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Article

Strictly Positive Functionals in Orlicz Spaces

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Abstract

In this paper, we do prove how to define strictly positive functionals in dual pairs of Orlicz Spaces. These Orlicz Spaces are endowed with the pointwise partial ordering. The Young functions that imply the definition of these dual pairs is significant for the difference between them. These strictly positive functionals imply that the No -Arbitrage pricing functionals in Incomplete Markets are well -defined. Orlicz Spaces are considered in Mathematical Finance, since they are related to form of the Utility Functions for the individuals -investors, as an Expected Utility Function.

Keywords: strictly positive functionals; asset pricing; orlicz spaces; expected utility

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1. Introduction

The recent evolution of the Asset Pricing Theory is established in the following Conditions, which may be summarized in the following way: In Frittelli (2004) *No -Market -Free Lunch* condition about pricing functionals was used to prove equivalence between *No-Free Lunch*, and the existence of equivalent (local) martingale measures $\mathbb{Q}$. In Klein (2004), Orlicz spaces define the property of No Market Free Lunch in L^∞, in order to deduce the equivalence between Kreps No-Free Lunch and the existence of equivalent (local) martingale measures $\mathbb{Q}$, relying on the topological results from Kusuoka (1993). We call Kreps No -Free Lunch the No-Free Luch condition as it appears in Delbaen and Schachermayer (2006), due to the seminal work of Kreps (1981). Here, we examine some separate families of Young functions in order to produce similar theorems, which are valid for both of dual pairs $\langle M^\Phi, L^\Psi \rangle$ and $\langle L^\Phi, L^\Phi \rangle$ of Orlicz spaces.

We denote by E^*, the norm -dual of some E being a norm -space. All of these spaces are subspaces of the $L^0(\Omega, \mathcal{F}, \mathbb{P})$. The last space is the following one :

$$L^0(\Omega, \mathcal{F}, \mathbb{P}) = \{X | X : \Omega \to \mathbb{R}\},$$

where ay X is $\mathcal{F}$-measurable. The probability space $(\Omega, \mathcal{F}, \mathbb{P})$ is such that $\mathbb{P}$ is non -atomic and complete. All of the spaces are partially ordered by the pointwise partial ordering, namely $X \geq Y$, if and only if $X(\omega) \geq Y(\omega), \mathbb{P}$ almost surely. E^*_+ denotes the positive cone of any such space E^* mentioned in this paper, namely $E^0_+ = \{f \in E^* | f(X) \geq 0\}$ for any $X \in E_+$. E_+ is the cone of the $\mathbb{P}$ -almost surely positive elements of E. The Orlicz space M^Φ, is the dual space of the Orlicz space L^Ψ, where Ψ is some N-Young function and Φ, Ψ are conjugate functions. Namely, Ψ is a N-function, then $(L^\Psi)^* = M^\Phi$. Let us suppose that L is a non-trivial vector space. K is supposed to be a non-empty set of L and $K \neq \{0\}$. The real number $f(x)$ whenever mentioned below, is the value of $x \in L$ under some linear functional f. K is a *wedge* of L, if $K + K \subseteq K$, $\lambda \cdot K = \{\lambda \cdot k, k \in K\} \subseteq K$, for any $\lambda \in \mathbb{R}_+$. $t \cdot x$ denotes the scalar product between any vector x of L and some real number t. K is a *cone* of L, if it is a wedge and moreover $K \cap (-K) = \{0\}$, where $0 \in L$ is the zero element of L.

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The Appendix contains some basic properties of Orlicz Spaces. About separation theorems and rest topological issues, the reader may look in Aliprantis and Border (2006).

The main issue of the paper is to prove the existence of *strictly positive functionals* in the case of the associate dual pairs of Orlicz Spaces. Strictly positive functionals are related to the definition of the Radon -Nikodym derivatives of some (local) martingale probability measure $\mathbb{Q}$ with respect to the nature probability measure $\mathbb{P}$. In general, strictly positive functionals are the pricing vectors in some commodity -price dual pair $\langle E, E^* \rangle$, where E is the commodity space and E^* is the price space. The reader may look in Aliprantis et al. (1990) which is a seminal reference about related notions, which are in general common in Mathematical Finance. If E is a partially ordered linear space, whose positive cone is E_+ the set of strictly positive linear functionals is the following subset E' of $E^* : f \in E'$ if and only if $f(X) > 0$ for any $X \in E_+ \setminus \{0\}$.

1.1. Financial Framework

As it is considered in (Delbaen and Schachermayer 2006, Ch.5), we assume a $\mathbb{R}^{d+1}$- valued stochastic process, based on and adapted to the filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F})_{t\geq 0}, \mathbb{P})$. The zero coordinate of this process S^0, denotes the risk -free asset and it is normalized to $S^0_t = 1, t \geq 0$. S is *locally bounded*, namely there exists a sequence of stopping times $\tau_n, n \in \mathbb{N}$, increasing to ∞, such that the stopped processes $S^{\tau_n}_t = S_{t\wedge\tau_n}$ are *uniformly bounded*. This corresponds to the fact that S is assumed to be a *cádlág* process with uniformly bounded jumps. Also, a standrard assumption about the filtration is that it is right continuous, and $\mathcal{F}_0$ contains the null sets of $\mathcal{F}_\infty$. Also, a simple trading strategy is defined as follows -see also (Delbaen and Schachermayer 2006, Def.5.1.1):

Definition 1.1. *A* ***simple trading strategy*** *H for a locally bounded stochastic process is an $\mathbb{R}^d$-valued stochastic process $H = (H_t)_{t\geq 0}$, being of the form*

$$H = \sum_{i=1}^{n} h_i I_{(\tau_{i-1},\tau_i]},$$

where $0 = \tau_0 \leq \tau_1 \leq ... \leq \tau_n < \infty$ are finite stopping times and h_i are $\mathcal{F}_{\tau_{i-1}}$-measurable, $\mathbb{R}^d$ -valued functions. The associated ***stochastic integral*** *$H \cdot S$ is the stochastic process*

$$(H \cdot S)_t = \sum_{i=1}^{n} (h_i, S_{\tau_i\wedge t} - S_{\tau_{i-1}\wedge t}) = \sum_{i=1}^{n}\sum_{j=1}^{d} (h^j_i, S^j_{\tau_i\wedge t} - S^j_{\tau_{i-1}\wedge t}), 0 \leq t \leq \infty.$$

Its terminal value is the random variable

$$(H \cdot S)_\infty = \sum_{i=1}^{n} h_i (S_{\tau_i} - S_{\tau_{i-1}}).$$

Definition 1.2. *H is* ***admissible****, if the functions h_i and stopped process S^{τ_n} are uniformly bounded.*

For the next definition, see also (Delbaen and Schachermayer 2006, Def.5.1.2):

Definition 1.3. *A probability measure $\mathbb{Q}$ on $\mathcal{F}$, which is equivalent (absolutely continuous) with respect to $\mathbb{P}$ is called an equivalent (absolutely continuous) local martingale measure, if S is a local martingale under $\mathbb{Q}$. We denote by $\mathcal{M}^e(S)$ the set of all such measures, and we say that S satisfies the condition of the existence of an equivalent local martingale measure (EMM), if $\mathcal{M}^e(S) \neq \emptyset$.*

For the next Lemma, see also (Delbaen and Schachermayer 2006, Lem.5.1.3):

Lemma 1.4. *Let $\mathbb{Q}$ be a probability measure on $\mathcal{F}$, which is absolutely continuous with respect to $\mathbb{P}$. A locally bounded stochastic process S is a local martingale with respect to $\mathbb{Q}$, if and only if*

$$\mathbb{E}_{\mathbb{Q}}((H \cdot S)_\infty) = 0,$$

for any admissible simple trading strategy H.

Having this Lemma in mind, we define the following subspace of $L^\infty(\Omega, \mathcal{F}, \mathbb{P})$:

Definition 1.5.

$$K^s = \{(H \cdot S)_\infty | H : simple, admissible\}$$

and by C^s the following convex cone of $L^\infty(\Omega, \mathcal{F}, \mathbb{P})$:

Definition 1.6. $C^s = K^s - L^\infty_+ = \{f - k | f \in K^s, k \in L^\infty, k \geq 0, \mathbb{P} - a.e.\}$

Compare the following Definition and (Delbaen and Schachermayer 2006, Def.5.1.4):

Definition 1.7. *S satisfies the* **No-Arbitrage** *condition* (NA^s) *with respect to simple trading strategies if*

$$K^s \cap L^\infty_+(\Omega, \mathcal{F}, \mathbb{P}) = \{0\},$$

(or equivalently

$$C^s \cap L^\infty_+(\Omega, \mathcal{F}, \mathbb{P}) = \{0\}).$$

Also, compare the following Proposition and (Delbaen and Schachermayer 2006, Pr.5.1.7):

Proposition 1.8. *The condition (EMM) of xistence of an equivalent local martingale measure implies the condition* (NA^s) *of no-arbitrage with respect to simple trading strategies, but not vice versa.*

Moreover, compare the following Definition and (Delbaen and Schachermayer 2006, Def.5.2.1). Also see Kreps (1981):

Definition 1.9. *S satisfies the condition of* **No free lunch** *(NFL), if the closure* $\overline{C}$ *of* C^s*, taken with respect to the weak-star topology of* $L^\infty(\Omega, \mathcal{F}, \mathbb{P})$ *satisfies:*

$$\overline{C} \cap L^\infty_+(\Omega, \mathcal{F}, \mathbb{P}) = \{0\}.$$

The consequence of NFL Condition is the Kreps-Yan Theorem
(compare with (Delbaen and Schachermayer 2006, Th.5.2.2)):

Theorem 1.10. *A locally bounded stochastic process S satisfies the condition of (NFL), if and only if the condition (EMM) of the existence of an equivalent local martingale measure is satisfied.*

The above Definitions of K^s, C^s, remind us of the space of stochastic processes mentioned in Jaschke and Küchler (2001). Specifically, the associated linear space L_{sm} of the stochastic processs, mentioned above, may be replaced by the following linear space of stochastic processs:

$$x(t, \omega) = \sum_{i=0}^{n} x_i I_{E_i}(\omega) I_{\tau(\omega),T]}(t),$$

denotes the space of the *simple, adapted processes* on the propability space $(\Omega, \mathcal{F}, \mathbb{P})$. The meaning of x is that such a *cash -stream* pays x_i if E_i occurs, at the time τ_i. The subspace K^s is a *natural* generalization of the elements of L_{sm} from the aspect of the extension of the time-horizon to the infinity and the extension of the payment elements x_i on bounded random variables, in the sense of L^∞. The above space odf stochastic processes may also considered to be the one, which arises in the case where x_i lie in some Orlicz Space.

2. Results on the Strictly Positive Functionals in Orlicz Spaces

Theorem 2.1. *Let* Ψ *is some N-function and* $\langle M^\Phi, L^\Psi \rangle$ *is the commodity-price duality, and K is the subspace of the replicated contingent claims, satisfying the property of No-Free Lunches:*

$$\overline{(K - M^\Phi_+)}^{w^*} \cap M^\Phi_+ = \{0\}.$$

This implies the existence of some $f_0 \in L^\Psi$*, such that* $f_0(x) > 0, x \in M^\Phi_+ \setminus \{0\}$.

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Proof.

$$\left\{f \in M^{\Phi} \mid f \in [0, \mathbf{1}]\right\}$$

is a weak-star compact set of M^{Φ} since it is convex, closed and bounded with respect to any norm being posed on M^{Φ}, Let δ_n be a sequence of real numbers, such that $0 < \delta_n < 1$, and we apply the separation theorem between the convex sets $C = \overline{(K - M^{\Phi}_{+})}^{w^*}$ and

$$B_{\delta_n} = \left\{f \in M^{\Phi} \mid \mathbb{E}(f) \geq \delta_n\right\}.$$

For these sets we have that $C \cap B_{\delta_n} = \emptyset$. Then there exists some $g_n \in L^{\Psi}$ such that

$$g_n(c) \leq 0 \leq \epsilon \leq g_n(f),$$

where $f \in B_{\delta_n}, c \in C$. If $a_n > 0, n \in \mathbb{N}$ such that $\sum_{n=1}^{\infty} a_n = 1$, while $\|g_n\| = b_n$, we take the functionals $\tilde{g_n} = \frac{1}{b_n} g_n$. If $\sum_{n=1}^{\infty} \frac{a_n}{b_n} < \infty$, then

$$\sum_{n=1}^{\infty} a_n \tilde{g_n} = f_0. \quad \square$$

The following Corollary arises from the fact that $(L^{\Phi})^* = L^{\Phi}$, if Φ is both Δ_2 and N function.

Corollary 2.2. *Let Φ is some Δ_2-function and $\langle L^{\Phi}, L^{\Phi}\rangle$ is the commodity-price duality, and K is the subspace of the replicated contingent claims, satisfying the property of No-Free Lunches:*

$$\overline{(K - L^{\Phi}_{+})}^{w^*} \cap L^{\Phi}_{+} = \{0\}.$$

This implies the existence of some $f_0 \in L^{\Phi}$, such that $f_0(x) > 0, x \in M^{\Phi}_{+} \setminus \{0\}$.

Proof.

$$\left\{f \in L^{\Phi} \mid f \in [0, \mathbf{1}]\right\}$$

is a weak-star compact set of M^{Φ} since it is convex, closed and bounded with respect to any norm being posed on M^{Φ}, Let δ_n be a sequence of real numbers, such that $0 < \delta_n < 1$, and we apply the separation theorem between the convex sets $C = \overline{(K - L^{\Phi}_{+})}^{w^*}$ and

$$B_{\delta_n} = \left\{f \in L^{\Phi} \mid \mathbb{E}(f) \geq \delta_n\right\}.$$

For these sets we have that

$$C \cap B_{\delta_n} = \emptyset.$$

Then there exists some $g_n \in L^{\Phi}$ such that

$$g_n(c) \leq 0 \leq \epsilon \leq g_n(f),$$

where $f \in B_{\delta_n}, c \in C$. If $a_n > 0, n \in \mathbb{N}$ such that $\sum_{n=1}^{\infty} a_n = 1$, while $\|g_n\| = b_n$, we take the functionals $\tilde{g_n} = \frac{1}{b_n} g_n$. If $\sum_{n=1}^{\infty} \frac{a_n}{b_n} < \infty$, then

$$\sum_{n=1}^{\infty} a_n \tilde{g_n} = f_0\,. \quad \square$$

Namely, we obtain the following theorem:

Theorem 2.3. *Either Ψ is some N-function and $\langle M^{\Phi}, L^{\Psi}\rangle$ is the commodity-price duality, or Φ is some Δ_2-function and $\langle L^{\Phi}, L^{\Phi}\rangle$ is the commodity-price duality, the existence of f_0 implies the existence of a probability measure $\mathbb{Q}$ such that $\frac{d\mathbb{Q}}{d\mathbb{P}} = f_0$, such that $C \cap E^*_{+} = \{0\}$, where $C = \overline{(K - E^*_{+})}^{\sigma(E^*, E)}$, $E^* = M^{\Phi}$ if Ψ is some*

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N-function, or $E^* = L^\Phi$, if Φ is some Δ_2-function. Conversely, given an equivalent probability measure to $\mathbb{P}$, defined on $(\Omega, \mathcal{F})$, such that

$$\frac{d\mathbb{Q}}{d\mathbb{P}} \in E_+ \setminus \{0\},$$

where $E = L^\Psi$ if Ψ is some N-function, or $E = L^\Phi$, if Φ is some Δ_2-function, then $C = \{x \in E \mid \mathbb{E}_{\mathbb{Q}}(x) \leq 0\}$ is $\sigma(E^*, E)$-closed and satisfies $C \cap E^*_+ = \{0\}$.

Proof. Let $(\Omega,\ \mathcal{F},\ \mathbb{P})$ be the initial probability space. We define:

$$\mathbb{Q}(A) = \int_A f_0 d\mathbb{P}, A \in \mathcal{F}.$$

The converse is alike to the (Delbaen and Schachermayer 2006, Th. 5.2.3). □

The class of Orlicz spaces $\{L^\Phi\}$, whose *quasi-interior* of the cone L^Φ_+ in a normed linear space L^Φ is non-empty, coincides with the class of the spaces $\{L^\Phi\}$, such that the space L^Φ is *separable*, the cone L^Φ_+ is closed and holds $(L^\Phi)^* = L^\Phi$. If Φ obtains the Δ_2-property, then $(L^\Phi)^* = L^\Phi$ (see (Krasnoselski and Sobolev 1955, Th. 1)). Some $x \in L_+$, where L is a partially ordered linear space by the cone L_+ is a *quasi-interior* point of the cone L_+ if $L = \overline{I}_x$, where closure is obtained by the norm topology and $I_x = \cup_{n=1}^{\{} infty][-n \cdot x, n \cdot x]$. The order interval $[a, b]$ is actually the subset $(b - L_+) \cap (a + L_+)$, where $a, b \in L$. By $t \cdot y$, we denote the scalar product between $y \in L$ and $t \in \mathbb{R}$. The subspace I_x of L defined in the associated way is usually called the *solid suspace* of L generated by x.

These spaces are reflexive if $\Phi \in \Delta_2 \cap \nabla_2$, and the above theorems are valid for weak topology, since weak star and weak topology coincide. This reflexivity point is of particular interest, since it provides the above difference between the dual pairs. The entire properties of Orlicz Spaces are presented in Rao and Ren (1991),

3. Some Further Implications of Study

The following two propositions imply a relation between the Orlicz Spaces are the $L^p(\Omega, \mathcal{F}, \mathbb{P})$ spaces.

Proposition 3.1. *We suppose that Φ is a Young function, such that $L^\Phi \neq \{0\}$. Then, if $X \in L^\Phi_+ \setminus \{0\}$, then $X \in L^1(\Omega, \mathcal{F}, \mathbb{P})_+ \setminus \{0\}$.*

Proof. If $X \in L^\Phi_+$, then by applying Jensen's Inequality, $\Phi(\mathbb{E}(X)) \leq \mathbb{E}(\Phi(X)) < \infty$. Thus, $\mathbb{E}(X) < \infty$, because since there exists some sequence $(X_n)_{n\in\mathbb{N}}$ of step functions, such that $\mathbb{E}(X_n) < \infty$ and $X_n \to X$, $\mathbb{P} - a.e.$ pointwise, if $a_n = \lim_{n\in\mathbb{N}} \mathbb{E}(X_n) = +\infty$, then since Φ is a Young function, we would have that $\lim_{n\in\mathbb{N}} \Phi(a_n) = +\infty$. But in this case, the Jensen's Inequality is violated, which is a contradiction. □

The Definition of an *Orlicz Heart* M^Φ is mentioned in the Appendix. It is in general a convex set. Any such set generates the following cone :

Definition 3.2. *Since M^Φ is a convex set of L^Φ.*

$$K^\Phi = cone(M^\Phi) := \{t \cdot X | X \in M^\Phi, t \in \mathbb{R}_+\},$$

*is called **cone generated by** M^Φ.*

Proposition 3.3. *If $X \in L^\Phi_+ \setminus \{0\}$, then $\Phi(\|X\|_1) \leq \|X\|_\Phi$, if we consider that L^Φ is a normed linear space, where $\|.\|_\Phi$ is the Luxemburg norm and M^Φ is the corresponding Orlicz Heart.*

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Proof. Since $X \in M^{\Phi}$, then for any $c > 0$, $\mathbb{E}(\Phi(c \cdot X)) < \infty$. Then the set

$$\{\lambda > 0 \ : \ \mathbb{E}_{\mathbb{P}}\left[\Phi\left(\frac{X}{\lambda}\right)\right] \leq 1\},$$

is non-empty. □

As a mattet of fact, the cones of the Orlicz Spaces which are naturally asscociated with them through the pointwise partial ordering, may be considered as cones of admissible contingent claims in some $L^p(\Omega, \mathcal{F}, \mathbb{P})$ space. This kind of embedding may be studied in a more detailed way, associated with the No- Arbitrage and No free lunch properties. This may be considered as a direction of further study in this topic.

4. Appendix

We call *Young function* any convex, even, continuous function Φ satisfing the relations $\Phi(0) = 0$, $\Phi(-x) = \Phi(x) \geq 0$ for any $x \in \mathbb{R}$ and

$$\lim_{x \to \infty} \Phi(x) = \infty \, .$$

The *conjugate function* of Φ is defined by

$$\Psi(y) = \sup_{x \geq 0} \{xy - \Phi(x)\} \, , \qquad \forall \, y \geq 0 \, .$$

For any Young function Φ, let us denote by L^{Φ} the following linear space, called *Orlicz space*

$$\left\{X \in L^0(\Omega, \, \mathcal{F}, \, \mathbb{P}) \mid \mathbb{E}_{\mathbb{P}}[\Phi(c\, X)] < \infty, \text{ for some } c > 0\right\} .$$

Any Orlicz space L^{Φ} admits two equivalent norms: The first, known as *Luxemburg norm* of X, is given by:

$$\|X\|_{\Phi} = \inf\left\{\lambda > 0 \ : \ \mathbb{E}_{\mathbb{P}}\left[\Phi\left(\frac{X}{\lambda}\right)\right] \leq 1\right\},$$

and the second, known as *Orlicz norm*, is defined by L^{Ψ} as follows:

$$\|X\|^*_{\Phi} = \sup\{\mathbb{E}_{\mathbb{P}}[XY] \mid \|Y\|_{\Psi} \leq 1\} \, .$$

For both norms, the point-wise partial ordering $\geq$-$\mathbb{P} - a.s.$ implies that the space L^{Φ} is a Banach lattice.

For some Orlicz space L^{Φ} given by the Young function Φ, the associated *Orlicz heart* M^{Φ} of the L^{Φ} is defined:

$$M^{\Phi} := \left\{X \in L^0(\Omega, \, \mathcal{F}, \, \mathbb{P}) \mid \mathbb{E}_{\mathbb{P}}[\Phi(c\, X)] < \infty \, , \quad \forall \, c > 0\right\},$$

and then we have the dual pair of Orlicz spaces, as $\langle M^{\Phi}, L^{\Psi} \rangle$

Let us denote $L^{\Phi}_{+} = \{X \in L^{\Phi} \mid X \geq 0\}$ the positive cone of L^{Φ}.

Definition 4.1 (Krasnoselski)**.** *We call N-Young function, a Young function Φ defined on $\mathbb{R}$, which satisfies the conditions:*

1.

$$\lim_{x \to 0} \frac{\Phi(x)}{x} = 0 \, ,$$

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2.

$$\lim_{x\to\infty} \frac{\Phi(x)}{x} = \infty\,,$$

3. *The relation* $\Phi(x) = 0$ *implies* $x = 0$.

Definition 4.2. *We say that a Young function* Φ *which satisfies the* Δ_2*-property if there exist a constant* $k > 0$ *and a* $x_0 \in \mathbb{R}$ *such that holds*

$$\Phi(2x) \leq k\Phi(x)\,, \quad \forall\, x \geq x_0\,.$$

Let us mention some examples of Young functions: $\Phi_0(x) = |x|$ is a Young function. $\Phi_1 = \frac{1}{2}|x|^2$ is a Young function, which satisfies both N and Δ_2 properties. If we would like to specify some Young function which is not of the type of $\Phi_p(x) = \frac{1}{p}|x|^p, p > 1$ and satisfies both N properties and Δ_2 properties, then we may mention $\Phi_\ell(x) = (1+|x|)log(1+|x|) - |x|$. Then, due to this example we show that there exist Orlicz spaces, which are different from L^p spaces, generated from Φ_1, Φ_p.

About the class ∇_2 of Young functions, see (Rao and Ren 1991, p.22): A Young function Φ is a ∇_2 -Young function, if

$$\Phi(x) \leq \frac{1}{2g}\Phi(x), x \geq x_0 > 0$$

for some $g > 1$. x_0 may be equal to zero. An example of ∇_2 Young function is the conjugate of Φ_ℓ, which is the function $\Psi(x) = e^{|x|} - |x| - 1$. h "A"—e.g., Figure A1, Figure A2, etc.

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