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Posted Date: 25 February 2026

doi: 10.20944/preprints202601.0650.v3

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Article

Measuring Velocity Using Moving Clocks—The Surprising Test of Tangherlini’s Theory

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Abstract

Motivated by the work of Matsas et al. (2024), which demonstrated that time can serve as the fundamental unit for physical quantities—removing the need for traditional length–mass–time (LMT) dimensions—this research evaluates the operational resolution of velocity within relativistic frameworks. Utilising a Lorentz transformation matrix approach, we first validate the Matsas three-clock protocol, confirming the derivation of distance as a function of three proper clock times in Minkowski spacetime and uncovering two novel velocity expressions derived solely from these temporal intervals. The investigation was extended to Tangherlini’s 4D spacetime framework (1958) to test the hypothesis that absolute velocity could be resolved through subluminal signalling. While the initial three-clock scenario resulted in the systematic cancellation of the base system’s absolute velocity, a breakthrough was achieved by applying the relativistic Doppler effect within the Tangherlini metric. This approach effectively circumvents the mathematical cancellations prevalent in standard relativistic “null” experiments. The findings reveal that the Tangherlini and Minkowski frameworks are closely connected; the former serves as a necessary complement to the special theory of relativity (STR) rather than an antagonist. This theoretical advancement suggests a plausible methodology for the measurement of absolute velocity without the requirement of instantaneous signals. By resolving the longitudinal Doppler shift within a preferred-frame geometry, this research provides a fresh impetus for the historical debate on absolute motion initiated by Poincaré and Einstein.

Keywords: absolute rest; absolute velocity; Tangherlini transformation; postulate of relativity; physical units; relativistic Doppler effect

1. Introduction

In a 2024 study, Matsas et al. [1] introduced a compelling conceptual framework in which time serves as the sole fundamental unit for all physical quantities, effectively superseding the traditional length–mass–time (LMT) dimensional system. A particularly significant implication of this framework is the ability to measure spatial distance using only three inertial clocks. This is made possible by employing the Unruh protocol¹. This unusual measurement can be achieved via a round-trip configuration involving one stationary clock C_3 and two relatively moving clocks C_1 and C_2 , as illustrated in Figure 1. While such a result is unattainable within a Galilean coordinate system, it becomes viable in Minkowski spacetime. This is due to the reduction in degrees of freedom caused by the implementation of Einstein’s light-speed isotropy postulate. The distance-measurement formula derived in [1] is based on the worldlines of three inertial clocks forming a triangle in a Minkowski diagram and is expressed as follows:

$$D = \frac{\sqrt{[(\tau_3^2 - \tau_1^2 - \tau_2^2)^2 - 4\tau_1^2\tau_2^2]}}{2\tau_3} \tag{1}$$

¹ Bill Unruh, undisclosed private communication, according to [1].

where τ_3 is the C_3 clock time at the stationary system origin, and τ_1 and τ_2 are the respective trip durations of moving clocks (C_1 and C_2) at unspecified velocities. This formula can be expanded to the following equivalent expression:

$$D = \frac{\sqrt{\tau_1^4 - 2\tau_1^2\tau_2^2 - 2\tau_1^2\tau_3^2 + \tau_2^4 - 2\tau_2^2\tau_3^2 + \tau_3^4}}{2\tau_3} \quad (2)$$

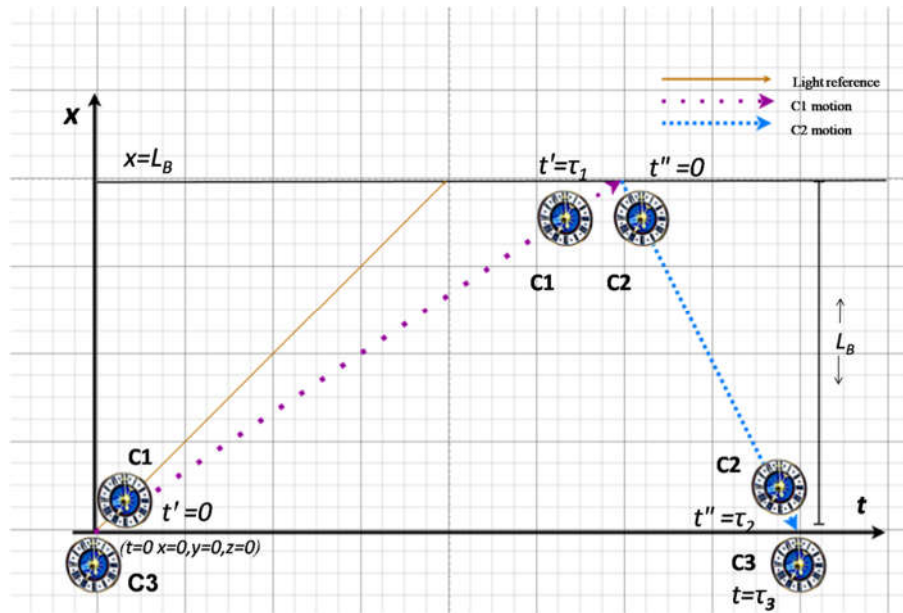


Figure 1. The three-clock scenario (angles not to scale).

1.1. Experimental Protocol

In this paper, we employ a motion protocol that differs slightly from the original arrangement in [1] but remains physically equivalent.

1. **Reference Frame:** The stationary laboratory frame is designated as system B (base platform).
2. **Clock C_1 Initiation:** Clock C_1 approaches from the negative x -axis at a constant, unknown velocity v_1 . Upon passing the origin, it is synchronised to $t' = 0$.
3. **Base Synchronisation:** The stationary clock C_3 is simultaneously reset to $t = 0$ at the moment of C_1 's departure from the origin.
4. **Clock C_2 Worldline:** Clock C_2 is launched from a distant point on the positive x -axis at velocity v_2 on a reciprocal heading towards the origin. To ensure a continuous trajectory, a negligible y -axis offset is assumed. Upon its encounter with C_1 at unknown position $x = L_B$, C_2 is synchronised to $t'' = 0$.
5. **Procedural Independence:** While this round-trip arrangement aligns with the triangular geometry in Minkowski spacetime used in [1], the individual legs of the trip can be executed sequentially in practice, as the recorded durations are independent of the specific epoch of initiation. The delay δ between the C_1 arrival and C_2 arrival can be measured by an additional unsynchronised clock, similar to C_3 , located at $x = \tau_2$. This increases the accumulated total duration on clock C_3 . In this case, the distance L_B must be fixed rather than emerging dynamically from the collision at random time. For derivation clarity, we assume that $\delta=0$ to maintain consistency with reference [1].
6. **Data Transmission C_1 :** At the point of encounter with C_2 , clock C_1 transmits its elapsed proper time τ_1 to the origin of B to support future calculations, while C_2 resets the time t'' to 0.
7. **Data Transmission C_2 :** Upon reaching the origin, clock C_2 communicates its recorded duration τ_2 to the base to support future calculations.

8. **Final Measurement:** The stationary clock C_3 records the total round-trip time τ_3 , which represents the sum of the durations for clock C_1 (τ_1) to reach the unknown distance L_B and for clock C_2 (τ_2) to reach the origin.

Through this protocol, we demonstrate that the acquired data τ_1, τ_2 , and τ_3 appear sufficient to resolve not only the unknown distance L_B but also the previously undetermined velocities v_1 and v_2 .

2. Distance and Velocity Calculations in Minkowski Spacetime

The following derivations implicitly assume the idealisations and simplifications of the physical reality typical of the STR [2]. The preferred method for these derivations is linear algebra, utilising the Lorentz transformation (LT) matrix defined as follows:

$$\Lambda_v = \begin{bmatrix} \gamma_v & -v\frac{\gamma_v}{c} & 0 & 0 \\ -v\frac{\gamma_v}{c} & \gamma_v & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

$$\gamma_v = 1/\sqrt{1 - v^2/c^2}$$

We assume that a stationary inertial base system B , is represented by a 4D Cartesian coordinate system. Within this frame, we analyse the motion of two inertial point masses, PM_1 and PM_2 , represented by clocks C_1 and C_2 , moving along the x -axis. The 4-vector XL_B represents an unknown, reference distance:

$$XL_B = \begin{bmatrix} ct \\ L_B \\ 0 \\ 0 \end{bmatrix}. \quad (4)$$

The transformation of the fixed XL_B in B to the local C_1 coordinates using the LT matrix Λ_{v1} multiplied by XL_B yields the moving point X_{LB1} approaching C_1 , which is in the origin of the inertial system designated as $B1$:

$$\Lambda_{v1} \begin{bmatrix} ct \\ L_B \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} c^2t - L_Bv_1 \\ \sqrt{c^2 - v_1^2} \\ L_B - v_1t \\ \sqrt{c^2 - v_1^2} \\ 0 \\ 0 \end{bmatrix} \equiv X_{LB1}. \quad (5)$$

The transformed vector X_{L1} is initially expressed in terms of the base system time t and must be converted to the local proper time t' of C_1 :

$$X_{LB1}[1]/c = t' \Rightarrow t = \frac{\sqrt{c^2 - v_1^2}ct + L_Bv_1}{c^2}. \quad (6)$$

The converted vector in the local primed coordinates is obtained by the substitution of the t expression from Equation (6) and subsequent simplifications:

$$X'_{LB1} = \begin{bmatrix} ct' \\ -c\sqrt{c^2 - v_1^2}v_1t' + L_Bc^2 - L_Bv_1^2 \\ c\sqrt{c^2 - v_1^2} \\ 0 \\ 0 \end{bmatrix}. \quad (7)$$

The time it takes for L_B to come in contact with the C_1 origin can be found when $X'_{LB1}[1] = 0$, which is calculated from the following equation:

$$-c\sqrt{c^2 - v_1^2}v_1t' + L_Bc^2 - L_Bv_1^2 = 0 \Rightarrow t' = \frac{L_B\sqrt{c^2 - v_1^2}}{v_1c} \equiv \tau_1, \quad (8)$$

where τ_1 is the proper time of clock C₁.

C₂ differs from C₁ only in terms of the magnitude of the velocity and sign; hence, it can be deduced from the Minkowski diagram and Figure 1. Then, by analogy,

$$c\sqrt{c^2 - v_2^2}v_2t'' + L_Bc^2 - L_Bv_2^2 = 0 \Rightarrow t'' = \frac{L_B\sqrt{c^2 - v_2^2}}{-v_2c} \equiv \tau_2. \quad (9)$$

In clock C₃, the duration of the round trip is the sum of the successive durations on each segment given by the following expression:

$$\tau_3 \equiv \frac{L_B}{v_1} + \frac{L_B}{-v_2}. \quad (10)$$

The minus sign in the denominator makes the C₂ clock duration positive.

Given that the clock times were measured, we have three equations and three unknowns: L_B , v_1 , and v_2 , as follows:

$$\begin{cases} \tau_1 = \frac{L_B\sqrt{c^2 - v_1^2}}{v_1c} \\ \tau_2 = \frac{L_B\sqrt{c^2 - v_2^2}}{-v_2c} \\ \tau_3 = \frac{L_B}{v_1} + \frac{L_B}{-v_2} \end{cases} \quad (11)$$

The solution of the nonlinear system of equations (11) with respect to L_B , v_1 , and v_2 is as follows:

$$\begin{cases} L_B = \frac{\pm c\sqrt{\tau_1^4 - 2\tau_1^2\tau_2^2 - 2\tau_1^2\tau_3^2 + \tau_2^4 - 2\tau_2^2\tau_3^2 + \tau_3^4}}{2\tau_3} \\ v_1 = \frac{\pm c\sqrt{\tau_1^4 - 2\tau_1^2\tau_2^2 - 2\tau_1^2\tau_3^2 + \tau_2^4 - 2\tau_2^2\tau_3^2 + \tau_3^4}}{\tau_1^2 - \tau_2^2 + \tau_3^2} \\ v_2 = \frac{\pm c\sqrt{\tau_1^4 - 2\tau_1^2\tau_2^2 - 2\tau_1^2\tau_3^2 + \tau_2^4 - 2\tau_2^2\tau_3^2 + \tau_3^4}}{\tau_1^2 - \tau_2^2 - \tau_3^2} \end{cases} \quad (12)$$

The expression for L_B in (12) is valid only for positive values. Although it appears algebraically distinct from the original Equation (1), it is equivalent to the expanded original form in Equation (2). Naturally, the constant speed of light c is absent in (2), because it was set to the dimensionless value of one in [1], in accordance with the Minkowski diagram convention.

While we reject negative L_B (unless it represents a coordinate on the negative side of x'), we need to decide which velocity variant to choose. In this scenario, all the roots not preceded by a minus sign are consistent. Another feature of this system should be noted. We can calculate the travel times of clocks C₁ and C₂ in B to $= L_B$ and $x = 0$, respectively.

$$\begin{aligned} \tau_{C1} &= \frac{L_B}{v_1} = \frac{\tau_1^2 - \tau_2^2 + \tau_3^2}{2\tau_3} \\ \tau_{C2} &= \frac{L_B}{v_2} = \frac{\tau_3^2 + \tau_2^2 - \tau_1^2}{2\tau_3} \end{aligned} \quad (13)$$

2.1. Conceptual Implications

The ability to measure L_B using clocks alone is a noteworthy discovery of Unruh and Matsas et al. [1]; however, the capacity to obtain velocities without using presynchronised clocks is arguably more significant. Conventionally, velocity measurement requires two distant, synchronised clocks by definition: $v = dx/dt$ (with some exceptions, such as Doppler methods or dual light-pulse round-trip measurements). Surprisingly, no explicit distance is necessary for the velocity expressions in Equation (12), as it is entirely factored into the temporal parameter arrangement. The three clocks,

moving relative to one another, indicate their proper times without regard for any specific coordinate system; clocks possess no inherent knowledge of other sensors or reference frames. Once we prove Equation (12), the result in Equation (13) is not unexpected. However, this simple and beautiful formula adds to the remarkable conclusion that three convention-independent invariant clock proper times can practically determine L_B, v_1, v_2, τ_{C1} , and τ_{C2} without any physical implementation of a coordinate system or pairwise synchronised clocks and, at most, by designating a distant point as L_B without actually measuring it. It may be a premature conclusion that Equation (13) shows nature's preference for a particular one-way velocity of light isotropy convention because τ_{C1} was measured at $x = L_B$ and all the equations leading to Equation (13) were derived using the STR framework. There are additional interesting consequences of Equation (13), which will be discussed in Section 0.

These findings suggest that the three-clock-based measurement result is a natural consequence of the spacetime geometry. This fact prompted an investigation into the possible implications of the three-clock scenario within Tangherlini spacetime, introduced in the 1958 doctoral thesis at Stanford University [3]. While the mathematical difference between the LT and the corresponding Tangherlini transformation (TT) matrices is subtle, the physical ramifications are vast. A similar scenario to that shown in Figure 1 can be applied by employing a hypothetical absolute rest frame concept and three moving clock frames in relative motion.

3. Methods

3.1. Background

Sections 1 and 2 presented a validation of the distance formula in Equation (1) using the widely accepted Lorentz Transformation (LT) matrix for relativistic kinematics. While the validation of distance was expected, the emergence of Tangherlini's velocity formulae in Equation (12) presented a distinct challenge. We recognised that the methodology must be adequate to the specific context in which Tangherlini's Theory (TT) is perceived. TT remains outside the mainstream; indeed, rejection is often declared *a priori*. Will [4] (pp 325,326), referencing the Mansouri-Sexl [5] generalisation of coordinates transformation, remarked that adopting the infinite signal speed required for absolute clock synchronisation is a "perverse choice." Such terminology can stifle objective scientific discussion. However, Tangherlini methodically derived the transformation from the Einstein field equations and he was fully aware of the instantaneous signal problem, expressing hope for absolute velocity detection via subluminal signals or rotational motion. Unlike earlier attempts—such as Eagle [6], requiring electrically controlled vibrating quartz rods, or Mansouri-Sexl [5], whose derivation appears to stem from a misinterpretation of LT time coordinates [5] (p 301 eq. 3.4)—the method adopted here seeks to ground the discussion in objective, theoretical, and potentially measurable evidence.

3.2. The Adopted Approach

The following rules characterise our approach:

1. We adopt the standard idealisations of Special Relativity (SR) [2], including the Einsteinian definition of inertial systems where Newtonian mechanics holds, and the use of ideal rigid rods within a Euclidean geometric framework.
2. We assume that TT applied to vectors results in physically valid quantities unless a contradiction is proven, granting TT the same initial mathematical confidence as LT.
3. Confidence in TT is further bolstered by the fact that, beyond Tangherlini's General Relativity (GR) approach, these transformations can be derived from the fundamental postulates presented in Appendix A.
4. To maintain a clear focus on the correspondence (or lack thereof) between these theories, we intentionally exclude Quantum Theory or complex GR metrics. Following the Rule of Analysis of Descartes in *Discourse on Method* [7] (p.35), we "divide each difficulty into as many parts as required" to achieve clarity.

5. Following Rule 2, transformed physical quantities are deemed potentially measurable. The failure to determine absolute velocity excludes experimental confirmation otherwise theory must first be deemed theoretically sound and worthy of experimental effort. Similarly to LT approach we implicitly assume the idealisations and simplifications of the physical reality typical of the STR [2] such as adopting Einstein definition of inertial systems where Newtonian laws of mechanics hold [2] (p38) and foundation of coordinate systems grid is an ideal rigid rod standard and Euclidean Geometry [2] (p38)

3.3. Notation Convention

In the following sections we will be dealing with four coordinate systems: *A* representing the absolute reference frame (ARF), *B* for the base system and *B1*, *B2* for clocks C1 and C2 respectively, which we recognise from the LT-based scenario. The notation used in this paper designates the last one or two symbols in relevant variable's suffix to determine in which frame the quantity is observed/measured, while one or two preceding characters may define the frame to which the quantity belongs; hence, v_{BA} denotes the velocity of system *B* as measured in *A*. Similarly, for the C2 clock's frame *B2*, we have v_{B2A} , which is the velocity of system *B2* in *A*. This allows clear distinction between similar quantities in different frames. Transformed vector symbols change suffix sequence accordingly however they often appear with one or more subscripts

4. Three Clocks in Tangherlini Spacetime

The derivation of the transformations by Tangherlini provided an analytical relativistic framework similar to STR [2]. Originally named the "absolute Lorentz transformation" (ALT), the TT was derived from the Einstein field equation in the absence of gravitational sources. We found that the same transformation can be derived from first principles based on fundamental postulates, including the assumption of an absolute rest frame (ARF), as well as the experimentally established isotropy of the round-trip average speed of light and the controversial invariance of the instantaneous signal hypothesis (see Appendix A for the exact formulation of the postulates). No relativistic effects were assumed *a priori*; time dilation and length contraction emerged naturally.

Unlike the STR convention, where absolute velocity is outside the scope of measurement, we treat the ARF as a reference inertial system where *t* is the absolute time variable; however, it is immeasurable. Thus far, according to the present consensus, absolute velocity suggestions appear fallacious, as once asserted by Eddington [8]. On the other hand, Tangherlini attempted to reason about the possibility of detecting absolute motion on the basis of the presented theory. First, he noted that if two distant clocks are not absolutely synchronised, it is not possible to calculate the one-way relative velocity of anything, because there is no way of correlating the time of arrival in terms of the time of departure [3] (p48). In Chapter 6 of [3] (p73–74), a claim is made that using subluminal signals, it would be possible to detect the absolute motion of the Earth. However, despite the focused, detailed analysis, no closed-form explicit solution or the exact details of the measurement method demonstrating this possibility are provided. Additionally, in the final chapter of [3] (p101), Tangherlini concluded that in the examples presented in the doctoral thesis, absolute velocity always cancels out when measurements are performed "*in the usual manner*". We assume that these methods do not depend on the prior absolute synchronisation of separated clocks, which is impossible without instantaneous signals.

After determining that velocities can be measured with only three clocks, as shown in Equation (12), the question emerged whether this method was sufficiently 'unusual' to prove absolute velocity. The Unruh three-clock protocol [1] requires only three measurable proper times, and no two distant clocks appear to be synchronised other than by coincidence of their positions in a predefined location. We attempted the proof on the basis of a methodology similar to that in Section 0, but with the TT matrix, which is represented as follows:

$$\Omega_{\infty}^v = \begin{bmatrix} 1/\gamma_v & 0 & 0 & 0 \\ -v\frac{\gamma_v}{c} & \gamma_v & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (14)$$

where $\gamma_v = 1/\sqrt{1 - v/c^2}$.

The infinity subscript in the TT matrix symbol emphasises the role of the instantaneous signal postulate. Matrix (14) differs from the LT matrix (3) because of the absence of the space-dependent time coordinate, which is now zero. While the impossibility of absolute synchronisation with infinitely fast signals appears to be a fundamental obstacle, the significance of the TT framework would be profound if such obstacles could be circumvented.

4.1. Three-Clock Thought Experiment in Tangherlini 4D Spacetime

The graphical representation of the scenario differs slightly from that of the previous case in terms of the symbols of the axes, as shown in Figure 2.

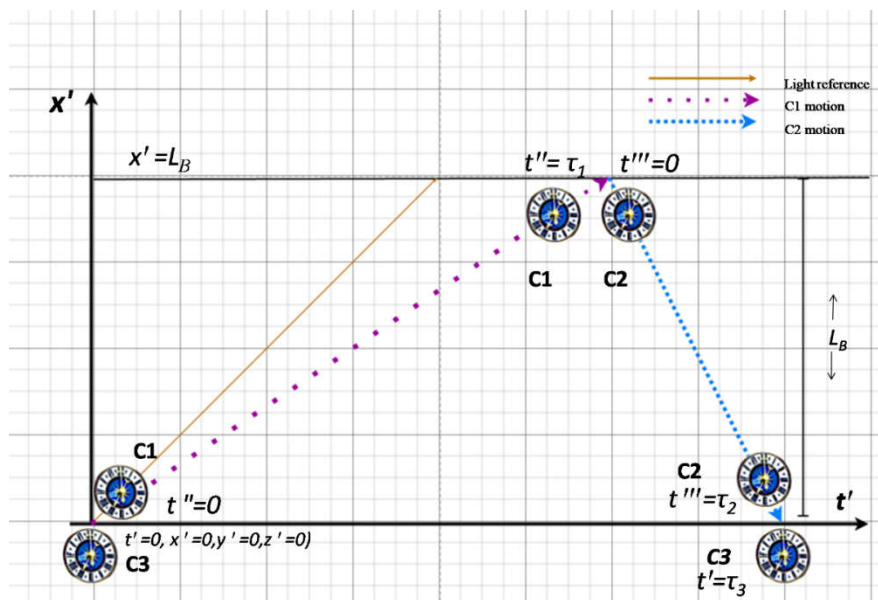


Figure 2. Two-clock round-trip scenario in system *B* (angles not to scale).

This shows the perspective of the base system denoted by *B* with the system clock *C*₃, which is an inertial moving system with respect to the hypothetical ARF denoted by *A*, which thus far, according to the present consensus, cannot be identified. However, it is treated here as a special purpose inertial system with time variable *t* where no measurement can be made because no reference points are known in empty space.

In partial agreement with the objections of Poincaré [9] regarding the absolute space coordinate axes², we instead consider the absolute rest state to be a unique property of the subclass of inertial systems out of the class of all inertial systems rather than the state of the ‘void’. Our position disagrees with Newton’s concept of absolute space, which “remains always similar and immovable.” [10], but aligns with Einstein’s remark on the ether: “the idea of motion may not be applied to it” [11].

² This idea came from Poincaré claiming that “absolute space is nonsense, and it is necessary for us to begin by referring space to a system of axes invariably bound to our body (which we must always suppose put back in the initial attitude).” With no features in space, the only way to bootstrap the derivation is to assume that at time *t*=0, some abstract coordinate system in *A* is momentarily aligned with the moving one and stays where it was as the distance increases. Absolute space then has no role other than being a passive container for inertial systems as far as this simple linear algebra model is concerned.

The inaccessibility of the featureless absolute space to measurement and the same with respect to any inertial absolute system A can be overcome by using the inverse transformation (TT^{-1}) from any inertial system where times and lengths are measurable and can formally be related to A on the basis of the presented theory. At present, absolute velocity remains hypothetical until such time as its measurability is proved.

4.2. Derivation and Mathematical Reconciliation

In system B , we designate a fixed distant point XL_B as a 4-vector at which the worldline of C_1 ends and that of C_2 begins:

$$XL_B = \begin{bmatrix} ct' \\ L_B \\ 0 \\ 0 \end{bmatrix}. \quad (15)$$

Instead of relative velocities as in the LT-based three-clock scenario, we look for absolute velocities with respect to the initially undefined absolute frame A . There is no obvious way to measure the relative velocity in Tangherlini spacetime; therefore, we need to introduce the base system's absolute velocity vector $\vec{v}_{BA}$ in A with an unknown magnitude v_{BA} . For simplicity, as in the STR standard configuration, it is aligned with the virtual x-axis of A and with the collinear x' -axis of B , as prescribed by the Tangherlini standard coordinate configuration (x-boost).

We can determine the vector equation of motion (EOM) of XL_B in A as X_{LBA} by applying the inverse TT matrix $(\Omega_{\infty}^{v_{BA}})^{-1}$.

$$(\Omega_{\infty}^{v_{BA}})^{-1} \begin{bmatrix} ct' \\ L_B \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{c^2 t'}{\sqrt{c^2 - v_{BA}^2}} \\ \frac{(v_{BA} t' + L_B) c^2 - L_B v_{BA}^2}{c \sqrt{c^2 - v_{BA}^2}} \\ 0 \\ 0 \end{bmatrix} \equiv X_{LBA}. \quad (16)$$

The variable t' in B needs to be eliminated from the transformed vector so that the absolute time t is consistently expressed in absolute coordinates.

$$\frac{X_{LBA}[1]}{c} = t \Rightarrow t' = \frac{t \sqrt{c^2 - v_{BA}^2}}{c}. \quad (17)$$

X_{LBA} can now be expressed in absolute time coordinates as follows:

$$X_{LBA} = \begin{bmatrix} ct \\ \frac{c \sqrt{c^2 - v_{BA}^2} v_{BA} t + L_B c^2 - L_B v_{BA}^2}{c \sqrt{c^2 - v_{BA}^2}} \\ 0 \\ 0 \end{bmatrix}. \quad (18)$$

The moving clock C_1 is associated with the symbol $B1$, which represents its local coordinate system, and X_{LBA} must be converted to this system, in which L_B is seen as a moving point towards the origin of $B1$. The absolute velocity of $B1$ in A is designated as v_{B1A} . Therefore, it needs to be transformed using the transformation matrix $\Omega_{\infty}^{v_{B1A}}$.

$$\Omega_{\infty}^{v_{B1A}} X_{LBA} = X_{LBB1} = \begin{bmatrix} \sqrt{c^2 - v_{B1A}^2} t \\ \frac{ct(v_{BA} - v_{B1A}) \sqrt{c^2 - v_{BA}^2} c^2 - L_B v_{BA}^2}{\sqrt{c^2 - v_{BA}^2} \sqrt{c^2 - v_{B1A}^2}} \\ 0 \\ 0 \end{bmatrix}. \quad (19)$$

The variable t needs to be eliminated from the transformed vector so that it is consistently expressed in t'', x'' coordinates.

$$\frac{X_{LBB1}[1]}{c} = t'' \Rightarrow t'' = \frac{ct}{\sqrt{c^2 - v_{B1A}^2}} \quad (20)$$

After the substitution, X_{LBB1} can be expressed in **B1** terms of the coordinate t'' as follows:

$$X''_{LBB1} = \begin{bmatrix} \frac{ct''}{L_B(c^2 - v_{BA}^2)\sqrt{c^2 - v_{B1A}^2} + c^2 t'' \sqrt{c^2 - v_{BA}^2}(v_{BA} - v_{B1A})} \\ \frac{0}{\sqrt{c^2 - v_{BA}^2}(c^2 - v_{B1A}^2)} \\ 0 \end{bmatrix}. \quad (21)$$

The X''_{LBB1} marker X''_{LBB1} in the **B1** frame, appears to move towards the **B1** origin. The time at which clock C₁ coincides with the marker is when its x'' coordinate is 0:

$$X''_{LBB1}[2] = 0 \Rightarrow t'' = \frac{L_B \sqrt{c^2 - v_{BA}^2} \sqrt{c^2 - v_{B1A}^2}}{c^2(-v_{BA} + v_{B1A})} \equiv \tau_1. \quad (22)$$

The trip duration τ_1 of clock C₁ is now determined. Because of the downwards worldline orientation, the duration τ_2 of clock C₂ follows the same formula (22), but with a different velocity symbol and with the sign inverted so that τ_2 remains positive.

$$X'''_{LBB2}[2] = 0 \Rightarrow t''' = -\frac{L_B \sqrt{c^2 - v_{BA}^2} \sqrt{c^2 - v_{B2A}^2}}{c^2(-v_{BA} + v_{B2A})} \equiv \tau_2. \quad (23)$$

We have determined the durations on paths from the perspective of moving clocks C₁ and C₂; now, we need to find the relative velocities v_1 and v_2 of these clocks in **B** and the time of the round trip τ_3 measured by clock C₃. The vector EOM of C₁ in **A** is given by the 4-vector X_{B1A} :

$$X_{B1A} = \begin{bmatrix} ct \\ v_{B1A}t \\ 0 \\ 0 \end{bmatrix}. \quad (24)$$

Applying TT to X_{B1A} yields the following:

$$\Omega_{\infty}^{v_{BA}} X_{B1A} = X_{B1B} = \begin{bmatrix} \sqrt{c^2 - v_{BA}^2}t \\ \frac{ct(-v_{BA} + v_{B1A})}{\sqrt{(c^2 - v_{BA}^2)}} \\ 0 \\ 0 \end{bmatrix}. \quad (25)$$

After (25) is converted to the local time t' , the relative EOM of C₁ is as follows:

$$X'_{B1B} = \begin{bmatrix} ct' \\ \frac{c^2 t' (-v_{BA} + v_{B1A})}{(c^2 - v_{BA}^2)} \\ 0 \\ 0 \end{bmatrix}. \quad (26)$$

Similarly, for **B2**:

$$X'_{B2B} = \begin{bmatrix} ct' \\ \frac{c^2 t' (-v_{BA} + v_{B2A})}{(c^2 - v_{BA}^2)} \\ 0 \\ 0 \end{bmatrix}. \quad (27)$$

The relative velocities v_{t1} and v_{t2} are then the same formulae except for v_{B2A} replacing v_{B1A} :

$$\begin{aligned} v_{t1} &= \frac{c^2(-v_{BA} + v_{B1A})}{(c^2 - v_{BA}^2)} \\ v_{t2} &= \frac{c^2(-v_{BA} + v_{B2A})}{(c^2 - v_{BA}^2)}, \end{aligned} \quad (28)$$

The round-trip time registered by clock C_3 is as follows:

$$\begin{aligned}\tau_3 &= \frac{L_B}{v_{t1}} - \frac{L_B}{v_{t2}} = \\ &= \frac{L_B(c^2 - v_{BA}^2)}{c^2(-v_{BA} + v_{B1A})} - \frac{L_B(c^2 - v_{BA}^2)}{c^2(-v_{BA} + v_{B2A})}.\end{aligned}\quad (29)$$

We obtain the system of the following three nonlinear equations from Equations (22), (23) and (29).

$$\begin{cases} \tau_1 = \frac{L_B \sqrt{c^2 - v_{BA}^2} \sqrt{c^2 - v_{B1A}^2}}{c^2(-v_{BA} + v_{B1A})} \\ \tau_2 = -\frac{L_B \sqrt{c^2 - v_{BA}^2} \sqrt{c^2 - v_{B2A}^2}}{c^2(-v_{BA} + v_{B2A})} \\ \tau_3 = \frac{L_B(c^2 - v_{BA}^2)}{c^2(-v_{BA} + v_{B1A})} - \frac{L_B(c^2 - v_{BA}^2)}{c^2(-v_{BA} + v_{B2A})} \end{cases} \quad (30)$$

From this system, we cannot calculate L_B because we have three unknown velocities and therefore four unknowns with only three equations. We cannot rely on the currently not practically feasible method described in Section 2. This is, however, not an obstacle because L_B is a free parameter that can be measured by traditional methods, particularly by using the return time of the light signal on the round trip: $L_B = c\Delta t_{ret}/2$.

The system solution was attempted using Maple™ 2019. Unfortunately, despite the unusual nature of the three-clock method, which does not explicitly rely on distant clock synchronisation, no solution was found due to usual cancellation.

While confirming and analysing the disappointing but widely expected null result, an important connection was found between the Minkowski and Tangherlini frameworks.

1. Using proper times τ_1, τ_2 , and τ_3 represented by Equations (30);
2. Substituting them into the positive root of the equation for L_B and to all velocity roots v_{11}, v_{12}, v_{21} , and v_{22} , from Equations (12); and
3. Assuming that $\{v_{BA} > 0, v_{B1A} > v_{BA}, v_{B2A} < v_{BA}, v_{BA} < c, v_{B1A} < c, \text{ and } c > 0\}$ and L_B is a real positive number, the result of algebraic simplification is as follows:

$$\begin{aligned}D &\equiv L_B = L_B \\ v_{11} &= \frac{c^2(-v_{BA} + v_{B1A})}{c^2 - v_{BA} v_{B1A}} \\ v_{12} &= \frac{c^2(v_{BA} - v_{B1A})}{c^2 - v_{BA} v_{B1A}} \\ v_{21} &= \frac{c^2(-v_{BA} + v_{B2A})}{c^2 - v_{BA} v_{B2A}} \\ v_{22} &= \frac{c^2(-v_{BA} + v_{B2A})}{c^2 - v_{BA} v_{B2A}}.\end{aligned}\quad (31)$$

This was as expected for L_B . The absolute velocities did cancel out; thus, L_B remained invariant. However, no cancellation can be seen for the STR relative velocities. One instance of the STR velocity can be the result of an unlimited number of combinations of v_{BA} and v_{B1A} . Measuring v_1 is insufficient to solve for an absolute velocity because of one extra degree of freedom. At this point, all the classic predictions seem to confirm the postulate of relativity as formulated by Poincaré [12] (drafted in June 5, 1905), placing the inability to detect the absolute movement of the Earth as the foundation (refer to the discussion on page). Poincaré reported that his principle, which is consistent with the Lorentz transformation, was thoroughly reviewed and rederived with full mathematical rigour [12]. This finding made it pointless for him and most of his successors to look elsewhere. However, the peculiar relationships in Equation (31) and their potential significance triggered further investigations.

4.3. Variable Speed of Light vs. Conventional Isotropy

The variable light pulse velocities in the standard coordinate configuration in the Tangherlini framework are given as follows:

$$c_{x+} = \frac{c^2}{c + v_{BA}}, c_{x-} = \frac{c^2}{c - v_{BA}}. \quad (32)$$

where c_{x+} and c_{x-} are the positive variable magnitudes of the velocity of light on the x' -axis in the positive and negative directions, respectively; hence, all relative velocities are functions of absolute velocities. First, we analyse the results in Equation (13). Using proper times τ_1, τ_2 , and τ_3 given in Equation (30) and substituting them into the first equation of Equation (13), we obtain the following:

$$\begin{aligned} \tau_{C1} &= \frac{\tau_1^2 - \tau_2^2 + \tau_3^2}{2\tau_3} = \frac{L_B(c^2 - v_{BA}v_{B1A})}{c^2(-v_{BA} + v_{B1A})} \Rightarrow \\ t &= \frac{x(c^2 - v_{BA}v_{B1A})}{c^2(-v_{BA} + v_{B1A})}, \end{aligned} \quad (33)$$

where t is the current time coordinate in the propagation of C_1 in B in the LT-based scenario in Figure 1 and x is the coinciding position of C_1 .

At the same physical location in the STR and Tangherlini frameworks, $x' = x$. Using the first equation of Equation (28), we obtain the following:

$$v_{t1}t' = x' = x = v_1t = \frac{c^2(-v_{BA} + v_{B1A})}{(c^2 - v_{BA}v_{B1A})}t. \quad (34)$$

Using Equations (33) and (34), we can solve the following system for t' and v_{B1A} :

$$\begin{cases} \frac{c^2(-v_{BA} + v_{B1A})}{(c^2 - v_{BA}v_{B1A})}t = x \\ \frac{c^2(-v_{BA} + v_{B1A})}{(c^2 - v_{BA}^2)}t' = x. \end{cases} \quad (35)$$

The solution of interest is $t' = f(t, x, v_{BA})$ with $x = x'$, while the other solution for v_{B1A} has no useful value:

$$t' = t + \frac{xv_{BA}}{c^2}. \quad (36)$$

This allows bidirectional conversions to/from the Tangherlini and STR frameworks because the x, x' -axes are identical when they statically coincide in B . This also shows that the irreducible degree of freedom with v_{B1A} being eliminated from the scope is the main reason behind all cancellations. It now appears that removing this freedom could occur only by implementing the nonexistent instantaneous signal synchronisation. Fortunately, this is not the case.

4.4. Closing the Gap

Our attention shifted towards identifying a missing equation that would allow for the recovery of the absolute velocity v_{BA} . Thus far, our derivation of the TT has assumed an empty, featureless, and passive space, focusing on the relative kinematics between a hypothetical privileged inertial system and any other inertial frame. The relativistic TT relation was derived from the empirical isotropy of the average round-trip speed of light, without assuming a specific physical cause for this behaviour. However, it is logical to conclude that this behaviour is not caused by the inertial systems themselves. This raised a critical question: is there an overlooked property of light that could resolve the absolute velocity? The most vital observation is that while light is emitted and absorbed by atoms to facilitate measurement, its orderly, causal propagation from point to point is a property of the vacuum and the electromagnetic field itself. Once emitted, a beam of light may exist independently of any inertial system, propagating as a coherent entity—much like a “rigid rod” of fixed length.

Considering light from a distant star that may no longer exist, it travels through the void and interacts with any inertial system it encounters. At the moment of interaction, the original source is irrelevant; only the freely propagating beam matters. While a stationary observer in the ARF would measure an intrinsic frequency, a moving observer in system B would measure a Doppler-shifted frequency. This invites us to look beyond the average round-trip speed of light and examine the Doppler effect as an additional fundamental property. We consider a monochromatic electromagnetic wave propagating along the x' -axis from the positive side towards a detector and clock C_3 at the origin of the base system B .

Let K be the wave 4-vector of the incoming wave in free space or rather in the ARF:

$$K_A = \begin{bmatrix} \frac{\omega_A}{c} \\ -\frac{\omega_A}{c} \\ 0 \\ 0 \end{bmatrix}. \quad (37)$$

To convert this vector to B , we apply TT as follows:

$$K_{AB} \equiv \begin{bmatrix} \frac{\omega_{AB}}{c} \\ k_{xB} \\ k_{yB} \\ k_{zB} \end{bmatrix} = \Omega_{\infty}^{v_{BA}} \begin{bmatrix} \frac{\omega_A}{c} \\ -\frac{\omega_A}{c} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{\omega_A \sqrt{c^2 - v_{BA}^2}}{c \cdot c} \\ -\frac{\omega_A (c + v_{BA})}{c \sqrt{c^2 - v_{BA}^2}} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{\omega_A}{c} \sqrt{1 - \beta^2} \\ -\frac{\omega_A}{c} \frac{(1 + \beta)}{\sqrt{1 - \beta^2}} \\ 0 \\ 0 \end{bmatrix}, \quad (38)$$

where $\beta = v_{BA}/c$.

The explicit presence of v_{BA} in Equation (38) determines the absolute velocity, assuming that the angular frequency ω_{AB} and the wave number k_{xB} can be accurately measured. The latter may be much more difficult to accomplish than the former. By shifting focus from the clocks to the light beam in transit, the missing equations are now identified. Unlike clocks, a freely propagating electromagnetic wave train has an intrinsic spatial and temporal periodicity (ω_{AB} and k_{xB}) that is governed by the vacuum itself, not the observer's synchronisation convention. By measuring the local frequency ω_{AB} and the local wavenumber k_{xB} (e.g., via the movable intensity sensor protocol), we can possibly determine two unknowns: the absolute velocity v_{BA} and the original ω_A , which can be obtained by solving the following system of equations:

$$\begin{cases} \frac{\omega_{AB}}{c} = \frac{\omega_A \sqrt{c^2 - v_{BA}^2}}{c^2} \\ k_{xB} = \frac{\omega_A (c + v_{BA})}{c \sqrt{c^2 - v_{BA}^2}}. \end{cases} \quad (39)$$

The solution of the above system is:

$$\boxed{\begin{cases} \omega_A = \sqrt{\frac{-\omega_{AB}}{2k_{xB} + \omega_{AB}}} c k_{xB} \\ v_{BA} = \frac{c(k_{xB} + \omega_{AB})}{k_{xB}}. \end{cases}} \quad (40)$$

This is quite interesting result. We once declared that there are no features in empty space, so we cannot find any reference point—anything to measure—except for light that never stops. Firstly, we can find not only the absolute velocity in an unknown ARF but also the so far unknown light angular frequency, where no clock and no coordinate system exist. The hypothetical ARF becomes potentially measurable. This would be an ultimate connection with absolute rest state which CMB might represent.

The above seems to be the most direct way of finding v_{BA} . Better still, is to exploit the property of the ratio $R = \omega_{AB}/ck_{xB}$.

$$\begin{aligned} R &= \frac{\omega_{AB}}{k_{xB}} = -\frac{c}{c - v_{BA}} \Rightarrow \\ v_{BA} &= -\frac{c(R + 1)}{R}. \end{aligned}$$

(41)

This equation is of ultimate simplicity but relies on the same direct measurement of ω_{AB} and k_{xB} . Note that if the ratio $R = -1$, as in the STR framework, $v_{BA}=0$. While this principle of measurement of absolute velocity appears to be valid, it is only a raw proof of concept requiring the physics community to generalise it to a 3D context such that the velocity vector can be fully identified and engineers to make the method practical and sensitive. The hope is that measurements of velocities with respect to the Cosmic Microwave Background (CMB) provide sufficient grounds to expect that adequate measurement method can be found in due time. By identifying the absolute velocity in Equation (41), we provide a potentially fundamental theoretical basis for CMB velocity measurements without contradicting STR, which remains valid for the vast majority of physical contexts where absolute motion does not alter local outcomes, consistent with the cancellations resulting from Equations (30). This finding may potentially closes the over-120-year gap in understanding the fundamental nature of flat spacetime, unaffected by strong gravitational fields.

There is however, one major problem that must be explained. The frequency $\omega_A/c \sqrt{1 - \beta^2}$ does not agree with overwhelming consensus. For example, in the publication of Drągowski, M., Włodarczyk [13] the authors demonstrated that Doppler effect under absolute transformation is indistinguishable from that derived from the Lorentz. We have expanded this research in order to reconcile the discrepancy and preliminary results look intriguing. There is, however, one major problem that must be explained. The frequency component $\omega_A/c \sqrt{1 - \beta^2}$ derived here does not agree with the overwhelming consensus. For example, Drągowski and Włodarczyk [13] demonstrated that the Doppler effect under absolute transformation should be indistinguishable from that derived from the Lorentz transformation. We have expanded upon this research to address the observed discrepancy; preliminary results are intriguing and suggest a path toward reconciling these two frameworks.

5. Discussion and Conclusions

Given the unresolved discrepancy in the Doppler shift results, which persists despite verified algebraic consistency, the Discussion and Conclusions sections are deferred pending final resolution of the underlying model. Ongoing work is addressing this problem, suggesting even deeper connection between the Tangherlini and Lorentz frameworks that promises their ultimate reconciliation.

5.1. Conclusions

To be continued.

Author Contributions: The author contributed to all aspects of the work.

Funding: This research received no funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: New data were generated from the mathematical model. Detailed Maple computation results may be available upon request.

Conflicts of Interest: The author declares no conflicts of interest.

Abbreviations

The following abbreviations are used in this manuscript:

ALT	Absolute Lorentz Transformation
AR	Absolute Rest
ARF	Absolute Rest Frame
CMB	Cosmic Microwave Background
EOM	Equation of Motion
GR	General Relativity
LT	Lorentz Transformation
STR	Special Theory of Relativity
TT	Tangherlini Transformation

Appendix A. On Absolute Rest, A Velocity and Tangherlini Transformation

Tangherlini’s derivation of the transformation applied in this paper was originally based on the Einstein field equation of general relativity (GR), resulting in transformation equations compatible in structure with the LT equations derived by Einstein, as presented in [2] (p48). The arrangement of the two relatively moving coordinate systems can be referred to as the standard configuration, where the relative velocity vector is aligned with the first spatial axis (usually x). The mathematical representations of the TT, the postulates leading to its derivation and resulting properties may be collectively referred to as the TT framework.

With respect to the TT framework, the absolute rest frame must be postulated *a priori* to allow derivation when using a method other than Tangherlini’s approach based on GR. For example, in Section 3, Selleri [14] defines the absolute frame as the one where the velocity of light is the same in any direction while not demanding moving frames to follow the same rule. The impossibility of absolute synchronisation with instantaneous signals—let alone without them—appears to be the fundamental obstacle to the practical use of TT on a larger scale. The TT framework seems to be the only sensible but impractical alternative to the STR framework. Both frameworks share the same foundation, which is the empirical law of isotropy of the round-trip average speed of light discovered by Michelson and Morley [15].

The TT framework can be derived from first principles of Newtonian kinematics *without* reference to noninertial rotational motion or forcing postulates of length contraction or time dilation, as was the case in some previous approaches. However, the frequently analysed Sagnac effect seems to be a legitimate approach.

The necessary and sufficient set of postulates/conditions for completely defining the TT are as follows:

Postulate 1: *There exists an absolute rest frame (ARF) represented by an inertial system A , with three Cartesian coordinate axes that can be bound (aligned) to preexisting axes of an inertial system M at time $t=0$ and $t'=0$. In that system, the one-way velocity of light is constant and equal to c in all directions, and there is only one system A (together with all its possible translations and rotations, all being at rest with it).*

Postulate 2: *The round-trip average speed of light in any inertial system is a constant, whose value $c=299792458$ m/s, and is independent of the relative direction and velocity of those systems.*

Postulate 3: *An instantaneous signal being represented as the limit of all signals moving in the same direction with ever increasing velocity $v_{\alpha A}$, observed in the absolute inertial system A as $S_{\infty A} \equiv t = x / \lim_{v_{\alpha A} \rightarrow \infty} v_{\alpha A}$, is invariant in all inertial frames such that when observed in an inertial moving system M of the absolute velocity v_{MA} , it is $S'_{\infty M} \equiv t' = x' / \lim_{v_{\alpha M} \rightarrow \infty} v_{\alpha M}$, where $v_{\alpha M} \neq v_{\alpha A}$.*

Constraint 1: *The spatial origin of system M moving in the system A coordinates transforms to the origin point in M at rest with itself as $M(0,0,0)$ at every instance of time t' .*

Constraint 2: The determinant of the linear coordinate transformation matrix Ω_{∞}^v is equal to +1, as in the case of the LT and Galilean transformation matrices.

The TT matrix Ω_{∞}^v meeting the postulates and constraints can be derived and is shown in Equation (14). Including the full derivation process in this paper would exceed a sensible publication word count limit.

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