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Temporal Efects of Surface Plasmon Polaritons in a Quantum Plasma Slab

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Abstract

Temporal effects associated with surface plasmon polaritons (SPP) in a quantum slab of a plasma material, such as a thin film of metal, semiconductor or a graphene plate, where the quantum dispersion effects, and in particular exchanges effects, are retained.

Keywords: plasmons; polaritons; time-refraction; surface waves; quantum materials

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1. Introduction

In recent years there has been an increasing interest on temporal optical processes, not only using conventional optics, but also new optical configurations and materials [1–4]. The concepts of time-reflection and time-refraction [5] play a central role in this area of optics.

Of particular relevance to this research activity is the rise of Plasmonics as a new optical branch [6], and the importance of Metamaterials in the field of spatio-temporal optics [7–11]. Surface plasmon polariton (SPP) modes are essential ingredients of plasmons and photonics, but the contribution of quantum effects in the surface materials is very often ignored. This is particularly surprising because semiconductors and metals, often used in plasmonics, can behave as quantum plasmas at room temperature.

Here we consider time-refraction of SPP modes in a slab of conductive quantum material (metal, graphene, semiconductor), which is generically described a quantum plasma. Exchange potentials describing quantum degeneracy are known to compete with thermal effects, and eventually lead to instabilities in the electrostatic limit. Here, we show that instabilities driven by exchange potentials are also possible in the electromagnetic quantum domain.

Our main focus is the occurrence of time-refraction and temporal optical phenomena using SPP propagation modes in a quantum plasma slab. The case of a single boundary due to the sudden occurrence of a plasma density increase was considered recently, but where quantum effects were ignored [12]. SPP quantum mode dispersion at a plasma boundary has been previously devrived by a number of authors [13,14], where the plasma dentisy was assumed constant.

Here, we extend the analysis to the case of a quantum plasma slab, where two vacuum-plasma boundaries have to be considered, and examine in detail the three distinct properties of the temporal plasma process, which are: i) the occurrence of time-reflection, ii) the wave frequency shift, and iii) the energy non-conservation and wave amplification. These three properties have been explored theoretically in the last decades and observed experimentally in a number of experimental configurations. We consider them now in the case of SPP modes. The relevance of exchange-driven instabilities is also discussed.

The content of the paper is the following. In section 2, we derive the dispersion relation of SPP modes in a slab of quantum material (metal, semiconductor or plasma). In section 3, we determine the frequency shift of these wave modes in the presence of a sharp time-boundary. In section 4, we derive the corresponding temporal reflection and transmission coefficients that determine the amplitude of time-refracted and time-reflected SPP modes. In section 5, we study the case of smooth

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time-boundaries and establish the range of validity of the sharp time-boundary model. Finally, in section 5, we state some conclusions.

2. SPP in a Plasma Slab

We consider a plasma slab of with ℓ, such as a metal or a semiconductor, surrounded by vacuum. We use the geometry represented in Figure 1, and assume wave propagation along the x direction. The wave fields $\mathbf{E}$ and $\mathbf{B}$ evolve in space and time as $\exp(i\mathbf{k}\cdot\mathbf{r} - i\omega t$, where the wavevector is $\mathbf{k} = k\mathbf{e}_x$. In such a geometry, it is well known that the TE mode cannot propagate [6]. We therefore assume a TM mode, with magnetic field $\mathbf{B} = B_y\mathbf{e}_y$, and $B_x = B_z = 0$. Using Maxwell's equation, we obtain for waves oscillating at a frequency ω the following relations for the electric and magnetic field components

$$\left(\frac{\partial E_x}{\partial z} - ikE_z\right) = i\omega B_y\,, \tag{1}$$

and

$$\frac{\partial B_y}{\partial z} = -i\frac{\omega}{c^2}\epsilon(\omega,k)E_x\,, \quad kB_y = -\frac{\omega}{c^2}\epsilon(\omega,k)E_z\,, \tag{2}$$

where we have assumed a non-magnetic material such that $\mathbf{B} = \mu_0\mathbf{H}$. The dielectric function $\epsilon(\omega,k)$ determines the displacement vector, as $\mathbf{D} = \epsilon_0\epsilon(\omega,k)\mathbf{E}$. In the surrounding vacuum region we simply have $\epsilon(\omega,k) = 1$. Describing the dense plasma slab as a quantum medium, we can use the following expression (see ([13–15])

$$\epsilon(\omega,k) = 1 - \frac{\omega_p^2}{\omega^2 - u^2k^2 - \hbar^2k^4/4m_e^2}\,, \tag{3}$$

where $\omega_p^2 = e^2n_0/\epsilon_0 m_e$ defines the plasma frequency. The velocity u can be defined as the difference between two quantum velocities, which are the Fermi velocity v_F and the exchange velocity v_{ex}, according to $u^2 = v_F^2/3 - v_{ex}^2/2$. They also depend on the electron plasma density n_0, as

$$v_F \equiv \frac{\hbar k_F}{m_e} = \left(3\pi^2\right)^{1/3}\frac{\hbar}{m_e}n_0^{1/3}\,, \quad v_{ex} \equiv \frac{\omega_p}{k_F} = \left(3\pi^2\right)^{-1/3}\frac{e}{\sqrt{\epsilon_0 m_e}}n_0^{1/6}\,. \tag{4}$$

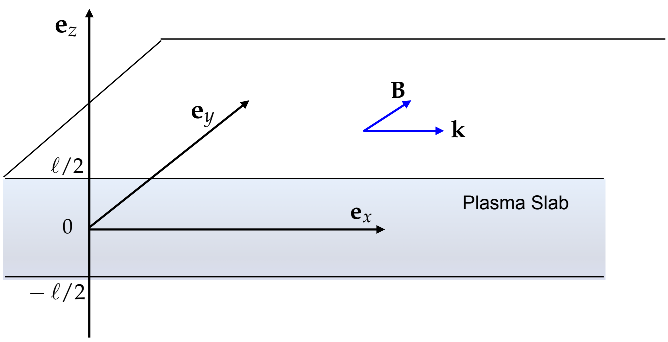

Figure 1. *Geometric configuration: SPP modes propagation in the y-direction, in a plasma slab (metal or semiconductor) with size ℓ.*

Given these definition, we realise that the quantity u^2 only depends on the plasma density n_0, and can change sign when we increase the plasma density such that the exchange term becomes dominant. This eventually leads to wave instabilities, as recently discussed in the electromagnetic and electrostatic regimes [15,16]. The same can occur in the electromagnetic regime, as shown below. In order to stress these properties, it is useful to write the dielectric function in normalised units to the *Fermi frequency*, $\omega_F \equiv \mathcal{E}_F/\hbar = \hbar k_F^2/2m_e$. Defining the dimensionless quantities

$$\Omega = \frac{\omega}{\omega_F}\,, \quad \Omega_p = \frac{\omega_p}{\omega_F}\,, \quad \kappa = \frac{kc}{\omega_F}\,, \tag{5}$$

and using $\nu_F = v_F/c$, we can rewrite eq. (3) as

$$\epsilon(\Omega,\kappa) = 1 - \frac{\Omega_p^2}{\Omega^2 - g(\Omega_p)\nu_F^2\kappa^2 - H_F^2\kappa^4}\,, \tag{6}$$

where we have used the additional quantities

$$g(\Omega_p) = \frac{1}{3}\left(1 - \frac{3}{8}\Omega_p^2\right), \quad H_F = \frac{\hbar\omega_F}{2m_e c^2}\,. \tag{7}$$

This expression shows that the term in κ^4 is usually negligible, and that the denominator diverges for $\Omega/\kappa^2 \equiv= \omega^2/k^2 = g(\Omega_p)\nu_F^2$.

Going back to eqs. (1)-(2) we can derive an evolution equation for the transverse field, of the form

$$\frac{\partial^2 B_y}{\partial z^2} - \beta^2(\omega,k)B_y = 0\,, \quad \beta^2(\omega,k) = k^2 - \frac{\omega^2}{c^2}\epsilon(\omega,k)\,. \tag{8}$$

For $\beta^2 > 0$, this leads to the following field solutions, evanescent along the z-direction. In the vacuum region, we should therefore have $\beta_0^2 \equiv k^2 - \omega^2/c^2 > 0$. The field solutions in the upper-vacuum region $(z > \ell/2)$ are then given by

$$B_y(z) = A\exp(-\beta_0 z) \quad \mathbf{E}(z) = A\frac{c^2}{\omega}(i\beta\mathbf{e}_x - k\mathbf{e}_z)\exp(-\beta_0 z)\,, \tag{9}$$

where the constant A is an amplitude. In the lower-vacuum region $(z < -\ell/2)$ this is replaced by

$$B_y(z) = B\exp(+\beta_0 z) \quad \mathbf{E}(z) = -B\frac{c^2}{\omega}(i\beta\mathbf{e}_x + k\mathbf{e}_z)\exp(+\beta_0 z)\,, \tag{10}$$

where B is another amplitude. Finally, in the intermediate region $(-\ell/2 < z < \ell/2)$, the evanescent field solutions become

$$B_y(z) = C\exp(i\beta z) + D\exp(-i\beta z)\,, \tag{11}$$

and

$$\mathbf{E}(z) = \frac{c^2}{\omega\epsilon(\omega,k)}(i\beta\mathbf{e}_x - k\mathbf{e}_z)\{-C\exp(+\beta z) + D\exp(+\beta z)\}\,, \tag{12}$$

with new amplitudes C and D. The continuity conditions between the fields B_y and E_x at the upper boundary $(z = \ell/2)$ will imply the following relations between the different amplitudes

$$A\,e^{-\beta_0\ell/2} = C\,e^{\beta\ell/2} + D\,e^{-\beta\ell/2}\,, \quad A\,e^{-\beta_0\ell/2} = -\frac{1}{\epsilon(\omega,k)}\left\{C\,e^{\beta\ell/2} + D\,e^{-\beta\ell/2}\right\}. \tag{13}$$

Similar relations can be established at the lower boundary $(z = -\ell/2)$, and lead to

$$B\,e^{-\beta_0\ell/2} = C\,e^{-\beta\ell/2} + D\,e^{\beta\ell/2}\,, \quad B\,e^{-\beta_0\ell/2} = \frac{1}{\epsilon(\omega,k)}\left\{C\,e^{\beta\ell/2} - D\,e^{\beta\ell/2}\right\}. \tag{14}$$

This implies a double equality relation between the internal field amplitudes C and D, of the form

$$C\,e^{\beta\ell} + D = -\frac{C}{\epsilon(\omega,k)}e^{\beta\ell} + \frac{D}{\epsilon(\omega,k)}\,, \quad C + D\,e^{\beta\ell} = -\frac{C}{\epsilon(\omega,k)} - \frac{D}{\epsilon(\omega,k)}e^{\beta\ell}\,, \tag{15}$$

This can also be written as

$$D(1-\epsilon(\omega,k)) = Ce^{\beta\ell}(1+\epsilon(\omega,k))\,, \quad C(1-\epsilon(\omega,k)) = De^{\beta\ell}(1+\epsilon(\omega,k))\,. \tag{16}$$

From here we obtain the dispersion relation

$$\exp(2\beta\ell) = \frac{(1-\epsilon(\omega,k))^2}{(1+\epsilon(\omega,k))^2}, \quad \beta = \pm\sqrt{k^2 - \frac{\omega^2}{c^2}\epsilon(\omega,k)}, \tag{17}$$

Or, in more explicit terms

$$\omega^2 = \frac{\omega_p^2}{2}\left(1 \pm e^{-\beta\ell}\right) + u^2k^2 + \frac{\hbar^2}{4m_e^2}k^4. \tag{18}$$

In the particular case of a large slab, such that $\beta\ell \gg 1$, this reduces to

$$\omega^2 = \frac{\omega_p^2}{2} + u^2k^2 + \frac{\hbar^2}{4m_e^2}k^4. \tag{19}$$

Notice that when the exchange corrections are neglected and u is reduced to the Fermi velocity v_F, this coincides with the well-known expression of the dispersion relation of a surface plasmon at a single boundary [13]. In the opposite case of a very thin film, where $\beta\ell \ll 1$, we can the identify the mode

$$\omega^2 = \frac{\omega_p^2}{2}\beta\ell + u^2k^2 + \frac{\hbar^2}{4m_e^2}k^4. \tag{20}$$

In terms of the dimensionless variables (5), this expression can be rewritten as

$$\Omega^2 = \frac{\Omega_p}{2}\kappa L\left[1 - \frac{\Omega^2}{\kappa^2}\epsilon(\Omega,\kappa)\right] + g(\Omega_p)\nu_F^2\kappa^2 + H_F^2\kappa^4. \tag{21}$$

Noting that the square-bracket needs to be positive in order to satisfy a evanescent mode in the z-direction, we realise that the mode frequency can become negative for $g(\Omega_p) \simeq 0$, which means for $\Omega_p^2 \simeq 8/3$, or $v_{ex} \simeq v_F$.

Furthermore, we should notice that, in the electrostatic limit, $c^2 \to \infty$, we can make the replacement $\beta = k$. Another useful limit corresponds to a slab with infinitesimal width $\ell \to 0$, which reduces to the case of a two-dimensional medium. But, this limit can never be attained because of the uncertainty principle, implies that $\ell \geq \ell_m$, with the minimum value determined by the uncertainty of the electron momentum Δp, or

$$\ell_m = \frac{\hbar}{\Delta p} = \frac{\hbar}{m_e v_F} = \frac{1}{k_F}, \tag{22}$$

This means that the dispersion relation of a surface plasmon in a two-dimensional plasma has to be given by

$$\omega^2 = \alpha k + u^2k^2 + \frac{\hbar^2}{4m_e^2}k^4, \quad \alpha = \frac{\omega_p^2}{2k_F}. \tag{23}$$

This expression has previously been determined by purely two-dimensional methods (see [16]). It can be seen that an instability can occur when the exchange effects dominate and u^2 become negative. In this case, a purely growing mode takes place for wavenumbers above a certain critical value, k_{cr}, with a growth rate

$$\gamma \equiv \Im(\omega) = \sqrt{|u^2|k^2 - \alpha k}, \quad k \geq k_{cr} \equiv \frac{\alpha}{|u^2|}. \tag{24}$$

The influence of the quartic term proportional to k^4 is usually negligible and was ignored here. Returning to eq. (20), we can see that the instability is also possible for electromagnetic modes in three-dimensions, but higher values of $k \geq k'_{cr}$ satisfying

$$k'_{cr} = \frac{\omega_p^2\ell}{2|u^2|} = (\ell k_F)k_{cr}. \tag{25}$$

Notice that the change in sign of u^2 occurs when the plasmon energy is equal to the Fermi energy $\hbar\omega_p \simeq \mathcal{E}_F$.

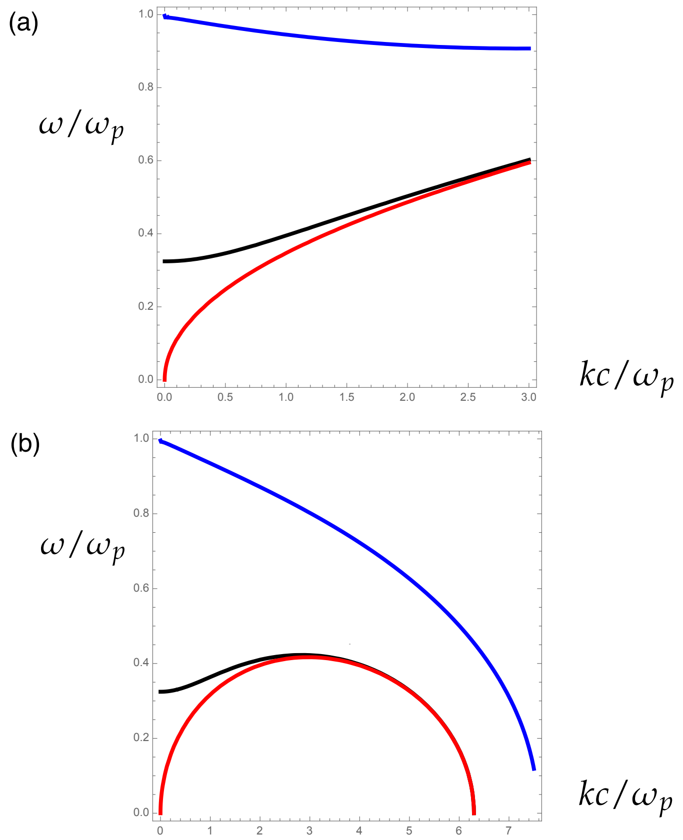

Figure 2. *Dispersion relation of SPP modes propagation along y, in a plasma slab of size ℓ. The normalised mode frequency ω/ω_p is represented as a function of the normalised wave number kc/ω_p. The electromagnetic SPP dispersion (18) is represented by blue (-) and black (+) curves. The electrostatic plasmon dispersion (23) is also represented for $\ell = 0.25$ in wavenumber units (red curve). The quantum square velocity is assumed: (a) positive $(u^2/c^2 = 0.01)$, and (b) negative $(u^2/c^2 = -0.01)$. The small k^4 term is ignored.*

3. Frequency Shifts

We now examine the occurrence of a temporal discontinuity, occurring at $t = t_0$, which can be described by a sudden exchange of the plasma frequency. This can be described by

$$\omega_p^2(t) = \omega_{pi}^2\{1 + \tilde{\epsilon}H(t - t_0)\}\,, \tag{26}$$

where $H(t - t_0)$ is the Heaviside function an ω_{pi} is the initial plasma frequency. Its final value, for $t < t_0$, is $\omega_{pf} = \sqrt{1 + \tilde{\epsilon}}$, where $\tilde{\epsilon} = n_f/n_0$ is the ratio between the initial and the final electron plasma density. This is what can be called a sharp sl time-boundary, which is similar to the space boundaries of the plasma slab, but exists in the temporal dimension. Notice that, at this temporal boundary, the Fermi and exchange velocities defined by eqs. (4) also suffer an abrupt shift.

Validity of Maxwell's equations at all times, including the instant $t = t_0$, imply time-boundary conditions that are distinct from the space-boundary conditions considered in the previous section. They now involve the fields $\mathbf{B}$ and $\mathbf{D}$, and can be stated as

$$\mathbf{B}(t_0 - \delta) = \mathbf{B}(t_0 + \delta)\,, \quad \mathbf{D}(t_0 - \delta) = \mathbf{D}(t_0 + \delta)\,, \tag{27}$$

for $\delta \to 0$. Given the invariance of the wavevector $\mathbf{k} = k\mathbf{e}_x$, the temporal boundary defined by eq. (26) implies the existence of a frequency shift associated with the same wave mode. For simplicity, we start with the electrostatic mode, valid in the quasi-two-dimensional limit. Using eq. (20) with $\beta = k$, we can see that the final frequency ω_f of a wave with initial frequency ω_i is determined by

$$\omega_f^2 - \omega_i^2 = \frac{k\ell}{2}\left(\omega_{pf}^2 - \omega_{pi}^2\right) + \left(u_f^2 - u_i^2\right)k^2 . \tag{28}$$

Or, equivalently

$$\omega_f^2 = \omega_i^2 + \frac{\tilde{\epsilon}}{2}k\ell\omega_{pi}^2 + \Delta u^2 k^2 , \tag{29}$$

where the quantity $\Delta u^2 = \left(u_f^2 - u_i^2\right)$ is a function of the initial plasma density, n_{0i} and $\tilde{\epsilon}$, and can eventually change sign due to the different density dependence of the Fermi and exchange velocities, v_F and v_{ex}. These results are now illustrated in Figure 3.

If we now use the electromagnetic description of the SPP modes, satisfying the dispersion relation (18), we arrive at a new value of the frequency shift, determined by

$$\omega_f^2 = \omega_i^2 + \frac{\tilde{\epsilon}}{2}\omega_{pi}^2 \pm \frac{1}{2}\left(\omega_{pf}^2 \pm e^{-\beta_f \ell} - \omega_{pi}^2 \pm e^{-\beta_i \ell}\right) + \Delta u^2 k^2 . \tag{30}$$

Here, we have used the initial and final values of the parameter β, defined by

$$\beta_j^2 = k^2 - \frac{\omega_j^2}{c^2}\left\{1 - \frac{\omega_{pj}^2}{\omega_j^2 - u_j^2 k^2 - \hbar^2 k^4/4m_e^2}\right\} , \tag{31}$$

for $j = (i, f)$. In contrast with the electrostatic case, we now have an implicit equation for the final wave frequency ω_f. Of course, in the electrostatic limit, $c^2 \to \infty$, we can use $\beta_i = \beta_f = k$ and recover the explicit equation (29).

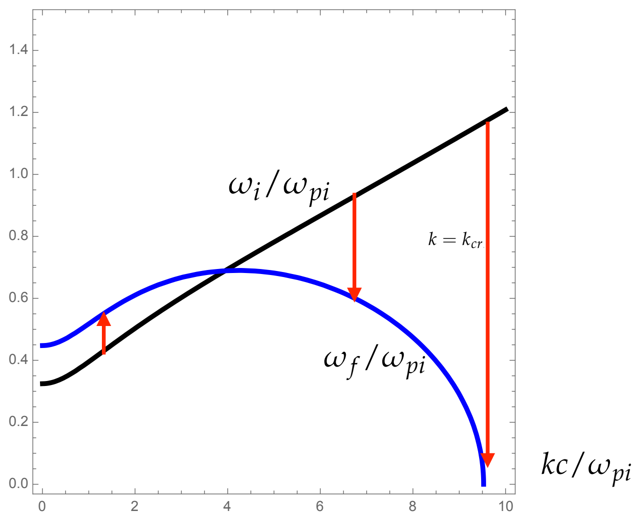

Figure 3. *Frequency of SPP modes, before ω_i/ω_{pi}, and after ω_f/ω_{pi} the temporal transsition, as a function of the normalised wavenumber kc/ω_{pi}. The possible occurrence of positive and negative frequency shifts is illustrated. The extreme case of the wave collapse into a static SPP crystal is indicated. The black (and blue) curve corresponds to the initial (and final) SPP electromagnetic dispersion.*

4. T and R Coefficients

The temporal boundary conditions of eq. (27) imply the creation of time-reflected signals and the eventual occurrence of wave amplification. This is can be described by the so-called time-transmission and time-reflected coefficients, T and R. In order to derive their expressions, we need to consider all the field modes associated with the same wavevector $\mathbf{k}$ and its reverse $(-\mathbf{k})$, as shown before [5,17]. The corresponding electric field can be written as

$$\mathbf{E}(\mathbf{r},t) = \left\{ \mathbf{E}(z)e^{-i\omega t} + \mathbf{E}'(z)e^{+i\omega t} \right\} e^{ikx} + c.c. \,, \tag{32}$$

where waves with field amplitudes $\mathbf{E}(z)$ and $\mathbf{E}'(z)$ assumed to propagate with the same wavenumber k along the x-axis, but in opposite directions.

In order to avoid ambiguities (that will be discussed at the end of this section), we consider the field continuity conditions at time $t_0 = 0$ and at specific positions $z_\pm = \pm\ell/2$, which correspond to the upper and lower boundaries of plasma slab. Using the temporal continuity conditions (27), this leads to the following relations between modes moving in opposite directions

$$\mathbf{B}_0(z_\pm) + \mathbf{B'}_0(z_\pm) = \mathbf{B}_f(z_\pm) + \mathbf{B'}_f(z_\pm) \,, \quad \mathbf{D}_0(z_\pm) + \mathbf{D'}_0(z_\pm) = \mathbf{D}_f(z_\pm) + \mathbf{D'}_f(z_\pm) \,, \tag{33}$$

where the subscripts $(0, f)$ pertain to $(t = t_0 - \delta)$ and $(t = t_0 + \delta)$, respectively, for $\delta \to 0$. Noting that $\mathbf{B}$ reduces to $B_y \mathbf{e}_y$ and using eqs. (3), we have

$$B_y = -\frac{\omega}{kc^2}\epsilon(\omega,k)E_z = -\frac{\omega}{\epsilon_0 kc^2} D_z \,, \tag{34}$$

This allows us to write the first of eqs. (33) in terms of the electric field as

$$\frac{\omega_i}{kc^2}\epsilon(\omega_i,k)\left(E_{z0} - E'_{zi}\right) = \frac{\omega_f}{kc^2}\epsilon(\omega_f,k)\left(E_{zf} - E'_{zf}\right) . \tag{35}$$

Similarly, from second of these equations, we obtain

$$\epsilon(\omega_i,k)\left(E_{zi} + E'_{zi}\right) = \epsilon(\omega_f,k)\left(E_{zf} + E'_{zf}\right) . \tag{36}$$

This can be rewritten as

$$\left(E_{zi} - E'_{zi}\right) = \alpha_z\left(E_{zf} - E'_{zf}\right) , \quad \left(E_{zi} + E'_{zi}\right) = \beta_z\left(E_{zf} + E'_{zf}\right) , \tag{37}$$

with

$$\alpha_z = \frac{\omega_f}{\omega_i}\beta_z \,, \quad \beta_z = \frac{\epsilon(\omega_f,k)}{\epsilon(\omega_i,k)} \,. \tag{38}$$

These relations are valid in the plasma slab region, in the close vicinity of the boundaries, $z = z_\pm + \epsilon$, with $\epsilon \to 0$. Similar relations could also be established for the external vacuum region, where we would have $\beta_z = 1$. We could also use the other electric field component, E_x. Using eqs. (1)-(2) we can see that it is related with E_z by

$$\frac{\partial E_x}{\partial z} = \frac{i}{k}\beta^2(\omega)E_z \,, \tag{39}$$

where $\beta^2(\omega)$ is defined in eq. (8). Noting that the field solutions evolve along z as $\exp[\pm\beta(\omega)z]$, we conclude that

$$\left|\frac{E_x}{E_z}\right| = \frac{1}{k}\beta(\omega) = \left(1 - \frac{\omega^2}{k^2c^2}\epsilon(\omega,k)\right)^{1/2} . \tag{40}$$

This means that we can characterise the electric field amplitude using E_z, and its transformation at the time-boundary $t = t_0$ using eqs. (37)-(38). Let us then solve them for the field amplitudes E_{zf} and E'_{zf}. We obtain

$$E_{zf} = \frac{(\alpha_z + \beta_z)}{2\alpha_z\beta_z}E_{zi} + \frac{(\alpha_z - \beta_z)}{2\alpha_z\beta_z}E'_{zi} \,, \tag{41}$$

and

$$E'_{zf} = \frac{(\alpha_z - \beta_z)}{2\alpha_z\beta_z}E_{zi} + \frac{(\alpha_z + \beta_z)}{2\alpha_z\beta_z}E'_{zi} \,, \tag{42}$$

These expressions show that the occurrence of a temporal discontinuity implies a change in the field amplitude of the time-transmitted wave mode, and the formation of a time-reflected signal. This is

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particularly obvious if we consider the case of $E'_{zi} = 0$, where initially there is no wave propagating in the negative x-direction. In this case, we can define transmission and reflection coefficients, T ad R, such that

$$T \equiv \frac{E_{zf}}{E_{zi}} = \frac{(\alpha_z + \beta_z)}{2\alpha_z \beta_z}, \quad R \equiv \frac{E'_{zf}}{E_{zi}} = \frac{(\alpha_z - \beta_z)}{2\alpha_z \beta_z}, \tag{43}$$

Using the explicit expressions for α_z and β_z, as given in (38), we get

$$T \equiv \frac{E_{zf}}{E_{zi}} = \frac{1}{2}\left(1 + \frac{\omega_i}{\omega_f}\right)\frac{\epsilon(\omega_i, k)}{\epsilon(\omega_f, k)}, \quad R \equiv \frac{E'_{zf}}{E_{zi}} = \frac{1}{2}\left(1 - \frac{\omega_i}{\omega_f}\right)\frac{\epsilon(\omega_i, k)}{\epsilon(\omega_f, k)}. \tag{44}$$

In the trivial case of $\omega_i = \omega_f$ we would get $T = 1, R = 0$, as expected. This result is illustrated in Figure 4, where these coefficients are presented as a function of the wavenumber k, for a sudden density perturbation. As can be seen from this Figure, the energy gain $W = R^2 + T^2$ due to a temporal boundary diverges for $k = k_{cr}$, when the initial wave mode can be transformed into a zero-frequency mode, such that $(\omega_i/\omega_f) \to \infty$. This occurs when $\omega_p \to \omega_F$, or $\Omega_p \to \sqrt{3/8}$. In this case, the formation of a static *SPP crystal* is expected to occur, with an amplitude that is determined by by available electromagnetic energy in the system.

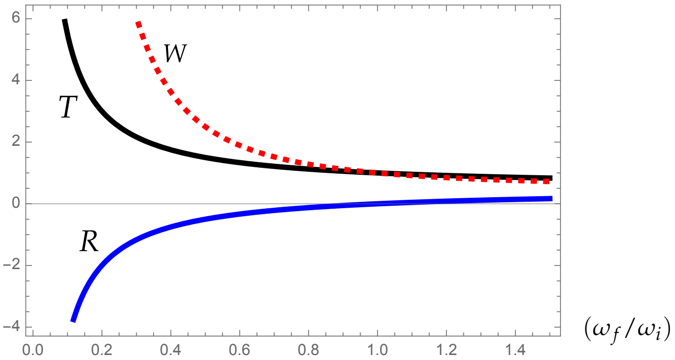

Figure 4. *Time-transmission and reflection coefficients, R and T for SPP modes, as function of the mode frequency ratio (ω_f/ω_i), as given by eqs. (44). The quantity $W = R^2 + T^2$, which represents the energy gain associated with time-reflexion and time-refraction.*

This completes our basic description of time-refraction of SPP modes in a quantum plasma slab. The derivation of the T and R coefficients was established at precise positions $z = z_\pm - \epsilon$, for $\epsilon \to 0$. But the temporal continuity conditions imposed by Maxwell's equations should stay valid everywhere, which means for all values of z. Our approach is correct in the vacuum region, but leads to deviations from continuity in the plasma side. Deviations depend on the z position and evolve as $\exp \pm[(\beta_i - \beta_f)z]$.

But, it can easily be realised that such deviations are spurious, and result from the sudden change of the medium at $t = t_0$. They disappear for a smooth time-boundary, where a more physically relevant boundary with a finite width is considered. For that reason, the apparent incongruence of the above method is ignored. In the next section, the case of a smooth time-boundary is briefly discussed.

5. Smooth Time-Boundary

Here, we consider the case where the Heaviside function appearing in eq. (26) is replaced by an hyperbolic sign function which provides a smooth temporal transition between the initial and final values of the electron plasma density. We therefore assume that

$$\omega_p^2(t) = \omega_{pi}^2 \left\{ 1 + \frac{\tilde{\epsilon}}{2} \tanh(\nu t) \right\}, \tag{45}$$

where the quantity $(1/\nu)$ determines the temporal width of the transition. We return to eqs. (41)-(42), and assume a time-varying dielectric function $\epsilon(t) \equiv \epsilon(\omega(t), k)$, where $\omega(t)$ is the instantaneous wave frequency. From here, we can then derive the differential evolution equations, which take the form

$$\frac{\partial E_z}{\partial t} = \delta A\, E_z + \delta B\, E_z' \exp[i\varphi(t)] \,, \quad \frac{\partial E_z}{\partial t} = \delta A\, E_z' + \delta B\, E_z \exp[-i\varphi(t)] \,, \tag{46}$$

where the phase is $\varphi(t) = 2\int^t \omega(t')dt'$, and the coupling coefficients are

$$\delta A = -\frac{1}{2}\frac{\partial}{\partial t} \ln\left[\omega(t)\epsilon^2(t)\right] , \quad \delta B = +\frac{1}{2}\frac{\partial}{\partial t} \ln\left[\omega(t)\epsilon^{-2}(t)\right] . \tag{47}$$

The coupled equations (46) can be formally integrated, using the new field variables

$$F_z = E_z \exp(-\delta A) \,, \quad F_z' = E_z' \exp(-\delta A) \,, \tag{48}$$

which allows us to write

$$\frac{\partial F_z}{\partial t} = \eta(t)\, F_z' \,, \quad \frac{\partial F_z'}{\partial t} = \eta^*(t)\, F_z \,, \qquad \eta(t) = \delta B \, \exp[-i\varphi(t)] \,. \tag{49}$$

For initial field values $F_z(0)$ and $F_z'(0)$, the solutions are

$$F_z(t) = \cosh[r(t)]F_z(0) - \sinh[r(t)]F_z'(0) \,, \quad F_z'(t) = \cosh[r(t)]F_z'(0) - \sinh[r(t)]F_z(0) \,, \tag{50}$$

where we have used the *squeezing parameter*, $r(t)$, defined as

$$r(t) = \int^t \eta(t')dt' \equiv \frac{1}{2}\int^t \frac{\partial}{\partial t} \ln\left[\omega(t')\epsilon^{-2}(t')\right] \exp[-i\varphi(t')]dt' \,. \tag{51}$$

Notice that this parameter can be explicitly written in terms of the derivative of the expression defined in eq. (45). In the particular case of a single wave mode propagating initially in the positive x-direction, we have $F_z'(0) = 0$. The solutions (50) can then be reduced to the following expressions for the electric fields

$$E_z(t) = T(t)E_z(0) \,, \quad E_z'(t) = R(t)F_z(0) \,, \tag{52}$$

where the instantaneous transmission and reflection coefficients $T(t)$ and $R(t)$ are determined by

$$T(t) = \cosh[r(t)] \exp[-\delta A(t) + \delta A(0)] \,, \quad R(t) = -\sinh[r(t)] \exp[-\delta A(t) + \delta A(0)] \,. \tag{53}$$

This completes our formal solution of the problem of a smooth temporal boundary. This solution also stays valid for the general case of an arbitrary temporal evolution of the plasma slab. See Figure 5 for three different values of the transition time scaale $1/\nu$. We can see that the formation of a reflected wave is stronger when $\nu \simeq 2\omega$, and nearly vanishes for slow time changes, when $\nu/2\omega \ll 1$. The simple sharp time-boundary model explored in sections 3 and 4 is valid when the temporal transition is faster than the period of the SPP mode, or $\nu \geq 2\omega$.

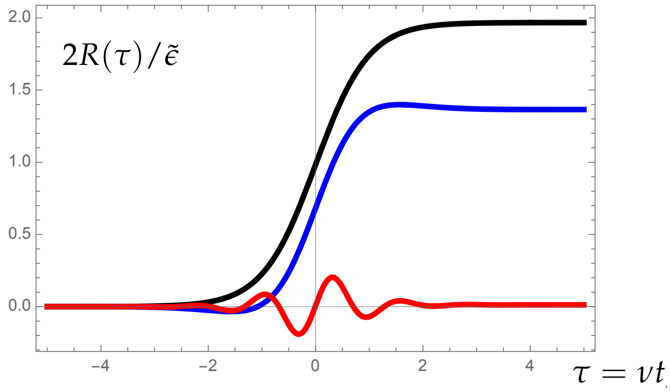

Figure 5. *Normalised time-reflection coefficient,* $2R(\tau)/\tilde{\epsilon}$ *of SPP modes, as a function of normalised time* $\tau = \nu t$*, for three distinct values of* $2\omega/\nu = 1$ *(black curve),* 3 *(blue) and* 10 *(red).*

6. Conclusions

Here, we have studied the problem of time-refraction for surface waves that can be excited in a quantum plasma slab. This problem involves field continuity conditions in space and in time, which are not identical to each other. Both electromagnetic and electrostatic wave modes were considered. The case of slabs with an infinitesimal width is reduced to a quasi two-dimensional problem, and was also studied in detail. We have derived the wave frequency shifts and the time-transmission and time-reflection coefficients, which are similar to those valid in spatially homogeneous media. The general case of smooth temporal transitions and arbitrary temporal variations of the slab density was formally solved.

Extension of this work to the case of temporal beam-splitters [17] and time-crystals [18] is straightforward, and can be derived from the preent results, for sharp as well as for smooth temporal boundaries. Another possible extension is related with superluminal fronts, using electron-ion plasmas [19,20] or metamaterials [3,21], which can be related to the present work by a simple Lorentz transformation [22]. Some of these scenarios will be examined in a separate work.

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