Philosophy, Art, Mathematics, and High-Frequency Applications of Triangular Shapes: from Plato to Sierpinski and beyond

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Abstract—Triangles have inspired several fields of interest depending on the discipline and potential applications. This peculiar shape has fascinated ancient philosophy, art, mathematics, and applied sciences. This paper will review triangles from different points of view, focusing on the equilateral shape and its internal complexity generated by the Sierpinski geometry, the symbolic meaning, and the perspective applications in microwave communications. Specifically, microwave planar components and antennas inspired by the Sierpinski geometry will be considered.

Keywords—Sierpinski, Microwaves, Planar Components, Antennas.

1. Introduction

This paper reviews the meaning of triangles from different perspectives and discusses their applications as resonating elements for microwave planar components and antennas.

Numbers initially motivated the Greek philosophers and mathematicians, trying to reduce the world's interpretation to specific numeric combinations and geometrical shapes. From this point of view, the triangles were one of the focal points of the mathematical and philosophical efforts of Pythagoras and Plato, originating the famous theorems and the dialogue called Timaeus [1][2][3][4]. Sierpinski also published about Pythagorean triangles in [5]. Of course, the number "3" is also related to triangles because of the three edges of the geometrical figure. It has been invoked for reasons pertaining to its magic and religious nature, encompassing Catholic culture and several examples since the early historical ages.

In many ancient traditions, "3" is the counterpart to "4", comparing the male and female principle or the complementarity of the sky and the earth. The number "4" has also been related to the four essential elements, i.e., water, air, earth, and fire. "3"+"4" originated the number "7," also considered "magic" for other reasons (one-fourth of the lunar cycle; the so-called "planets" of antiquity, including the moon and sun, the Pleiades, etc.). [6]

Concerning art, reutilizing small marble pieces from the destruction of old Roman temples, mainly columns and wall or floor slides, a medieval Roman family of marble workers invented the so-called Cosmatesque style, whose products are visible in many catholic churches in Italy, but sometimes also abroad [7][8][9]. Geometrical representations with squares, circles, or spirals were manufactured from the XI to the XIII Century by the Cosmati family, and triangles were available in many compositions as building blocks of picturesque wide floors looking like mosaics. More ancient manufactures are available in other places. It is interesting to see a handmade decoration from the Museum of Malta, La Valletta, which can be considered a preliminary fractal geometry (see Fig. 1).

The Sierpinski triangles are figures belonging to the more general group of fractals [10][11]. They are obtained by a progressive subdivision of whole triangles into many internal triangles of decreasing size. From a mathematical point of view, many publications, software tools, and potential applications have been considered in the past decades [12][13][14][15][16]. Triangles are possible elementary cells in the finite element method (FEM) of calculation for

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Fig. 1 Piece of a decoration exhibited in the Archeological Museum of La Valletta (Malta) with inner triangles embedded in a triangular frame (900 B.C., approximately). Personal Photo.

electromagnetic 2D and 3D simulations, where each side can represent a lumped element [17]. They are building blocks in manufacturing processes like 3D printing, contributing to mechanically stable structures, especially when combined in hexagons [18].

Finally, equilateral triangles can be seen as regular shapes resonating at specific frequencies when adequately sized and excited by a feeding system. A key point about resonators and antennas for high frequencies is the feeding network necessary for obtaining an effective resonant response and good radiative performance for the antennas. This is especially important when the triangles are characterized by an additional internal complexity, as the Sierpinski geometry [19][20]. Like any other resonator, the coupling degree (electrical matching) determines the quality factor and bandwidth. In the case of planar components or antennas, the specific difference between them is the necessity for substrates compatible with the manufacturing processes for electronic components (high dielectric constant) or radiative elements (low dielectric constant). In both cases, the intrinsic frequency is determined by the size and the substrate choice. Moreover, triangles are building blocks suitable for applications in resonating or radiative arrays, properly combining several of them [20].

In this paper, the triangle is presented from a complementary viewpoint rather than applications, and some configurations inspired by the Sierpinski geometry will also be proposed for specific configurations in high-frequency signal processing.

2. Philosophy, Mathematics and Art

Early philosophical currents have always considered mathematics and geometry interrelated and able to bring an inner meaning to understanding the natural order through symbols and specific shapes. This initial thinking was usual when no specialization was present among the scientific disciplines, and a unitary vision of the world was proposed. Such an attitude is also typical of oriental philosophy, but in that case, another, more spiritual vision is considered. In ancient Greece, a logical and scientific approach puzzled the scientists and philosophers, who were able to open schools with the ambition to select people able to understand matters considered “esoteric”, i.e., reserved to a selected group of students. This definition has nothing to do with other currents inspired by the magic meaning of numbers and shapes, which was trendy, especially in the XIX century.

A. Plato

The Timaeus essay of Plato, dating back to 360 B.C., treats the triangle as a building block figure originating solid figures. His understanding of reality was based on the World of Ideas and Forms, such that the original theory of the four elements evolved considering the presence of basic shapes, which are regular geometrical solids: Tetrahedron (fire), Octahedron (air), Icosahedron (water), and Cube (earth). Following this approach, the shapes correspond one by one to the four fundamental elements introduced in previous times to model reality. All the natural manifestations resemble their counterparts in the World of

Ideas, with the possibility of exchanging their fundamental nature and being transformed into another shape. These perfect polyhedral shapes comprise triangular faces with internal angles of 30-60-90 and 45-45-90 degrees, respectively. In detail, half of a square and half of an equilateral triangle are considered by Plato to be the fairest geometrical figures, able to originate all the others. See, for instance, [21].

B. Ancient Art and Cosmatesque Decorations

In the introduction, it was underlined that despite their symbolic meaning, triangles are part of many artistic decorations. Crossing lines was probably one of the initial ways humans created shapes, together with other preliminary techniques, continuing with artistic hunting scenes and everyday life pictures in the caves. After that, many authors have described the utilization of numbers and shapes with specific intents over the centuries [22][23], with alchemical meanings or to indicate that you belong to a group. The triangle is the Catholic symbol that means the triple God identity (Father, Son, and Saint Spirit). That symbol, with the same meaning, is even used in the American one-dollar banknote.

After the fall of the Roman Empire, several ancient buildings were destroyed by external populations, conquering the previous domain of Rome. Marbles from the Mediterranean area, initially used for columns, floors, and walls in buildings belonging to the emperor and rich families, were abandoned and no longer maintained. Then, pieces of precious marble could be reutilized in decorations, arranging fragments in original designs. In this framework, the Cosmati family invented a style based on geometrical configurations used mainly for church floors upon the requirement of the Pope [24]. A mathematical approach with a detailed description of Cosmatesque triangles and carpets interpreted using the Sierpinski theory can be found in [25][26]. A few examples of Cosmatesque representations, including Sierpinski triangles, are shown in Fig. 2.

3. Mathematics and geometry of Sierpinski fractals

Triangles are subdivided into different groups owing to their shape. Equilateral, isosceles, and rectangular triangles are all suitable for arrangement in a more complicated planar structure. Still, the equilateral ones are easier to subdivide and combine in an array, especially for applicative purposes.

Sierpinski triangles can be considered a fractal geometry derived from creating a series of internal triangles with decreasing size. Starting from the initial one, you can subdivide it by considering empty or full triangles and a frame with a specific thickness surrounding all the created sub-triangles. Independently of the above choice, the total number of triangles created by the subdivision can be computed as:

$$N_t = 3^n \qquad (1)$$

Where n is the complexity level, and N_t is the number of sub-triangles generated. So far, n=0 means an entire single triangle, and n=1

Fig. 2 Roman church of "San Lorenzo fuori le mura" (outside the walls). Two examples of Sierpinski-like floor decoration made by fragments of old imperial material. Medieval age, around 1200. Personal Photo.

means three sub-triangles plus an empty triangular shape, and so on. The situation is represented in Fig. 3 [27]. The "negative" subdivision is complementary and gives back empty triangles surrounded by a frame. Choosing "positive" or "negative" (empty) sub-triangles is fundamental in a high-frequency application. It corresponds to a metalized or empty area photolithographically obtained onto the substrate. In this paper, we shall consider only the "positive" sub-triangles.

Sierpinski's contribution was essential in obtaining a mathematical approach to formalize a theory supporting this specific shape, even if this geometry was known, as discussed before, for different reasons. Since then, a mathematical formulation of the number of positive and negative sub-triangles and the area as a function of the internal complexity is available.

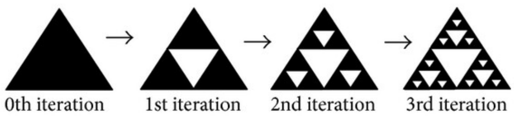

Fig. 3 Iterations to obtain the 3rd level of complexity in the Sierpinski triangle [27].

4. High Frequency Applications

The utilization of equilateral triangles and their fractal evolution, namely the Sierpinski ones, in antennas and resonators have been documented since the 1980's of the past century, up to evolutions for carpet Sierpinski geometries [28].

Several configurations have been studied, including different feeding solutions, trying to take advantage of the peculiar characteristic to modulate the geometry for obtaining multiple frequencies. Most of the literature is focused on the equilateral triangle, but modifications are suggested to calibrate the frequency and control the spectrum. This task is not trivial because the analytical approach to predicting the resonance frequency is already complicated, and electromagnetic simulations must often support it to have a direct comparison. The main problem is accounting for a structure presenting three significant electromagnetic discontinuities in the simple triangle and an increasing complexity when the internal figures are created. For this reason, some papers are missing the spectra, probably because they match the expected resonances but are poorly excited [29], and in other papers, it was possible to make a comparison using simulations and experiments but not formulae [30]. To the best of my knowledge, only in [31][32] a numerical approach supported by measures was successful in the spectrum prediction. Most other papers discuss comparisons between different methods, but they are only sometimes supported by an experiment, even if suggesting valuable modifications of the original theory. An interesting approach is given in [33], and additional considerations can be found in [34][35][36].

A. *Antennas*

Like any other geometrical figure manufactured for planar high-frequency applications, a triangle is a structure with its resonance frequencies determined by size. Additional properties can be considered if magnetic (like Permalloy [37] or the classical garnets and ferrites [38]) or ferroelectric [39] materials are used for tunability. As discussed above, the feeding network is essential to enhance the antenna's radiation capabilities. A pretty accurate design procedure for triangular antennas and arrays is described in [36][40]. The reason for having a microstrip excitation on the opposite side of the antenna is the necessity for not having metal radiative contributions on the same side, especially for substrates having a lower dielectric constant, and the microstrip could be broad and comparable to the antenna size. Another contribution in predicting the resonance frequencies for Sierpinski antennas is given in [41]. The fractal dimension helps obtain a multi-band response, which is enhanced compared to the simple triangle. The internal complexity of the Sierpinski geometry or a combination in an array can also modulate the expected resonance frequency. Still, we shall see that it can affect other radiator properties.

An example of the frequency response of the sequence from C0 to C3 (from the 0^{th} to the 3^{rd} iteration in the internal complexity) of a Sierpinski antenna is given in the following simulations, where the substrate is a commercially available material Rogers 5880 (RO5880), with dielectric constant ε=2.2 and dielectric losses tanδ=0.0009 for frequencies higher than 10 GHz [43]. A low dielectric constant impedes the excitation of substrate modes, otherwise lowering the overall antenna performance. The substrate thickness has been fixed at d=1.575 mm. The metal thickness is typically t=35 µm, which is also better for power applications, and the antenna is grounded. A frame of 200 µm surrounds the entire structure and each individual sub-triangle. A bowtie configuration with an internal feeding has been chosen to show how the spectrum can be complicated by the internal sub-triangles of the antenna, whose feeding might be optimized depending on the frequency and the application. In Fig. 4, the simulated configurations and the spectrum of the antennas are plotted, with evidence of a complicated response, with an increase in the number of the excitation modes as a function of the sub-divisions, even if identifying the modes to be compared is still subject to interpretation, as the frequency shift appears to be not monotone with the internal complexity. In fact, the excited mode strictly depends on the coupling solution.

The radiation pattern at resonance exhibits a narrow lobe when measured in the middle of the two wings. It is broader in the 90° position, as shown in Fig. 5 plot at resonance for C0, i.e., at approximately 26 GHz. The expected antenna gain is in the order of 7 dB. Qualitatively, the same radiation pattern is predicted for all the configurations at resonance. Nevertheless, better matching is obtained for C1, which exhibits a gain of around 7.8 dB at 20 GHz and enhanced sidelobes suppression, as shown in Fig. 6. The presence of a substrate and the ground condition alters the response of the naked

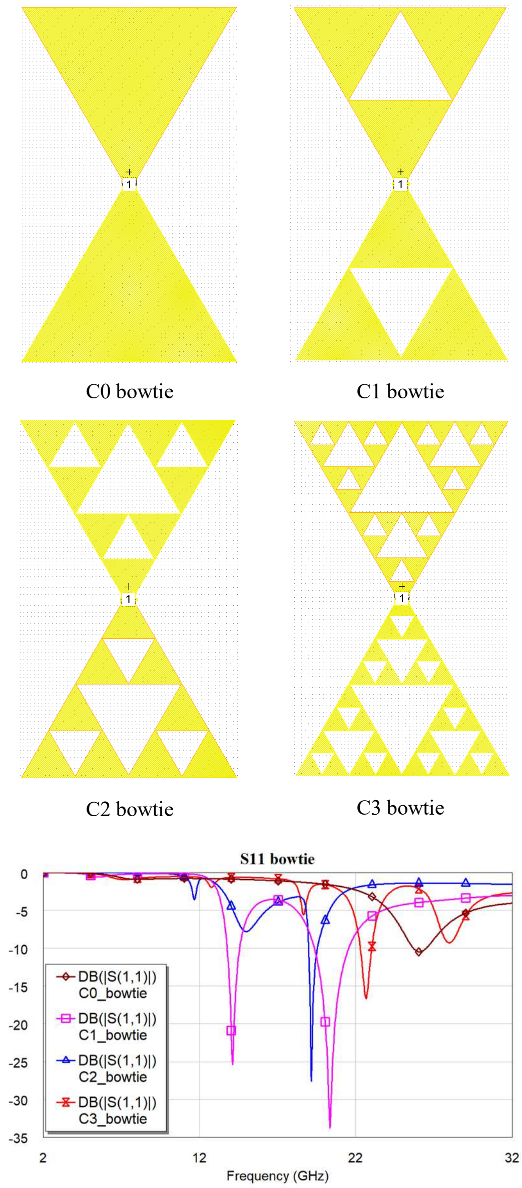

Fig. 4 Configurations and spectrum (in dB) of the Sierpinski bowties. Simulation by Microwave Office 17.1.

structure. Still, it is necessary to simulate a situation closer to the experimental one, where a feeding line should be present, and the ground helps improve the antenna gain.

B. Resonators

The same fractal configurations used to present antenna applications are suitable for resonating structures. Planar filters based on triangles have been studied in [42]. This section will present some examples of resonators for notch filters fed by microstrips.

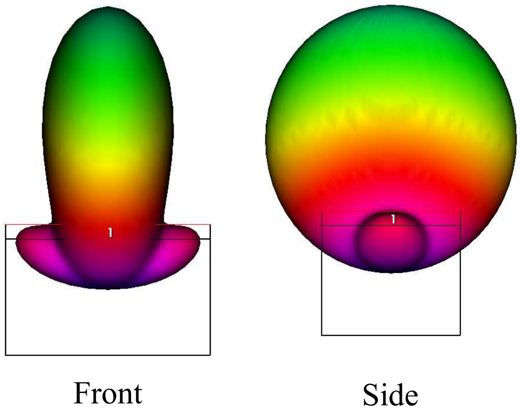

Fig. 5 Front and side view of the radiation pattern for the C0 configuration at 26 GHz.

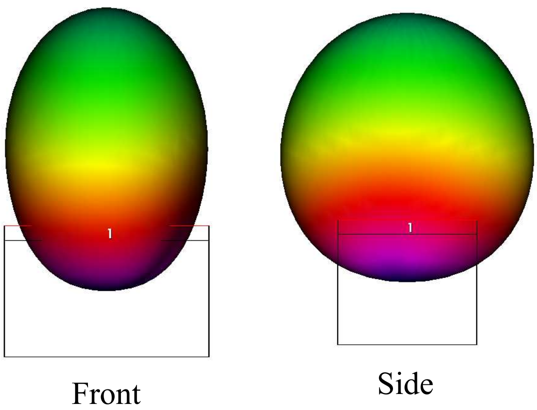

Fig. 6 Front and side view of the radiation pattern for the C1 configuration at 20.3 GHz.

The possibility of using triangles in coplanar waveguide configurations has already been proposed in [20], and their possible interpretation in terms of metamaterial structures has been discussed in [44].

Following the same idea, simulations about microstrip excited Sierpinski triangles will be analyzed in this section. The planar structure is now minded for planar signal processing. Unlike antennas, the high-frequency signal must not be radiated, and the substrate will have a high dielectric constant to confine the electromagnetic field along the propagation path. A standard configuration is proposed here using a 525 µm thick high-resistivity silicon wafer, for which a 420 µm wide microstrip is utilized to get a 50-ohm impedance for the feeding line. For the sake of simplicity, we shall use the same triangles previously introduced to propose K-band antennas with an edge 6 mm long.

Fig. 7 shows the simulated structures, with a 5 µm metallization shape suitable to be manufactured by photolithography on the Si substrate. For completeness, asymmetric and symmetric configurations have been studied to show the best response regarding the excited modes and their electrical matching. Fig. 8 shows the simulated spectrum for the asymmetric and symmetric structures.

Compared to the CPW excited resonators, where the separation from the central conductor was also 50 µm, i.e., the separation between the microwave path and the side coupled resonating structure, the results are confirmed, with deeper notches when the symmetric device is considered. Nevertheless, asymmetric configurations also achieve acceptable performance, which must be considered when space occupancy is limited.

It is worth noting that it is easier to electrically match the microstrip-fed band-stop structure in comparison with the coplanar excitation [20], with a very low loss for the microstrip transducer outside the resonance peaks. The selectivity of the entire triangle C0 compared to the multi-resonance of the C1-C2-C3 structures gives another exciting characteristic of the modes in the investigated band. Table 1 shows the frequency position and separation for all the simulated devices in the asymmetric and symmetric configurations within the X-band.

Table 1. Frequency of resonance for the simulated structures (asymmetric and symmetric)

Resonator	**Resonance frequencies [GHz]**		
	Fres1	***Fres2***	***Difference***
C0 asym	9.46	---	---
C1 asym	4.42	7.98	3.56
C2 asym	5.66	9.22	3.56
C3 asym	7.18	8.94	1.76
C0 sym	9.62	---	---
C1 sym	4.34	7.82	3.48
C2 sym	5.54	9.30	3.76
C3 sym	7.38	8.98	1.60

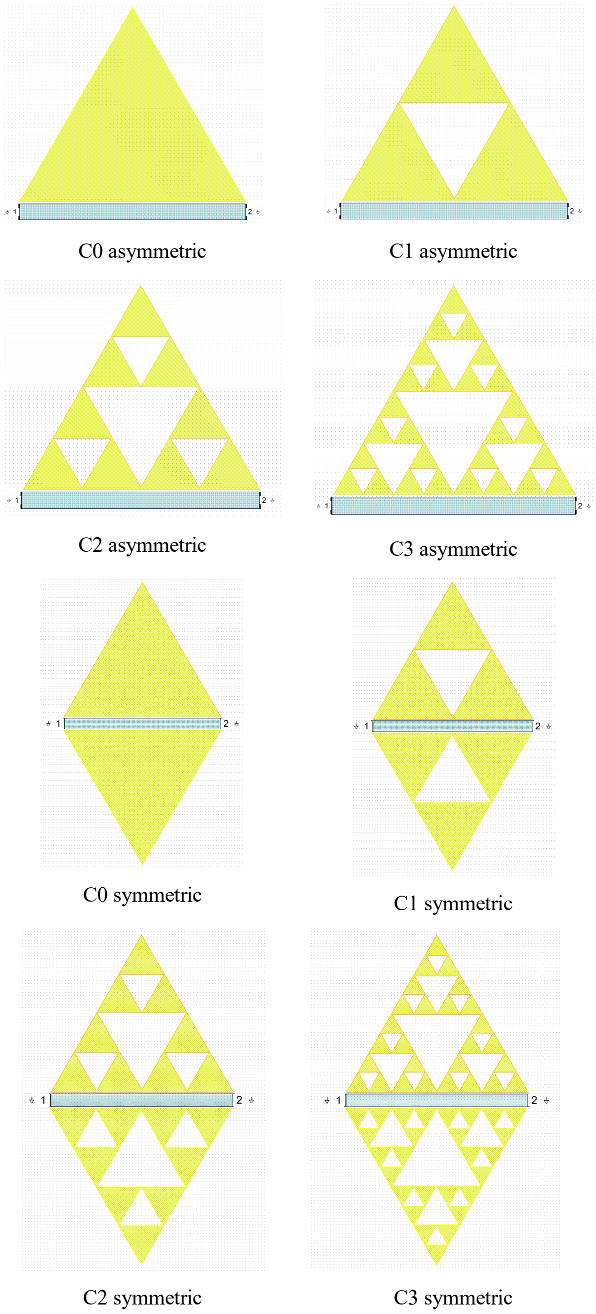

Fig. 7 Simulated resonating structures. Above, the four asymmetric configurations are shown, whereas below, the four symmetric ones are seen. The blue line is the feeding microstrip, 50 µm far from the triangle edge. Metallization is always 5µm.

When two peaks close to each other are present, the more intense one is chosen. Further investigations are needed to interpret the simulated spectrum correctly.

Since all the proposed configurations are also suitable for higher-frequency resonances, this can also help in different microwave bands.

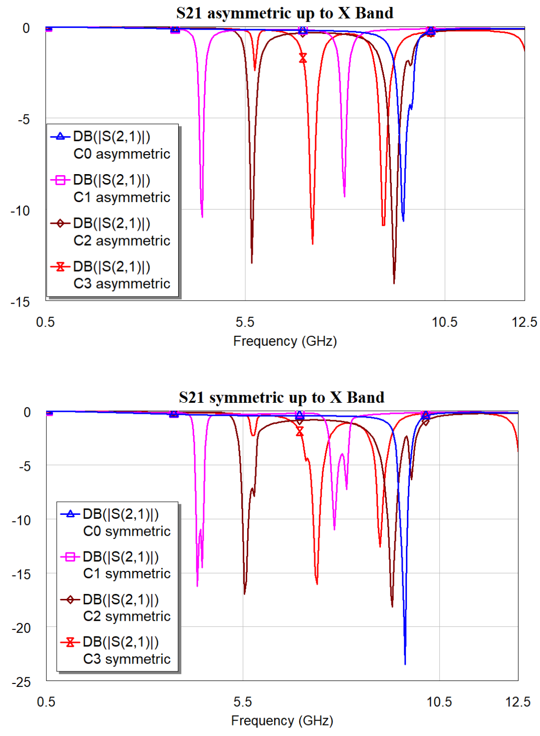

Fig. 8 Simulated response of the S_{21} trasmission parameter (in dB) for the asymmetric and symmetric resonators. Modes excited up to the X-Band (12 GHz) are shown in both cases. Simulation by Microwave Office 17.1.

Still, the spectrum is more difficult to interpret or manipulate for a specific application. This is evident in Fig. 9, which shows the predicted full spectrum of the C3 configuration, with promising results, especially in the K-band.

So far, Sierpinski triangles can easily be considered for a multi-resonance response, and a proper, non-canonical reorganization of the internal triangles, even renouncing the

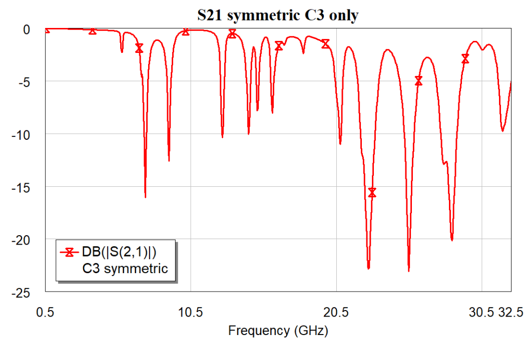

Fig. 9 Simulated spectral response for the S21 parameter (in dB) of the C3 resonator in the symmetric configuration up to 32 GHz.

classical equilateral shape for the building block, can help calibrate the desired resonance frequencies.

5. Conclusion

The triangle figure, with a focus on the equilateral one and its Sierpinski variants, has been discussed in this contribution from different points of view. It was always an intriguing figure, animating the discussion in philosophy and mathematics about its relationships with elemental constituents of reality. A few examples of Cosmati art were also presented to show its use for original and unique decorations. Then, microwave engineering applications for planar signal processing and antennas have been proposed with a very basic design, exhibiting promising responses in different frequency bands. The possibility of obtaining a multi-frequency response has been stressed thanks to the internal complexity generated by the Sierpinski geometry. Additional non-canonical variants of the internal geometry can adequately tailor the excited resonance modes for specific multi-resonance applications of antennas and resonators.

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