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RBFNN-Based Secure Tracking Control for a Class of Strict-Feedback Nonlinear Systems with Asymmetric Output Constraints and Its Application to UAVs

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Abstract

This paper investigates a tracking control problem for a class of strict-feedback nonlinear systems with time delays, asymmetric output constraints, and deception attacks on the controller. First, by introducing a novel error transformation techniques, any non-zero and bounded initial state is converted into zero. Second, a barrier function with the asymmetric output constraints is designed, which convert the problem of satisfying the tracking control problem of nonlinear systems under output constraints boils down to ensuring the boundedness. In additional, the radial basis function neural networks (RBFNNs) are utilized to handle both unknown uncertain term and deception attacks simultaneously. By utilizing the new asymmetric delayed barrier function error together with a RBFNNs technique, the tracking controller is designed to achieve asymptotic tracking, regardless of presence or absence of output constraints. Finally, the effectiveness of the proposed strategy is verified through its simulation on the unmanned aerial vehicle's (UAVs) systems.

Keywords: asymmetric output constraints; error-shifting function; barrier function; deception attacks; neural networks; unmanned aerial vehicle

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1. Introduction

Most practical systems are inevitably subject to various operational constraints. For example, when UAVs along the desired trajectory, it must maintain a certain pitch angle to avoid obstacles. The output constrain problem of nonlinear systems has garnered widespread attention in numerous fields [1–8]. For such problems, in controller design the optimal control [9] are achieves optimal tracking for a continuous-time boiler turbine system with asymmetric constraints, [10] ensures that the system state always satisfies the predetermined bounds, while tube-based model predictive control (MPC) scheme is found in [11]. However, these methods often come with a significant online computational burden or can only be applied to linear systems. In contrast, methods based on barrier functions (BF) and barrier Lyapunov functions (BLF) not only require less computation but also can address the output constraint problems of nonlinear systems. Therefore, it has attracted hot research [12–22].

In the majority of existing works, constraints can be divided into static and time-varying. Static constraints based on barrier functions can be found in [23–25]. However, static constraints are difficult to apply to complex scenarios. In [26–28], time-varying constraints are addressed, but the lower bound must be negative in [26]. While [27] relaxes the requirement of the output constraints in [26], which allowing the lower bound of the output be positive. The time-varying constraint functions in [28] can be directly defined by users, but their first and second derivatives need continuous and bounded. It should be noted that in the aforementioned work, the values of the upper and lower bounds of the constraints must be known. It is inevitably increases the computational burden. Additionally,

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the constrains exist throughout the entire operation of the system, and the control scheme cannot be directly applied to the constrain-free case without switching.

In general, asymmetric constraints can be categorized as undelayed and delay. The study of undelayed asymmetric can be found in [5,29–32], and delay asymmetric can be found in [14,33,34]. Such as, for [31,32] a unified state feedback control scheme is proposed for nonlinear systems subject to asymmetric output constraints, which can be applied to scenarios with or without constraints, without the need to modify or switch its structure. It should be noted that the problem of delay constraint is not considered in [31,32]. To cope with the delay constrain problem, a novel adaptive tracking control scheme is proposed in [14], even if the initial state constraints are not satisfied. However, the transformation function in [14] is non-smooth, which can easily cause output overshoot or oscillation during the switching phase of the constraints. Compared with [14], the constraint switching in [33] is smoother, but it fails to extend it to the control problem under cyber attacks. Thus, dealing with such delay constraints inherently demands that the proposed method be uniformly applicable, regardless of the presence or absence of constraints.

Inspired by the above research, in this paper a neural network-based output control scheme is proposed for the tracking problem of strict-feedback nonlinear systems with delayed asymmetric constraints, multiplicative attacks, additive attacks, and disturbances. The main contributions of this paper are as follows:

(1) Compared with [14,35] the error transformation function constructed in this article has two advantages: first, by introducing an exponential term, it ensures the smoothness of the transition phase before the constraint is activated; second, by introducing the parameter b, the transition speed when the constraint is activated can be flexibly adjusted.

(2) Barrier function constructed based on error transformation function, which not only satisfy the delayed asymmetric constraints, but also allows the controller to be applicable regardless of the presence of delay constraints, without changing its structure.

(3) Different from existing studies [9–11,32], the upper and lower output constraint boundaries are not to be known, in this paper the proposed strategy only assumes that the existence of constraint boundaries. It makes the control algorithm require less computation and easier to generalize to more application scenarios.

2. Problem Formulation and Preliminaries

In this paper, we consider the following strict-feedback nonlinear systems:

$$\begin{cases} \dot{\xi}_i = \varphi_i(\bar{\xi}_i)\xi_{i+1} + \psi_i(\bar{\xi}_i) + d_i(\bar{\xi}_i, t),\ i = 1, \ldots, n-1 \\ \dot{\xi}_n = \varphi_n(\bar{\xi}_n)[\kappa(t, t_\kappa)u + \chi(t, t_\chi)] + \psi_n(\bar{\xi}_n) + d_n(\bar{\xi}_n, t) \\ y = \xi_1 \end{cases} \tag{1}$$

where $\xi_i = [\xi_{i_1}, \ldots, \xi_{i_d}]^T \in \mathbb{R}^d$, $i = 1, \ldots, n$, denote system states, $\bar{\xi}_i = [\xi_1, \ldots, \xi_i]^T \in \mathbb{R}^{id}$ is the state vector; $u = [u_1, \ldots, u_d]^T \in \mathbb{R}^d$ represent control input; $\varphi_i(\cdot) \in \mathbb{R}^{d\times d}$, $\psi_i(\cdot) \in \mathbb{R}^d$ are unknown smooth nonlinear functions; the multiplicative attack signal $\kappa(t, t_\kappa)$ and additive attack signal $\chi(t, t_\chi)$ are continuous time-varying deception attack signals that attempt to disrupt the normal controller u, with starting instants t_κ and t_χ, respectively.

The systems subject to time-varying output constraints defined by:

(1) For $t \in [0, t_s)$, $y_k(t)$ is unconstrained;

(2) For $t \in [t_s, \infty)$, $y_k \in \Omega_{1_k} := \{(t, y_k) \in \Re_+ \times \Re \mid C_{l_k}(t) < y_k < C_{h_k}(t)\}$.

where $t_s \in \mathbb{R}$ is the time instant when output must begin to conform to the constraints; $C_{l_k}(t)$ and $C_{h_k}(t)$ represent the lower and upper asymmetric constraints of output $y_k(t)$, respectively, some or all of them may not exist in some time intervals, where it can be considered as $\mp\infty$.

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Control Objective: For the strict-feedback nonlinear systems subject to delay asymmetric constraints, the aim is to design a new neural network tracking control scheme such that the reference trajectory $y_{r_k}(t)$ should be tracked by the system output $y_k(t)$ which meaning the tracking error can be made arbitrarily small.

The following assumptions and lemmas are needed.

Assumption 1. *([31]) The constraint functions $C_{l_k}(t)$, $C_{h_k}(t)$ and the desired trajectory $y_{r_k}(t)$ are all piecewise differentiable, and their first derivatives are bounded but unknown. Specifically, there exist unknown finite positive constants $\bar{C}_{l_k}$, $\bar{C}_{h_k}$, and $\bar{y}_{r_k}$ such that $\|\dot{C}_{l_k}\| < \bar{C}_{l_k}$, $\|\dot{C}_{h_k}\| < \bar{C}_{h_k}$, and $\|(\dot{f}y_{r_k} + f\dot{y}_{r_k})\| < \bar{y}_{r_k}$. In addition, there exist positive constants $\underline{\epsilon}_0$ and $\bar{\epsilon}_0$ such that the desired trajectory $y_{r_k}(t)$ satisfies $y_{r_k}(t) \in \Omega_d := \{(t, y_{r_k}) \in \Re_+ \times \Re \mid C_{l_k}(t) + \underline{\epsilon}_0 \leq y_{r_k} \leq C_{h_k}(t) - \bar{\epsilon}_0\}$.*

Assumption 2. *([20]) The control coefficients $\varphi_i(\bar{\xi}_i)$, $i = 1, \ldots, n$ are unknown and time-varying but bounded away from zero, i.e., there exist certain unknown constants $\underline{\varphi}_i$ and $\bar{\varphi}_i$ such that $0 < \underline{\varphi}_i \leq ||\varphi_i(\bar{\xi}_i)|| \leq \bar{\varphi}_i < \infty$. In general, it is further assumed that all of the signs of $\varphi_i(\bar{\xi}_i)$ are positive.*

Assumption 3. *([36]) The unknown multiplicative attack signal $\kappa(t, t_k)$ and additive attack signal $\chi(t, t_k)$ are both boundary. In other words, the positive scalars $\underline{\kappa}$, $\bar{\kappa}$ and $\bar{\chi}$ exist such that $\underline{\kappa} \leq ||\kappa(t, t_k)|| \leq \bar{\kappa}$ and $||\chi(t, t_k)|| \leq \bar{\chi}$.*

Lemma 1. *([37]) On a compact set $\Omega_U \subset \mathbb{R}^l$, radial basis function neural networks can realize online approximation for any continuous function $F(U)$, and the approximation accuracy can meet any preset requirement. Its approximation relationship can be expressed as: $\bar{F}(U) = W^{*T}\eta(U) + \varepsilon$. Let $\omega_i = \|W_i^*\|^2$, $\forall i = 1, \ldots, n$, where ω_i is an unknown constant. The definitions and properties of each parameter in the above expression are as follows:*

(1) Input and structural parameters: $U \in \mathbb{R}^l$ represents the input vector of the neural network; $g > 1$ is the number of network basis functions, corresponding to the weight vector $W^T = [W_1, W_2, \ldots, W_g]^T \in \mathbb{R}^g$ and the basis function vector $\eta(U) = [\eta_1(U), \eta_2(U), \ldots, \eta_g(U)]^T \in \mathbb{R}^g$, and the basis function vector satisfies the norm constraint $\|\eta(U)\|^2 \leq q$.

(2) Form of basis functions: Each basis function $\eta_i(U)$ ($i = 1, \ldots, q$) adopts a Gaussian function structure, and its specific form is $\eta_i(U) = \exp\left[-\frac{(U-k_i)^T(U-k_i)}{l_i^2}\right]$ where k_i is the center parameter of the Gaussian function, and l_i is its width parameter.

(3) Approximation error property: $\varepsilon(U)$ denotes the error generated during the approximation process, and this error satisfies the boundedness condition $|\varepsilon(U)| \leq \varepsilon_N$ (where ε_N is a positive constant). Furthermore, the error $\varepsilon(U)$ can be adjusted to a minimal value by selecting the ideal weight vector W^*. The definition of this ideal weight vector is: $W^* = \arg\min_{W \in \mathbb{R}^g} \sup_{U \in \Omega_U} \left|\bar{F}(U) - W^T\eta(U)\right|$, which is the weight vector that minimizes the "maximum deviation between the function $\bar{F}(U)$ and the neural network output $W^T\eta(U)$" on the compact set Ω_U.

2.1. Error Shifting Function

To address the delay output constraint problem, the following transformation function is proposed:

$$f(t) = \begin{cases} 1 - \left(\frac{t_s - t}{t_s}\right)^{n+2} \exp\left(-\frac{t^{2n}}{2b^{2n}}\right), & 0 \leq t < t_s \\ 1, & t \geq t_s \end{cases} \quad (2)$$

with $t_s > 0$ being a prespecified regulation time, n being the system order and b is a positive constant.

The novel error shifting function can make any nonzero and bounded initial state is converted into zero, as show in Figure 1:

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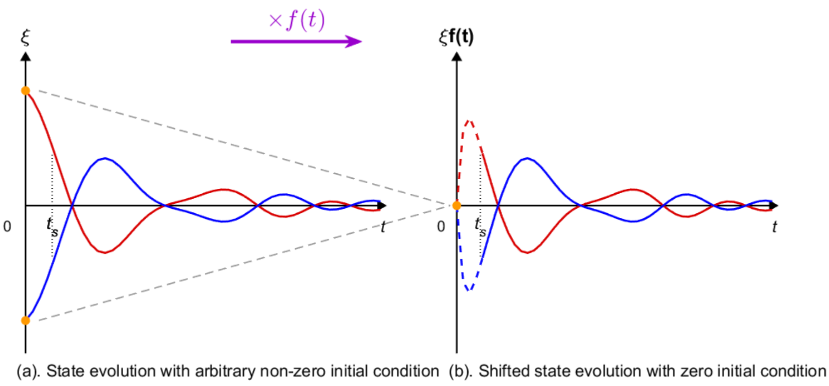

Figure 1. The role of the shifting function $f(t)$.

Remark 1. *Compared with [35], the error transformation function proposed in this paper has two distinct advantages: first, by introducing the exponential term* $\exp\left(-\frac{t^{2n}}{2b^{2n}}\right)$*, the proposed error transformation function can transition more smoothly when the constraint is activated; second, by introducing the parameter b, the transition rate of constraint activation can be flexibly adjusted as shown in Figure 2.*

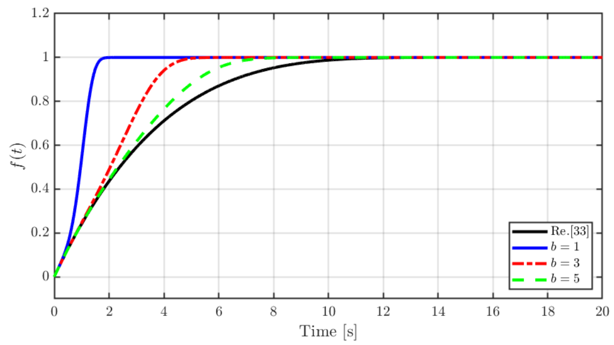

Figure 2. the response of $f(t)$ with different b values and in Re.[35].

Lemma 2. *([14]) As shown in (2), the proposed error shifting function $f(t)$ has the following key properties:*

(1) $f(t)$ is monotonically increasing for $t \in [0, t_s]$; for any $t \geq 0$, $f(t) \in [0,1]$ and $f(0) = 0$;

(2) $f(t)$ attains its maximum value of 1 at $t = t_s$, and remains 1 for $t \geq t_s$;

(3) $f^{(i)}(t)$, $i = 0, 1, \ldots, n+1$, are C^{n-i+1} and boundary for $t \in [0, \infty)$.

2.2. Barrier Function

To address the problem of delay asymmetric constraints, based on the error shifting function (2) a class of BFs is introduced as follows:

$$H(fx_k, C_{l_k}, C_{h_k}) = \frac{P(fx_k)}{fx_k - C_{l_K}} + \frac{Q(fx_k)}{C_{h_k} - fx_k} + R(fx_k)$$
$$x_k \in \{\xi_{1_k}, y_{r_k}\}, k = 1, \ldots, d \tag{3}$$

with the initial condition $\xi_{1_k}(0) \in \Omega_{1_k}$,where $P(fx_k)$, $Q(fx_k)$ and $R(fx_k)$ satisfy the following criteria: For $\forall x_k \in \Re$, 1) $P(fx_k) < 0$, $0 < \frac{dP}{dx_k} < +\infty$; 2) $Q(fx_k) > 0$, $0 < \frac{dQ}{dx_k} < +\infty$; and 3) $0 < \frac{dR}{dx_k} < +\infty$, $\lim_{x_k \to +\infty} = \bar{R}$, $\lim_{x_k \to -\infty} = \underline{R}$ with $\bar{R}$ and $\underline{R}$ being constants.

Remark 2. *Compared with the barrier function in [31], in this work, a novel unified BF is proposed, which allows the controller to be applicable regardless of the presence of delay constraints, without changing its structure.*

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Remark 3. *A neural network-based tracking control scheme is proposed for systems with output delay asymmetric constraints such that not only make sure that all the tracking errors become arbitrarily small, but also guarantee the output obey delay asymmetric constraints in fixed time even if the initial restriction conditions are not satisfied.*

Lemma 3. *[31] For any $\xi_{1_k}(0) \in \Omega_{1_k}$, $H(fx_k, C_{l_k}, C_{h_k})$, $x_k \in \{\xi_{1_k}, y_{r_k}\}$, has the following properties:*

$$\begin{cases} \lim\limits_{C_{l_k} \to -\infty} H(fx_k, C_{l_k}, C_{h_k}) = \dfrac{Q(fx_k)}{C_{h_k} - x_k} + R(fx_k) \\ \lim\limits_{C_{h_k} \to +\infty} H(fx_k, C_{l_k}, C_{h_k}) = \dfrac{P(fx_k)}{x_k - C_{l_k}} + R(fx_k) \\ \lim\limits_{\substack{C_{l_k} \to -\infty \\ C_{h_k} \to +\infty}} H(fx_k) = R(fx_k) \end{cases} \tag{4}$$

Therefore, it holds that, the absence of either C_{l_k} or C_{h_k}, does not affect the normal preservation of the remaining constraints. For any $x_k \in \Omega_{1_k}$, the external constraints C_{l_k} and C_{h_k} are not violated as long as $H(fx_k, C_{l_k}, C_{h_k})$ is bounded. Moreover, the nonlinear BF $H(fx_k, C_{l_k}, C_{h_k})$, will be reduced to $R(fx_k)$ when both C_{l_k} and C_{h_k} do not exist.

Lemma 4. *([31]) Even if the system (1) is potentially affected by delay asymmetric constraints, the constructed barrier function $H(fx_k, C_{l_k}, C_{h_k})$, can always maintain the controllability of the original system.*

Proof. Taking the derivative of $x_k \in \{\xi_{1_k}, y_{r_k}\}$ with respect to (w.r.t.) time yields:

$$\dot{H}(fx_k, C_{l_k}, C_{h_k}) = \mu_1(fx_k)(\dot{f}x_k + f\dot{x}_k) + \mu_2(fx_k)\dot{C}_{l_k} - \mu_3(fx_k)\dot{C}_{h_k} \tag{5}$$

where $\mu_1(fx_k) = \frac{\frac{dP}{dx_k}(fx_k - C_{l_k}) - fP(fx_k)}{(fx_k - C_{l_k})^2} + \frac{\frac{dQ}{dx_k}(C_{h_k} - fx_k) + Q(fx_k)}{(C_{h_k} - fx_k)^2} + \frac{dR}{dx_k}$, $\mu_2(fx_k) = \frac{P(fx_k)}{(fx_k - C_{l_k})^2}$, $\mu_3(fx_k) = \frac{Q(fx_k)}{(C_{h_k} - fx_k)^2}$.

If constraints are not violated, then $C_{l_k} < \xi_{1_k} < C_{h_k}$ holds, and we can readily obtain that $\mu_1(\xi_{1_k}) > 0$ for $\forall \xi_{1_k} \in (C_{l_k}, C_{h_k})$, which ensures that the gain term $\mu_1(\xi_{1_k})$ of subsystem (5) is always not equal to zero, further implying that the original system can always remain controllable. □

2.3. Error analysis

To proceed, for the sake of promoting the development of the adaptive tracking control framework, the tracking error $\beta_i, i = 1, \ldots, n$ is defined as follows:

$$\begin{aligned} &\beta_1 = \xi_1 - y_r \\ &\beta_i = \xi_i - \vartheta_{i-1}, \ i = 2, \ldots, n \end{aligned} \tag{6}$$

with ϑ_i being the virtual controller to be designed later.

Subsequently, to address the problem of delay asymmetric output constraints, on the basis of barrier function (3), a new error $z_i, i = 1, \ldots, n$ is defined as follows:

$$\begin{aligned} &z_1 = H(f\xi_1) - H(fy_r) \\ &z_i = \xi_i - \vartheta_{i-1}, \quad i = 2, \ldots, n \end{aligned} \tag{7}$$

Lemma 5. *([31]) If constraints are not violated, then $z_{1_k} = 0$ if and only if $\beta_{1_k} = 0$.*

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Proof. In accordance with(3), (7) can be expressed:

$$\begin{aligned} z_{1_k} =& \frac{P(f\xi_{1_k})}{f\xi_{1_k} - C_{l_k}} - \frac{P(fy_{r_k})}{fy_{r_k} - C_{l_k}} + \frac{Q(f\xi_{1_k})}{C_{h_k} - f\xi_{1_k}} - \frac{Q(fy_{r_k})}{C_{h_k} - fy_{r_k}} \\ &+ R(f\xi_{1_k}) - R(fy_{r_k}) \end{aligned} \tag{8}$$

It is readily verified that $z_{1_k} = 0$ when $\beta_{1_k} = 0$, if z_{1_k} are bounded and the initial condition is satisfied for any. To prove that $z_{1_k} = 0$ only when $\beta_{1_k} = 0$, we take the derivative of z_{1_k} w.r.t. ξ_{1_k} and then have $\frac{\partial z_{1_k}}{\partial \xi_{1_k}} = \mu_1(\xi_{1_k}) > 0$ for $\forall x_{1_k} \in (C_{l_k}, C_{h_k})$. Therefore, we can draw the conclusion that $z_{1_k} = 0$ if and only if $\beta_{1_k} = 0$. □

3. Main Results

3.1. Controller Design

The virtual control laws ϑ_i, actual controller u and adaptation laws $\dot{\hat{\omega}}$ are designed as:

$$\vartheta_i = -m_i z_i - K_i, i = 1, \ldots, n-1 \tag{9}$$

$$u = \frac{1}{\underline{\kappa}}(-m_n z_n - K_n) \tag{10}$$

$$\dot{\hat{\omega}} = \frac{\gamma_i}{4a_i^2} z_i^2 q_i - h_i \hat{\omega}_i, i = 1, \ldots, n \tag{11}$$

the values of the above parameters are shown later.

3.2. Stability Analysis

Theorem 1. *In terms of the nonlinear (1), suppose Assumptions 1-4 are true. Then, the actual controller (10), together with the virtual controller (9) and adaptation laws (11) for $i = 1, 2, \ldots, n$, can guarantee the ultimately uniformly bounded (UUB) tracking of the output $y(t)$ to the reference $y_r(t)$. Moreover, even if the initial output constraint is violated, the output $y(t) = \xi_1(t)$ of the resulting closed-loop system obeys the delayed asymmetric output constraints in preset-time t_s, i.e., $C_l(t) < y(t) = \xi_1(t) < C_h(t) \quad (\forall t \geq t_s)$. All the internal signals in closed-loop system are ensured to be bounded.*

Proof. Step 1 : Construct the Lyapunov function:

$$V_1 = \frac{1}{2} z_1^2 + \frac{1}{2\gamma_1} \tilde{\omega}_1^2 \tag{12}$$

where $\tilde{\omega}_1 = \omega_1 - \hat{\omega}_1$ represents the estimation error. As a result, differentiating (12) obtains:

$$\begin{aligned} \dot{V}_1 &= z_1 \dot{z}_1 - \frac{1}{\gamma_1} \tilde{\omega}_1 \dot{\hat{\omega}}_1 \\ &= z_1(\dot{H}(f\xi_1) - \dot{H}(fy_r)) - \frac{1}{\gamma_1} \tilde{\omega}_1 \dot{\hat{\omega}}_1 \end{aligned} \tag{13}$$

For

$$\begin{aligned} \dot{H}(f\xi_1) &= \mu_1(f\xi_1)(\dot{f}\xi_1 + f\dot{\xi}_1) + \mu_2(f\xi_1)\dot{C}_l - \mu_3(f\xi_1)\dot{C}_h \\ &= \mu_1(f\xi_1)[\dot{f}\xi_1 + f(\varphi_1 \xi_2 + \psi_1 + d_1)] + \mu_2(f\xi_1)\dot{C}_l - \mu_3(f\xi_1)\dot{C}_h \\ &= \mu_1(f\xi_1)[\dot{f}\xi_1 + f(\varphi_1 z_2 + \varphi_1 \vartheta_1 + \psi_1 + d_1)] + \mu_2(f\xi_1)\dot{C}_l - \mu_3(f\xi_1)\dot{C}_h \end{aligned} \tag{14}$$

similarly,

$$\dot{H}(fy_r) = \mu_1(fy_r)(\dot{f}y_r + f\dot{y}_r) + \mu_2(fy_r)\dot{C}_l - \mu_3(fy_r)\dot{C}_h \tag{15}$$

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Substituting (14) and (15) into (13), we obtain:

$$\begin{aligned}\dot{V}_1 = z_1\big[&\mu_1(f\xi_1)\big(\dot{f}\xi_1 + f(\varphi_1 z_2 + \varphi_1\vartheta_1 + \psi_1 + d_1)\big) + \mu_2(f\xi_1)\dot{C}_l - \mu_3(f\xi_1)\dot{C}_h \\ &- \mu_1(fy_r)\big(\dot{f}y_r + f\dot{y}_r\big) - \mu_2(fy_r)\dot{C}_l + \mu_3(fy_r)\dot{C}_h\big] - \frac{1}{\gamma_1}\tilde{\omega}_1\dot{\hat{\omega}}_1\end{aligned} \tag{16}$$

Based on Lemma 1, choosing:

$$\mu_1(f\xi_1)f\psi_1 + \mu_1(f\xi_1)fd_1 = W_1^{*T}\eta_1(U_1) + \varepsilon_1 \tag{17}$$

then

$$\begin{aligned}\dot{V}_1 =& z_1\big[\mu_1(f\xi_1)\big(\dot{f}\xi_1 + f\varphi_1 z_2 + f\varphi_1\vartheta_1\big) + \mu_2(f\xi_1)\dot{C}_l - \mu_3(f\xi_1)\dot{C}_h - \mu_1(fy_r)\big(\dot{f}y_r + f\dot{y}_r\big) \\ &- \mu_2(fy_r)\dot{C}_l + \mu_3(fy_r)\dot{C}_h\big] + z_1 W_1^{*T}\eta_1(U_1) + z_1\varepsilon_1 - \frac{1}{\gamma_1}\tilde{\omega}_1\dot{\hat{\omega}}_1\end{aligned} \tag{18}$$

Based on Young's inequality, Lemma 1, and Assumptions 1 and 2 and adaptation laws (11), we have:

$$\begin{aligned}&z_1\big[(\mu_2(f\xi_1) - \mu_2(fy_r))\dot{C}_l + (\mu_3(fy_r) - \mu_3(f\xi_1))\dot{C}_h\big] \\ &\le \alpha_1 z_1^2\Big[(\mu_2(f\xi_1) - \mu_2(fy_r))^2 + (\mu_3(fy_r) - \mu_3(f\xi_1))^2\Big] + \frac{\bar{C}_l^2 + \bar{C}_h^2}{4\alpha_1}\end{aligned} \tag{19}$$

$$z_1\big[-\mu_1(fy_r)\big(\dot{f}y_r + f\dot{y}_r\big)\big] \le \alpha_1 z_1^2\mu_1(fy_r)^2 + \frac{\bar{y}_r^2}{4\alpha_1} \tag{20}$$

$$z_1\mu_1(f\xi_1)\dot{f}\xi_1 \le \underline{\varphi}_1 c_1 z_1^2\mu_1^2(f\xi_1)\dot{f}^2\xi_1^2 + \frac{1}{4\underline{\varphi}_1 c_1} \tag{21}$$

$$z_1\mu_1(f\xi_1)f\varphi_1 z_2 \le \underline{\varphi}_2 c_2 f^2 z_1^2\mu_1^2(f\xi_1)z_2^2 + \frac{\bar{\varphi}_1^2}{4\underline{\varphi}_2 c_2} \tag{22}$$

$$z_1(W_1\eta_1(U_1) + \varepsilon_1) \le \frac{1}{4a_1^2}z_1^2\omega_1 q_1 + a_1^2 + \frac{1}{4}z_1^2 + \varepsilon_N^2 \tag{23}$$

$$\frac{h_1}{\gamma_1}\tilde{\omega}_1\hat{\omega}_1 \le \frac{h_1}{2\gamma_1}\omega_1^2 - \frac{h_1}{2\gamma_1}\tilde{\omega}_1^2 \tag{24}$$

where c_1 and c_2 are positive control design parameters. Based on (19) to (24), (18) can be rewritten as:

$$\dot{V}_1 \le z_1\mu_1(f\xi_1)f\Big(\varphi_1\vartheta_1 + \underline{\varphi}_1 K_1\Big) + \alpha_1 z_1^2\Phi_1 + \underline{\varphi}_2 c_2 f^2 z_1^2\mu_1^2(f\xi_1)z_2^2 + \Gamma_1 + \frac{1}{4}z_1^2 - \frac{h_1}{2\gamma_1}\tilde{\omega}_1^2 \tag{25}$$

where $K_1 = c_1 z_1\mu_1(f\xi_1)\dot{f}^2\xi_1^2$, $\Gamma_1 = \frac{1}{4\underline{\varphi}_1 c_1} + \frac{\bar{\varphi}_1^2}{4\underline{\varphi}_2 c_2} + \frac{\bar{y}_r^2 + \bar{C}_l^2 + \bar{C}_h^2}{4\alpha_1} + \varepsilon_N^2 + a_1^2 + \frac{h_1}{2\gamma_1}\omega_1^2$ and $\Phi_1 = (\mu_2(f\xi_1) - \mu_2(fy_r))^2 + (\mu_3(fy_r) - \mu_3(f\xi_1))^2 + \mu_1^2(fy_r)$. Since K_1, Γ_1 and Φ_1 are both calculated, we design the following virtual controller:

$$\vartheta_1 = -m_1 z_1 - K_1 \tag{26}$$

with m_1 denoting the positive control design parameter. Then:

$$\begin{aligned}z_1\mu_1(f\xi_1)f\varphi_1\vartheta_1 &= -m_1 z_1^2\mu_1(f\xi_1)f\varphi_1 - z_1\mu_1(f\xi_1)f\varphi_1 K_1 \\ &\le -m_1 z_1^2\mu_1(f\xi_1)f\underline{\varphi}_1 - z_1\mu_1(f\xi_1)f\underline{\varphi}_1 K_1\end{aligned} \tag{27}$$

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Substituting (27) into (25) yields:

$$\dot{V}_1 \leq -m_1\mu_1(f\xi_1)f\underline{\varphi}_1 z_1^2 + \frac{1}{4}z_1^2 - \frac{h_1}{2\gamma_1}\tilde{\omega}_1^2 + \underline{\varphi}_2 c_2 z_1^2 \mu_1^2(f\xi_1)z_2^2 + \alpha_1\Phi_1 z_1^2 + \Gamma_1 \tag{28}$$

Step i $(2 \leq i \leq n-1)$: At the current i th step, it should be noticed that $\beta_i = \xi_i - \vartheta_{i-1}$ and ϑ_{i-1} denotes a function of ξ_{i-1}, $y_r^{(i-1)}$, $\hat{\omega}_{i-1}$, $f^{(i-1)}$, $C_l^{(i-j)}$ and $C_h^{(i-j)}$ with $j = 1,2,\ldots,i-1$, where $\bar{\xi}_{i-1} = [\xi_1,\xi_2,\ldots,\xi_{i-1}]^T$, $\bar{y}_r^{(i-1)} = [y_r,\ldots,y_r^{(i-1)}]^T$, $\bar{\omega}_{i-1} = [\hat{\omega}_1,\ldots,\hat{\omega}_{i-1}]^T$, $\bar{f}^{(i-1)} = [f,\ldots,f^{(i-1)}]^T$, $\bar{C}_l^{(i-j)} = [C_l,\ldots,C_l^{(i-j)}]^T$ and $\bar{C}_h^{(i-j)} = [C_h,\ldots,C_h^{(i-j)}]^T$ with $j = 1,2,\ldots,i-1$. Ergo, we can derive:

$$\dot{\vartheta}_{i-1} = \sum_{j=1}^{i-1}\frac{\partial\vartheta_{i-1}}{\partial\xi_j}\left(\varphi_j\xi_{j+1} + \psi_j + d_j\right) + \Gamma_{\vartheta_{i-1}} \tag{29}$$

$$\begin{aligned}\Gamma_{\vartheta_{i-1}} = &\sum_{j=0}^{i-1}\frac{\partial\vartheta_{i-1}}{\partial y_r^{(j)}}y_r^{(j+1)} + \sum_{j=1}^{i-1}\frac{\partial\vartheta_{i-1}}{\partial\hat{\omega}_j}\dot{\hat{\omega}}_j + \sum_{j=1}^{i-1}\frac{\partial\vartheta_{i-1}}{\partial f^{(j)}}f^{(j+1)} \\ &+ \sum_{j=1}^{i-1}\sum_{p=0}^{i-j}\frac{\partial\vartheta_{i-1}}{\partial C_l^{(p)}}C_l^{(p+1)} + \sum_{j=1}^{i-1}\sum_{p=0}^{i-j}\frac{\partial\vartheta_{i-1}}{\partial C_h^{(p)}}C_h^{(p+1)}\end{aligned} \tag{30}$$

Construct the Lyapunov function:

$$V_i = V_{i-1} + \frac{1}{2}z_i^2 + \frac{1}{2\gamma_i}\tilde{\omega}_i^2 \tag{31}$$

where $\tilde{\omega}_i = \omega_i - \hat{\omega}_i$ represents the estimation error. We can calculate the derivative of z_i w.r.t. time as:

$$\begin{aligned}\dot{z}_i &= \dot{\xi}_i - \dot{\vartheta}_{i-1} \\ &= \left(\varphi_i(\bar{\xi}_i)\xi_{i+1} + \psi_i(\bar{\xi}_i) + d_i(\bar{\xi}_i,t)\right) \\ &\quad - \left(\sum_{j=1}^{i-1}\frac{\partial\vartheta_{i-1}}{\partial\xi_j}\left(\varphi_j(\bar{\xi}_j)\xi_{j+1} + \psi_j(\bar{\xi}_j) + d_j(\bar{\xi}_j,t)\right) + \Gamma_{\vartheta_{i-1}}\right)\end{aligned} \tag{32}$$

Based on Lemma 1, choosing:

$$\left(\psi_i(\bar{\xi}_i) + d_i(\bar{\xi}_i,t)\right) - \sum_{j=1}^{i-1}\frac{\partial\vartheta_{j-1}}{\partial\xi_j}\left(\psi_j(\bar{\xi}_j) + d_j(\bar{\xi}_j,t)\right) = W_i^{*T}\eta_i(U_i) + \varepsilon_i \tag{33}$$

then

$$\begin{aligned}\dot{V}_i &= \dot{V}_{i-1} + z_i\varphi_i(\bar{\xi}_i)\xi_{i+1} - z_i\left(\sum_{j=1}^{i-1}\frac{\partial\vartheta_{i-1}}{\partial\xi_j}\varphi_j(\bar{\xi}_j)\xi_{j+1} + \Gamma_{\vartheta_{i-1}}\right) + z_iW_i^{*T}\eta_i(U_i) + z_i\varepsilon_i - \frac{1}{\gamma_i}\tilde{\omega}_i\dot{\hat{\omega}}_i \\ &= \dot{V}_{i-1} + z_i\left(\varphi_i(\bar{\xi}_i)z_{i+1} + \varphi_i(\bar{\xi}_i)\vartheta_i\right) - z_i\left(\sum_{j=1}^{i-1}\frac{\partial\vartheta_{i-1}}{\partial\xi_j}\varphi_j(\bar{\xi}_j)\xi_{j+1} + \Gamma_{\vartheta_{i-1}}\right) + z_iW_i^{*T}\eta_i(U_i) \\ &\quad + z_i\varepsilon_i - \frac{1}{\gamma_i}\tilde{\omega}_i\dot{\hat{\omega}}_i\end{aligned} \tag{34}$$

Based on Young's inequality, Lemma 1, and Assumptions 1 and 2, adaptation laws (12), we have:

$$z_i\varphi_i(\bar{\xi}_i)z_{i+1} \leq \underline{\varphi}_{i+1}c_{i+1}z_{i+1}^2 z_i^2 + \frac{\bar{\varphi}_i^2}{4\underline{\varphi}_{i+1}c_{i+1}} \tag{35}$$

$$-z_i\Gamma_{\vartheta_{i-1}} \leq \underline{\varphi}_i c_i z_i^2\Gamma_{\vartheta_{i-1}}^2 + \frac{1}{4\underline{\varphi}_i c_i} \tag{36}$$

$$-z_i\sum_{j=1}^{i-1}\frac{\partial\vartheta_{i-1}}{\partial\xi_j}\varphi_j\xi_{j+1}\le \underline{\varphi}_i c_i z_i^2\left(\sum_{j=1}^{i-1}\frac{\partial\vartheta_{i-1}}{\partial\xi_j}\right)^2\xi_{j+1}^2+\sum_{j=1}^{i-1}\frac{\bar{\varphi}_j^2}{4\underline{\varphi}_i c_i} \tag{37}$$

$$z_i(W_i\eta_i(U_i)+\varepsilon_i)\le\frac{1}{4a_i^2}z_i^2\omega_i q_i+a_i^2+\frac{1}{4}z_i^2+\varepsilon_N^2 \tag{38}$$

$$\frac{h_i}{\gamma_i}\tilde{\omega}_i\hat{\omega}_i\le\frac{h_i}{2\gamma_i}\omega_i^2-\frac{h_i}{2\gamma_i}\tilde{\omega}_i^2 \tag{39}$$

where c_i and c_{i+1} are positive control design parameters. Accordingly, we can get:

$$\begin{aligned}\dot{V}_i\le&-m_1\mu_1(f\xi_1)f\underline{\varphi}_1 z_1^2+\frac{1}{4}z_1^2-\frac{h_1}{2\gamma_1}\tilde{\omega}_1^2+\alpha_1\Phi_1 z_1^2+\Gamma_1+\sum_{j=2}^{i-1}\left(-m_i\underline{\varphi}_j z_j^2+\frac{1}{4}z_j^2-\frac{h_j}{2\gamma_j}\tilde{\omega}_j^2+\Gamma_j\right)\\&+z_i\left(\varphi_i\vartheta_i+\underline{\varphi}_i K_i\right)+\Gamma_i+\underline{\varphi}_{i+1}c_{i+1}z_{i+1}^2 z_i^2+\frac{1}{4}z_i^2-\frac{h_i}{2\gamma_i}\tilde{\omega}_i^2\end{aligned} \tag{40}$$

where $K_i=c_iz_iz_{i-1}^2+c_iz_i\left(\sum_{j=1}^{i-1}\frac{\partial\vartheta_{i-1}}{\partial\xi_j}\right)^2\xi_{j+1}^2+c_iz_i\Gamma_{\vartheta_{i-1}}^2$, $\Gamma_i=\frac{1}{4\underline{\varphi}_i c_i}+\sum_{j=1}^{i-1}\frac{\bar{\varphi}_j^2}{4\underline{\varphi}_i c_i}+\frac{\bar{\varphi}_i^2}{4\underline{\varphi}_{i+1}c_{i+1}}+\varepsilon_N^2+a_i^2+\frac{h_i}{2\gamma_i}\omega_i^2$. Since K_i and Γ_i are both calculated, we design the following virtual controller:

$$\vartheta_i=-m_iz_i-K_i \tag{41}$$

with m_i denoting the positive control design parameter. Then:

$$z_i\varphi_i\vartheta_i=-m_iz_i^2\varphi_i-z_i\varphi_iK_i\le-m_iz_i^2\underline{\varphi}_i-z_i\underline{\varphi}_iK_i \tag{42}$$

Substituting (42) into (40):

$$\begin{aligned}\dot{V}_i\le&-m_1\mu_1(f\xi_1)f\underline{\varphi}_1 z_1^2-\sum_{j=2}^{i}\left(m_j\underline{\varphi}_j z_j^2\right)+\underline{\varphi}_{i+1}c_{i+1}z_{i+1}^2z_i^2\\&+\alpha_1\Phi_1z_1^2+\sum_{j=1}^{i}\left(\frac{1}{4}z_j^2-\frac{h_j}{2\gamma_j}\tilde{\omega}_j^2+\Gamma_j\right)\end{aligned} \tag{43}$$

Step n : Construct the Lyapunov function:

$$V_n=V_{n-1}+\frac{1}{2}z_n^2+\frac{1}{2\gamma_n}\tilde{\omega}_n^2 \tag{44}$$

where $\tilde{\omega}_n=\omega_n-\hat{\omega}_n$ represents the estimation error. We can calculate the derivative of z_n w.r.t. time as:

$$\begin{aligned}\dot{z}_n&=\dot{\xi}_n-\dot{\vartheta}_{n-1}\\&=\left(\varphi_n(\bar{\xi}_n)[\kappa(t,t_k)u+\chi(t,t_\chi)]+\psi_n(\bar{\xi}_n)+d_n(\bar{\xi}_n,t)\right)\\&\quad-\left(\sum_{j=1}^{n-1}\frac{\partial\vartheta_{n-1}}{\partial\xi_j}\left(\varphi_j(\bar{\xi}_j)\xi_{j+1}+\psi_j(\bar{\xi}_j)+d_j(\bar{\xi}_j,t)\right)+\Gamma_{\vartheta_{n-1}}\right)\end{aligned} \tag{45}$$

Based on Lemma 1, choosing:

$$\left(\psi_n(\bar{\xi}_n)+d_n(\bar{\xi}_n,t)\right)+\varphi_n(\bar{\xi}_n)\chi(t,t_\chi)-\sum_{j=1}^{n-1}\frac{\partial\vartheta_{n-1}}{\partial\xi_j}\left(\psi_j(\bar{\xi}_j)+d_j(\bar{\xi}_j,t)\right)=W_n^{*T}\eta_n(U_n)+\varepsilon_n \tag{46}$$

then:

$$\begin{aligned}\dot{V}_n &= \dot{V}_{n-1} + z_n\dot{z}_n - \tilde{\omega}_n\dot{\hat{\omega}}_n \\ &\le -m_1\mu_1(f\xi_1)f\underline{\varphi}_1 z_1^2 - \sum_{j=2}^{n-1}\left(m_j f\underline{\varphi}_j z_j^2\right) + \underline{\varphi}_n c_n z_n^2 z_{n-1}^2 + \alpha_1\Phi_1 z_1^2 + \sum_{j=1}^{n-1}\left(\frac{1}{4}z_j^2 - \frac{h_j}{2\gamma_j}\tilde{\omega}_j^2 + \Gamma_j\right) \\ &\quad + z_n f\varphi_n(\bar{\xi}_n)\kappa(t,t_k)u - z_n\sum_{j=1}^{n-1}\frac{\partial\vartheta_{i-1}}{\partial\xi_j}\left(\varphi_j(\bar{\xi}_j)\xi_{j+1} + \Gamma_{\vartheta_{n-1}}\right) \\ &\quad + z_n W_n^T\eta_n(U_n) + z_n\varepsilon_n - \frac{1}{\gamma_i}\tilde{\omega}_i\dot{\hat{\omega}}_i\end{aligned} \tag{47}$$

Based on Young's inequality, Lemma 1, and Assumptions 1 and 2, adaptation laws (11), we have:

$$-z_n\sum_{j=1}^{n-1}\frac{\partial\vartheta_{n-1}}{\partial\xi_j}\varphi_j\xi_{j+1} \le \underline{\varphi}_n c_n z_n^2\left(\sum_{j=1}^{n-1}\frac{\partial\vartheta_{n-1}}{\partial\xi_j}\right)^2\xi_{j+1}^2 + \sum_{j=1}^{n-1}\frac{\bar{\varphi}_j^2}{4\underline{\varphi}_n c_n} \tag{48}$$

$$-z_n\Gamma_{\vartheta_{n-1}} \le \underline{\varphi}_n c_n z_n^2\Gamma_{\vartheta_{n-1}}^2 + \frac{1}{4\underline{\varphi}_n c_n} \tag{49}$$

$$z_n\left(W_n^{*T}\eta_n(U_n) + \varepsilon_n\right) \le \frac{1}{4a_n^2}z_n^2\omega_n q_n + a_n^2 + \frac{1}{4}z_n^2 + \varepsilon_N^2 \tag{50}$$

$$\frac{h_n}{\gamma_n}\tilde{\omega}_n\hat{\omega}_n \le \frac{h_n}{2\gamma_n}\omega_n^2 - \frac{h_n}{2\gamma_n}\tilde{\omega}_n^2 \tag{51}$$

where c_n is positive control design parameter. Accordingly, we can get:

$$\begin{aligned}\dot{V}_n &\le -m_1\mu_1(f\xi_1)f\underline{\varphi}_1 z_1^2 - \sum_{j=2}^{n-1}\left(m_j\underline{\varphi}_j z_j^2\right) + \alpha_1\Phi_1 z_1^2 + \sum_{j=1}^{n}\left(\frac{1}{4}z_j^2 - \frac{h_j}{2\gamma_j}\tilde{\omega}_j^2 + \Gamma_j\right) \\ &\quad + z_n\left(\varphi_n\kappa(t,t_k)u + \underline{\varphi}_n K_n\right)\end{aligned} \tag{52}$$

where $K_n = c_n z_n z_{n-1}^2 + c_n z_n + c_n z_n\left(\sum_{j=1}^{n-1}\frac{\partial\vartheta_{n-1}}{\partial\xi_j}\right)^2\xi_{j+1}^2 + c_n z_n\Gamma_{\vartheta_{n-1}}^2$, $\Gamma_n = \frac{1+\sum_{j=1}^{n-1}\bar{\varphi}_j^2}{4\underline{\varphi}_n c_n} + \varepsilon_N^2 + a_n^2 + \frac{h_n}{2\gamma_n}\omega_n^2$. Since K_n and Γ_n are both calculated, the controller is designed as:

$$u = \frac{1}{\underline{\kappa}}(-m_n z_n - K_n) \tag{53}$$

with m_n denoting the positive control design parameter. Then:

$$\begin{aligned}z_n\varphi_n\kappa(t,t_k)u &= -m_n z_n^2\varphi_n - z_n\varphi_n K_n \\ &\le -m_n z_n^2\underline{\varphi}_n - z_n\underline{\varphi}_n K_n\end{aligned} \tag{54}$$

$$\begin{aligned}\dot{V}_n &\le -m_1\mu_1(f\xi_1)f\underline{\varphi}_1 z_1^2 - \sum_{j=1}^{n}(m_j\underline{\varphi}_j z_j^2) + \gamma_1\Phi_1 z_1^2 + \sum_{j=1}^{n}\left(\frac{1}{4}z_j^2 - \frac{h_j}{2\gamma_j}\tilde{\omega}_j^2 + \Gamma_j\right) \\ &\le -\mu V_n + \nu\end{aligned} \tag{55}$$

where $\mu = \min\left\{m_1\mu_1\underline{\varphi}_1 f, m_i\underline{\varphi}_i, \frac{1}{4}, \frac{h_1}{2\gamma_1}, \frac{h_i}{2\gamma_i} \mid i = 2,3,\ldots,n\right\}$, $\nu = \sum_{j=1}^{n}\Gamma_i + \gamma_1\Phi_1 z_1^2$, we can infer from (55) that:

$$0 \le V_n(t) \le \frac{v}{\mu} + \left(V_n(0) - \frac{v}{\mu}\right)e^{-\mu t} \tag{56}$$

$$\frac{1}{2}z_1^2 \le V_n(t) \le \frac{v}{\mu} + \left(V_n(0) - \frac{v}{\mu}\right)e^{-\mu t} \tag{57}$$

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$$|z_1| \leq \sqrt{\frac{2v}{\mu} + 2\left(V_n(0) - \frac{v}{\mu}\right)e^{-\mu t}} \tag{58}$$

When $t \to \infty$, $|z_1| \leq \sqrt{\frac{2\nu}{\mu}}$, and the tracking error can be made arbitrarily small by adjusting μ and ν, satisfying the UUB tracking of the output $y(t)$ to $y_r(t)$.

$$\dot{V}_n \leq -\iota \sum_{j=1}^{n} z_j^2 + \nu \tag{59}$$

where $l = \min\left\{m_j \underline{\mu}_j \underline{\varphi}_j f, m_j \underline{\varphi}_j f \mid j = 2, 3, \ldots, n\right\}$. For the sake of convenience, we denote $z = [z_1, z_2, \ldots, z_n]^T$ and $\beta = [\beta_1, \beta_2, \ldots, \beta_n]^T$, then (59) can be reconstructed as follows:

$$lz^T z + \dot{V}_n \leq \nu \tag{60}$$

By means of integrating (60) and utilizing the fact that V_n is bounded, we can get:

$$\lim_{T_v \to \infty} \frac{1}{T_v} \int_0^{T_v} z^T z dt \leq \frac{\nu}{\iota} \tag{61}$$

notice that:

$$\lim_{T_v \to \infty} \frac{1}{T_v} \int_0^{T_v} \beta^T \beta dt = \lim_{T_v \to \infty} \frac{1}{T_v} \int_0^{T_v} z^T z dt \tag{62}$$

Similarly, we can further obtain:

$$\lim_{T_v \to \infty} \frac{1}{T_v} \int_{t_s}^{T_v} z^T z dt = \lim_{T_v \to \infty} \frac{1}{T_v} \int_0^{T_v} z^T z dt \tag{63}$$

According to (61), (62) and (63), one has:

$$\lim_{T_v \to \infty} \frac{1}{T_v} \int_0^{T_v} \beta_i^2 dt \leq \lim_{T_v \to \infty} \frac{1}{T_v} \int_0^{T_v} \beta^T \beta dt \leq \frac{\nu}{\iota} \tag{64}$$

The above analysis indicates that ν is a key parameter affecting the mean-square tracking error and virtual tracking errors. If ν is chosen sufficiently small, the value of β_i will decrease accordingly.

From the boundedness of $V(t)$, it follows that z_i and $\tilde{\omega}_i$ are bounded; Combining $z_i = f(t)\beta_i$, $\beta_i = \xi_i - \vartheta_{i-1}$, and the virtual controller $\vartheta_i = -m_i z_i - K_i$ (where K_i is bounded), we can conclude that ξ_i and ϑ_i are bounded; The actual controller $u = \frac{1}{\underline{\kappa}}(-m_n z_n - K_n)$, since z_n, K_n, and $\underline{\kappa}$ are bounded, u is bounded; In summary, all closed-loop signals such as ξ_i, u, $\hat{\omega}_i$, and z_1 are uniformly bounded. □

4. Numerical Simulation

In order to attest the efficacy of the proposed control framework in this section, we give an emblematic application example. The attitude control of the quadrotor UAV is described in [38] and a part of parameters are designed as (65):

$$\begin{cases} \dot{x}_{1,1} = x_{1,2} \\ \dot{x}_{1,2} = l_1 x_{2,2} x_{3,2} - l_2 \Omega_r x_{2,2} - l_3 x_{1,2} + b_1 u_\varphi + d_{\dot{\varphi}}(t) \\ \dot{x}_{2,1} = x_{2,2} \\ \dot{x}_{2,2} = l_4 x_{1,2} x_{3,2} - l_5 \Omega_r x_{1,2} - l_6 x_{2,2} + b_2 u_\theta + d_{\dot{\theta}}(t) \\ \dot{x}_{3,1} = x_{3,2} \\ \dot{x}_{3,2} = l_7 x_{1,2} x_{2,2} - l_8 x_{3,2} + b_3 u_\psi + d_{\dot{\psi}}(t) \end{cases} \tag{65}$$

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where $x = (\varphi, \dot{\varphi}, \theta, \dot{\theta}, \psi, \dot{\psi})$, $u = \left[u_\varphi, u_\theta, u_\psi\right]^T$ denotes the input vector. $b_1 = \frac{d_q}{I_x}$, $b_2 = \frac{d_q}{I_y}$, $b_3 = \frac{1}{I_z}$, $l_1 = \frac{I_y - I_z}{I_x}$, $l_2 = \frac{I_r}{I_x}$, $l_3 = \frac{k_\varphi}{I_x}$, $l_4 = \frac{I_z - I_x}{I_y}$, $l_5 = \frac{I_r}{I_y}$, $l_6 = \frac{k_\theta}{I_y}$, $l_7 = \frac{I_x - I_y}{I_z}$, $l_8 = \frac{k_\psi}{I_z}$, $\Omega_r = \omega_1 - \omega_2 + \omega_3 - \omega_4$,

Table 1. Quadrotor physical parameters.

Parameter(s)	Value(s)
d_q	0.205 cm
J_q	2.8×10^{-5} kg m
I_x, I_y	9.3×10^{-2} kg m
I_z	8.2×10^{-2} kg m
K_φ, K_θ	5.56×10^{-3} N m s/rad
K_ψ	6.35×10^{-3} N m s/rad
Ω_r	100 rad/s

The ϕ, θ, and ψ denote the roll, pitch, and yaw angles, respectively. The (I_x, I_y, I_z) denote the body inertia, $(k_\varphi, k_\theta, k_\psi)$ denote aerodynamic coefficients, J_r, b, k, d_q denote rotor inertia, thrust factor, drag factor, the distance from the quadcopter's center of mass to the rotor shaft respectively, and ω_i for i ranging from 1 to 4 denote the angular velocity of each rotor i respectively.

Our control goal is to make the reference trajectory $y_r = [\phi_r, \theta_r, \psi_r]^T$ should be tracked by the system output $y = [\phi, \theta, \psi]^T$.

Define the following vector to represent the attitude of the UAV: $\xi_1 = [\varphi, \theta, \psi]^T$, $\xi_2 = [\dot{\varphi}, \dot{\theta}, \dot{\psi}]^T$. Then, the attitude control system of the UAV can be rewritten as:

$$\begin{cases} \dot{\xi}_1 = \xi_2 \\ \dot{\xi}_2 = g_2[\kappa u + \chi] + \psi_2 + d_2 \\ y = \xi_1 \end{cases} \tag{66}$$

where $g_2 = \begin{bmatrix} b_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & b_3 \end{bmatrix} = \begin{bmatrix} \frac{d}{I_x} & 0 & 0 \\ 0 & \frac{d}{I_y} & 0 \\ 0 & 0 & \frac{1}{I_z} \end{bmatrix}$, $\psi_2 = \begin{bmatrix} l_1\dot{\theta}\dot{\psi} - l_2\Omega_r\dot{\theta} - l_3\dot{\varphi} \\ l_4\dot{\varphi}\dot{\theta} - l_5\Omega_r\dot{\varphi} - l_6\dot{\theta} \\ l_7\dot{\varphi}\dot{\theta} - l_8\dot{\psi} \end{bmatrix}$, $d_2 = \begin{bmatrix} d_\varphi(t) \\ d_\theta(t) \\ d_\psi(t) \end{bmatrix} = \begin{bmatrix} 0.5\sin(2t) \\ 0.5\cos(2t) \\ 0.2\sin(3t) \end{bmatrix}$.

The parameter selection is determined as follows: The initial state of Case I is $\xi(0) = [1.2, 1.33, 1.5]^T$, the initial state of Case II is $\xi(0) = [0.8, 0.5, 0.6]^T$. The upper bound of the output constraint is $C_{h_k}(t) = 1 + 0.1\sin(4t)$, the lower bound of the output constraint is $C_{l_k}(t) = -2 - \sin(2t)$. The multiplicative deception attack signals
$\kappa(t, t_\kappa) = \mathrm{diag}(0.01 + 0.01\sin(20t),\ 0.01 + 0.3\sin(30t),\ 0.01 + 0.03\sin(20t))$, the additive deception attack signals $\chi(t, t_\chi) = [\chi_{\mathrm{roll}}, \chi_\theta, \chi_\psi]^T = [50.0sin(4t), 60.0sin(3t), 70.0sin(3t)]^T$, there are the multiplicative deception attack and the additive deception attack on the controller when $t_\kappa = 7.0s$ and $t_\chi = 3.0s$, respectively. The reference signal is selected as $y_r = [(0.3 + 0.3\tanh(2.0t))\sin t, (0.3 + 0.3\tanh(2.0t))\cos t, (0.3 + 0.3\tanh(2.0t))0.8\sin t]^T$. The output should be constrained in $-2 - \sin(2t) < \xi_{1_k} < 1 + 0.1\sin(4t)$ after $t = 10$s. The design parameters are selected as $m_1 = 3.0$, $m_2 = 2.0$.

Case I: The output restrictions are not obeyed at the initial time instant. The simulation results are presented in Figures 3–8.

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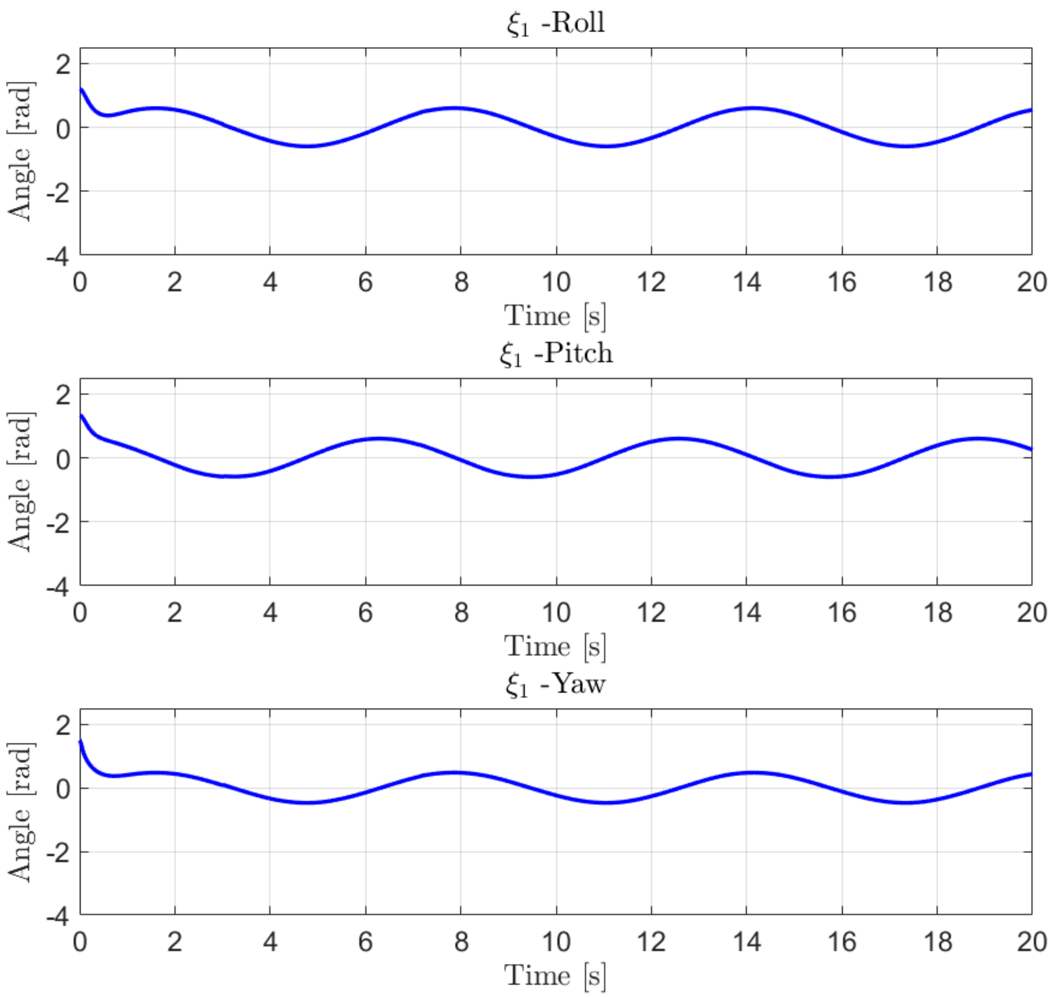

Figure 3. The trajectory of ξ_1 of case I

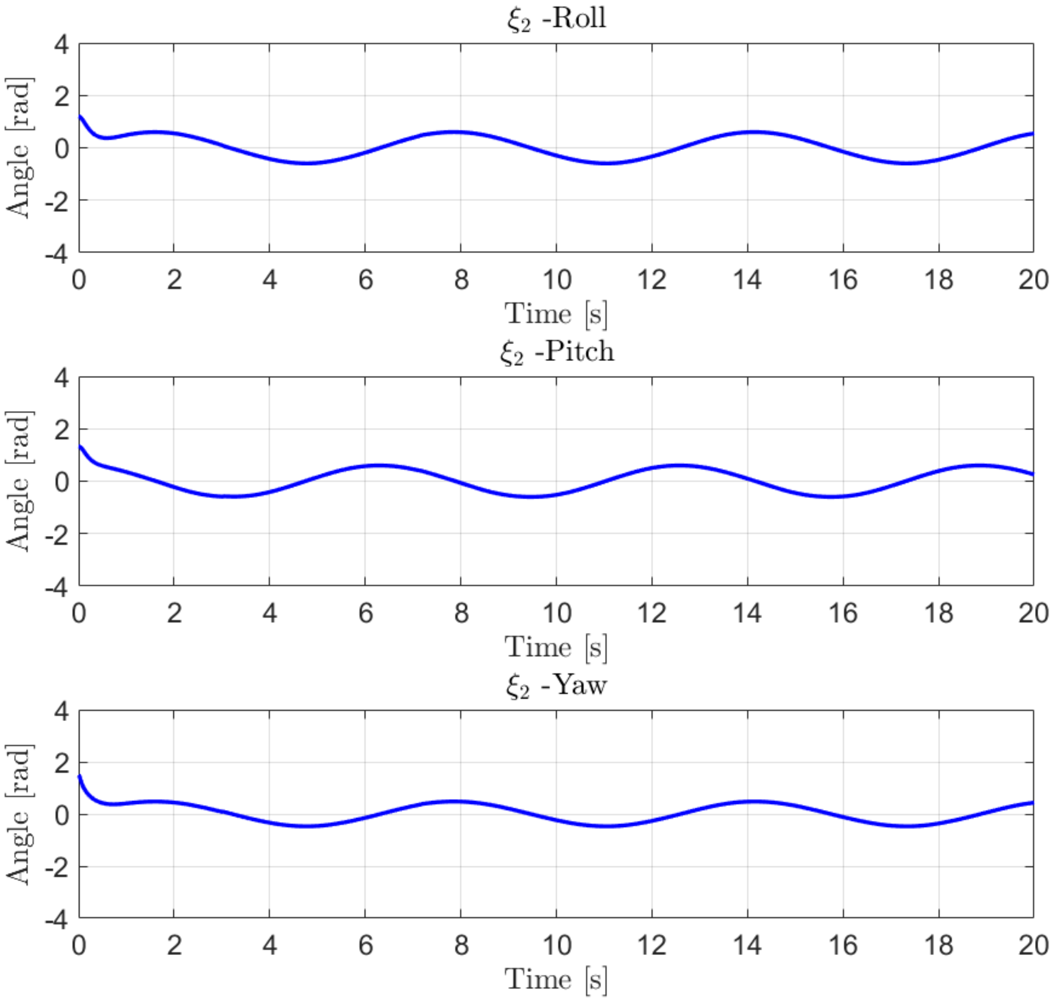

Figure 4. The trajectory of ξ_2 of case I

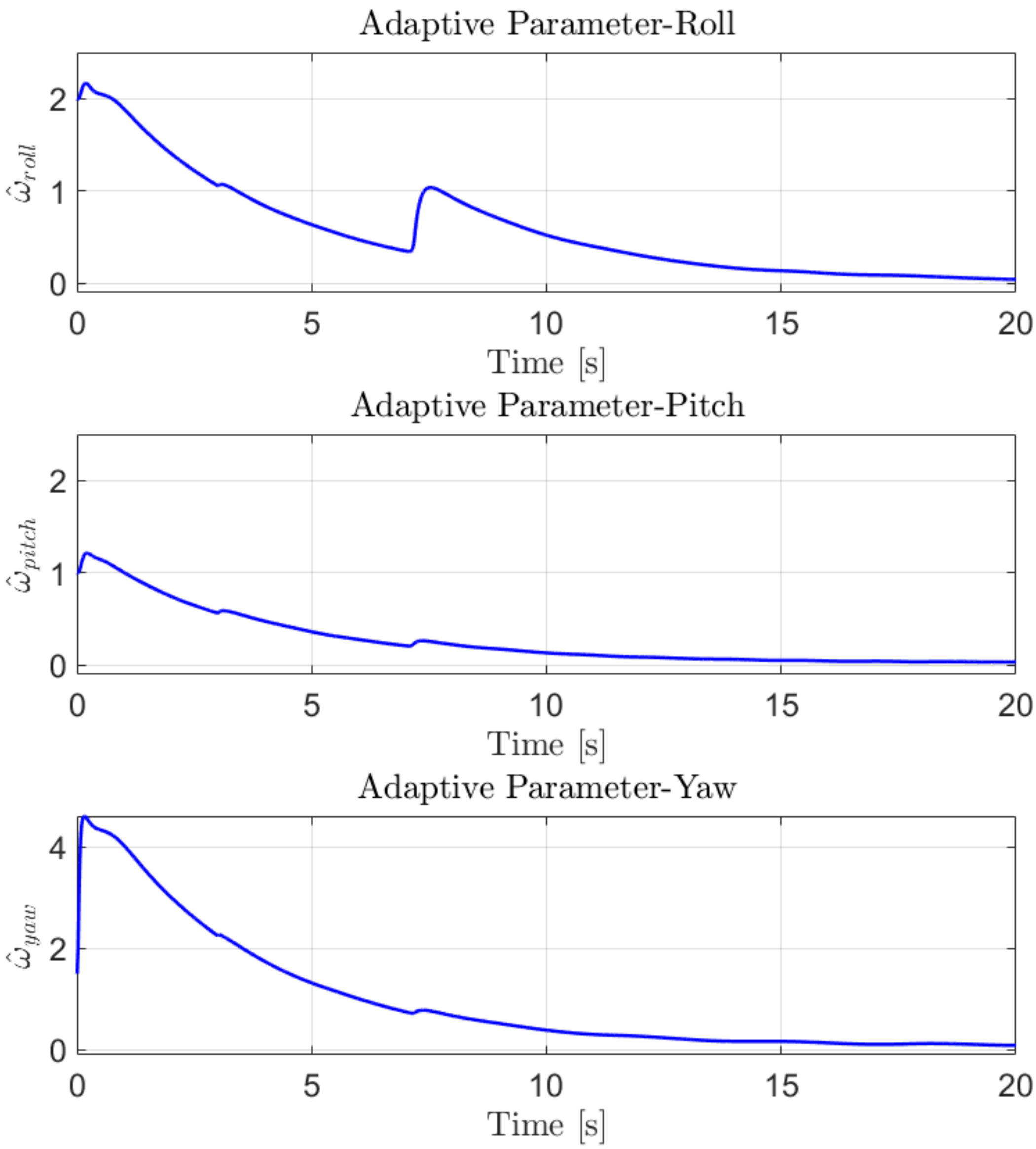

Figure 5. The trajectory error of case I

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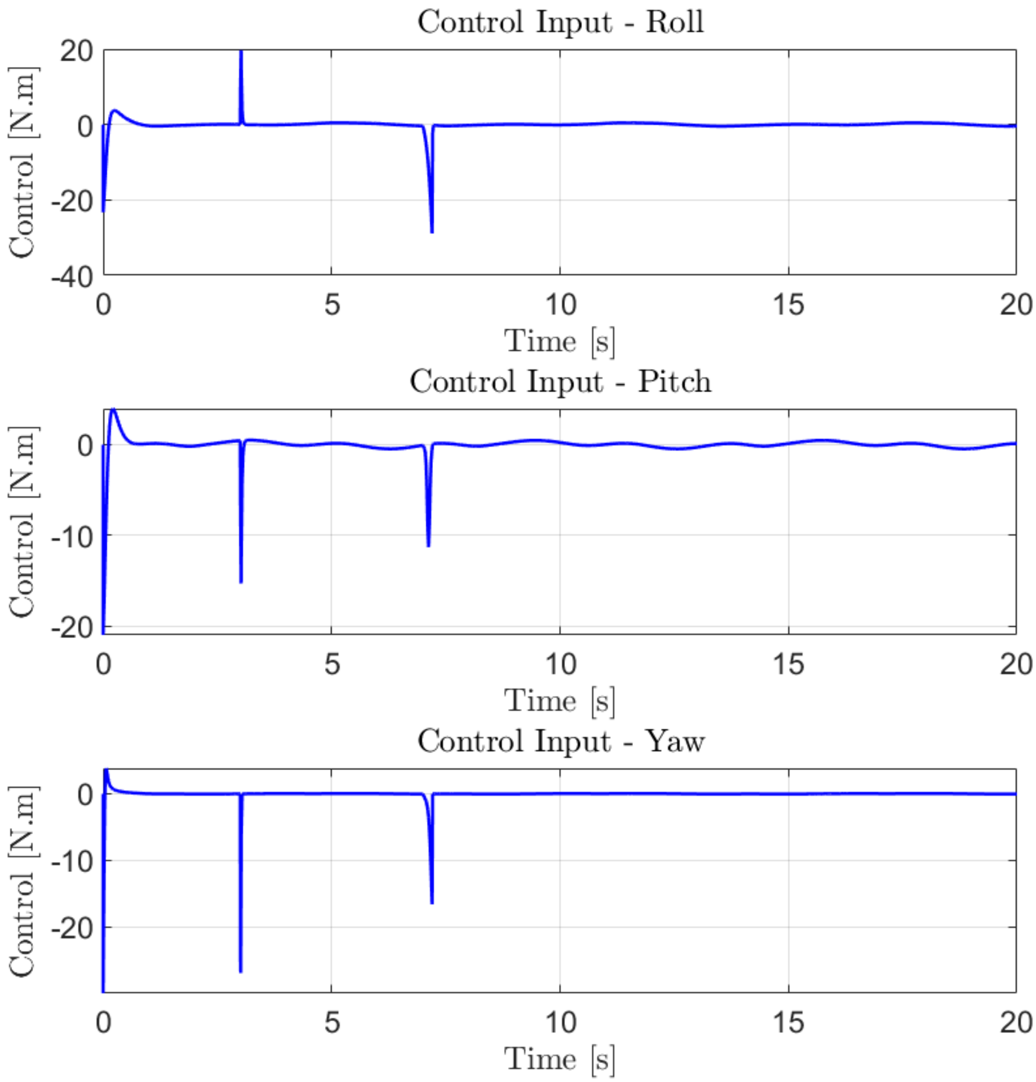

Figure 6. The control input of case I

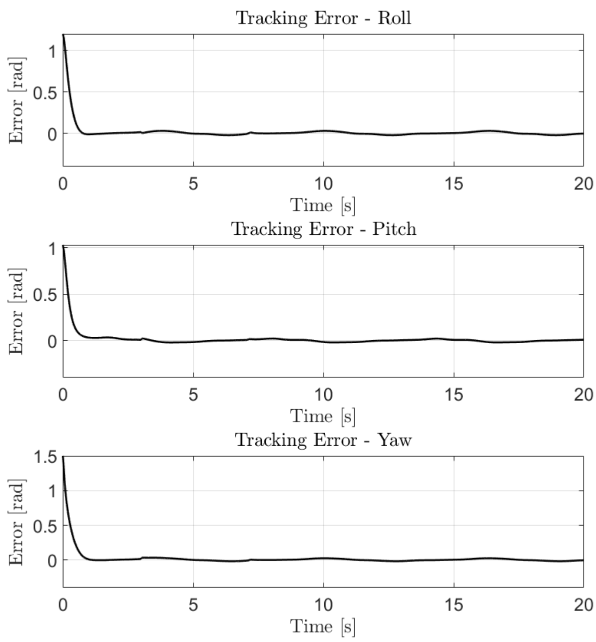

Figure 7. Case I: the evolution of tracking error β_1.

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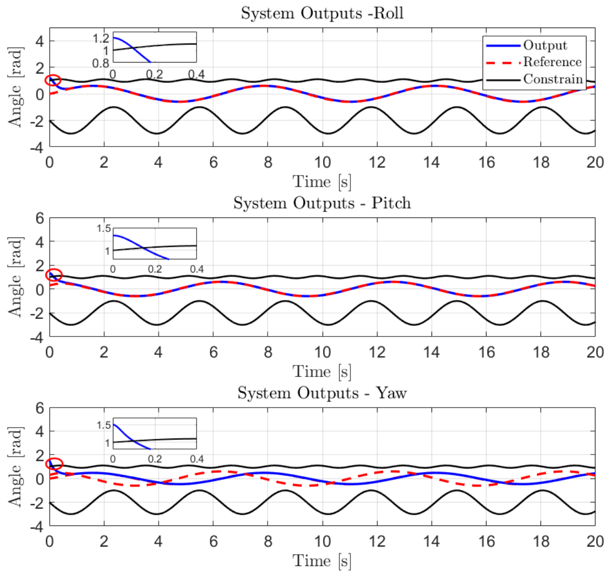

Figure 8. The output y and reference output y_r of case I

Case II: The output conform to the constraint conditions initially and are free of restraint for $t \in [0, t_s)$. When $t \geq t_s$, the output obey the corresponding constraints. The simulation results are presented in Figures 9–14.

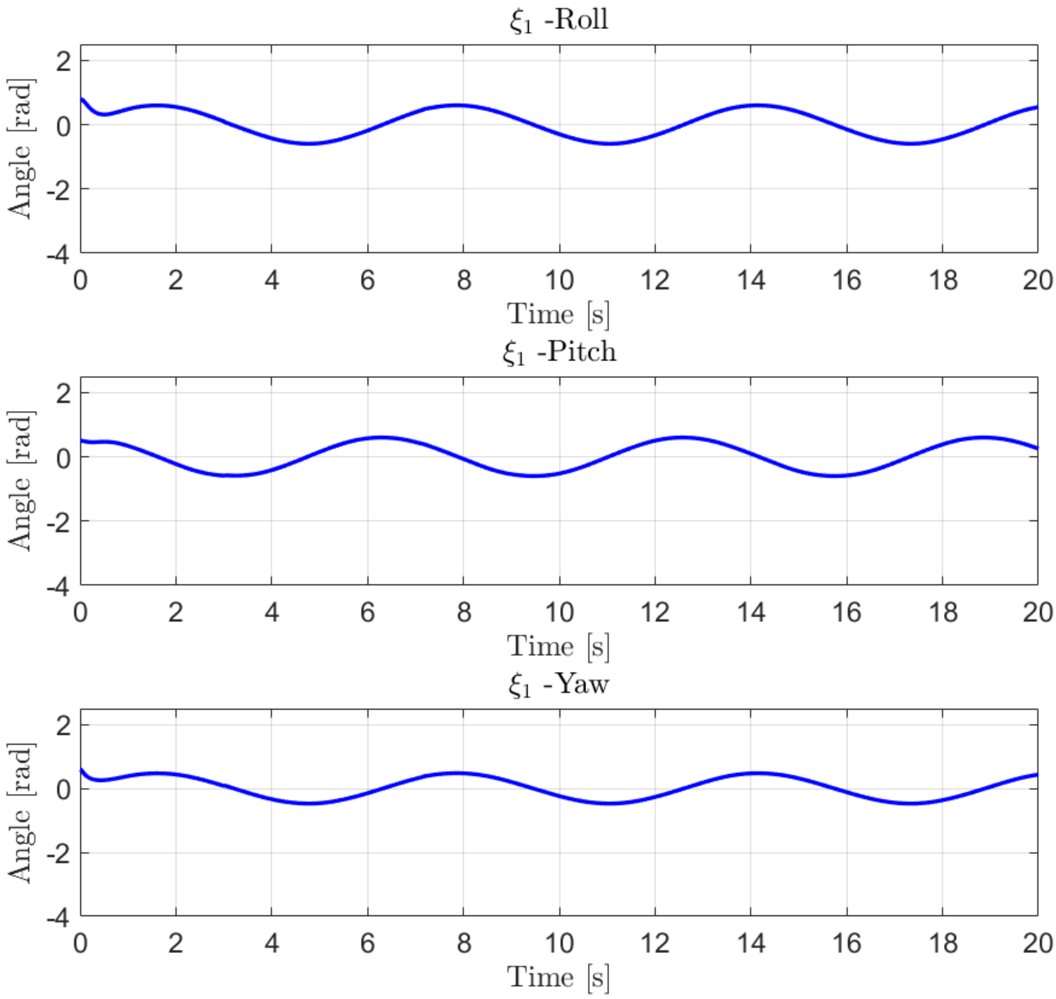

Figure 9. The trajectory of ξ_1 of case II

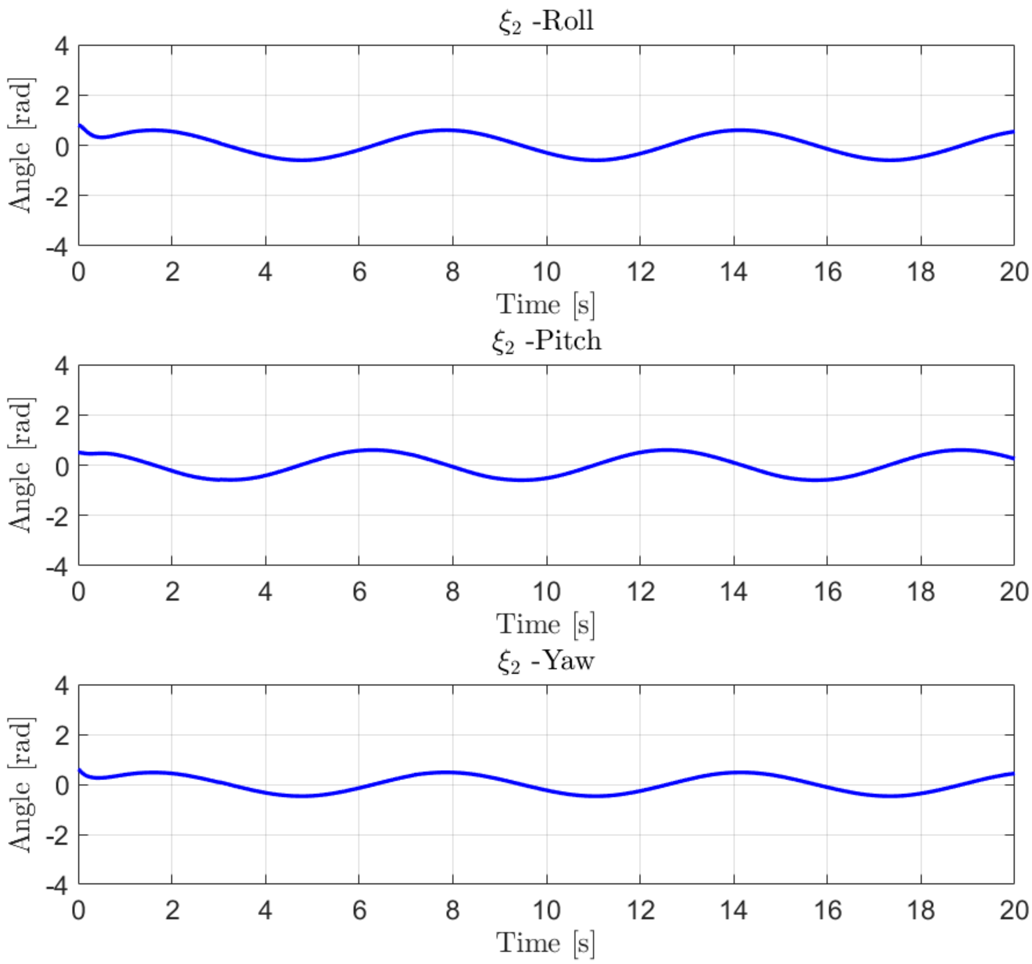

Figure 10. The trajectory of ξ_2 of case II

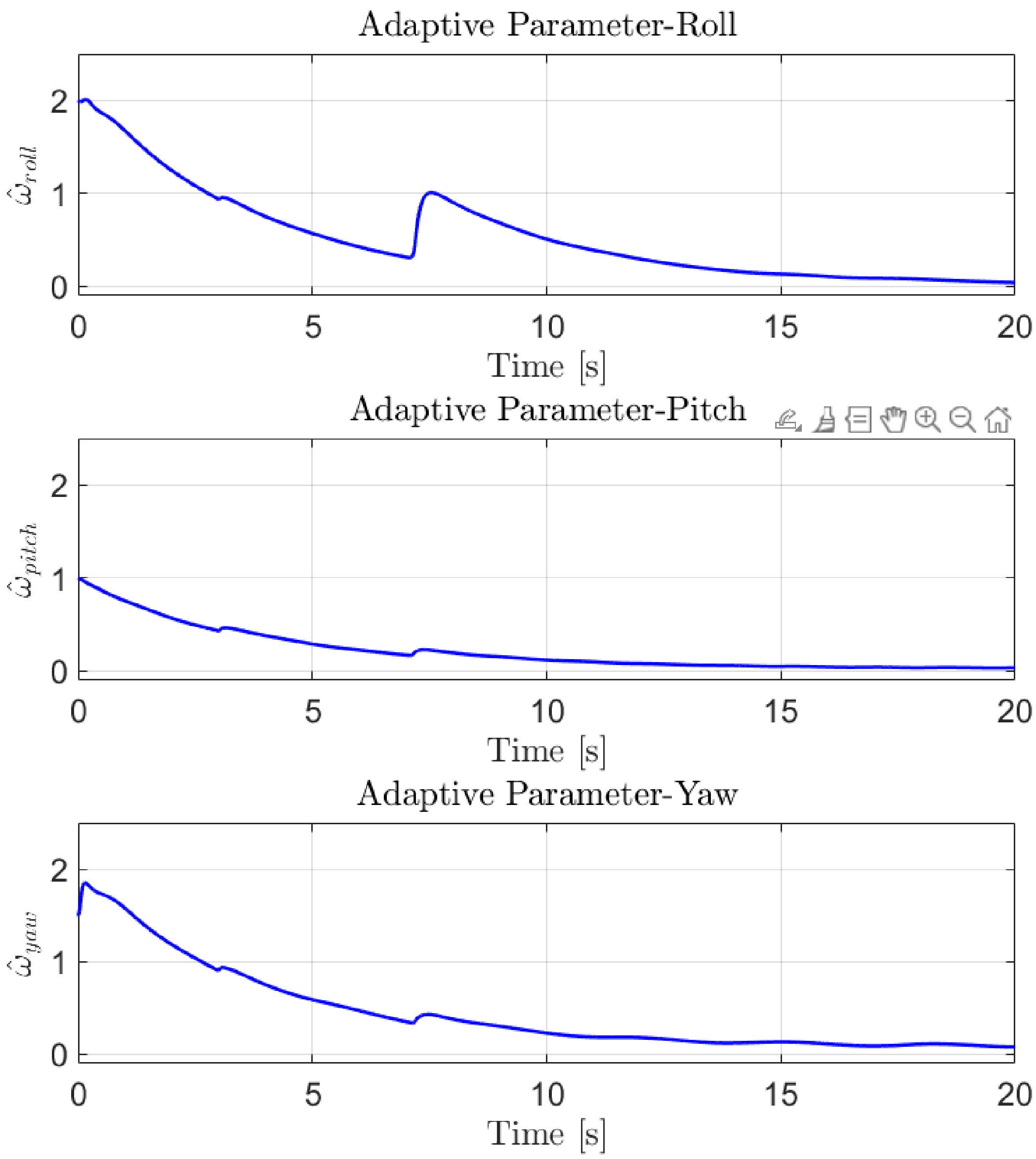

Figure 11. The trajectory error of case II

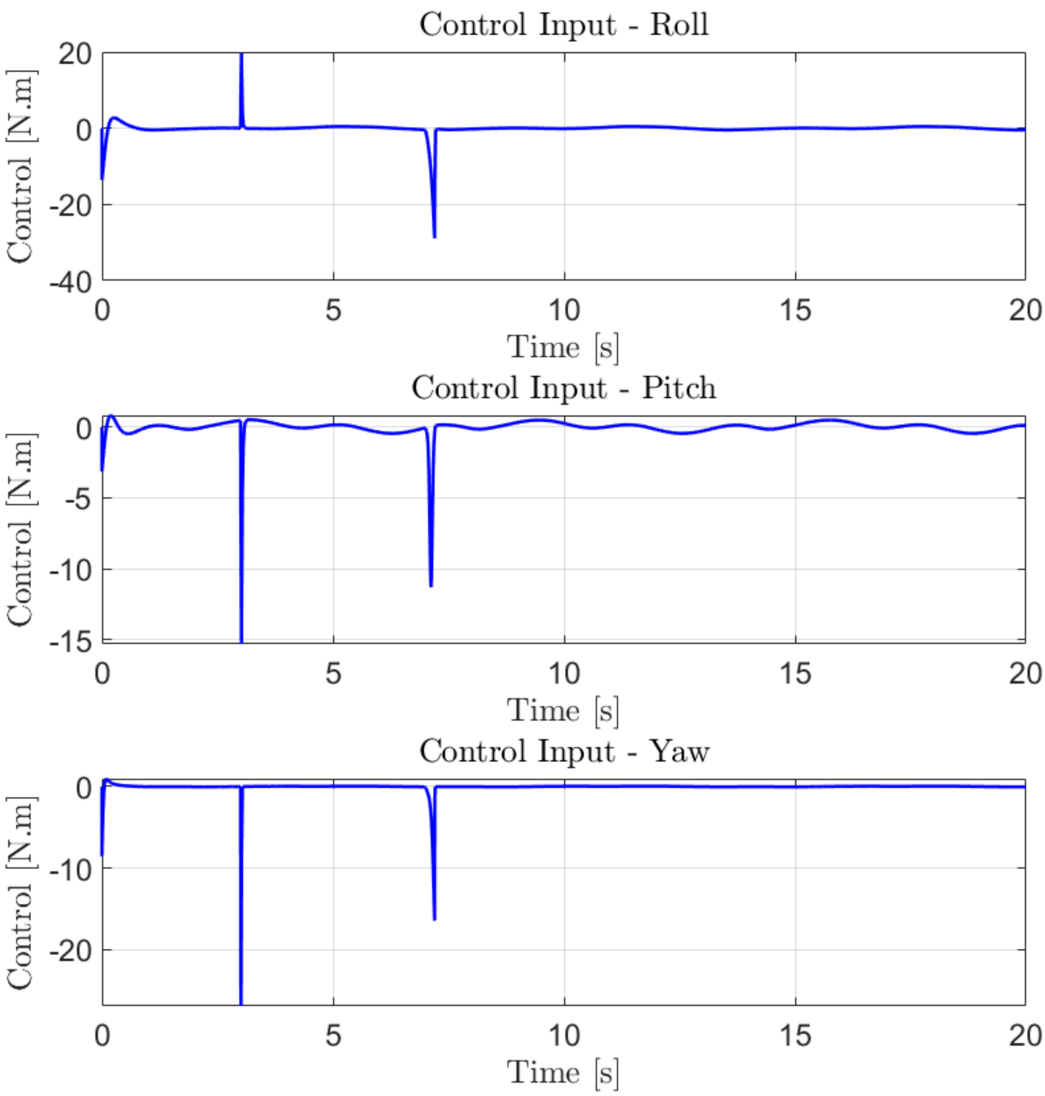

Figure 12. The control input of case II

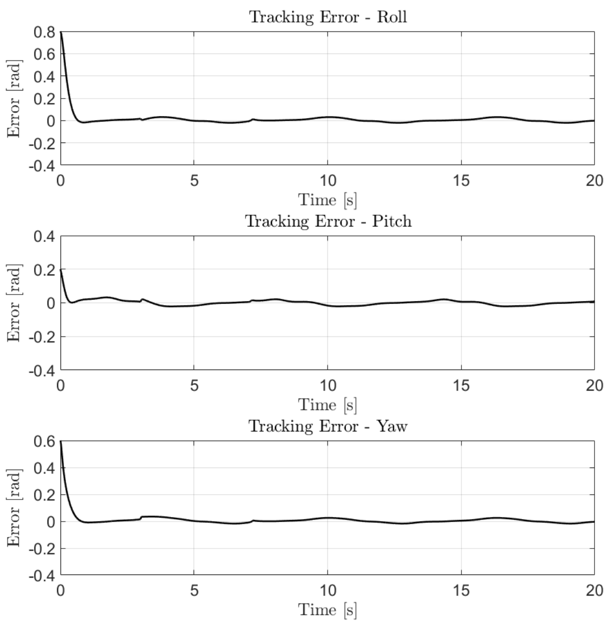

Figure 13. Case II: the evolution of tracking error β_1.

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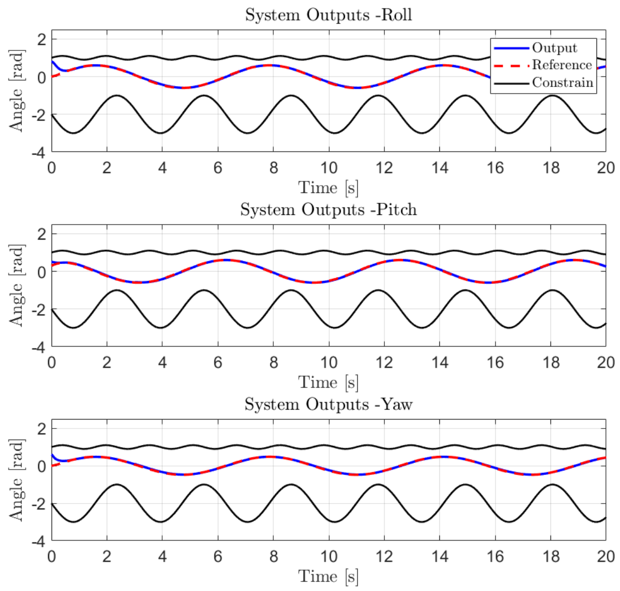

Figure 14. The output y and reference output y_r of case II

Figs. 3 and Figs. 4 describe the profile of the state ξ_1, ξ_2 under case I. Figs. 9 and Figs. 10 describe the profile of the state ξ_1, ξ_2 under case II. It can be seen that all signals in the closed-loop systems are UUB. The control input and the adaptive laws $\hat{\omega}$ are shown in Figs. 5 and 6 under case I. The control input and the adaptive laws $\hat{\omega}$ are shown in Figs. 11 and 12 under case II. It is observe that the adaptive and controller have a good performance under the multiplicative attack at 7s and additive attack at 3s. Figure 7 Figs. 13 illustrate the profile of tracking error under case I and under II. Figs. 8 and Figs. 14 present the trajectories of y and y_r. Through the above simulation results, we can observe that the good tracking performance can be derived and the system output y satisfies its restriction condition just after the preset time ts = 10s whether the initial constraint conditions are observed or not.

5. Conclusions

In this work, a unified output feedback control method is proposed for strict-feedback nonlinear systems subject to delay asymmetric output constraints, deception attacks and disturbances. Specifically, by introducing a novel error shifting function and designing a novel barrier function, the issue of delay asymmetric output constraints is effectively addressed, and the designed controller is applicable regardless of the presence or absence of delay asymmetric constraints, without the need for changes or switching. A key advantage lies in that the method does not require knowledge of the maximum and minimum values of constraint functions, which effectively lowers the threshold for algorithm design and brings convenience to practical implementation. Eventually, the effectiveness of the proposed control scheme have been illustrated by a unmanned aerial vehicle application example.

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