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Peter Chew Sum: Beyond Newton Sum in Quadratic Root Function Computations

Peter Chew *

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Article

Peter Chew Sum: Beyond Newton Sum in Quadratic Root Function Computations

Peter Chew [1]

[1] PCET Multimedia Education Malaysia; peterchew999@hotmail.my

Abstract: This study provides a comprehensive comparison of the Peter Chew Sum method and Newton's Sum in Quadratic Root Function Computations. Both methods offer significant advancements by alleviating the need for memorization of multiple formulas. However, when confronted with higher-order quadratic root functions such as the calculation of power 33. Newton's Sum method proves cumbersome due to its requirement of all previous answers for final computation, leading to increased steps and error risks. In contrast, the Peter Chew Sum method simplifies the computational process by requiring only a few previous answers, resulting in a more straightforward series of operations. We contextualize our analysis within the historical progression of mathematical techniques, highlighting the transformative leap represented by the Peter Chew Sum method in quadratic root function computations. Peter Chew Sum addressing the limitations of Newton's Sum , especially concerning higher-order functions such as **the calculation of power 256**. Furthermore, study demonstrates that the innovative approach of the Peter Chew Sum method not only simplifies complex computations but also fosters enhanced conceptual understanding among students. In conclusion, study findings underscore the profound impact of the Peter Chew Sum method on quadratic root function computations.

Keywords: Peter Chew Sum; Newton's Sum; Quadratic Root Function Computations

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1. Introduction

1.1. Algebra Formula: Symmetric ***Function***[1,2,3] *Of A Quadratic's Roots*

i) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

Prove: $\alpha^2 + \beta^2 = \alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta$

$= (\alpha+\beta)^2 - 2\alpha\beta$

ii) $\alpha^3 + \beta^3 = (\alpha+\beta)[(\alpha+\beta)^2 - 3\alpha\beta]$

Prove:

$\alpha^3 + \beta^3 = (\alpha+\beta)[\alpha^2 + \beta^2 - \alpha\beta]$

$= (\alpha+\beta)[\alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta - \alpha\beta]$

$= (\alpha+\beta)[(\alpha+\beta)^2 - 3\alpha\beta]$

Or $\boldsymbol{\alpha^3+\beta^3} = (\alpha+\beta)^3 \boldsymbol{- 3\alpha\beta}(\alpha+\beta)$

iii) $\alpha^4 + \beta^4 = [(\alpha+\beta)^2 - 2(\alpha\beta)]^2 - 2(\alpha\beta)^2$

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Prove: $\alpha^4 + \beta^4 = \alpha^4 + \beta^4 + 2\,\alpha^2\beta^2 - 2\,\alpha^2\beta^2$

$= (\alpha^2 + \beta^2)^2 - 2\,\alpha^2\beta^2$

$=[(\alpha+\beta)^2 - 2\alpha\beta]^2 - 2\,(\alpha\beta)^2$ [From ii]

iv) $\alpha^5 + \beta^5$

$= [(\alpha+\beta)^2 - 2\alpha\,\beta]\{(\alpha+\beta)[(\alpha+\beta)^2 - 3\alpha\,\beta]\} - (\alpha\beta)^2(\alpha+\beta)$

Prove:

$\alpha^5 + \beta^5$

$= (\alpha^2+\beta^2)(\alpha^3+\beta^3) - \alpha^2\beta^3 - \beta^2\alpha^3$

$= (\alpha^2+\beta^2)(\alpha^3+\beta^3) - \alpha^2\beta^2(\beta+\alpha)$

$= [(\alpha+\beta)^2 - 2\alpha\,\beta]\{(\alpha+\beta)[(\alpha+\beta)^2 - 3\alpha\,\beta]\} - (\alpha\beta)^2(\alpha+\beta)$

[From ii and iii]

v) $\alpha^6 + \beta^6 = \{(\alpha+\beta)\,[(\alpha+\beta)^2 - 3\alpha\,\beta]\}^2 - 2\,(\alpha\beta)^3$

Prove:

$\alpha^6 + \beta^6$

$= \boldsymbol{\alpha^6 + \beta^6 + 2\,\alpha^3\beta^3} - 2\,\alpha^3\beta^3$

$= (\boldsymbol{\alpha^3 + \beta^3})^{\boldsymbol{2}} - 2\,\alpha^3\beta^3$

$= \{(\alpha+\beta)\,[(\alpha+\beta)^2 - 3\alpha\,\beta]\}^2 - 2\,(\alpha\beta)^3$ [From iii]

vi) $\alpha^7 + \beta^7 = [(\alpha+\beta)^{\boldsymbol{3}} - \boldsymbol{3}\alpha\beta(\alpha+\beta)]\,\{[(\alpha+\beta)^2 - 2\,(\alpha\,\beta)]^2 - [2\,(\alpha\beta)^2]\} - \alpha^3\beta^3(\beta+\alpha)$

Prove:

$\alpha^7 + \beta^7$

$= (\alpha^3+\beta^3)(\alpha^4+\beta^4) - \alpha^3\beta^4 - \beta^3\alpha^4$

$= [(\alpha+\beta)^{\boldsymbol{3}} - \boldsymbol{3}\alpha\beta(\alpha+\beta)]\,\{[(\alpha+\beta)^2 - 2\,(\alpha\,\beta)]^2 - [2\,(\alpha\beta)^2]\} - \alpha^3\beta^3(\beta+\alpha)$ From (ii) and (iii)

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vii) $\alpha^8 + \beta^8 = \{ [(\alpha+\beta)^2 - 2(\alpha\beta)]^2 - 2(\alpha\beta)^2\}^2 - 2(\alpha\beta)^4$

Prove: $\alpha^8 + \beta^8$

$= \alpha^8 + \beta^8 + 2\alpha^4\beta^4 - 2\alpha^4\beta^4$

$= (\alpha^4 + \beta^4)^2 - 2\alpha^4\beta^4$

$= \{ [(\alpha+\beta)^2 - 2(\alpha\beta)]^2 - 2(\alpha\beta)^2 \}^2 - 2(\alpha\beta)^4$ [From iii]

1.2. Example Quadratic Root function Calculation [Algebra Formula]

Example[4]: If α, β are the zeroes of the polynomial $x^2 - 2x + 2$. find the value of i) $\alpha^2+\beta^2$, ii) $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$,iii) $\alpha^3+\beta^3$, iv) $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}$ v) $\alpha^4+\beta^4$

Solution

Sum of the roots = $\alpha + \beta = -(-\frac{2}{1}) = 2$, Product of the roots = $\alpha\beta = \frac{c}{a} = 2$

$i)\ \alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = 2^2 - 2\times2 = 4 - 4 = 0$

ii) $\frac{\alpha}{\beta}+\frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{0}{2} = 0$

iii) $\alpha^3+\beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

$= (2)^3 - 3(2)(2)$ **from (i)**

$= 8 - 12$

$= -4$

iv) $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}$

$= \frac{\alpha^3+\beta^3}{\alpha\beta}$

$= \frac{-4}{2}$ from (iii)

$= -2$

v) $\alpha^4 + \beta^4 = (\alpha^2+\beta^2)^2 - 2(\alpha\beta)^2$

$= 0 - 2(\alpha\beta)^2$ from (i)

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$= -2\ (\mathbf{2})^{2}$

$= -8$

2. Peter Chew Sum.

$\mathbf{P}_n = \mathbf{P}_a\ \mathbf{P}_b$ - $(\boldsymbol{\alpha}\ \boldsymbol{\beta})^{a}\ \mathbf{P}_{b-a}$, where $\boldsymbol{a} + \boldsymbol{b} = \boldsymbol{n}$, b $\geq$ a

If a = b or n = $\mathbf{2}\boldsymbol{a}$, $\mathbf{P}_n = (\mathbf{P}_a)^{2}$ - 2 $(\boldsymbol{\alpha}\ \boldsymbol{\beta})^{a}$

2.1. Prove [2,3] of Peter Chew Sum

$$\mathbf{Let}\ \mathbf{P}_n = \boldsymbol{\alpha}^{n} + \boldsymbol{\beta}^{n}\ \ \boldsymbol{and}\ \boldsymbol{a} + \boldsymbol{b} = \boldsymbol{n}\ , \mathbf{b} \geq \mathbf{a}$$

$$\boldsymbol{\alpha}^{n} + \boldsymbol{\beta}^{n} = \boldsymbol{\alpha}^{a+b} + \boldsymbol{\beta}^{a+b}$$

$$= (\boldsymbol{\alpha}^{a} + \boldsymbol{\beta}^{a})(\boldsymbol{\alpha}^{b} + \boldsymbol{\beta}^{b}) - \boldsymbol{\alpha}^{a}\boldsymbol{\beta}^{b} - \boldsymbol{\beta}^{a}\boldsymbol{\alpha}^{b}$$

$$= \mathbf{P}_a\ \mathbf{P}_b - (\boldsymbol{\alpha}\ \boldsymbol{\beta})^{a}\left(\frac{\boldsymbol{\beta}^{b}}{\boldsymbol{\beta}^{a}} + \frac{\boldsymbol{\alpha}^{b}}{\boldsymbol{\alpha}^{a}}\right)$$

$$= \mathbf{P}_a\ \mathbf{P}_b - (\boldsymbol{\alpha}\ \boldsymbol{\beta})^{a}\left(\boldsymbol{\beta}^{b-a} + \boldsymbol{\alpha}^{b-a}\right)$$

$$= \mathbf{P}_a\ \mathbf{P}_b - (\boldsymbol{\alpha}\ \boldsymbol{\beta})^{a}\ \mathbf{P}_{b-a}\ \ \text{(Prove)}$$

If a = b , n = $\boldsymbol{a} + \boldsymbol{a} = \mathbf{2}\boldsymbol{a}$.

$$\mathbf{P}_n = \mathbf{P}_a\ \mathbf{P}_a - (\boldsymbol{\alpha}\ \boldsymbol{\beta})^{a}\ \mathbf{P}_{a-a}$$

$$= (\mathbf{P}_a)^{2} - (\boldsymbol{\alpha}\ \boldsymbol{\beta})^{a}\ \mathbf{P}_0$$

$$= (\mathbf{P}_a)^{2} - 2\ (\boldsymbol{\alpha}\ \boldsymbol{\beta})^{a}$$

2.2. Example using Peter Chew Sum Method.

Example : If the roots of the equation x^2 - 3x + 2 =0 **are** α **and** β , find the $\boldsymbol{\alpha}^{8}+\boldsymbol{\beta}^{8}$.

Solution: $\boldsymbol{P}_i = \boldsymbol{\alpha}^{i}+\boldsymbol{\beta}^{i}$, So, $\boldsymbol{P}_0 = \boldsymbol{\alpha}^{0}+\boldsymbol{\beta}^{0}$= 2. $\boldsymbol{P}_1 = \boldsymbol{\alpha}^{1}+\boldsymbol{\beta}^{1}$= 3 , αβ = 2.

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$P_2 = (\mathbf{P_1})^2 - 2\,(\boldsymbol{\alpha\,\beta})^1$

$= (\mathbf{3})^2 - 2\,(\mathbf{2})^1$

$= \mathbf{5}$

$P_4 = (\mathbf{P_2})^2 - 2\,(\boldsymbol{\alpha\,\beta})^2$

$= (\mathbf{5})^2 - 2\,(\mathbf{2})^2$

$= \mathbf{17}$

$P_8 = (\mathbf{P_4})^2 - 2\,(\boldsymbol{\alpha\,\beta})^4$

$= (\mathbf{17})^2 - 2\,(\mathbf{2})^4$

$= \mathbf{257}$

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3. Newton Sum (**Newton's Identities**)[5,6]

Newton's identities, also known as **Newton-Girard formulae**, is an efficient way to find the power sum of roots of polynomials without actually finding the roots. If $x_1, x_2, \ldots\ldots x_n$ are the roots of a polynomial equation, then Newton's identities are used to find the summations like

$$\sum_{i=1}^{n} x_i^k = x_1^k + x_2^k + \cdots + x_n^k.$$

It is mainly used in conjunction with **Vieta's formula** while working with the (complex) roots (say $a_1, a_2, \ldots\ldots a_k$) of a k^{th} degree polynomial. The main idea is that the **elementary symmetric polynomials** form an algebraic basis to produce all symmetric polynomials. Newton's identity gives us the calculation via a recurrence relation with known coefficients.

Newton's Identities for a Quadratic Polynomial

Suppose that you have a quadratic polynomial $P(x)$ with (complex) roots α_1 and α_2. Now, you are asked to find the value of $\alpha_1{}^2 + \alpha_2{}^2$.

This seems very easy since you can use **Vieta's formula** along with the identity $(a+b)^2 = a^2 + b^2 + 2ab$ to find the required result. But what if you need to find ($\alpha_1{}^{10} + \alpha_2{}^{10}$) ? This would take a while if you were to simply use algebraic manipulations. But there's a clever way, using Newton's sums.

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Let P(x) = ax^2 + bx + c . Then using Vieta`s formula,

we can get $\alpha_1 + \alpha_2 = -\frac{b}{a}$ and $\alpha_1 \alpha_2 = \frac{c}{a}$.

Denote P_i *as the* i^{th} *power sum of the roots, namely*

$$P_i = \alpha_1{}^i + \alpha_2{}^i \ .$$

Then we can obtain P_i *recursively as follows.*

$$P_0 = \alpha_1{}^0 + \alpha_2{}^0$$

$$= 2$$

$$P_1 = \alpha_1{}^1 + \alpha_2{}^1 = -\frac{b}{a}$$

$$P_2 = \alpha_1{}^2 + \alpha_2{}^2$$

$$= (\alpha_1 + \alpha_2)(\alpha_1{}^1 + \alpha_2{}^1) - 2\,\alpha_1\alpha_2$$

$$= -\frac{b}{a}\ P_1 - \frac{c}{a}\ P_0$$

$$P_3 = \alpha_1{}^3 + \alpha_2{}^3$$

$$= (\alpha_1 + \alpha_2)(\alpha_1{}^2 + \alpha_2{}^2) - \alpha_1\alpha_2\,(\alpha_1 + \alpha_2)$$

$$= -\frac{b}{a}\ P_2 - \frac{c}{a}\ P_1$$

$$P_i = \alpha_1{}^i + \alpha_2{}^i$$

$$= (\alpha_1 + \alpha_2)(\alpha_1{}^{i-1} + \alpha_2{}^{i-1}) - \alpha_1\alpha_2\,(\alpha_1{}^{i-2} + \alpha_2{}^{i-2})$$

$$= -\frac{b}{a}\ P_{i-1} - \frac{c}{a}\ P_{i-2}$$

This is a linear recurrence relation that gives us the i^{th} power sum. Note that solving this recurrence to get a closed-form solution is equivalent to finding the roots of the quadratic polynomial.

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THEOREM

For a quadratic polynomial $f(x) = ax^2 + bx + c$ with (complex) roots α_1, α_2, we denote the i^{th} power sum of the roots as $P_i = \sum_{k=1}^{2} \alpha_k^i$ and the sum and product of the roots as A and B, respectively. Then we have

$$\begin{aligned} P_1 &= A \\ P_2 &= AP_1 - 2B \\ &\vdots \\ P_i &= AP_{i-1} - BP_{i-2} \ \forall i \geq 2. \end{aligned}$$

EXAMPLE

Let a polynomial $P(x)$ be defined as $P(x) = x^2 - 2x + 6$ with its (complex) roots a and b. Then what is the value of $a^{10} + b^{10}$?

By Vieta's formula, the sum of the roots is $2 = A$ (say) and the product of the roots is $6 = B$ (say). Now, using the recurrence relation found above and Newton's identities, we have

$$P_i = AP_{i-1} - BP_{i-2} = 2P_{i-1} - 6P_{i-2} \ \forall i \geq 2, \quad P_0 = 2, \quad P_1 = A = 2.$$

Now, we can simply use this recurrence relation obtained repeatedly to find $P_2, P_3, \ldots, P_9$ and then finally P_{10} which is the required answer. The answer comes out to be $2(-3808) - 6(-2528)$.

Hence, our final answer is 7552. $\square$

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Full Calculation;

P(x) = x^2 - 2x + 6 , A=2 , B= 6.

$$P_i = A\ P_{i-1} - B\ P_{i-2}$$

$$= 2\ P_{i-1} - 6\ P_{i-2}$$

$$P_0 = \alpha_1{}^0 + \alpha_2{}^0 = 2$$

$$P_1 = \alpha_1 + \alpha_2 = A = 2$$

$$P_2 = 2\ P_{2-1} - 6\ P_{2-2}$$

$$= 2\ P_1 - 6\ P_0$$

$$= 2(2) - 6(2)$$

$$= -8$$

$$P_3 = 2\ P_{3-1} - 6\ P_{3-2}$$

$$= 2\ P_2 - 6\ P_1$$

= 2(-8) - 6(2)

= - 28

$$P_4 = 2\ \mathrm{P}_{4-1} - 6\,\mathrm{P}_{4-2}$$

$$= 2\ \mathrm{P}_3 - 6\,\mathrm{P}_2$$

= 2(-28) - 6(-8)

= - 8

$$P_5 = 2\ \mathrm{P}_{5-1} - 6\,\mathrm{P}_{5-2}$$

$$= 2\ \mathrm{P}_4 - 6\,\mathrm{P}_3$$

= 2(-8) - 6(-28)

= 152

$$P_6 = 2\ \mathrm{P}_5 - 6\,\mathrm{P}_4$$

= 2(152) - 6(-8)

= 352

$$P_7 = 2\ \mathrm{P}_6 - 6\,\mathrm{P}_5$$

= 2(352) - 6(152)

= -208

$$P_8 = 2\ \mathrm{P}_7 - 6\,\mathrm{P}_6$$

= 2(-208) - 6(352)

= -2528

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$P_9 = 2\ \mathrm{P}_8 - 6\ \mathrm{P}_7$

= 2(-2528) - 6(-208)

= -3808

$P_{10} = 2\ \mathrm{P}_9 - 6\ \mathrm{P}_8$

= 2(-3808) - 6(-2528)

= 7 552

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4. Peter Chew Sum and Newton Sum free students from the arduous task of memorizing various formulas for Calculating Quadratic Root Functions

4.1. Real Root Quadratic Equation

Example : If the roots of the equation x^2 - 6x + 5 = 0 **are** α **and** β , find the value of i) $\alpha^2+\beta^2$, ii) $\alpha^3+\beta^3$, iii) $\alpha^4+\beta^4$ iv) $\alpha^5+\beta^5$, v) $\alpha^6+\beta^6$, vi) $\alpha^7+\beta^7$ and vii) $\alpha^8+\beta^8$,

Solution:

Sum of the roots = α + β

$= -\frac{b}{a}$

$= -\left(\frac{-6}{1}\right)$

= 6

Product of the roots = αβ

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$= \frac{c}{a}$

$= \frac{5}{1}$

$= 5$

Let A= α + β = 6 and B = αβ = 5

$P_n = \alpha^n + \beta^n$.

$P_0 = \alpha^0 + \beta^0 = 2$.

$P_1 = \alpha^1 + \beta^1 = 6$.

Newton Sum Method

$Let\ P_i = A\ P_{i-1} - B\ P_{i-2}$

$= 6\ P_{i-1} - 5\ P_{i-2}$

Peter Chew Sum Method

$P_n = P_a P_b - (\alpha\beta)^a P_{b-a}$, where $a + b = n$, $b \geq a$.

i) $\alpha^2 + \beta^2$,

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a)Algebraic formulas Method:

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$\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\,\beta$ **from 1.1**

$= 6^2 - 2\times(5)$

$= 36 - 10$

$= 26$

$\therefore \alpha^2+\beta^2 = 26$

---

b) Newton's Sum method

$P_2 = 6\ P_1 - 5\ P_0$

$= 6\ (6) - 5\ (2)$

$= 26$

$\therefore \alpha^2+\beta^2 = 26$

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c) Peter Chew Sum method ,

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$\mathbf{P}_{n} = \mathbf{P}_{a}\ \mathbf{P}_{b} - (\boldsymbol{\alpha}\ \boldsymbol{\beta})^{a}\ \mathbf{P}_{b-a}$, where $\boldsymbol{a} + \boldsymbol{b} = \boldsymbol{n}$, **b** $\geq$ **a .**

$\boldsymbol{P}_{2} = \mathbf{P}_{1}\ \mathbf{P}_{1} - (\boldsymbol{\alpha}\ \boldsymbol{\beta})^{1}\ \mathbf{P}_{1-1}$

= 6 (6) – 5 (2)

= 26

$\therefore \boldsymbol{\alpha}^{2}+\boldsymbol{\beta}^{2}= \mathbf{26}$

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ii) $\boldsymbol{\alpha}^{3}+\boldsymbol{\beta}^{3}$,
a)Algebraic formulas Method:

$\boldsymbol{\alpha}^{3}+\boldsymbol{\beta}^{3}=(\alpha + \beta)[(\alpha+\beta)^{2} - 3\alpha\,\beta]$ **from 1.1**

$= (\mathbf{6})[(\mathbf{6})^{2} - \mathbf{3}(\mathbf{5})]$

= 6 (36 - 15)

= 126

$\therefore \boldsymbol{\alpha}^{3}+\boldsymbol{\beta}^{3}= \mathbf{126}$

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b) Newton's Sum method

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$P_3 = 6\,P_2 - 5\,P_1$

$= 6\,(26) - 5(6)$

$= 126$

$\therefore \alpha^3 + \beta^3 = 126$

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c) Peter Chew Sum method

$P_n = P_a\,P_b - (\alpha\,\beta)^a\,P_{b-a}$, where $a + b = n$, $b \geq a$.

$P_3 = P_1\,P_2 - (\alpha\,\beta)^1\,P_{2-1}$

$= 6\,(26) - 5\,(6)$

$= 126$

$\therefore \alpha^3 + \beta^3 = 126$

iii) $\alpha^4 + \beta^4$,

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a)Algebraic formulas Method:

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$\alpha^4+\beta^4 = [(\alpha+\beta)^2 - 2(\alpha\beta)]^2 - 2(\alpha\beta)^2$ **from 1.1**

$= [(6)^2 - 2(5)]^2 - 2(5)^2$

$= [26]^2 - 50$

$= 626$

$\therefore \alpha^4+\beta^4 = 626$

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b) Newton's Sum method

$P_4 = 6\ P_3 - 5\ P_2$

$= 6\ (126) - 5\ (26)$

$= 82$

$\therefore \alpha^4+\beta^4 = 626$

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c) Peter Chew Sum method

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$\mathbf{P}_n = \mathbf{P}_a\ \mathbf{P}_b - (\alpha\,\beta)^a\ \mathbf{P}_{b-a}$, where $\boldsymbol{a} + \boldsymbol{b} = \boldsymbol{n}$, $\mathbf{b} \geq \mathbf{a}$.

$P_4 = \mathbf{P}_2\ \mathbf{P}_2 - (\alpha\,\beta)^2\ \mathbf{P}_{2-2}$

$= 26\ (26) - 5^2\ (2)$

$= 626$

$\therefore \alpha^4 + \beta^4 = 626$

iv) $\alpha^5 + \beta^5$,

a)Algebraic formulas Method:

$\alpha^5 + \beta^5$

$= [(\alpha + \beta)^2 - 2\alpha\,\beta]\{(\alpha + \beta)[(\alpha + \beta\}^2 - 3\alpha\,\beta]\} - (\alpha\beta)^2(\alpha + \beta)$

[From 1.1]

$= [(6)^2 - 2(5)]\{(6)[(6)^2 - 3(5)]\} - (5)^2(6)$

$= [26][(6)21] - 150$

$= 3\ 126$

$\therefore \alpha^5 + \beta^5 = 3\ 126$

b) Newton's Sum method

$P_5 = 6\ \mathbf{P}_4 - 5\ \mathbf{P}_3$

$= 6\ (626) - 5(126)$

$= 3\ 126$

$\therefore \alpha^5 + \beta^5 = 3\ 126$

c) Peter Chew Sum method

$\mathbf{P}_n = \mathbf{P}_a\ \mathbf{P}_b - (\alpha\,\beta)^a\ \mathbf{P}_{b-a}$, where $\boldsymbol{a} + \boldsymbol{b} = \boldsymbol{n}$, $\mathbf{b} \geq \mathbf{a}$.

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$P_5 = P_2\,P_3 - (\alpha\,\beta)^2\,P_{3-2}$

$= 26\,(126) - 5^2\,(6)$

$= 3\,126$

$\therefore \alpha^5 + \beta^5 = 3\,126$

v) $\alpha^6+\beta^6$,

a)Algebraic formulas Method:

$\alpha^6 + \beta^6$

$= \{\,(\alpha+\beta)\,[\,(\alpha+\beta)^2 - 3\alpha\,\beta]\,\}^2 - 2\,(\alpha\beta)^3$ [From 1.1]

$= \{\,(6)\,[\,(6)^2 - 3(5)]\,\}^2 - 2\,(5)^3$

$= \{\,(6)\,[\,21\,]\,\}^2 - 250$

$= 15\,626$

$\therefore \alpha^6+\beta^6 = 15\,626$

b) Newton's Sum method

$P_6 = 6\,P_5 - 5\,P_4$

$= 6\,(3\,126) - 5\,(626)$

$= 15\,626$

$\therefore \alpha^6+\beta^6 = 15\,626$

c) Peter Chew Sum method

$P_n = P_a\,P_b - (\alpha\,\beta)^a\,P_{b-a}$, where $a + b = n$, $b \geq a$.

$P_6 = P_3\,P_3 - (\alpha\,\beta)^3\,P_{3-3}$

$= \mathbf{126\ (126) - 5^3\ (2)}$

$= 15\ 626$

$\therefore \boldsymbol{\alpha^6 + \beta^6} = 15\ 626$

vi) $\boldsymbol{\alpha^7 + \beta^7}$,

a)Algebraic formulas Method:

$\alpha^7 + \beta^7 = [(\alpha+\beta)^3 - \mathbf{3}\alpha\beta(\alpha+\beta)]\{[(\alpha+\beta)^2 - \mathbf{2}(\alpha\beta)]^2 - [\mathbf{2}(\alpha\beta)^2]\} - \alpha^3\beta^3(\beta + \alpha)$ $\mathbf{[From\ 1.1]}$

$= [(\mathbf{6})^3 - \mathbf{3}(\mathbf{5})(\mathbf{6})]\{[(\mathbf{6})^2 - \mathbf{2}(\mathbf{5})]^2 - [\mathbf{2}(\mathbf{5})^2]\} - \mathbf{5}^3(\mathbf{6})$

$= [\mathbf{126}]\{[\mathbf{26}]^2 - [\mathbf{50}]\} - \mathbf{750}$

$= \mathbf{78\ 126}$

$\therefore \boldsymbol{\alpha^7 + \beta^7 = 78\ 126}$

b) Newton's Sum method

$\boldsymbol{P_7 = 6\ P_6 - 5\ P_5}$

$= \mathbf{6}\ (15\ 626) - \mathbf{5\ (3\ 126)}$

$= \mathbf{78\ 126}$

$\therefore \boldsymbol{\alpha^7 + \beta^7 = 78\ 126}$

c) Peter Chew Sum method

$\mathbf{P}_n = \mathbf{P}_a \mathbf{P}_b - (\alpha\beta)^a \mathbf{P}_{b-a}$, where $\boldsymbol{a} + \boldsymbol{b} = \boldsymbol{n}$, $\mathbf{b} \geq \mathbf{a}$.

$\boldsymbol{P}_7 = \mathbf{P}_3 \mathbf{P}_4 - (\alpha\beta)^3 \mathbf{P}_{4-3}$

$= 126\,(626) - 5^3\,(6)$

$= 78\,126$

$\therefore \alpha^7 + \beta^7 = 78\,126$

vii) $\alpha^8 + \beta^8$,

a)Algebraic formulas Method:

$\alpha^8 + \beta^8 = \{[(\alpha+\beta)^2 - 2(\alpha\beta)]^2 - 2(\alpha\beta)^2\}^2 - 2(\alpha\beta)^4$

[From 1.1]

$= \{[(6)^2 - 2(5)]^2 - 2(5)^2\}^2 - 2(5)^4$

$= \{[26]^2 - 50\}^2 - 1\,250$

$= 390\,626$

$\therefore \alpha^8 + \beta^8 = 390\,626$

b) Newton's Sum method

$P_8 = 6\ \mathrm{P}_7 - 5\,\mathrm{P}_6$

$= 6\,(78\,126) - 5(15\,626)$

$= 390\,626$

$\therefore \alpha^8 + \beta^8 = 390\,626$

c) Peter Chew Sum method

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$\mathbf{P}_n = \mathbf{P}_a \mathbf{P}_b - (\alpha\,\beta)^a\, \mathbf{P}_{b-a}$, where $\boldsymbol{a} + \boldsymbol{b} = \boldsymbol{n}$, b $\geq$ a .

$P_8 = \mathbf{P}_4\, \mathbf{P}_4 - (\alpha\,\beta)^4\, \mathbf{P}_{4-4}$

$= 626\,(626) - 5^4\,(2)$

$= 390\ 626$

$\therefore \alpha^8 + \beta^8 = 390\ 626$

Note: The significant advantages of Newton's Sum and Peter Chew Sum method are obvious, such as freeing students from the burden of memorizing a large number of formulas. As shown above, if the algebraic formula method is used to calculate the quadratic root function, students need to remember a large number of formulas.

4.2. Complex Root Quadratic Equation

Example : If the roots of the equation x^2 - 4x + 5 = 0 are α and β , find the value of i) $\boldsymbol{\alpha}^2$+$\boldsymbol{\beta}^2$, ii) $\boldsymbol{\alpha}^3$+$\boldsymbol{\beta}^3$, iii) $\boldsymbol{\alpha}^4$+$\boldsymbol{\beta}^4$ iv) $\boldsymbol{\alpha}^5$+$\boldsymbol{\beta}^5$, v) $\boldsymbol{\alpha}^6$+$\boldsymbol{\beta}^6$, vi) $\boldsymbol{\alpha}^7$+$\boldsymbol{\beta}^7$ and vii) $\boldsymbol{\alpha}^8$+$\boldsymbol{\beta}^8$.

Solution:

Sum of the roots = $\alpha + \beta$

$= -\frac{b}{a}$

$= -\left(\frac{-4}{1}\right)$

$= 4$

Product of the roots = $\alpha\beta$

$= \frac{c}{a}$

$= \frac{5}{1}$

$= 5$

Let A= $\alpha + \beta = 4$ and B = $\alpha\beta = 5$

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$\boldsymbol{P_n} = \boldsymbol{\alpha^n} + \boldsymbol{\beta^n}$.

$\boldsymbol{P_0} = \boldsymbol{\alpha^0}+\boldsymbol{\beta^0} = 2.$

$\boldsymbol{P_1} = \boldsymbol{\alpha^1}+\boldsymbol{\beta^1} = 4$.

Newton Sum Method

$\boldsymbol{Let\ P_i = \mathrm{A}\ \mathrm{P}_{i-1} - B\ \mathrm{P}_{i-2}}$

$\boldsymbol{= 4\ \mathrm{P}_{i-1} - 5\ \mathrm{P}_{i-2}}$

Peter Chew Sum Method

$\mathbf{P}_n = \mathbf{P}_a\ \mathbf{P}_b$ - $(\boldsymbol{\alpha\ \beta})^a\ \mathbf{P}_{b-a}$, where $\boldsymbol{a + b = n,}$ $\mathbf{b} \geq \mathbf{a}$ **.**

$\boldsymbol{i}$) $\boldsymbol{\alpha^2}+\boldsymbol{\beta^2}$,

a) Algebraic formulas Method:

$\boldsymbol{\alpha^2}+\boldsymbol{\beta^2} = (\alpha + \beta)^2 - 2\alpha\ \beta$ $\quad \mathbf{from\ 1.1}$

$= (\mathbf{4})^2 - 2\times(5)$

$= 16 - 10$

$= 6$

$\therefore \boldsymbol{\alpha^2 + \beta^2 = 6}$

b) Newton's Sum method

$\boldsymbol{P_2 = 4\ \mathrm{P}_1 - 5\ \mathrm{P}_0}$

$\mathbf{= 4\ (4) – 5(2)}$

$\mathbf{= 6}$

$\therefore \boldsymbol{\alpha^2+\beta^2 = 6}$

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c) Peter Chew Sum method ,

$\mathbf{P}_n = \mathbf{P}_a\ \mathbf{P}_b$ - $(\alpha\ \beta)^a\ \mathbf{P}_{b-a}$, where $\boldsymbol{a} + \boldsymbol{b} = \boldsymbol{n}$, **b** $\geq$ **a .**

$\boldsymbol{P}_2 = \mathbf{P}_1\ \mathbf{P}_1$ - $(\alpha\ \beta)^1\ \mathbf{P}_{1-1}$

= 4 (4) – 5 (2)

= 6

$\therefore \alpha^2+\beta^2 = 6$

---

ii) $\alpha^3+\beta^3$,

a)Algebraic formulas Method:

$\alpha^3+\beta^3=(\alpha+\beta)[(\alpha+\beta)^2-3\alpha\beta]$ **from 1.1**

$= (4)[(4)^2 - 3(5)]$

= 4(16 - 15)

= 4

$\therefore \alpha^3+\beta^3 = 4$

b) Newton's Sum method

$\boldsymbol{P}_3 = 4\ \mathbf{P}_2 - 5\ \mathbf{P}_1$

= 4 (6) – 5 (4)

= 4

$\therefore \alpha^3+\beta^3 = 4$

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c) Peter Chew Sum method ,

$\mathbf{P}_n = \mathbf{P}_a \, \mathbf{P}_b - (\alpha\,\beta)^a \, \mathbf{P}_{b-a}$, where $\boldsymbol{a+b=n}$, b $\geq$ a .

$P_3 = \mathbf{P}_1 \, \mathbf{P}_2 - (\alpha\,\beta)^1 \, \mathbf{P}_{2-1}$

$= 4\,(6) – 5\,(4)$

$= 4$

$\therefore \alpha^3+\beta^3 = 4$

---

iii) $\alpha^4+\beta^4$,

a)Algebraic formulas Method:

$\alpha^4+\beta^4 = [(\alpha + \beta)^2 - 2\,(\alpha\,\beta)]^2 - 2\,(\alpha\beta)^2$ **from 1.1**

$= [(4)^2 - 2\,(5)]^2 - 2\,(5)^2$

$= [6]^2 - 50$

$= -14$

$\therefore \alpha^4+\beta^4 = -14$

b) Newton's Sum method

$P_4 = 4\, \mathbf{P}_3 - 5\, \mathbf{P}_2$

$= 4\,(4) – 5(6)$

$= -14$

$\therefore \alpha^4+\beta^4 = -14$

c) Peter Chew Sum method ,

$\mathbf{P}_{n} = \mathbf{P}_{a}\ \mathbf{P}_{b} - (\boldsymbol{\alpha}\ \boldsymbol{\beta})^{a}\ \mathbf{P}_{b-a}$, where $\boldsymbol{a}+\boldsymbol{b}=\boldsymbol{n}$, $\mathbf{b} \geq \mathbf{a}$.

$\boldsymbol{P}_4 = \mathbf{P}_2\ \mathbf{P}_2 - (\boldsymbol{\alpha}\ \boldsymbol{\beta})^2\ \mathbf{P}_{2-2}$

$= \mathbf{6\ (6) - (5)^2\ (2)}$

$= \mathbf{-14}$

$\therefore \boldsymbol{\alpha}^4 + \boldsymbol{\beta}^4 = \mathbf{-\ 1\ 4}$

---

iv) $\boldsymbol{\alpha}^5 + \boldsymbol{\beta}^5$,

a)Algebraic formulas Method:

$\alpha^5 + \beta^5$

$= [(\alpha+\beta)^2 - 2\alpha\,\beta]\{(\alpha+\beta)[(\alpha+\beta\}^2 - 3\alpha\,\beta]\} - (\alpha\beta)^2(\alpha+\beta)$

$[\mathbf{Fr}\boldsymbol{om}\ \mathbf{1.1}]$

$= [(\mathbf{4})^2 - \mathbf{2}(\mathbf{5})]\{(\mathbf{4})[(\mathbf{4})^2 - \mathbf{3}(\mathbf{5})]\} - (\mathbf{5})^2(\mathbf{4})$

$= [\mathbf{6}][(\mathbf{4})(\mathbf{1})] - \mathbf{100}$

= - 76

$\therefore \boldsymbol{\alpha}^5 + \boldsymbol{\beta}^5 = \mathbf{-76}$

b) Newton's Sum method

$\boldsymbol{P}_5 = \mathbf{4\ P_4 - 5\ P_3}$

$= \mathbf{4(-14) - 5\ (4)}$

$= \mathbf{-76}$

$\therefore \boldsymbol{\alpha}^5 + \boldsymbol{\beta}^5 = \mathbf{-76}$

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c) Peter Chew Sum method ,

$\mathbf{P}_n = \mathbf{P}_a\ \mathbf{P}_b - (\alpha\ \beta)^a\ \mathbf{P}_{b-a}$, where $\boldsymbol{a} + \boldsymbol{b} = \boldsymbol{n}$, **b** $\geq$ **a .**

$P_5 = \mathbf{P}_2\ \mathbf{P}_3 - (\alpha\ \beta)^2\ \mathbf{P}_{3-2}$

$= 6\ (4) - (5)^2\ (-14)$

$= -74$

$\therefore \alpha^5 + \beta^5 = -76$

v) $\alpha^6 + \beta^6$,

a)Algebraic formulas Method:

$\alpha^6 + \beta^6$

$= \{ (\alpha + \beta) [(\alpha + \beta)^2 - 3\alpha\ \beta] \}^2 - 2\ (\alpha\beta)^3$ **[From 1.1]**

$= \{ (4) [(4)^2 - 3(5)] \}^2 - 2\ (5)^3$

$= \{ (4) [1] \}^2 - 250$

$= -234$

$\therefore \alpha^6 + \beta^6 = -234$

b) Newton's Sum method

$P_6 = 4\ \mathrm{P}_5 - 5\ \mathrm{P}_4$

$= 4(-76) - 5(-14)$

$= -234$

$\therefore \alpha^6 + \beta^6 = -234$

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c) Peter Chew Sum method ,

$\mathbf{P}_n = \mathbf{P}_a\ \mathbf{P}_b$ - $(\boldsymbol{\alpha}\ \boldsymbol{\beta})^a\ \mathbf{P}_{b-a}$, where $\boldsymbol{a + b = n}$, **b** $\geq$ **a .**

$\boldsymbol{P_6 = P_3\ P_3 - (\alpha\ \beta)^3\ P_{3-3}}$

$= \mathbf{4\ (4) - (5)^3\ (2)}$

$= \mathbf{-234}$

$\therefore \boldsymbol{\alpha^6+\beta^6 = -234}$

vi) $\boldsymbol{\alpha^7+\beta^7}$,

a)Algebraic formulas Method:

$\alpha^7 + \beta^7 = [(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)]\{[(\alpha+\beta)^2 - 2(\alpha\beta)]^2 - [2(\alpha\beta)^2]\} - \alpha^3\beta^3(\beta + \alpha)$ $\mathbf{[From\ 1.1]}$

$= [(4)^3 - 3(5)(4)]\{[(4)^2 - 2(5)]^2 - [2(5)^2]\} - 5^3(4)$

$= [4]\{[6]^2 - 50\} - 500$

$= \mathbf{-556}$

$\therefore \boldsymbol{\alpha^7+\beta^7 = -556}$

b) Newton's Sum method

$\boldsymbol{P_7 = 4\ P_6 - 5\ P_5}$

$= \mathbf{4(-234) - 5(-76)}$

$= \mathbf{-556}$

$\therefore \boldsymbol{\alpha^7+\beta^7 = -556}$

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c) Peter Chew Sum method ,

$\mathbf{P}_n = \mathbf{P}_a \mathbf{P}_b - (\alpha\beta)^a \mathbf{P}_{b-a}$, where $a + b = n$, b $\geq$ a .

$P_7 = \mathbf{P}_3\ \mathbf{P}_4 - (\alpha\beta)^3\ \mathbf{P}_{4-3}$

$= 4\ (-14) - (5)^3\ (4)$

$= -556$

$\therefore \alpha^7+\beta^7 = -556$

vii) $\alpha^8+\beta^8$,

a)Algebraic formulas Method:

$\alpha^8 + \beta^8 = \{[(\alpha+\beta)^2 - 2(\alpha\beta)]^2 - 2(\alpha\beta)^2\}^2 - 2(\alpha\beta)^4$

[From 1.1]

$= \{[(4)^2 - 2(5)]^2 - 2(5)^2\}^2 - 2(5)^4$

$= \{[6]^2 - 50\}^2 - 1250$

$= -1054$

$\therefore \alpha^8+\beta^8 = -1054$

b) Newton's Sum method

$P_8 = 4\ \mathbf{P}_7 - 5\ \mathbf{P}_6$

$= 4\ (-556) - 5(-234)$

$= -1054$

$\therefore \alpha^8+\beta^8 = -1054$

c) Peter Chew Sum method ,

$\mathbf{P}_n = \mathbf{P}_a \mathbf{P}_b - (\alpha\beta)^a \mathbf{P}_{b-a}$, where $a + b = n$, b $\geq$ a .

$P_8 = \mathbf{P}_4\ \mathbf{P}_4 - (\alpha\beta)^4\ \mathbf{P}_{4-4}$

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$= (-14)\,(-14) - \quad (5)^4\ (2)$

$= -1054$

$\therefore \alpha^8 + \beta^8 = -1054$

Note: The significant advantages of Newton's Sum and Peter Chew Sum method are obvious, such as freeing students from the burden of memorizing a large number of formulas. As shown above, if the algebraic formula method is used to calculate the quadratic root function, students need to remember a large number of formulas.

5. The Peter Chew Sum method is more simple (fewer computational steps) than Newton Sum to calculate higher order quadratic root functions.

Example 1: If the roots of the equation x^2 - 4x + 3 = 0 are α and β , find the value α^{16} + β^{16} **by using Newton's Sum method and Peter Chew Sum Method.**

Solution: i)Newton's Sum method. Let A= $\alpha + \beta = -\frac{-4}{1} = 4$ and B = $\alpha\beta = \frac{3}{1} = 3$

$P_i = \alpha^i + \beta^i$, So, $P_0 = \alpha^0 + \beta^0 = 2$. $P_1 = \alpha^1 + \beta^1 = 3$

$P_i = A\ P_{i-1} - B\ P_{i-2} = 4\ P_{i-1} - 3\ P_{i-2}$

$P_2 = 4\ P_1 - 3\ P_0$

$= 4\ (4) - 3(2)$

$= 10$

$P_3 = 4\ P_2 - 3\ P_1$

$= 4\ (10) - 3\ (4)$

$= 28$

$P_4 = 4\ P_3 - 3\ P_2$

$= 4\ (28) - 3\ (10)$

$= 82$

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$P_5 = 4\ \mathrm{P}_4 - 3\ \mathrm{P}_3$

= 4 (82) – 3 (28)

= 244

$P_6 = 4\ \mathrm{P}_5 - 3\ \mathrm{P}_4$

= 4 (244) – 3 (82)

= 730

$P_7 = 4\ \mathrm{P}_6 - 3\ \mathrm{P}_5$

= 4 (730) – 3(244)

= 2 188

$P_8 = 4\ \mathrm{P}_7 - 3\ \mathrm{P}_6$

= 4 (2 188) – 3 (730)

= 6 562

$P_9 = 4\ \mathrm{P}_8 - 3\ \mathrm{P}_7$

= 4(6 562) – 3(2 188)

= 19 684

$P_{10} = 4\ \mathrm{P}_9 - 3\ \mathrm{P}_8$

= 4 (19 684) – 3 (6 562)

= 59 050

$P_{11} = 4\ \mathrm{P}_{10} - 3\ \mathrm{P}_9$

= 4 (59 050) – 3 (19 684)

= 177 148

$P_{12} = 4\ \mathrm{P}_{11} - 3\ \mathrm{P}_{10}$

= 4 (177 148) – 3 (59 050)

= 531 442

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$P_{13} = 4\ P_{12} - 3\ P_{11}$

= 4 (531 442) – 3 (177 148)

= 1 594 324

$P_{14} = 4\ P_{13} - 3\ P_{12}$

= 4 (1 594 324) – 3 (531 442)

= 4 782 970

$P_{15} = 4\ P_{14} - 3\ P_{13}$

= 4 (4 782 970) – 3 (1 594 324)

= 14 348 908

$P_{16} = 4\ P_{15} - 3\ P_{14}$

= 4 (14 348 908) – 3 (4 782 970)

= 43 046 722

Note: As shown above, Newton's Sum is a historical calculation method that relies on all previous results for subsequent calculations. This common historical basis reveals a limitation, particularly when dealing with higher-order quadratic root functions involving exponents such as $\alpha^{16} + \beta^{16}$. For higher-order quadratic root functions, using Newton's Sum involves many steps that can lead to calculation errors.

ii) Peter Chew Sum Method. If a = b 0r n = $2a$. $P_n = (P_a)^2 - 2\,(\alpha\,\beta)^a$

$P_i = \alpha^i + \beta^i$, So, $P_0 = \alpha^0 + \beta^0 = 2$. $P_1 = \alpha^1 + \beta^1 = 4$, $\alpha\beta = 3$.

$P_2 = (P_1)^2 - 2\,(\alpha\,\beta)^1$

$=(4)^2 - 2\,(3)^1$

= **16** – 6

= 10

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$P_4 = (P_2)^2 - 2\,(\alpha\,\beta)^2$

$= (10)^2 - 2\,(3)^2$

$= 82$

$P_8 = P_4\ P_4 - (\alpha\,\beta)^4\ P_{4-4}$

$= (82)\,(82) - (3)^4\ (2)$

$= 6562$

$P_8 = P_4\ P_4 - (\alpha\,\beta)^4\ P_{4-4}$

$= (82)\,(82) - (3)^4\ (2)$

$= 6562$

$P_{16} = P_8\ P_8 - (\alpha\,\beta)^8\ P_{8-8}$

$= (6562)\,(6562) - (3)^8\ (2)$

$= 43\ 046\ 722$

Note: As shown above, using Peter Chew Sum involves a few steps compare using Newton Sum. This can prevent calculation errors.

Example 2: If the roots of the equation x^2 - x - 2 = 0 are α and β , find the value $\alpha^{33} + \beta^{33}$ **by using Newton's Sum method and Peter Chew Sum Method.**

Solution:

i)Newton's Sum method. Let A= α + β =- $\frac{-1}{1}$ = 1 and B = αβ = $\frac{-2}{1}$ = -2

$P_i = \alpha^i + \beta^i$, So, $P_0 = \alpha^0 + \beta^0 = 2$. $P_1 = \alpha^1 + \beta^1 = 1$

$P_i = A\ P_{i-1} - B\ P_{i-2} = P_{i-1} + 2\,P_{i-2}$

$P_2 = P_1 + 2\,P_0$

$= 1 + 2(2)$

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$= 5$

$P_3 = P_2 + 2P_1$

$= 5 + 2(1)$

$= 7$

$P_4 = P_3 + 2P_2$

$= 7 + 2(5)$

$= 17$

$P_5 = P_4 + 2P_3$

$= 17 + 2(7)$

$= 31$

$P_6 = P_5 + 2P_4$

$= 31 + 2(17)$

$= 65$

$P_7 = P_6 + 2P_5$

$= 65 + 2(31)$

$= 127$

$P_8 = P_7 + 2P_6$

$= 127 + 2(65)$

$= 257$

$P_9 = P_8 + 2P_7$

$= 257 + 2(127)$

$= 511$

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$P_{10} = P_9 + 2P_8$

$= 511 + 2(257)$

$= 1\,025$

$P_{11} = P_{10} + 2P_9$

$= 1\,025 + 2(511)$

$= 2\,047$

$P_{12} = P_{11} + 2P_{10}$

$= 2\,047 + 2(1\,025)$

$= 4\,097$

$P_{13} = P_{12} + 2P_{11}$

$= 4\,097 + 2(2\,047)$

$= 8\,191$

$P_{14} = P_{13} + 2P_{12}$

$= 8\,191 + 2(4\,097)$

$= 16\,385$

$P_{15} = P_{14} + 2P_{13}$

$= 16\,385 + 2(8\,191)$

$= 32\,767$

$P_{16} = P_{15} + 2P_{14}$

$= 32\,767 + 2(16\,385)$

$= 65\,537$

$P_{17} = P_{16} + 2P_{15}$

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$= 65\ 537 + 2\ (32\ 767)$

$= 131\ 071$

$P_{18} = P_{17} + 2\ P_{16}$

$= 131\ 071 + 2\ (65\ 537)$

$= 262\ 145$

$P_{19} = P_{18} + 2\ P_{17}$

$= (262\ 145) + 2(131\ 071)$

$= 524\ 287$

$P_{20} = P_{19} + 2\ P_{18}$

$= (524\ 287) + 2(262\ 145)$

$= 1\ 048\ 577$

$P_{21} = P_{20} + 2\ P_{19}$

$= (1048577) + 2(524\ 287)$

$= 2\ 097\ 151$

$P_{22} = P_{21} + 2\ P_{20}$

$= (2\ 097\ 151) + 2(1\ 048\ 577)$

$= 4194305$

$P_{23} = P_{22} + 2\ P_{21}$

$= (4\ 194\ 305) + 2(2\ 097\ 151)$

$= 8\ 388\ 607$

$P_{24} = P_{23} + 2\ P_{22}$

$= (8\ 388\ 607) + 2(4194305)$

$= 16\ 777\ 217$

$P_{25} = P_{24} + 2\ P_{23}$

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$= (16\ 777\ 217) + 2\ (8\ 388\ 607)$

$= 33\ 554\ 431$

$P_{26} = P_{25} + 2\ P_{24}$

$= (33\ 554\ 431) + 2\ (16\ 777\ 217)$

$= 67\ 108\ 865$

$P_{27} = P_{26} + 2\ P_{25}$

$= (67\ 108\ 865) + 2(33\ 554\ 431)$

$= 134\ 217\ 727$

$P_{28} = P_{27} + 2\ P_{26}$

$= (134\ 217\ 727) + 2(67\ 108\ 865)$

$= 268\ 435\ 457$

$P_{29} = P_{28} + 2\ P_{27}$

$= (268\ 435\ 457) + 2(134\ 217\ 727)$

$= 536\ 870\ 911$

$P_{30} = P_{29} + 2\ P_{28}$

$= (536\ 870\ 911) + 2(268\ 435\ 457)$

$= 1\ 073\ 741\ 825$

$P_{31} = P_{30} + 2\ P_{31}$

$= (1\ 073\ 741\ 825) + 2(536\ 870\ 911)$

$= 2\ 147\ 483\ 647$

$P_{32} = P_{31} + 2\ P_{30}$

$= (2\ 147\ 483\ 647) + 2\ (1\ 073\ 741\ 825)$

$= 4\ 294\ 967\ 297$

$P_{33} = P_{32} + 2\ P_{31}$

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= (4 294 967 297)+ 2 (2 147 483 647)

= 8 589 934 591

Note: As shown above, Newton's Sum is a historical calculation method that relies on all previous results for subsequent calculations. For higher-order quadratic root functions such as $\alpha^{33}+\beta^{33}$, using Newton's Sum involves many steps that can lead to calculation errors.

ii) Peter Chew Sum Method. If a = b 0r n = $\boldsymbol{2a}$. $\mathbf{P}_{\boldsymbol{n}} = (\mathbf{P}_{\boldsymbol{a}})^{\mathbf{2}} - 2\,(\boldsymbol{\alpha}\,\boldsymbol{\beta})^{\boldsymbol{a}}$

$\boldsymbol{if}$ **b** ≥ **a** , $\mathbf{P}_{\boldsymbol{n}} = \mathbf{P}_{\boldsymbol{a}}\ \mathbf{P}_{\boldsymbol{b}}$ - $(\boldsymbol{\alpha}\,\boldsymbol{\beta})^{\boldsymbol{a}}\ \mathbf{P}_{\boldsymbol{b-a}}$, where $\boldsymbol{a}+\boldsymbol{b}=\boldsymbol{n}$.

$\boldsymbol{P_i} = \boldsymbol{\alpha}^{\boldsymbol{i}}+\boldsymbol{\beta}^{\boldsymbol{i}}$, So, $\boldsymbol{P_0} = \boldsymbol{\alpha}^{\mathbf{0}}+\boldsymbol{\beta}^{\mathbf{0}}$= 2. $\boldsymbol{P_1} = \boldsymbol{\alpha}^{\mathbf{1}}+\boldsymbol{\beta}^{\mathbf{1}}$= 1, αβ = $\frac{\mathbf{-2}}{\mathbf{1}}$ = -2

$\boldsymbol{P_2} = (\mathbf{P_1})^{\mathbf{2}} - 2\,(\boldsymbol{\alpha}\,\boldsymbol{\beta})^{\mathbf{1}}$

$= (\mathbf{1})^{\mathbf{2}} - 2\,(\mathbf{-2})^{\mathbf{1}}$

$= 5$

$\boldsymbol{P_3} = \mathbf{P_1}\ \mathbf{P_2} - (\boldsymbol{\alpha}\,\boldsymbol{\beta})^{\mathbf{1}}\ \mathbf{P_{2-1}}$

$= (\mathbf{1})(\mathbf{5}) - (\mathbf{-2})^{\mathbf{1}}\ (1)$

$= 7$

$\boldsymbol{P_4} = (\mathbf{P_2})^{\mathbf{2}} - 2\,(\boldsymbol{\alpha}\,\boldsymbol{\beta})^{\mathbf{2}}$

$= (\mathbf{5})^{\mathbf{2}} - 2\,(\mathbf{-2})^{\mathbf{2}}$

$= 17$

$\boldsymbol{P_4} = (\mathbf{P_2})^{\mathbf{2}} - 2\,(\boldsymbol{\alpha}\,\boldsymbol{\beta})^{\mathbf{2}}$

$= (\mathbf{5})^{\mathbf{2}} - 2\,(\mathbf{-2})^{\mathbf{2}}$

$= 17$

$\boldsymbol{P_7} = \mathbf{P_3}\ \mathbf{P_4} - (\boldsymbol{\alpha}\,\boldsymbol{\beta})^{\mathbf{3}}\ \mathbf{P_{4-3}}$

$= (\mathbf{7})(\mathbf{17}) - (\mathbf{-2})^{\mathbf{3}}\ (1)$

$= 127$

$\boldsymbol{P_8} = (\mathbf{P_4})^{\mathbf{2}} - 2\,(\boldsymbol{\alpha}\,\boldsymbol{\beta})^{\mathbf{4}}$

$= (\mathbf{17})^{\mathbf{2}} - 2\,(\mathbf{-2})^{\mathbf{4}}$

$= 257$

$\boldsymbol{P_9} = \mathbf{P_1}\ \mathbf{P_8} - (\boldsymbol{\alpha}\,\boldsymbol{\beta})^{\mathbf{1}}\ \mathbf{P_{8-1}}$

$= (\mathbf{1})(\mathbf{257}) - (\mathbf{-2})^{\mathbf{1}}\ (127)$

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= 511

$P_{16} = (\mathbf{P_8})^2 - 2\,(\boldsymbol{\alpha\,\beta})^8$

= $(\mathbf{257})^2 - 2\,(\mathbf{-2})^8$

= 65 537

$P_{17} = \mathbf{P_8\ P_9}$ - $(\boldsymbol{\alpha\,\beta})^8\ \mathbf{P_{9-8}}$

= $(\mathbf{257})(\mathbf{511})$ - $(\mathbf{-2})^8$ (1)

= 131 071

$P_{33} = \mathbf{P_{16}\ P_{17}}$ - $(\boldsymbol{\alpha\,\beta})^{16}\ \mathbf{P_{17-16}}$

= $(\mathbf{65\ 537})(\mathbf{131\ 071})$ - $(\mathbf{-2})^{16}$ (1)

= 8 589 934 591.

Note: Peter Chew Sum Method involves fewer steps than using Newton Sum Method, preventing calculation errors.

6. Limitation **Newton's Sum**

Example : If the roots of the equation x^2 - 2x + 2 = 0 are α and β , find the value α^{256} + β^{256} **by using Newton's Sum method and Peter Chew Sum Method.**

Solution: i) **Newton's Sum method , not suitable .**

ii) Peter Chew Sum Method. If a = b 0r n = $2a$. $\mathbf{P}_n = (\mathbf{P}_a)^2 - 2\,(\boldsymbol{\alpha\,\beta})^a$

$P_i = \alpha^i + \beta^i$, So, $P_0 = \alpha^0 + \beta^0 = 2$. $P_1 = \alpha^1 + \beta^1 = 2$, αβ = 2.

$P_2 = (\mathbf{P_1})^2 - 2\,(\boldsymbol{\alpha\,\beta})^1$

$= (\mathbf{2})^2 - 2\,(\mathbf{2})^1$

= 0

$P_4 = (\mathbf{P_2})^2 - 2\,(\boldsymbol{\alpha\,\beta})^2$

= $(\mathbf{0})^2 - 2\,(\mathbf{2})^2$

= - 8

$P_8 = (\mathbf{P_4})^2 - 2\,(\boldsymbol{\alpha\,\beta})^4$

= $(\mathbf{-8})^2 - 2\,(\mathbf{2})^4$

= 32

$P_{16} = (\mathbf{P_8})^2 - 2\,(\boldsymbol{\alpha\,\beta})^8$

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$= (\mathbf{32})^2 - 2\,(\mathbf{2})^{8}$

$= 512$

$P_{32} = (P_{16})^2 - 2\,(\alpha\,\beta)^{16}$

$= (\mathbf{512})^2 - 2\,(\mathbf{2})^{16}$

$= 131072$

$P_{64} = (P_{32})^2 - 2\,(\alpha\,\beta)^{32}$

$= (\mathbf{131072})^2 - 2\,(\mathbf{2})^{32}$

$= 8\,589\,934\,592$

$P_{128} = (P_{64})^2 - 2\,(\alpha\,\beta)^{64}$

$= (\mathbf{8\,589\,934\,592})^2 - 2\,(\mathbf{2})^{64}$

$= 36893488147419103232$

$P_{256} = (P_{128})^2 - 2\,(\alpha\,\beta)^{128}$

$= (\mathbf{36893488147419103232})^2 - 2\,(\mathbf{2})^{128}$

$= 680564733841876926926749214863536422912$

Note: Newton's sum method is not suitable for solving this problem because it involves too much steps. The Peter Chew Sum Method involves fewer steps so this problem can be calculated.

6. Conclusion

In conclusion, this study provides a thorough examination and comparison of the Peter Chew Sum method and Newton's Sum in quadratic root function computations. It is evident that both methods offer advancements by eliminating the need for memorization of numerous formulas. However, when faced with higher-order quadratic root functions, such as $\alpha^{33}+\beta^{33}$. Newton's Sum method becomes cumbersome due to its reliance on all previous answers for final computation, resulting in increased steps and error risks.

In contrast, the Peter Chew Sum method streamlines the computational process by requiring only a few previous answers, leading to a more straightforward series of operations. Through contextualizing our analysis within the historical progression of mathematical techniques, we underscore the transformative leap represented by the Peter Chew Sum method in quadratic root function computations.

Furthermore, study demonstrates the Peter Chew Sum method's ability to address the limitations of Newton's Sum, particularly concerning higher-order functions like $\alpha^{256}+\beta^{256}$.

In summary, the findings of this study emphasize the profound impact of the Peter Chew Sum method on quadratic root function computations, highlighting its potential to revolutionize mathematical education and problem-solving approaches in this domain.

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