the "normal" state by an electronic transition, emitting a gamma photon. They state that in the collision arrangements the angles of the recoils were at both 0º and 180º, - that is, longitudinally. They found that in the factor $(1-\beta^2)^A$, the index A was confirmed to be -0.5 to an accuracy of 3.5% for the observed frequency change, moving versus at rest, justifying a relativistic interpretation.

The same situation exists here as for the previous longitudinal Doppler experiments: with a γ of 1.00125, the ratio comparing (3) to (1), is 1 to within 0.125%, which is well within the stated accuracy. Therefore, the relativistic interpretation is not justified.

2.4 A comment

It must be said that the assertion that the conclusions of these reports are not justified is only possible because of the low values of β employed in the experiments.

3. Saturation spectroscopy. References [11], [12], [13], [14]

In saturation spectroscopy, photons from a tuneable laser are absorbed by atoms moving on the same line as the lasers, and an electron is raised into a higher-energy orbit, - the opposite of what happens in normal longitudinal Doppler. In principle, therefore, one could use the same process of comparing (3) and (1), but the information on beam velocities is scarce, so that the same analysis cannot be applied. Other methods of evaluation will be used.

3.1 Examining ref. [11]

In [11] the analysis is done in terms of relations of the Doppler shifted wave numbers σ_+ (parallel beams) and σ_- (antiparallel beams). Their development of those relations is

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referred to in an unpublished document, so that, unfortunately, proper examination cannot be carried out.

3.2 Examining ref. [12]

In [12] the authors say, using the classical relation (5) " the laser frequency measured in the rest frame of an atom is given by special relativity as $\nu_\pm = \gamma\nu_L(1 \pm \upsilon/c)$. Here + (-) refers to the laser beam traveling in the opposite (same) direction as the atom." That is, $\theta = 0$ or π. The laser frequency is ν_L in the laboratory frame. It is clear that $\nu_\pm$ represents the frequency of the absorbed photon, that the atom is the receiver of the photon (i.e. the atom is the observer) and that the source is the laser. Therefore, the measurement - made in the rest frame of the observer - should be expressed classically, as (5). However, they have used (6), which is wrong.

3.3 Examining ref. [13]

The authors of [13] say **"**The laser frequency measured in the atomic rest frame is $\nu_{atom} = \gamma\nu_L(1 - \upsilon\cos\theta/c)$". This is the same error as in [12]. Equation (5) is relevant, but they have used (6), (as an aside, there is an extra error in the sign of $\upsilon\cos\theta/c$.)

3.4 Examining ref. [14]

In [14] the authors say "the transition frequency ν_i of an atom at rest in an inertial system S′, which is moving with a constant velocity $\beta = \upsilon/c$ in the laboratory system S, is related to the frequency ν measured by an observer at rest in S by $\nu_i = \nu\gamma(1 - \beta\cos\theta)$ where θ is the observation angle with respect to the atom velocity." Since the angle θ is defined wrt the atom's (i.e. the observer's) velocity, they are measuring in the observer's frame. Again, they have used (6) instead of (5).

4. Transverse Doppler: references [15] and [16].

The authors of [15] pass a steam of H^o atoms essentially perpendicularly through a UV laser, noting accurately the actual angle, and using the angle-dependant Doppler relation. In [16] the authors simply looked to view the light from a stream of rectilinearly moving emitters, but at 90° to their motion.

4.1. Examining ref. [15]

In [15], there are some serious errors as they develop their version of the transverse relation:- for example, they express the relativistic time and position relations as

$$T = a_o t + (\upsilon/c^2) a_1 x \qquad (7)$$

$$X = \upsilon a_o t + a_1 x, \qquad (8)$$

where $a_o = \gamma g_o$ and $a_1 = \gamma g_1$, with upper case referring to values in a frame at rest with the observer and lower case to values in a frame moving at υ. However, when these latter relations are entered into (8), the authors say, for a particular event 1,

$$T_1 = (g_o/\gamma)t_1 + (\upsilon/c^2)X_1, \qquad (9)$$

The substitution error for a_o is obvious, and it is quite unclear why $a_1 x$ should be replaced by X, since (8) says $a_1 x = X - \upsilon a_o t$.

There are subsequent repercussions: they go on to say, continuing with the erroneous (9), "If we assume that the transition takes place in the motionless frame, so that the atom is at rest in this frame, then $X_I - X_2 = 0$. In this case $\Delta T_{atom} = \Delta T_{lab} g_o/\gamma$, or $\nu_{atom} = \nu_{lab}\gamma/g_o$. This is the relativistic transverse-Doppler-effect formula. The nonrelativistic Doppler effect, where $E = E_o(1+\beta\cos\theta)$, is the low-velocity limit of the entire formula, suggesting the final energy relation:

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$$E = (E_o/g_o)\gamma(1+ \beta\cos\theta) \quad (10)”$$

[end of quotation].

Since “the atom is at rest in this [the motionless] frame” then we take it that it is as measured in the atom’s (the observer’s) frame. Equation [10] is incorrect, being according, again, to (6) whereas it should accord with (5).

Now, in their first paragraph the authors say as follows, “Consider an atom with velocity υ, colliding at an angle θ, with a photon, whose energy in the lab is E_o, The relativistic Doppler formula for the photon energy as seen by the atom is

$$E = (E_o/g_o)\gamma(1+ \beta\cos\theta), \quad (11)$$

with $\beta = \upsilon/c$, $\gamma = (1 - \beta^2)^{-½}$, and E_o the rest-frame energy of the photon.” [end of quotation].

However, although (10) and (11) appear to be consistent, that is incorrect: Equation (10) is couched “in the rest-frame energy of the photon”. Since the rest-frame of a photon is travelling at *c*, what does that mean? Do the authors mean in the rest-frame of the laser - which is the source? In which case (11) is correct. However, for (10) they say “If we assume that the transition takes place in the motionless frame, so that the atom is at rest in this frame”. The atom is then at rest, and for this case (the stationary observer) the Doppler relation is different from the case where the observer is moving (“as seen by the moving atom”). The Doppler relations for the stationary observer and for the moving observer are different. But they get the same relation - (10) and (11). By dint of using the erroneous (9) they have arrived at the same Doppler relation for conditions where they should not be the same.

It is clear that this reference is seriously flawed, and its conclusions cannot be relied upon.

4.2 Examining ref. [16]

The workers here simply looked to view the light from a stream of rectilinearly moving emitters, but at 90° to their motion. This experiment is recognized to be particularly difficult to perform due to the possible intrusion of the first order effect if the angle of viewing is not very close to 90° to the moving stream of emitters. Typically, a viewing angle difference of 1° will allow sufficient intrusion of the first-order effect as to affect the result. They do claim an uncertainty $90° \le \theta < 91°$ in the angle between the emitting beam and the viewing system. But the most interesting feature is that, in the four cases illustrated, there is not only a “flat-top” nature to the reported wavelength distribution with velocity, especially at the higher speeds (up to $\beta = 0.03$), but a large increase in the spread of the wavelengths. For example, there is a large increase in FWHM, from approximately $\delta\lambda \approx$ 1nm at $\beta = 0$, to $\delta\lambda \approx$ 2.4nm at $\beta = 0.03$ (that is estimated ignoring the flat-top, and drawing a putative “normal” type of distribution; otherwise FWHM would be about 2.8nm). That spread is specifically attributed in the report to be due to Doppler broadening. If we take it that, as intended, $\theta = 90°$, then all broadening due to relativistic transverse Doppler must be according to the correct expression (4)

$$\nu = \nu_o/\gamma \quad (12)$$

and, expressed in terms of λ,

$$\lambda = \lambda_o(1 - \beta^2)^{-½} \quad (13)$$

where λ_o is the wavelength from an emitter at rest and λ is the observed wavelength corresponding to some value of β in the thermally-broadened velocity distribution.

There is a particular problem with the results for H_2^+ ions that the authors illustrate, in that if the transverse Doppler relation (13) were applicable (and they themselves use it), it would not be possible, since β is always < 1, for a value of λ to be less that λ_o so that any Doppler broadening would have to be one-sided, with $\lambda > \lambda_o$. The reported values of λ

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below λ_o cannot therefore be due to transverse Doppler and must be due to intrusion of the first-order effect. The authors use the centres of gravity of their "Doppler-broadened" lines to derive their "[transverse] Doppler shifts" so those values cannot be correct. They are using the centre of what is actually a one-sided distribution.

We can estimate the extent of the intrusion by considering the minimum value of $\lambda \approx 654.9$nm as illustrated in the report, and the mean value at $\beta = 0$ which is the H_α line at 656.3nm. It occurs with the ions at 900keV, given as $\beta = 0.03$. Writing the general expression in terms of λ, it is, in general, for stationary observer viewing the moving source, and θ measured in the observer's frame, from (5),

$$\lambda = \lambda_o (1-\beta^2)^{-1/2}(1 \pm \beta\cos\theta) \qquad (19)$$

which gives, ignoring powers of β above 2 and powers of $\cos\theta$ above 1 (since θ has to be very close to 90°)

$$\lambda \approx \lambda_o + \tfrac{1}{2}\lambda_o\beta^2 \ldots \pm \lambda_o\beta\cos\theta \ldots.$$

or

$$\cos\theta \approx \frac{\lambda - \lambda_o - \tfrac{1}{2}\lambda_o\beta^2}{\pm\lambda_o\beta} \qquad (20)$$

which, using the above figures for the extreme shift to $\lambda = 654.9$nm at $\beta = 0.03$, and $\lambda_o = 656.3$, gives $\theta = 85.1^o$ or 94.9^o - which is clearly not an acceptable angle.

These figures are well outside of the necessary limit of 89°- 91°, and the overwhelming effect is due to intrusion of first order Doppler. These values for θ are consistent with the acceptance angle which would be estimated from examination of the more technical diagram of the apparatus as given in ref. 21 of their paper.

It is clear that this experiment cannot be taken as a demonstration of the transverse Doppler relation, due to intrusion from the first order.

5. Conclusion

It has been shown that reference [3] cannot say anything about SR. For those that involve longitudinal Doppler, involving electron transition and photon emission. It is seen that the ratio between the two relations (SR and non-SR) is always 1 *within the experimental limitations*, This situation pertains because of the low values of the emitter speeds involved. Considerably higher speeds would be required before the question could be answered with confidence.

In the cases involving saturation spectroscopy, it is seen that there is considerable confusion concerning the appropriate classical Doppler relation to be used, and in some cases other serious flaws. Unfortunately, in one case, an examination is not possible due to the lack of publicly available information.

In the two cases referenced for transverse Doppler, the conclusions cannot be justified, because the arguments are flawed and/or the first order effect intrudes.

What the above arguments have shown is that the considerable number of experiments reported to confirm relativistic Doppler, using light as emitted by transitioning electrons or as absorbed by an atom, have given inconclusive results. That is because of the experimental conditions pertaining, - in particular the low velocity of the emitter, – together with confusion regarding the appropriate Doppler relation, and serious errors in the theoretical analyses.

None of the reports examined can be said to confirm the relativistic Doppler relation.

6. Financial interest

There is no financial interest associated with this work

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7. Conflict of interest

The author is not aware of any conflict of interest associated with this work.

8. References

1. Møller, C.: *The Theory of Relativity*. 2nd edn. (Oxford, 1972), p60.
2. Einstein, A., Zur electrodynamik bewegter korper. *Ann. Phys.* **17**, 891-921 (1905). Translated Perret, W., Jeffrey, G.B.: (in *The Principle of Relativity*. Dover Publications, New York, 1952)
3. Pound, R.V. & Rebka, G.A. Jr., Variation with Temperature of the Energy of Recoil-Free Gamma Rays from Solids. *Phys.Rev.Lett*. **4**, 274-275, (1960).
4. Mandelberg, H.I. & Witten, L., Experimental verification of the relativistic Doppler effect. *J. Opt. Soc. Am.* **52,** 529-536, (1962).
5. Mandelberg, H.I., PhD thesis, The Johns Hopkins University, Baltimore, Maryland, (1960)
6. Otting, G., Der quadrische Dopplereffekt. *Z.Phys.* **40**, 681-687, (1939).
7. Ives H.E. & Stilwell, G.R., An experimental study of the rate of a moving atomic clock. *Opt. Soc. Am.* **28,** 215-226, (1941).
8. Ives H.E. & Stilwell, G.R., An experimental study of the rate of a moving atomic clock II. *J. Opt. Soc. Am.* **31,** 369-374, (1941).
9. Olin, A., Alexander, T.K., Häusser, O., McDonald, A.B. & Ewan, G.T., Measurement of the relativistic Doppler effect using 8.6-MeV capture γ rays. *Phys. Rev. D.* **8**, 1633-1639, (1973)
10. Einstein, A. *Relativity the Special and the General Theory.* 15^{th} edition, (Crown, New York, 1961), p61.
11. Juncar P. *et al*., New Method to Measure the Relativistic Doppler Shift: First Results and Proposal, *Phys. Rev. Lett*., **54**, (1), 11-13, (1985).
12. Kaivola, M., Poulsen, O., Riis, E. & Lee, S.A., Measurement of the relativistic Doppler shift in neon. *Phys.Rev.Lett*. **54**, 255-258, (1985).
13. McGowan, R.W., Giltner, D.M.., Sternberg, S.J., Lee, S.A., New Measurement of the Relativistic Doppler Shift in Neon, *Phys. Rev. Lett*. **70**, (3), 251-254, (1993).
14. Boterman, B., Bing D., Geppert, C., Gwinner,G., Hänsch, T.W. Huber, G., Karpuk,S., Krieger, A., Kühl, T., Nörtershäuser,W., Novotny, C.,Reinhardt, S., Sánchez, R., Schwalm D., Stöhlker, T., Wolf, A., Saathoff, G., Test of Time Dilation Using Stored Li^+ Ions as Clocks at Relativistic Speed, *Phys. Rev. Lett*., **113**, 120401-5 (2014).
15. MacArthur D. *et al.*, Test of the Special-Relativistic Doppler Formula at $\beta = 0.84$, *Phys.Rev. Lett*., **56**, (4), 282-285, (1986).
16. Hasselkamp, D., Mondry, E. & Scharmann,A., Direct observation of the transverse Doppler-shift. *Z. Phys. A.* **289**,151-155, (1979)

17a,b,c. Konopinski, E.J., *Electromagnetic Fields and Relativistic Particles*, McGraw-Hill, New York, (1981), a) p316, b) p430, c)p432.