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Gauge Couplings of the Standard Model from First Principles in the Octonionic Framework

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Abstract

We present a self-contained gauge-sector account of the octonionic programme, starting from the underlying trace-dynamics Lagrangian and ending with closed-form expressions for the strong and electromagnetic couplings, together with a brief review of the weak mixing angle. The derivation has three steps. First, inside the visible bosonic sector we derive the broken-phase relation

$$\frac{\alpha_s}{\alpha_{\rm em}} = 16,$$

from a single visible Yang–Mills coupling before symmetry breaking. The mechanism combines the standard visible charge-trace factor $8/3$ with a six-direction support factor 6 on the real octonionic ladder space H_6. Second, we recall the 2022 *Eur. Phys. J. Plus* paper [1], where the minimal visible charge quantum $q_0 = 1/3$ fixes the exponential seed

$$A := \exp\left[q_0\left(q_0 - \sqrt{\frac{3}{8}}\right)\right] = \exp\left[\frac{1}{3}\left(\frac{1}{3} - \sqrt{\frac{3}{8}}\right)\right].$$

Combining this seed with the charged-sector datum $3/8$ gives

$$\alpha_s^{\rm th}(M_Z) = \frac{9}{64}\exp\left[\frac{2}{3}\left(\frac{1}{3} - \sqrt{\frac{3}{8}}\right)\right] = 0.11675418,$$

while the broken-phase factor 16 then yields

$$\alpha_{\rm em}^{\rm th}(0) = \frac{9}{1024}\exp\left[\frac{2}{3}\left(\frac{1}{3} - \sqrt{\frac{3}{8}}\right)\right] = 0.00729713629.$$

Third, we briefly review the earlier spinorial derivation of the weak mixing angle [2], which leads to

$$1 = \frac{\sqrt{\cos(\theta_W/2)}}{2} + \sqrt{\sin(\theta_W/2)}, \qquad \sin^2\theta_W^{\rm th} = 0.24969776.$$

A key conceptual point is that the seed is attached to the *minimal visible charge quantum* $q_0 = 1/3$, not to a specific particle species. The electron, whose charge is $1 = 3q_0$, is not omitted: its contribution enters explicitly through the electromagnetic charge trace $k_{\rm em} = 8/3$. In this form the derivation of $\alpha_{\rm em}$ is conceptually sharper than in the earlier *Eur. Phys. J. Plus* presentation [1], because the factor $1/16$ is no longer hidden in a length-identification step but is derived directly from the visible broken-phase gauge structure.

Keywords: standard model; Gauge couplings; octonionic unification; QCD gauge coupling; fine structure constant; weak mixing angle

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1. Introduction

The octonionic programme aims at a pre-quantum, pre-spacetime description of elementary physics in which quantum field theory and classical spacetime both emerge from an underlying matrix-valued dynamics. In this approach, the fundamental variables are matrices living on an octonionic or split-bioctonionic spinor space and evolving in Connes time. The corresponding framework is a version of trace dynamics: one writes a trace Lagrangian for matrix degrees of freedom and does *not* impose canonical commutators at the outset. Instead, ordinary quantum theory is expected to emerge in an appropriate thermodynamic limit.

This framework has led to a sequence of concrete phenomenological claims. The 2022 *Eur. Phys. J. Plus* paper [1] proposed a derivation of the low-energy fine-structure constant together with fermion mass ratios from Jordan eigenvalues and the octonionic state structure. The 2022 bosonic-Lagrangian paper [2] derived the weak mixing angle from a spinorial rotation between the (B, W_3) and (γ, Z^0) sectors. More recently, the visible gauge-sector relation $\alpha_s/\alpha_{\rm em} = 16$ has emerged as the missing piece that connects the earlier *Eur. Phys. J. Plus* fine-structure seed to the strong coupling.

The purpose of the present paper is to make that gauge-sector chain readable in one place. The emphasis is not on presenting new ultraviolet dynamics, but on giving a clear and pedagogical account of how the various ingredients fit together. In particular, we want to answer three questions.

1. Where does the factor 16 come from inside the visible bosonic sector?
2. How does one go from the earlier *Eur. Phys. J. Plus* seed [1] to separate formulas for $\alpha_s(M_Z)$ and $\alpha_{\rm em}(0)$?
3. Why is it legitimate to anchor the seed on the charge quantum $q_0 = 1/3$ even though the electron carries charge $1 = 3q_0$?

Roadmap. Section 2 recalls the trace-dynamics gauge-sector background and the earlier *Eur. Phys. J. Plus* seed [1]. Section 3 derives the broken-phase ratio $\alpha_s/\alpha_{\rm em} = 16$. Section 4 combines that ratio with the *Eur. Phys. J. Plus* seed to obtain the individual strong and electromagnetic couplings and explains why the present organization is conceptually cleaner than the earlier *Eur. Phys. J. Plus* bookkeeping [1]. Section 5 reviews the older weak-mixing-angle derivation [2]. Section 6 assembles the three Standard-Model gauge couplings and compares the numerical results with current reference values.

2. Trace-Dynamics Background and the Earlier *Eur. Phys. J. Plus* Seed

The gauge-sector story begins with the bosonic part of the trace-dynamics Lagrangian. In the notation of Ref. [2], one introduces bosonic and fermionic matrix variables Q_B and Q_F, and then writes the bosonic velocity in the form

$$\dot{Q}_B = \frac{1}{L}(i\alpha q_B + L\dot{q}_B). \tag{1}$$

Here L is a characteristic length scale and α is a dimensionless parameter generated when scale invariance is broken. At the schematic level, the visible bosonic block takes the form

$$\mathcal{L}_{\rm vis} \sim \frac{L_P^2}{2L^2}\,{\rm Tr}\Big(\dot{q}_B^\dagger + \frac{i\alpha}{L}q_B^\dagger\Big)\Big(\dot{q}_B + \frac{i\alpha}{L}q_B\Big). \tag{2}$$

When this is expanded, one finds three qualitatively different structures:

1. a pure visible block $q_B^\dagger q_B$, from which the photon and the gluons are identified;
2. a mixed block $q_B^\dagger \dot{q}_B$, from which the weak bosons are identified;
3. fermionic terms and mixed boson-fermion terms, not needed for the present note.

This split is important. In the present gauge-sector package, color and electromagnetism are read from the same visible block, whereas the weak bosons are associated with the mixed sector.

The earlier *Eur. Phys. J. Plus* paper [1] then identifies the dimensionless coefficient of the charged visible terms as

$$C \equiv \alpha^2\frac{L_P^4}{L^4}. \tag{3}$$

To avoid confusion with the gauge couplings α_s and $\alpha_{\rm em}$, we denote the earlier exponential seed by A rather than by α. In the *Eur. Phys. J. Plus* paper [1], the anti-down sector is used to define

$$\ln A = \lambda_{ad} q_{ad}, \qquad q_{ad} = \frac{1}{3}, \qquad \lambda_{ad} = \frac{1}{3} - \sqrt{\frac{3}{8}}, \tag{4}$$

so that

$$A = \exp\left[\frac{1}{3}\left(\frac{1}{3} - \sqrt{\frac{3}{8}}\right)\right]. \tag{5}$$

The same *Eur. Phys. J. Plus* paper [1] then arrives at the low-energy fine-structure expression

$$\alpha_{\rm em}^{\rm EPJP}(0) = A^2 \left(\frac{3}{32}\right)^2 = \frac{9}{1024} \exp\left[\frac{2}{3}\left(\frac{1}{3} - \sqrt{\frac{3}{8}}\right)\right]. \tag{6}$$

Numerically, this works remarkably well. Conceptually, however, the origin of the extra factor $1/16$ inside $(3/32)^2$ is not as transparent as it could be, because in the original *Eur. Phys. J. Plus* presentation [1] it is tied to a length-identification step. The aim of the present paper is to separate that bookkeeping into two cleaner pieces:

$$\left(\frac{3}{32}\right)^2 = \left(\frac{3}{8}\right)^2 \times \frac{1}{16}. \tag{7}$$

We will interpret $3/8$ as the primary charged-sector datum and $1/16$ as the visible broken-phase gauge normalization.

3. Broken-Phase Derivation of $\alpha_s/\alpha_{\rm em} = 16$

3.1. *The Standard Visible Charge-Trace Factor* $8/3$

Before introducing any octonionic support hypothesis, one first recalls a standard one-generation normalization. For visible matter with quark charges $2/3$ and $-1/3$ and charged-lepton charge -1, the electromagnetic charge trace is

$$k_{\rm em} = 3\left[\left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2\right] + 1 = \frac{8}{3}. \tag{8}$$

The factor 3 counts color, and the final $+1$ is precisely the charged-lepton contribution. If the visible bosonic sector carries a single Yang–Mills coupling g before symmetry breaking, then the canonically normalized abelian coupling at equal support is

$$e_0 = \frac{g}{\sqrt{k_{\rm em}}} = \sqrt{\frac{3}{8}}\, g, \tag{9}$$

so that

$$\frac{\alpha_s}{\alpha_{\rm em}^{(0)}} = \frac{g^2}{e_0^2} = \frac{8}{3}. \tag{10}$$

This $8/3$ factor is conventional. The new step is the broken-phase support factor.

3.2. *The Six Real Ladder Directions and the Support Factor* 6

The visible ladder operators employed in the octonionic construction are [2,3]

$$\alpha_1 = \frac{-e_5 + ie_4}{2}, \qquad \alpha_2 = \frac{-e_3 + ie_1}{2}, \qquad \alpha_3 = \frac{-e_6 + ie_2}{2}. \tag{11}$$

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They single out the real six-dimensional space

$$H_6 := \mathrm{span}_{\mathbb{R}}\{e_1, e_2, e_3, e_4, e_5, e_6\}. \tag{12}$$

In the visible bosonic block, the gluon representatives are built from antisymmetric bilinears involving exactly these six directions. This suggests treating H_6 as the relevant broken-phase support space for the visible gauge sector.

We now make the effective broken-phase hypothesis used throughout this paper.

Broken-phase support hypothesis. After symmetry breaking, the unbroken color sector is localized on one effective support direction of H_6, while the unbroken visible electromagnetic mode is the democratic trace direction on H_6.

Choose an orthonormal support basis $\chi_1, \ldots, \chi_6$ for H_6. The normalized abelian trace mode is then

$$\psi_{\mathrm{em}} = \frac{1}{\sqrt{6}} \sum_{A=1}^{6} \chi_A, \tag{13}$$

whereas a minimal localized color mode and the relevant matter zero mode may be taken as

$$\psi_c = \chi_1, \qquad \eta = \chi_1. \tag{14}$$

Modeling the effective couplings by support overlaps gives

$$g_s \propto g\,\langle \eta, \psi_c \rangle, \qquad e \propto e_0\,\langle \eta, \psi_{\mathrm{em}} \rangle. \tag{15}$$

Using Equations (13) and (14), one finds

$$\langle \eta, \psi_c \rangle = 1, \qquad \langle \eta, \psi_{\mathrm{em}} \rangle = \frac{1}{\sqrt{6}}, \tag{16}$$

so that

$$g_s = g, \qquad e = \frac{e_0}{\sqrt{6}}. \tag{17}$$

This is the origin of the support factor 6.

3.3. The Ratio 16

Combining Equations (9) and (17) gives

$$e = \sqrt{\frac{3}{8}}\,\frac{g}{\sqrt{6}} = \frac{g}{4}, \tag{18}$$

so that

$$\boxed{\frac{\alpha_s}{\alpha_{\mathrm{em}}} = 16.} \tag{19}$$

Equivalently,

$$\frac{e^2}{g^2} = \frac{3}{8} \times \frac{1}{6} = \frac{1}{16}. \tag{20}$$

The factor $3/8$ is the standard visible charge-trace normalization, and the factor $1/6$ is the broken-phase support dilution of the democratic electromagnetic mode.

It is important to be honest about the status of this result. The derivation of Equation (19) is *conditional*: the support factor 6 is not yet derived from a microscopic localization operator. What has been achieved is a precise and geometrically motivated broken-phase mechanism under which the factor 16 follows.

Comparison with experiment.

Using the current PDG/CODATA reference values [4–6],

$$\alpha_s(M_Z) = 0.1180 \pm 0.0009, \qquad \left[\hat{\alpha}^{(5)}(M_Z^2)\right]^{-1} = 127.930 \pm 0.008, \qquad \alpha^{-1}(0) = 137.035999177(21), \tag{21}$$

one obtains

$$\frac{\alpha_s(M_Z)}{\hat{\alpha}^{(5)}(M_Z^2)} = 15.096 \pm 0.115, \qquad \frac{\alpha_s(M_Z)}{\alpha(0)} = 16.170 \pm 0.123. \tag{22}$$

The first comparison is the conservative same-scale benchmark; the second is the mixed-scale comparison that turns out to be especially relevant in the present framework.

4. From the Earlier *Eur. Phys. J. Plus* Seed to $\alpha_s(M_Z)$ and $\alpha_{\rm em}(0)$

4.1. *The Charged-Sector Datum* 3/8

The present reorganization keeps the exponential seed A of the earlier *Eur. Phys. J. Plus* paper [1], but it interprets the geometric number 3/8 more directly. In the charged Furey-type first-generation states, two orthogonal components of magnitude 1/4 give a norm 1/8 per charged slot. Summing the three charged generation slots gives

$$\|\psi_{\rm charged}\|^2 = 3 \times \frac{1}{8} = \frac{3}{8}. \tag{23}$$

This is the number that now enters the color-sector seed.

4.2. *The Strong Coupling*

Using the earlier *Eur. Phys. J. Plus* seed A from Equation (5) together with the charged-sector datum (23), we define

$$\alpha_s^{\rm th}(M_Z) = A^2 \left(\frac{3}{8}\right)^2. \tag{24}$$

Hence

$$\boxed{\alpha_s^{\rm th}(M_Z) = \frac{9}{64} \exp\left[\frac{2}{3}\left(\frac{1}{3} - \sqrt{\frac{3}{8}}\right)\right] = 0.11675418.} \tag{25}$$

The corresponding QCD gauge coupling is

$$\boxed{g_3^{\rm th}(M_Z) = \sqrt{4\pi\alpha_s^{\rm th}(M_Z)} = 1.21127.} \tag{26}$$

4.3. *The Electromagnetic Coupling*

Once Equation (19) is available, the electromagnetic coupling follows immediately:

$$\alpha_{\rm em}^{\rm th}(0) = \frac{1}{16}\,\alpha_s^{\rm th}(M_Z). \tag{27}$$

Substituting Equation (25) gives

$$\boxed{\alpha_{\rm em}^{\rm th}(0) = \frac{9}{1024} \exp\left[\frac{2}{3}\left(\frac{1}{3} - \sqrt{\frac{3}{8}}\right)\right] = 0.00729713629.} \tag{28}$$

Equivalently,

$$\boxed{\alpha_{\rm em}^{\rm th}(0)^{-1} = 137.04006064, \qquad e^{\rm th}(0) = \sqrt{4\pi\alpha_{\rm em}^{\rm th}(0)} = 0.30281763.} \tag{29}$$

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Thus the earlier *Eur. Phys. J. Plus* formula (6) is recovered exactly, but now as the end of a more structured chain:

$$\text{primitive seed } A \longrightarrow \alpha_s(M_Z) \longrightarrow \alpha_{\text{em}}(0) = \frac{1}{16}\alpha_s(M_Z).$$

4.4. Why the Present Electromagnetic Derivation is Conceptually Sharper

This reorganization makes the electromagnetic derivation more credible than in the earlier *Eur. Phys. J. Plus* presentation [1] in a specific and limited sense. In the *Eur. Phys. J. Plus* paper [1], the numerical factor $1/16$ is packaged inside the step $L_P^2/L^2 = 3/32$, so that the fine-structure constant is obtained directly as $A^2(3/32)^2$. The numerical result is excellent, but the gauge-sector origin of the extra factor is not manifest.

In the present paper the bookkeeping is separated into two distinct ingredients:

$$\alpha_s^{\text{th}}(M_Z) = A^2\left(\frac{3}{8}\right)^2, \qquad \alpha_{\text{em}}^{\text{th}}(0) = \frac{1}{16}\alpha_s^{\text{th}}(M_Z). \tag{30}$$

This is conceptually cleaner for three reasons.

1. The primary charged-sector datum remains $3/8$, with no hidden extra factor.
2. The factor $1/16$ is derived from the visible broken-phase gauge structure, namely the combination $8/3 \times 6$.
3. The charged-lepton contribution is explicit, because the $+1$ in $k_{\text{em}} = 8/3$ is precisely the electron/charged-lepton term.

So "more credible" does *not* mean that all microscopic questions are solved. The support factor 6 is still a hypothesis. What it means is that the earlier numerical success of the *Eur. Phys. J. Plus* formula [1] is now attached to a cleaner and more transparent gauge-sector interpretation.

5. Why the Seed Uses $q_0 = 1/3$ Although the Electron Has Charge 1

A natural concern is the following. If the electric charge of the electron is three times that of the down quark, why should the seed be tied to $q_0 = 1/3$ rather than to 1?

The answer is that the seed is not attached to a particular particle species. It is attached to the *minimal visible charge quantum*. In the octonionic/Furey construction, the charge operator is normalized as [3]

$$Q = \frac{N}{3}, \tag{31}$$

so the visible charge lattice is

$$Q \in \left\{0, \frac{1}{3}, \frac{2}{3}, 1\right\}. \tag{32}$$

The electron therefore does not carry a different primitive charge; it carries three quanta of the same basic unit $q_0 = 1/3$.

Just as importantly, the electron is not left out of the derivation. Its contribution enters explicitly in the electromagnetic trace factor

$$k_{\text{em}} = 3\left[\left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2\right] + 1 = \frac{8}{3}, \tag{33}$$

where the final $+1$ is the charged-lepton contribution. The logic is therefore split between two levels:

1. the primitive algebraic seed is fixed by the minimal nonzero charge quantum $q_0 = 1/3$;
2. the fact that the electron has charge $1 = 3q_0$ enters later through the electromagnetic normalization already used in deriving the factor 16.

This is why using $q_0 = 1/3$ is not in conflict with the existence of the unit-charged electron.

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Conceptual takeaway. The exponential seed is attached to the minimal visible charge quantum $q_0 = 1/3$, not to a specific observed particle species. The electron, with charge $1 = 3q_0$, is already taken into account through the electromagnetic charge trace $k_{\rm em} = 8/3$ that enters the derivation of the factor 16.

6. Review of the Weak Mixing Angle Derivation

The derivation of the weak mixing angle belongs to the 2022 bosonic-Lagrangian paper [2], not to the present gauge-ratio mechanism. Since the present paper aims to be self-contained, it is useful to summarize that argument briefly.

The starting point is the claim that the transformation from (B, W_3) to (γ, Z^0) is a rotation on the *spinorial* octonionic gauge space. Because this is a spinorial space, the physical gauge-space angle is twice the octonionic one. Thus one writes [2]

$$\begin{pmatrix} \gamma \\ Z^0 \end{pmatrix} = \begin{pmatrix} \cos(\theta_W/2) & \sin(\theta_W/2) \\ -\sin(\theta_W/2) & \cos(\theta_W/2) \end{pmatrix} \begin{pmatrix} B \\ W_3 \end{pmatrix}. \tag{34}$$

In the asymptotic flat limit $q_B \to kL_P$, the antisymmetric Z^0 contribution is taken to vanish, and one obtains

$$B = Q^2 \cos(\theta_W/2), \qquad W_3 = Q^2 \sin(\theta_W/2), \tag{35}$$

where Q^2 is the fine-structure coupling in that limit.

The next step is to use the standard electroweak relation

$$Q = \frac{Y_W}{2} + T_3, \tag{36}$$

together with the additional identifications

$$B \propto Y_W^2, \qquad W_3 \propto T_3^2. \tag{37}$$

These imply

$$Y_W \to Q\sqrt{\cos(\theta_W/2)}, \qquad T_3 \to Q\sqrt{\sin(\theta_W/2)}. \tag{38}$$

Substituting Equation (38) into Equation (36) gives the governing trigonometric equation

$$1 = \frac{\sqrt{\cos(\theta_W/2)}}{2} + \sqrt{\sin(\theta_W/2)}. \tag{39}$$

The nontrivial solution is

$$\theta_W^{\rm th} = 29.98^\circ, \qquad \boxed{\sin^2\theta_W^{\rm th} = 0.24969776.} \tag{40}$$

This derivation is elegant, but it is not on exactly the same footing as the strong and electromagnetic chain. Beyond the half-angle identification, it uses the asymptotic flat limit, drops the antisymmetric Z^0 term, and assumes the quadratic identifications in Equation (37). In addition, the weak mixing angle is quoted experimentally in several different schemes, so any numerical comparison is inherently more delicate. For that reason the weak-angle part of the present paper should be read as a brief review of Ref. [2], not as the most rigid part of the gauge-sector package.

7. The Three Standard-Model Gauge Couplings and comparison with Experiment

Once Equations (25), (28), and (40) are in hand, the three Standard-Model gauge couplings follow as

$$g_3^{\rm th}(M_Z) = \sqrt{4\pi\alpha_s^{\rm th}(M_Z)} = 1.21127, \qquad e^{\rm th}(0) = \sqrt{4\pi\alpha_{\rm em}^{\rm th}(0)} = 0.30281763, \tag{41}$$

$$g_2^{\rm th} = \frac{e^{\rm th}(0)}{\sin\theta_W^{\rm th}} = 0.60600, \qquad g_Y^{\rm th} = \frac{e^{\rm th}(0)}{\cos\theta_W^{\rm th}} = 0.34959. \tag{42}$$

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The first two numbers, g_3 and e, are tied directly to the closed-form formulas above. The latter two inherit the more tentative status of the weak-angle derivation and should therefore be interpreted with the same caution.

Strong and electromagnetic sectors

Using the current PDG and NIST/CODATA reference values [4–6],

$$\begin{aligned} \alpha_s(M_Z) &= 0.1180 \pm 0.0009, \\ \alpha_{\rm em}^{-1}(0) &= 137.035999177(21), \\ \alpha_{\rm em}(0) &= 0.0072973525643(11), \\ \hat{\alpha}^{(5)}(M_Z^2)^{-1} &= 127.930 \pm 0.008, \end{aligned}$$

we obtain the comparison shown in Table 1. The experimental value for $g_3(M_Z)$ inferred from the PDG world average is 1.21772 ± 0.00464.

Table 1. Strong and electromagnetic sectors versus experiment.

Quantity	Theory	Experiment	Deviation
$\alpha_s(M_Z)$	0.11675418	0.1180 ± 0.0009	−1.06%
$g_3(M_Z)$	1.21127	1.21772 ± 0.00464	−0.53%
$\alpha_{\rm em}(0)$	0.00729713629	0.0072973525643(11)	−0.002964%
$\alpha_{\rm em}^{-1}(0)$	137.04006064	137.035999177(21)	+0.002964%
$e(0)$	0.30281763	0.30282212	−0.00148%
$\alpha_s(M_Z)/\hat{\alpha}^{(5)}(M_Z^2)$	16.0000	15.096 ± 0.115	+5.99%
$\alpha_s(M_Z)/\alpha_{\rm em}(0)$	16.0000	16.170 ± 0.123	−1.05%

Weak-Angle Benchmarks

Because the weak mixing angle is scheme dependent, we compare Equation (40) with a small set of standard reference values rather than with a single unique experimental number. The current PDG electroweak review quotes [5]

$$\sin^2\theta_W^{\rm on-shell} = 0.22342 \pm 0.00009, \qquad \hat{s}_Z^2 = 0.23122 \pm 0.00006, \qquad \sin^2\theta_{\ell,\rm eff} = 0.23148 \pm 0.00013. \tag{43}$$

As a low-Q^2 benchmark we also quote the SLAC E158 determination [7],

$$\sin^2\theta_W^{\rm eff}(Q^2 = 0.026\,{\rm GeV}^2) = 0.2397 \pm 0.0013. \tag{44}$$

The comparison is summarized in Table 2.

Table 2. Weak-mixing-angle benchmarks versus the octonionic value $\sin^2\theta_W^{\rm th} = 0.24969776$.

Benchmark	Experiment	Deviation
On-shell s_W^2	0.22342 ± 0.00009	+11.76%
$\overline{\rm MS}$ quantity $\hat{s}_Z^2$	0.23122 ± 0.00006	+7.99%
Effective leptonic angle $\sin^2\theta_{\ell,\rm eff}$	0.23148 ± 0.00013	+7.87%
SLAC E158 low-Q^2 effective angle	0.2397 ± 0.0013	+4.17%

The pattern is clear. The strong and electromagnetic sectors are numerically successful at the percent level or better, while the weak-angle result is substantially more tentative. It lies well above the standard on-shell and collider-effective values, though it comes somewhat closer to the low-Q^2 benchmark.

8. Summary

The present note reorganizes the gauge sector of the octonionic framework into a single self-contained chain.

1. Starting from a common visible Yang–Mills coupling in the trace-dynamics Lagrangian, the standard visible normalization 8/3 combines with a six-direction support factor to give the broken-phase relation
$$\frac{\alpha_s}{\alpha_{\rm em}} = 16.$$
2. The earlier *Eur. Phys. J. Plus* seed [1] and the charged-sector datum 3/8 then give
$$\alpha_s(M_Z) = \frac{9}{64}\exp\left[\frac{2}{3}\left(\frac{1}{3} - \sqrt{\frac{3}{8}}\right)\right], \qquad \alpha_{\rm em}(0) = \frac{9}{1024}\exp\left[\frac{2}{3}\left(\frac{1}{3} - \sqrt{\frac{3}{8}}\right)\right].$$
3. The present organization is conceptually sharper than the earlier *Eur. Phys. J. Plus* bookkeeping [1], because the factor 1/16 is no longer hidden in a length-identification step but is derived directly from the visible broken-phase gauge structure.
4. The use of the primitive charge quantum $q_0 = 1/3$ is consistent because it fixes the algebraic seed, whereas the charged-lepton contribution enters explicitly through the electromagnetic charge trace $k_{\rm em} = 8/3$.
5. The older weak-angle derivation [2] can be summarized compactly, but it remains the least settled part of the package, both conceptually and phenomenologically.

What remains open is the microscopic origin of the support factor 6, i.e. the dynamical reason why the broken visible electromagnetic mode should be democratic on H_6 while the color mode remains localized. Until that step is derived, the result should be read as a concrete broken-phase mechanism together with its phenomenological continuation.

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