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Algebraic Chrono–Dynamics: Stratified Covariant Phase Space and Boundary Algebra

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Abstract

We construct a fully self-contained covariant phase-space formulation of a complex scalar field theory whose canonical structure becomes stratified under a diffeomorphism-invariant local trigger condition. Beginning from a well-posed variational principle on a globally hyperbolic spacetime with boundary, we derive the presymplectic current, establish hypersurface independence through explicit boundary symplectic augmentation, and perform presymplectic reduction to obtain the physical phase space. We then construct integrable boundary charges and prove that their algebra is represented up to a cocycle determined by the boundary symplectic structure. A local invariant functional partitions spacetime into regular and dense strata. In the dense stratum, admissible variations are restricted, enlarging the presymplectic kernel and suppressing the boundary cocycle. The resulting theory exhibits a mathematically controlled transition in algebraic structure without modification of Euler–Lagrange dynamics. Quantization is shown to preserve consistency on both strata. The work provides a complete algebraic closure independent of prior physical interpretation.

Keywords: covariant phase space; presymplectic geometry; boundary charge algebra; stratified field theory

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I. Introduction

Covariant phase-space methods provide a geometric framework for defining canonical structure in classical and quantum field theory without reliance on coordinate-dependent canonical splitting [1–3]. In the presence of boundaries, the naive presymplectic form fails to be hypersurface independent due to symplectic flux. This obstruction is well known in gauge theory and gravity and is resolved by introducing boundary symplectic contributions or edge modes [4,5]. The associated boundary charge algebras — and their central extensions — have been studied extensively in the covariant formalism [3,6,7].

A natural generalisation arises when the admissible variation class itself changes across spacetime, controlled by a diffeomorphism-invariant scalar condition. Physical situations in which a local invariant — such as a curvature scalar or energy density — crosses a critical threshold provide concrete realisations of this structure. When the variation class changes, the presymplectic kernel and reduced phase-space dimension change with it, producing a *stratified* covariant phase space. Despite the naturalness of this setting, a fully covariant mathematical treatment in the scalar field context has not previously been given.

The present paper provides that treatment. The result is a general theorem: *any* scalar field theory on a globally hyperbolic spacetime with boundary, equipped with a diffeomorphism-invariant stratification functional, admits a well-defined stratified covariant phase space whose boundary charge algebra undergoes a controlled algebraic transition — from centrally extended to non-extended — across the stratification surface, without any modification of the bulk equations of motion.

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The construction is entirely self-contained. Three minimal geometric assumptions, stated precisely in Sec. II, are all that is required. No appeal to any specific physical programme is necessary, and all results follow from standard variational field theory and differential geometry.

The main result is the following theorem, proved in Sec. XXIV:

Theorem (Main Theorem — Stratified Covariant Phase Structure)**.** *Let $(\mathcal{M}, g_{\mu\nu})$ be a smooth, oriented, globally hyperbolic Lorentzian four-manifold with boundary, carrying a complex scalar field Φ satisfying Assumptions 1–3 of Sec. II. Then:*

- *(i)* *The augmented presymplectic form $\Omega_\Sigma^{\mathrm{aug}}$ is hypersurface independent and descends to a nondegenerate symplectic form on the reduced phase space $\mathcal{P}$ on each stratum.*
- *(ii)* *The boundary charge algebra satisfies $\{Q_\xi, Q_\eta\} = Q_{[\xi,\eta]} + K(\xi,\eta)$ with K a non-trivial 2-cocycle on $\mathcal{P}_{\mathrm{reg}}$.*
- *(iii)* *On $\mathcal{P}_{\mathrm{dense}}$, the cocycle vanishes identically: $K_{\mathrm{dense}} = 0$, and the reduced phase-space dimension strictly decreases.*
- *(iv)* *The phase boundary $\mathcal{H}$ is a degeneracy surface of $\Omega_\Sigma^{\mathrm{aug}}$: the unique locus where the presymplectic kernel enlarges and the central extension of the boundary algebra simultaneously vanishes.*
- *(v)* *These transitions occur without modification of the Euler–Lagrange equations of motion.*

Outline of proof. The construction proceeds in three stages. *Stage 1* (Secs. II–XI): we establish the standard covariant phase space on a spacetime with boundary, resolving the hypersurface-independence obstruction via an explicit boundary symplectic augmentation and constructing integrable boundary charges. *Stage 2* (Secs. XII–XIII): we derive the boundary charge algebra and identify the cocycle $K(\xi,\eta)$ through its explicit polar-decomposition form. *Stage 3* (Secs. XIV–XXIV): we introduce the stratification trigger, define dense-time admissibility, and prove by direct computation that the admissibility condition $\delta\theta = 0$ simultaneously enlarges the presymplectic kernel (Theorem XIX.1) and suppresses the cocycle (Theorem XXI.1). The Phase Boundary Characterisation Theorem XXII.1 then unifies these two results. □

The purpose of this work is to:

1. State the minimal geometric assumptions governing phase-stratified spacetimes.
2. Construct a fully rigorous covariant phase space with boundary.
3. Derive the associated boundary charge algebra and identify the cocycle structure explicitly.
4. Introduce a diffeomorphism-invariant local trigger functional and prove it induces a stratified presymplectic structure.
5. Prove the Phase Boundary Characterisation Theorem connecting kernel enlargement, cocycle suppression, and spacetime geometry.
6. Demonstrate that quantization remains algebraically consistent on both strata.

Remark I.1 (Relation to broader context)**.** *The stratified structure constructed here has natural applications wherever a local invariant partitions a field theory into dynamically distinct regimes: strong-gravity interiors, dense-matter phases, and analogous condensed-matter systems with phase boundaries. The boundary charge algebra transition proved in Theorem XXIV.1 in particular provides a coordinate-free characterisation of such phase boundaries that is independent of any specific dynamical model.*

II. Minimal Geometric Assumptions

The entire construction of this paper rests on three assumptions, stated here precisely. No further input from any specific physical theory is required.

Assumption 1 — Phase Stratification. Spacetime $(\mathcal{M}, g_{\mu\nu})$ carries a scalar functional

$$P[\Phi, g](x),$$

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constructed *locally* from the metric $g_{\mu\nu}$, the field Φ, and a *finite* number of their covariant derivatives at x. In particular, $P(x)$ depends only on the values of $(g_{\mu\nu}, \Phi)$ and their derivatives evaluated at the single point x; it is not an integral or nonlocal functional. P is smooth wherever $(g_{\mu\nu}, \Phi)$ are smooth, and is strictly diffeomorphism invariant: $P[\varphi^*\Phi, \varphi^*g](x) = P[\Phi, g](\varphi(x))$ for all diffeomorphisms φ. This functional partitions $\mathcal{M}$ into two open regions:

$$\mathcal{M} = \mathcal{M}_{\text{reg}} \cup \mathcal{H} \cup \mathcal{M}_{\text{dense}},$$

where $\mathcal{M}_{\text{reg}} = \{P < P_\star\}$ is the *oscillatory stratum*, $\mathcal{M}_{\text{dense}} = \{P \geq P_\star\}$ is the *suppressed stratum*, and $\mathcal{H} = \{P = P_\star\}$ is the phase boundary, for some fixed threshold $P_\star > 0$.

Assumption 2 — Smooth Phase Boundary. The gradient $\nabla_\mu P$ does not vanish on $\mathcal{H}$, so that $\mathcal{H}$ is a smooth codimension-one hypersurface by the regular level-set theorem.

Assumption 3 — Stratified Variation Class. On $\mathcal{M}_{\text{reg}}$, fields satisfy standard variational principles with no restriction on the admissible variations $\delta\Phi$. On $\mathcal{M}_{\text{dense}}$, the admissible variation class is restricted: writing $\Phi = \sqrt{\hat{t}}\, e^{i\theta}$ in polar form, only amplitude variations $\delta\hat{t}$ are admissible; phase variations satisfy $\delta\theta = 0$. The Euler–Lagrange equations are unchanged on both strata.

These three assumptions are the complete logical input for all theorems that follow. The physical origin of P and the mechanism enforcing Assumption 3 are not assumed; they may arise from a variety of underlying dynamics. Concrete illustrations are provided in Sec. XIV.

III. Geometric and Functional Setup

A. Spacetime

Let $(\mathcal{M}, g_{\mu\nu})$ be a smooth, oriented, globally hyperbolic Lorentzian manifold of dimension four, possibly with smooth boundary $\partial\mathcal{M}$.

Let $\Sigma \subset \mathcal{M}$ be a smooth Cauchy hypersurface with boundary $\partial\Sigma \subset \partial\mathcal{M}$.

B. Configuration Space

Definition III.1 (Configuration Space)**.**

$$\mathcal{C} = \left\{ \Phi : \mathcal{M} \to \mathbb{C} \,\middle|\, \Phi \in C^\infty(\mathcal{M}), \quad \Phi|_{\partial\mathcal{M}} \in H^k(\partial\mathcal{M}), \quad k \geq 1 \right\}.$$

Definition III.2 (Admissible Variations)**.**

$$\delta\Phi \in C^\infty(\mathcal{M}), \quad \delta\Phi|_{\partial\mathcal{M}} \in H^k(\partial\mathcal{M}).$$

These conditions guarantee all subsequent integrals are well defined.

IV. Action Principle

A. Lagrangian

$$\mathcal{L} = \nabla_\mu \Phi^\dagger \nabla^\mu \Phi - V(|\Phi|^2), \qquad V \in C^\infty(\mathbb{R}^+).$$

The action is

$$S[\Phi] = \int_{\mathcal{M}} \mathcal{L} \sqrt{-g}\, d^4x.$$

B. First Variation

$$\delta S = \int_{\mathcal{M}} E(\Phi) \delta\Phi \sqrt{-g} + \int_{\partial\mathcal{M}} \theta,$$

where

$$E(\Phi) = -\nabla_\mu \nabla^\mu \Phi + V'(|\Phi|^2)\Phi.$$

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The presymplectic potential current is

$$\theta_\mu = \nabla_\mu \Phi^\dagger \delta\Phi + \nabla_\mu \Phi \delta\Phi^\dagger.$$

V. Covariant Phase Space

Definition V.1 (Solution Space)**.**

$$\mathcal{S} = \{\Phi \in \mathcal{C} \mid E(\Phi) = 0\}.$$

Definition V.2 (Presymplectic Current)**.**

$$\omega^\mu = \delta_1 \theta^\mu(\delta_2) - \delta_2 \theta^\mu(\delta_1).$$

Explicitly,

$$\omega^\mu = \delta_1(\nabla^\mu \Phi^\dagger)\delta_2\Phi - \delta_2(\nabla^\mu \Phi^\dagger)\delta_1\Phi + \text{c.c.}$$

Definition V.3 (Presymplectic Form)**.**

$$\Omega_\Sigma = \int_\Sigma \omega^\mu n_\mu.$$

Theorem V.1. *On shell,*

$$\nabla_\mu \omega^\mu = 0.$$

Proof. Follows directly from linearization of the Euler–Lagrange operator. □

Corollary V.1. *Ω_Σ is independent of smooth deformations of Σ in the absence of boundary flux.*

VI. Boundary Flux Obstruction

Let Σ_1 and Σ_2 bound a spacetime region $\mathcal{R}$ such that

$$\partial\mathcal{R} = \Sigma_2 - \Sigma_1 + \mathcal{B},$$

where $\mathcal{B}$ is the timelike boundary segment.

Using $\nabla_\mu \omega^\mu = 0$ and Stokes' theorem,

$$\Omega_{\Sigma_2} - \Omega_{\Sigma_1} = -\int_{\mathcal{B}} \omega^\mu n_\mu.$$

Proposition VI.1. *If*

$$\omega^\mu n_\mu|_{\mathcal{B}} \neq 0,$$

then Ω_Σ depends on hypersurface choice.

Thus hypersurface independence fails in the presence of boundary symplectic flux.

VII. Boundary Symplectic Augmentation

To restore well-posedness, we introduce a boundary 2-form.

Definition VII.1 (Edge Symplectic Form)**.** *Let*

$$\widetilde{\Omega}_{\partial\Sigma} = \int_{\partial\Sigma} \alpha(\Phi; \delta_1, \delta_2),$$

where α is bilinear, antisymmetric, and satisfies

$$\delta\widetilde{\Omega}_{\partial\Sigma} = \int_{\partial\Sigma} \omega^\mu n_\mu.$$

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Remark VII.1 (Explicit Construction of α)**.** *Writing $\Phi = \sqrt{\hat{t}}\, e^{i\theta}$ in polar form, the presymplectic current restricted to $\partial\Sigma$ decomposes into amplitude and phase sectors. A boundary symplectic density satisfying the required condition is given explicitly by*

$$\alpha(\Phi; \delta_1, \delta_2) = \delta_1\theta\, \delta_2\hat{t} - \delta_2\theta\, \delta_1\hat{t}.$$

One verifies directly that $\delta\alpha = \omega^\mu n_\mu|_{\partial\Sigma}$, so this choice resolves the boundary flux obstruction identified in Sec. 5. The decomposition into θ- and $\hat{t}$-sectors will be essential for the cocycle analysis in Sec. XII.

Define the augmented form:

$$\Omega_\Sigma^{\text{aug}} = \Omega_\Sigma + \widetilde{\Omega}_{\partial\Sigma}.$$

Theorem VII.1. *$\Omega_\Sigma^{\text{aug}}$ is hypersurface independent.*

Proof. Under deformation,

$$\delta\Omega_\Sigma = -\int_{\partial\Sigma} \omega^\mu n_\mu,$$

which is cancelled by

$$\delta\widetilde{\Omega}_{\partial\Sigma}.$$

□

Remark VII.2. *The augmentation is not optional but mathematically necessary whenever boundary flux is non-vanishing.*

VIII. Presymplectic Reduction

Definition VIII.1 (Kernel)**.**

$$\ker \Omega_\Sigma^{\text{aug}} = \{\delta_\lambda \mid \Omega_\Sigma^{\text{aug}}(\delta_\lambda, \delta) = 0\ \forall\delta\}.$$

The global $U(1)$ symmetry

$$\Phi \mapsto e^{i\lambda}\Phi$$

generates infinitesimal variations

$$\delta_\lambda\Phi = i\lambda\Phi.$$

Boundary augmentation cancels pure boundary degeneracy, so remaining null directions correspond to true symmetries.

Definition VIII.2 (Reduced Phase Space)**.**

$$\mathcal{P} = \mathcal{S} / \ker \Omega_\Sigma^{\text{aug}}.$$

Theorem VIII.1. *$\Omega_\Sigma^{\text{aug}}$ descends to a nondegenerate symplectic form on $\mathcal{P}$.*

Proof. Standard quotient-by-kernel argument. □

Remark VIII.1 (Consistency with standard covariant phase space)**.** *When $P(x) < P_\star$ everywhere — that is, when $\mathcal{M}_{\text{dense}} = \emptyset$ and Assumption 3 imposes no restriction — the construction of this paper reduces identically to the standard covariant phase space formalism of Lee–Wald [2] and Iyer–Wald [3]. In particular, $\mathcal{P}_{\text{reg}}$ carries the full phase-wave degrees of freedom and the boundary charge algebra retains its central extension $K \neq 0$. All known results for ordinary scalar field theory on a spacetime with boundary are therefore recovered as the $P_\star \to \infty$ limit of the present framework. The stratification is a strict extension of the standard theory, not a replacement of it.*

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IX. Poisson Structure

Definition IX.1. *For $F : \mathcal{P} \to \mathbb{R}$, define X_F by*

$$\iota_{X_F} \Omega_\Sigma^{\text{aug}} = \delta F.$$

Definition IX.2 (Poisson Bracket)**.**

$$\{F, G\} = \Omega_\Sigma^{\text{aug}}(X_F, X_G).$$

Proposition IX.1. *The bracket satisfies bilinearity, antisymmetry, Leibniz rule, and Jacobi identity.*

Proof. Follows from closure and nondegeneracy. □

X. Equal-Time Canonical Structure

On a coordinate choice where

$$\pi = \partial_t \Phi^\dagger,$$

one obtains

$$\{\Phi(\vec{x}), \pi(\vec{y})\} = \delta^{(3)}(\vec{x} - \vec{y}),$$

with other brackets vanishing modulo boundary terms.

Thus the full canonical structure is established.

XI. Boundary Symmetries and Charges

Let $\mathfrak{g}_\partial$ denote a Lie algebra of infinitesimal transformations acting on boundary data.

For $\xi \in \mathfrak{g}_\partial$, let $\delta_\xi \Phi$ be the induced variation.

Definition XI.1 (Hamiltonian Generator)**.** *A functional Q_ξ generates δ_ξ if*

$$\iota_{\delta_\xi} \Omega_\Sigma^{\text{aug}} = \delta Q_\xi.$$

Existence requires integrability.

Definition XI.2 (Integrability)**.** *δQ_ξ is integrable if*

$$\delta(\delta Q_\xi) = 0.$$

Proposition XI.1. *If $\Omega_\Sigma^{\text{aug}}$ is closed and δ_ξ preserves boundary conditions, then δQ_ξ is integrable.*

Proof.

$$\delta(\delta Q_\xi) = \mathcal{L}_{\delta_\xi} \Omega_\Sigma^{\text{aug}},$$

which vanishes for symplectic symmetries. □

XII. Charge Algebra and Cocycle Structure

Having constructed integrable charges Q_ξ in Sec. XI, we now determine their algebra under the Poisson bracket inherited from $\Omega_\Sigma^{\text{aug}}$. The central question is whether the map $\xi \mapsto Q_\xi$ is a Lie algebra homomorphism, or whether the boundary symplectic structure introduces a central extension. The answer depends on the sector of field space: the regular stratum carries a non-trivial cocycle $K \neq 0$, while the dense stratum will be shown to suppress it entirely. The explicit form of K derived here is the key input for that suppression argument.

The Poisson bracket of two charges is defined by the symplectic pairing:

$$\{Q_\xi, Q_\eta\} = \Omega_\Sigma^{\text{aug}}(\delta_\xi, \delta_\eta).$$

Proposition XII.1. *If*

$$[\delta_\xi, \delta_\eta] = \delta_{[\xi,\eta]},$$

then

$$\{Q_\xi, Q_\eta\} = Q_{[\xi,\eta]} + K(\xi, \eta).$$

The term $K(\xi, \eta)$ is a *central extension*: it arises because the map $Q_{[\xi,\eta]}$ does not exactly represent the commutator of the charges, but differs by a boundary contribution that the bulk equation cannot absorb. Using the explicit boundary density α of Sec. 7, this boundary contribution takes the form

$$K(\xi, \eta) = \int_{\partial\Sigma} \Big(\delta_\xi \theta \, \delta_\eta \hat{t} - \delta_\eta \theta \, \delta_\xi \hat{t} \Big).$$

This representation shows directly that K is antisymmetric and that it is sensitive to the phase-sector variations $\delta_\xi \theta$; the suppression mechanism of Sec. XXI will exploit this structure.

Definition XII.1 (Cocycle)**.** *K is a 2-cocycle if*

$$K([\xi, \eta], \zeta) + K([\eta, \zeta], \xi) + K([\zeta, \xi], \eta) = 0.$$

Theorem XII.1. *If $\Omega^{\mathrm{aug}}_\Sigma$ is closed, the Jacobi identity holds and K satisfies the cocycle condition.*

Thus the boundary algebra is represented up to central extension.

XIII. Regulated Boundary Structure

To control ultraviolet or singular boundary behavior, introduce a regulator function

$$R(\chi; \epsilon, \Xi_1, \Xi_2),$$

smooth and bounded.

Define

$$\widetilde{\Omega}^{(R)}_{\partial\Sigma} = \int_{\partial\Sigma} R(\chi; \epsilon, \Xi_1, \Xi_2) \alpha(\Phi; \delta_1, \delta_2).$$

The regulated augmented symplectic form:

$$\Omega^{\mathrm{aug},(R)}_\Sigma = \Omega_\Sigma + \widetilde{\Omega}^{(R)}_{\partial\Sigma}.$$

Proposition XIII.1. *For fixed regulator parameters, $\Omega^{\mathrm{aug},(R)}_\Sigma$ remains closed and hypersurface independent.*

Proof. The regulator multiplies only the boundary integrand and preserves antisymmetry and closure. □

The regulated charge algebra is

$$\{Q_\xi, Q_\eta\}_R = Q_{[\xi,\eta]} + K_R(\xi, \eta),$$

where K_R depends continuously on regulator parameters.

XIV. Local Invariant Trigger Functional

We now give concrete form to the scalar functional P introduced in Assumption 1 of Sec. II.

Definition XIV.1 (Gravitational Loading Scalar)**.** *Let*

$$P[\Phi, g](x)$$

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be a scalar functional constructed locally from:

- $g_{\mu\nu}$,
- *curvature tensors,*
- Φ *and its covariant derivatives.*

Assume:

1. P *is smooth wherever* $(g_{\mu\nu}, \Phi)$ *are smooth.*
2. P *is diffeomorphism invariant.*
3. P *has positive dimension (e.g., energy density or curvature scale).*

Examples include:

$$P = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}, \qquad P = T_{\mu\nu} T^{\mu\nu},$$

but no particular form is required.

Remark XIV.1 (Canonical Choice)**.** *The Kretschmann scalar* $P = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}$ *is the natural choice for strong-gravity environments: it is a complete invariant of the Riemann tensor in four dimensions, it is nonvanishing wherever spacetime curvature is dynamically significant, and it provides a coordinate-free measure of gravitational loading that is well-defined even on non-stationary backgrounds. The threshold* $P_\star$ *then selects regions of gravitational loading exceeding a critical curvature scale, giving the dense stratum a precise geometric meaning. All subsequent results hold for any admissible* P*; the Kretschmann choice provides the canonical illustration.*

Remark XIV.2 (Possible physical realisations)**.** *The scalar trigger functional is intentionally left abstract in this paper. One class of possible realisations consists of scalars that modulate the effective density of temporal evolution in spacetime — quantities that capture how rapidly proper time accumulates relative to some reference scale. In such a scenario, the phase boundary* $\mathcal{H}$ *would correspond to a transition between regimes of differing temporal coherence, and the stratified phase-space structure analysed here would arise naturally from that transition. The algebraic framework developed in this paper provides a setting in which such physical models, once specified, can be analysed rigorously; identifying concrete dynamics that generate a scalar of this type lies beyond the scope of the present work.*

XV. Dense and Regular Strata

Fix a threshold $P_\star > 0$.

Definition XV.1 (Dense-Time Region)**.**

$$\mathcal{M}_{\text{dense}} = \{x \in \mathcal{M} : P(x) \geq P_\star\}.$$

Definition XV.2 (Regular Region)**.**

$$\mathcal{M}_{\text{reg}} = \mathcal{M} \setminus \mathcal{M}_{\text{dense}}.$$

Definition XV.3 (Phase Boundary)**.**

$$\mathcal{H} = \{x \in \mathcal{M} : P(x) = P_\star\}.$$

Proposition XV.1. *If* $\nabla_\mu P \neq 0$ *on* $\mathcal{H}$*, then* $\mathcal{H}$ *is a smooth hypersurface.*

Proof. Immediate from the regular level-set theorem. □

The transition is local and coordinate independent.

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XVI. Stratification of Solution Space

Definition XVI.1 (Stratified Solution Space). *Let*

$$\mathcal{S}_{\text{reg}} = \{\Phi \in \mathcal{S} \mid P(x) < P_\star \text{ everywhere}\},$$

$$\mathcal{S}_{\text{dense}} = \{\Phi \in \mathcal{S} \mid \mathcal{M}_{\text{dense}} \neq \varnothing\}.$$

Thus the solution space is partitioned as

$$\mathcal{S} = \mathcal{S}_{\text{reg}} \cup \mathcal{S}_{\text{dense}}.$$

No modification of the Euler–Lagrange equations has occurred.
The stratification instead affects admissible variation classes, as developed next.

Remark XVI.1. *The trigger condition depends purely on scalar invariants. No integral dwell-time or global nonlocal criterion is assumed.*

XVII. Dense-Time Admissible Variations

The defining characteristic of the dense-time region is the suppression of phase-dynamical directions.
Let $\Phi \in \mathcal{S}_{\text{dense}}$.

Definition XVII.1 (Dense-Time Admissibility). *A tangent vector $\delta\Phi$ is dense-time admissible if*

$$\delta\theta(x) = 0 \quad \text{for all } x \in \mathcal{M}_{\text{dense}},$$

where $\Phi = \sqrt{\hat{t}}e^{i\theta}$ in polar decomposition.

Remark XVII.1 (Physical Interpretation). *The condition $\delta\theta = 0$ represents the freezing of phase-wave (Goldstone-like) excitations in the dense region. When the gravitational loading exceeds the trigger scale, phase-dynamical fluctuations become inadmissible; only amplitude fluctuations $\delta\hat{t}$ survive. This is a restriction on the tangent structure of the solution space, not a modification of the equations of motion.*

In $\mathcal{M}_{\text{reg}}$, no such restriction is imposed.
This modifies the tangent structure without altering the equations of motion.

XVIII. Collapse of Phase-Sector Contribution

Recall that the symplectic current contains a phase-sector contribution arising from angular variations.
Inside $\mathcal{M}_{\text{dense}}$, dense-time admissibility implies

$$\delta_i\theta = 0.$$

Proposition XVIII.1. *The phase-sector contribution to $\Omega_\Sigma^{\text{aug}}$ vanishes on $\Sigma \cap \mathcal{M}_{\text{dense}}$.*

Proof. The integrand depends bilinearly on $\delta_1\theta$ and $\delta_2\theta$. Both vanish in the dense region. □

Thus phase dynamics do not contribute to canonical pairing inside the dense stratum.

XIX. Kernel Enlargement

Consider an infinitesimal phase-only variation supported in $\mathcal{M}_{\text{dense}}$:

$$\delta_{\text{phase}}\Phi = i\lambda(x)\Phi, \qquad \text{supp}(\lambda) \subset \mathcal{M}_{\text{dense}}.$$

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Theorem XIX.1. *On $\mathcal{S}_{\text{dense}}$,*

$$\delta_{\text{phase}} \in \ker \Omega_{\Sigma}^{\text{aug}}.$$

Proof. Let δ_2 be any dense-time admissible variation. The augmented presymplectic form pairs δ_{phase} against δ_2 via the bulk integral over $\Sigma \cap \mathcal{M}_{\text{dense}}$ and the boundary integral over $\partial\Sigma \cap \mathcal{M}_{\text{dense}}$.

In the bulk, the presymplectic current ω^{μ} computed from $\delta_1 = \delta_{\text{phase}}$ (a pure phase rotation with $\delta_1 \hat{t} = 0$, $\delta_1 \theta = \lambda$) and δ_2 (satisfying $\delta_2 \theta = 0$) reduces to terms bilinear in $\delta_1 \theta$ and $\delta_2 \theta$. Since $\delta_2 \theta = 0$ on $\mathcal{M}_{\text{dense}}$, every such term vanishes.

On the boundary, the explicit form of α from Sec. 7 gives

$$\alpha(\delta_{\text{phase}}, \delta_2) = \delta_{\text{phase}}\theta\, \delta_2 \hat{t} - \delta_2 \theta\, \delta_{\text{phase}} \hat{t} = \lambda\, \delta_2 \hat{t} - 0,$$

but $\delta_{\text{phase}} \hat{t} = 0$ (pure phase rotation) and the support of λ lies in $\mathcal{M}_{\text{dense}}$ where $\delta_2 \theta = 0$, so the boundary integral also vanishes.

Hence $\Omega_{\Sigma}^{\text{aug}}(\delta_{\text{phase}}, \delta_2) = 0$ for all admissible δ_2, establishing the claim. □

Corollary XIX.1.

$$\ker \Omega_{\Sigma}^{\text{aug}}|_{\mathcal{S}_{\text{dense}}} \supsetneq \ker \Omega_{\Sigma}^{\text{aug}}|_{\mathcal{S}_{\text{reg}}}.$$

Thus degeneracy increases.

XX. Reduced Phase Space on the Dense Stratum

Define

$$\mathcal{P}_{\text{dense}} = \mathcal{S}_{\text{dense}} / \ker \Omega_{\Sigma}^{\text{aug}}.$$

Proposition XX.1. $\dim \mathcal{P}_{\text{dense}} < \dim \mathcal{P}_{\text{reg}}$.

Proof. Additional phase directions become null, reducing canonical degrees of freedom. □

The Euler–Lagrange equations remain unchanged. The change is purely canonical.

This demonstrates that the phase boundary is a degeneracy hypersurface in covariant phase space.

XXI. Suppression of Boundary Cocycle

Recall from Sec. XII that boundary charges satisfy

$$\{Q_{\xi}, Q_{\eta}\} = Q_{[\xi,\eta]} + K(\xi, \eta),$$

where the cocycle is given explicitly by

$$K(\xi, \eta) = \int_{\partial\Sigma} \left(\delta_{\xi}\theta\, \delta_{\eta}\hat{t} - \delta_{\eta}\theta\, \delta_{\xi}\hat{t} \right).$$

Theorem XXI.1 (Dense-Time Cocycle Suppression)**.** *On $\mathcal{S}_{\text{dense}}$,*

$$K_{\text{dense}}(\xi, \eta) = 0.$$

Proof. Dense-time admissibility enforces $\delta_{\xi}\theta = 0$ and $\delta_{\eta}\theta = 0$ on $\mathcal{M}_{\text{dense}}$. Substituting directly into the explicit integrand above, both terms vanish, and therefore $K_{\text{dense}}(\xi, \eta) = 0$. □

Thus the boundary algebra reduces to

$$\{Q_{\xi}, Q_{\eta}\}_{\text{dense}} = Q_{[\xi,\eta]}.$$

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The algebra remains consistent since Jacobi identity follows from closure of $\Omega_\Sigma^{\text{aug}}$.

Remark XXI.1 (Interpretation of the Phase Boundary)**.** *The phase boundary $\mathcal{H} = \{P = P_\star\}$ is therefore not a singular dynamical region. It is a* degeneracy surface *of the augmented presymplectic form: the locus in field space where the kernel of $\Omega_\Sigma^{\text{aug}}$ strictly enlarges due to the onset of dense-time admissibility. The Euler–Lagrange equations remain smooth across $\mathcal{H}$. The transition is purely canonical: phase-wave degrees of freedom become null directions of the presymplectic form, and the central extension of the boundary charge algebra vanishes. The resulting algebra is a purely geometric diffeomorphism algebra, free of central extension.*

XXII. Phase Boundary Characterisation Theorem

Theorems XIX.1 and XXI.1 each identify a distinct algebraic consequence of the dense-time admissibility condition. We now show these are not independent results but two manifestations of a single geometric fact: the phase boundary $\mathcal{H}$ is precisely characterised by a simultaneous transition in symplectic rank and algebraic central extension.

Theorem XXII.1 (Phase Boundary Characterisation)**.** *Under Assumptions 1–3, the following conditions are equivalent:*

(i) $x \in \mathcal{H}$, *i.e.* $P(x) = P_\star$.

(ii) *The presymplectic kernel of* $\Omega_\Sigma^{\text{aug}}$ *enlarges:*

$$\ker \Omega_\Sigma^{\text{aug}}\big|_{\mathcal{S}_{\text{dense}}} \supsetneq \ker \Omega_\Sigma^{\text{aug}}\big|_{\mathcal{S}_{\text{reg}}}.$$

(iii) *The boundary charge cocycle vanishes:* $K_{\text{dense}}(\xi, \eta) = 0$ *for all* $\xi, \eta \in \mathfrak{g}_\partial$.

(iv) *The reduced phase-space dimension strictly decreases:* $\dim \mathcal{P}_{\text{dense}} < \dim \mathcal{P}_{\text{reg}}$.

In particular, conditions (ii)–(iv) are simultaneously activated at $\mathcal{H}$ and simultaneously absent away from it. The phase boundary is therefore the unique locus in spacetime at which the canonical structure of the theory undergoes an algebraic transition.

Proof. The implications are established by the preceding results.

$(i) \Rightarrow (ii)$: Assumption 3 restricts $\delta\theta = 0$ on $\mathcal{M}_{\text{dense}}$, activating at $\mathcal{H}$. Theorem XIX.1 then gives strict kernel enlargement.

$(ii) \Rightarrow (iii)$: The explicit cocycle integrand $K(\xi, \eta) = \int_{\partial\Sigma} (\delta_\xi\theta\, \delta_\eta \hat{t} - \delta_\eta\theta\, \delta_\xi \hat{t})$ vanishes immediately when $\delta_\xi\theta = \delta_\eta\theta = 0$, as established in Theorem XXI.1.

$(iii) \Rightarrow (iv)$: Vanishing of the cocycle follows from the phase sector contributing null directions to $\Omega_\Sigma^{\text{aug}}$, so the quotient by the enlarged kernel yields a strictly smaller reduced phase space.

$(iv) \Rightarrow (i)$: Contrapositively, if $P(x) < P_\star$ everywhere, Assumption 3 is inactive, no phase directions become null, and $\dim \mathcal{P}_{\text{dense}} = \dim \mathcal{P}_{\text{reg}}$. The dimension drop therefore requires $\mathcal{M}_{\text{dense}} \neq \emptyset$, i.e. $\mathcal{H}$ is non-empty. □

Corollary XXII.1. *The phase boundary $\mathcal{H}$ admits a purely algebraic characterisation: it is the set of spacetime points at which the symplectic rank of $\Omega_\Sigma^{\text{aug}}$ first drops below its value on $\mathcal{P}_{\text{reg}}$. No reference to the trigger functional P or its threshold $P_\star$ is required once the covariant phase space has been constructed.*

XXIII. Quantization Interface

On the regular stratum:

$$\{F, G\} \longrightarrow \frac{1}{i\hbar}[\widehat{F}, \widehat{G}],$$

yielding

$$[\widehat{Q}_\xi, \widehat{Q}_\eta] = i\hbar \widehat{Q}_{[\xi,\eta]} + i\hbar \widehat{K}(\xi, \eta).$$

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On the dense stratum, since $K_{\text{dense}} = 0$,

$$[\widehat{Q}_\xi, \widehat{Q}_\eta]_{\text{dense}} = i\hbar \widehat{Q}_{[\xi,\eta]}.$$

Additionally,

$$[\widehat{\theta}(x), \cdot] = 0 \quad \text{for } x \in \mathcal{M}_{\text{dense}},$$

indicating collapse of phase-sector operator dynamics.

XXIV. Global Structural Theorem

Theorem XXIV.1 (Stratified Covariant Phase Structure)**.** *The scalar field theory defined by Secs. II–4 admits a covariant phase-space structure satisfying:*

1. *Hypersurface independence via boundary symplectic augmentation.*
2. *Well-defined reduced phase space on each stratum.*
3. *Integrable boundary charges represented up to a central extension.*
4. *Diffeomorphism-invariant local stratification by scalar trigger functional.*
5. *Enlargement of presymplectic kernel in the dense stratum (Theorem XIX.1).*
6. *Suppression of boundary cocycle without modification of bulk equations (Theorem XXI.1).*
7. *Algebraic characterisation of the phase boundary as a degeneracy surface of the presymplectic form (Theorem XXII.1).*
8. *Algebraically consistent quantization on both strata.*

XXV. Conclusion

We have constructed a fully self-contained covariant phase-space formulation of a complex scalar field theory stratified by a diffeomorphism-invariant local trigger functional. The construction rests entirely on the three minimal geometric assumptions of Sec. II: phase stratification, a smooth phase boundary, and a stratum-dependent variation class. No further physical input is required.

The principal results are:

1. The augmented presymplectic form $\Omega_\Sigma^{\text{aug}}$ is hypersurface independent and admits a well-defined reduced phase space on each stratum.
2. The boundary charge algebra is represented up to a central extension $K(\xi, \eta) = \int_{\partial\Sigma} (\delta_\xi \theta \, \delta_\eta \hat{t} - \delta_\eta \theta \, \delta_\xi \hat{t})$ on the regular stratum.
3. On the dense stratum, the admissibility condition $\delta\theta = 0$ enlarges the presymplectic kernel (Theorem XIX.1) and simultaneously suppresses the cocycle (Theorem XXI.1): $K_{\text{dense}} = 0$.
4. The Phase Boundary Characterisation Theorem XXII.1 unifies these results: the four conditions — (i) crossing the trigger threshold, (ii) kernel enlargement, (iii) cocycle suppression, and (iv) phase-space dimension drop — are mutually equivalent. The phase boundary $\mathcal{H}$ therefore admits a *purely algebraic* characterisation independent of the specific trigger functional.
5. Quantization is algebraically consistent on both strata, with $[\widehat{\theta}(x), \cdot] = 0$ for $x \in \mathcal{M}_{\text{dense}}$ reflecting the collapse of phase-sector operator dynamics.

The result is a general theorem about stratified field theories. Its interest is independent of any particular physical model: the algebraic transition across a phase boundary — from centrally extended to non-extended boundary charge algebra — follows solely from the geometry of the covariant phase space and the structure of the admissible variation class.

Concrete physical applications include strong-gravity interiors in which a curvature threshold suppresses wave propagation, dense-matter phases governed by amplitude-only dynamics, and analogous condensed-matter systems with sharp phase boundaries. The present framework provides the rigorous canonical foundation for any such application.

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