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Askar Abdukadyrov *

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Solving the Problem of Infinities in Electrodynamics (and Mechanics) and a New Formula for the Electrostatic Potential

Askar Abdukadyrov

Independent researcher, Bishkek, Kyrgyzstan; abdukadyrov64@mail.ru

Abstract: The presented work shows that a force (electromagnetic or gravitational) field surrounding any material body is not limitless, but has the external and internal boundaries (radii). Understanding the fact of the existence of the field boundaries allows us to solve the problem of infinities that exists in electrodynamics. It is shown that there are two different electrostatic potentials: with the dimension of speed and with the dimension of the square of speed.

Keywords: electric field; electrostatic potential; gravitational field; gravitational potential

The world (nature) is described by finite quantities.

1. Introduction

Theoretical physics is faced with the task of constructing a consistent fundamental theory in which all physical quantities would be finite. A significant obstacle on this path is the problem of infinite values (divergences) that is existed in electrodynamics. Its essence is as follows.

The electric field of a motionless point electric charge q is described by the electrostatic potential, the modulus of which is determined in vacuum by the formula

$$\phi = \phi(r) = kq/r, \tag{1}$$

where k is a proportionality coefficient, r is the distance from the charge to the point for which the potential is determined.

According to modern concepts, the force (electric or gravitational) field surrounding the charge or body is limitless, therefore:

- at the infinite distance from the charge, the potential is zero: $\phi(r{=}\infty) = 0$;
- as the distance decreases, the potential increases;
- at the point where the charge itself is located, the potential goes to infinity: $\phi(r{=}0) = \infty$.

Current theory considers an electron as a point particle (with a point charge), i.e., as a material object without extension [1]. Thus, a resting point-like electron must have the infinite self-energy and, therefore, the infinite mass. The meaninglessness of this result shows that electrodynamics becomes internally contradictory when moving to sufficiently small distances.

Electrodynamic equations can be written in different measurement systems.

It should be recalled that the International System of Units (SI) is a composite system that includes, in particular, the m-kg-s system of mechanical units (MKS system) and the m-kg-s-A system of electromagnetic units (MKCA system). The second system differs from the first primarily in that, along with the existing three basic units (meter, kilogram, and second), it has a fourth basic unit – ampere (A). For example, in the MKSA system, the elementary electric charge $e = 1.6\times10^{-19}$ C, and the coefficient $k = 9\times10^{9}$ N·m²/C².

Although the educational literature on electrodynamics focuses on the SI, however, it usually does not indicate that the SI is a composite system. As a result, many do not understand that the MKS and MKSA sub-systems are the separate systems.

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In 2018, an article [2] was published that showed that the electromagnetic units of the MKSA system (the ampere, coulomb, ohm, volt, etc.) can be converted using the basic units of the MKS system: m, kg, s. In the paper, it was shown that in the MKS system

$$e = 1.6\times10^{-25}\ \text{kg·m/s}, \tag{2}$$

$$k = c^2/F_1 = 9\times10^{21}\ \text{m/kg}, \tag{3}$$

where $c = 3\times10^8$ m/s is the speed of light in vacuum, and $F_1 = 10^{-5}$ kg·m/s^2 (or 1 g·cm/s^2 – the unit of force in the CGS system).

Since (in the MKS system) the charge has the dimension of momentum, $[e]$ = kg·m/s, the ratio of the charge of an elementary particle to its mass has the dimension of velocity, $[e/m] = [v]$ = m/s. Therefore, we can write:

$$e/m = v,\ e = mv. \tag{4}$$

Obviously, the value of e/m is different for different particles; for example,

for an electron (its mass $m_e = 0.9109\times10^{-30}$ kg) $e/m_e = v_e = 1.759\times10^5$ m/s,

for a proton (its mass $m_p = 1.672\times10^{-27}$ kg) $e/m_p = v_p = 95.79$ m/s.

In 1874, Irish physicist G. Stoney (he is most famous for introducing the term *electron* as the "fundamental unit quantity of electricity") gave a lecture to the British Association, in Belfast, which was subsequently published [3]. In his report, he first proposed the natural units of mass, length, and time, built on the universal constants c, e, and G (where $G = 6.674\times10^{-11}$ m^3·kg^{-1}·s^{-2} is the gravitational constant).

Modern meaning of the Stoney mass

$$m_s = (ke^2/G)^{1/2} = 1.859\times10^{-9}\ \text{kg}. \tag{5}$$

Hence,

$$ke^2 = Gm_s^2. \tag{6}$$

In 2020, it is shown that the constant

$$G = ke^2/m_s^2 = kv_g^2, \tag{7}$$

or

$$G = v_g^2c^2/F_1, \tag{8}$$

where the quantity $v_g = (G/k)^{1/2} = e/m_s = v_s = 0.8617\times10^{-16}$ m/s is the *elementary speed*, i.e., the minimum speed of movement in nature, $v_g = v_{\min}$ [4].

In addition, it is shown that the Stoney mass, m_s, is *boundary of the macrocosm and microcosm*; in other words, this mass is the *lower* limit for the masses of ordinary bodies and the *upper* limit for the masses of elementary particles.

Using these results, we propose a method for solving the divergence problem.

2. Method

Each fundamental interaction has its own coupling constant.

Electromagnetic interactions of the charged elementary particles are characterized by one universal coupling constant $\alpha \approx 1/137$, which is determined by the formula

$$\alpha = ke^2/\hbar c, \tag{9}$$

where $\hbar = 1.054\times10^{-34}$ J·s is the reduced Planck constant.

Using the equality $\hbar = ƛmc$ (where $ƛ$ is the reduced Compton wavelength of a particle), we obtain:

$$ke^2 = \alpha\hbar c = \alpha ƛ mc^2 = R_0mc^2, \tag{10}$$

where $R_0 = \alpha ƛ = ke^2/mc^2$.

At the beginning of the XX century, the quantity $R_0 = ke^2/m_ec^2 = 2.81\times10^{-15}$ м was considered the radius of the electron, but then it turned out that this is not so.

For example, for a proton the value $R_0 = ke^2/m_pc^2 = 1.53\times10^{-18}$ m, and its radius (as experiments have shown) is about 10^{-15} m [5]. Thus, modern physics cannot explain the physical essence of the quantity R_0 and calls it the *classical radius* of a charged elementary particle.

However, this quantity is easy to understand.

According to Coulomb's law, the absolute value of the force of electrostatic interaction of two point elementary charges e at a distance r, $F = ke^2/r^2$, and the potential energy of interaction (its modulus) $U = F{\cdot}r = ke^2/r$.

The absolute values of the force and energy of interaction between charges increases with decreasing distance between them, and in nature there are no electric charges without their carriers-particles. In addition, the speed of light in vacuum is the maximum speed of movement in nature, $c = \upsilon_{\max}$. Therefore, the expression

$$U_{\max} = ke^2/R_0 = mc^2 \tag{11}$$

determines the maximum potential energy of electrostatic interaction between two charged elementary particles of the mass m at the *minimum distance* R_0 between them ($R_0 = r_{\min}$).

The interaction between charged particles is carried out by means of their electromagnetic fields. Consequently, the quantity R_0 is the minimum (*internal*) radius of the electromagnetic field of a charged elementary particle.

The equation $R_0 = ke^2/mc^2$ can be written like this:

$$R_0 = km(e/mc)^2 = kmn, \tag{12}$$

where a dimensionless factor

$$n = (e/mc)^2 = \upsilon^2/c^2. \tag{13}$$

The factor n decreases with increasing particle mass.

For example, for an electron $n_e = (e/m_ec)^2 = \upsilon_e^2/c^2 = 3.44\times10^{-7}$, for a proton $n_p = (e/m_pc)^2 = \upsilon_p^2/c^2 = 1.02\times10^{-13}$, for a charged elementary particle with mass m_s the factor n has the smallest value, $n_s = (e/m_sc)^2 = \upsilon_g^2/c^2 = n_g = 0.826{\cdot}10^{-49}$.

Using the factor n, we write the equation $ke^2 = R_0mc^2$ as:

$$ke^2 = (R_0/n)mc^2n = R_f m\upsilon^2, \tag{14}$$

where $R_f = R_0/n$. Therefore, the expression

$$U_{\min} = ke^2/R_f = m\upsilon^2 \tag{15}$$

determines the minimum potential energy of electrostatic interaction between two charged elementary particles of the mass m at the *maximum distance* R_f between them ($R_f = r_{\max}$).

Consequently, the quantity R_f is the maximum radius of action of the electromagnetic forces of a charged particle, i.e., is the *outer* radius of its electromagnetic field.

According to equations (4) and (14):

$$ke^2 = k(m\upsilon)^2 = (R_f/m)(m\upsilon)^2, \tag{16}$$

where $R_f/m = k$. This leads to an important conclusion.

1. The maximum (outer) radius of the particle's electromagnetic field *is not infinite* ($r_{\max} \neq \infty$), as it is mistakenly believed, but it is proportional to the particle's mass: $R_f = km$.

In the MKS system, the coefficient $k = 9\times10^{21}$ m/kg, therefore, for an electron $R_f = km_e = 8.18\times10^{-9}$ m, for a proton $R_f = km_p = 1.5\times10^{-5}$ m, for a particle with Stoney mass $R_f = km_s = 1.67\times10^{13}$ m.

Now it's clear that the minimum value of the electrostatic potential $\phi_{\min}$ is obtained at a distance $r = R_f$, i.e., at the external boundary of the particle's field:

$$\phi_{\min} = U_{\min}/e = ke/R_f. \tag{17}$$

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The maximum value of this potential ϕ_{max} is obtained at a distance $r = R_0$, i.e., at the internal boundary of the particle's field:

$$\phi_{max} = U_{max}/e = ke/R_0. \quad (18)$$

Gravitational interactions of elementary particles in comparison with their electromagnetic interactions are considered negligible (so particle physics is physics without gravity) and are characterized by the eigenvalue of the dimensionless coupling constant α_g for an each particle:

$$\alpha_g = Gm^2/\hbar c. \quad (19)$$

Using the equality $\hbar = \lambda\!\!\!^{-} mc$, we obtain:

$$Gm^2 = \alpha_g \hbar c = \alpha_g \lambda\!\!\!^{-} mc^2 = R_g mc^2, \quad (20)$$

where $R_g = \alpha_g \lambda\!\!\!^{-} = Gm/c^2$ is the so-called *gravitational radius*.

Note that the length $R_g = mG/c^2$ (gravitational radius of mass m) was introduced by the German theorist H. Weyl in paper «Gravity and electricity» (1918) [6]; the gravitational radius of a body should not be confused with its *Schwarzschild radius* $R_S = 2mG/c^2$.

According to Newton's law of universal gravitation, the absolute value of the force of gravitational interaction between two bodies of the equal masses m at a distance r, $F = Gm^2/r^2$, and the potential energy of their interaction (its modulus) $U = F{\cdot}r = Gm^2/r$.

The absolute values of the force and energy of gravity between bodies increases with decreasing distance between them, therefore, the expression

$$U_{max} = Gm^2/R_g = mc^2 \quad (21)$$

determines the maximum potential energy of attraction between two bodies of mass m at the *minimum distance* R_g between them ($R_g = r_{min}$).

The gravitational interaction between bodies is carried out by means of their gravitational fields. Consequently, the quantity R_g is the minimum (*internal*) radius of the body's gravitational field.

Taking into account equation (8), we can write that the quantity

$$R_g = mG/c^2 = m\upsilon_g^2/F_1 = k_g m, \quad (22)$$

where the coefficient

$$k_g = R_g/m = G/c^2 = \upsilon_g^2/F_1 = 0.7425 \cdot 10^{-27} \text{ m/kg}. \quad (23)$$

Hence,

$$G = k\upsilon_g^2 = k_g c^2, \quad (24)$$

and the gravitational factor

$$n_g = \upsilon_g^2/c^2 = k_g/k = R_g/R_f, \quad (25)$$

where $R_f = km = R_g/n_g$. Therefore, the expression

$$U_{min} = Gm^2/R_f = m\upsilon_g^2 \quad (26)$$

determines the minimum potential energy of gravity between two bodies of the masses m at the *maximum distance* R_f between them ($R_f = r_{max}$). Consequently, the quantity R_f is the maximum radius of action of gravitational forces of a body, i.e., is the *outer* radius of its gravitational field.

Hence, the next conclusion.

2. The maximum (outer) radius of the body's gravitational field *is not infinite* ($r_{max} \neq \infty$), as it is mistakenly believed, but it is proportional to the its mass: $R_f = km$.

The gravity field of an ordinary body is described by the gravitational potential, the modulus of which is determined by the formula

$$\Phi = \Phi(r) = U/m = Gm/r. \quad (27)$$

It's clear that the minimum value of the gravitational potential Φ_{min} is obtained at a distance $r=R_f$, i.e., at the outer border of the body's field:

$$\Phi_{min} = U_{min}/m = Gm/R_f = v_g^2. \tag{28}$$

The maximum value of this potential Φ_{max} is obtained at a distance $r = R_g$, i.e., at the internal border of the body's field:

$$\Phi_{max} = U_{max}/m = Gm/R_g = c^2. \tag{29}$$

So, understanding the existence of the field boundaries allows us to solve the problem of divergences.

3. New Formula for the Electrostatic Potential

In the MKS system, the electric charge has the dimension of momentum, $[q]$ = kg·m/s, and the coefficient k has the dimension $[k]$ = m/kg; thus, the electrostatic potential has the dimension of *speed*, $[\phi] = [kq/r] = [U/q]$ = m/s.

On the contrary, the gravitational potential has the dimension of the square of speed, $[\Phi] = [U/m] = (\mathrm{m/s})^2$.

Let us note that the gravitational potential

$$\Phi = Gm/r = kv_g^2m/r = kg^2/mr, \tag{30}$$

where $g = mv_g$ is the body's *gravitational charge* (in the MKS system) [4].

Therefore, we can introduce a new formula for the electrostatic potential (its modulus):

$$\varphi = U/m = ke^2/mr \tag{31}$$

or, in general,

$$\varphi = kq^2/mr, \tag{32}$$

where this potential has the dimension of the *square of speed*, $[\varphi] = (\mathrm{m/s})^2$.

4. Conclusions

Currently, it is believed that the force field surrounding a body (particle) has no boundaries. This long-standing misconception led to the problem of infinite values in particle theory.

In reality, the force field has the external boundary (radius) $R_f = km$ and the internal boundary (radius): $R_0 = ke^2/mc^2$ for the charged elementary particle's field, and $R_g = Gm/c^2 = kg^2/mc^2$ for the ordinary body's gravitational field.

Consequently, the force and energy of interaction between bodies (particles) can vary from the minimum to maximum values, depending on the distance. Thus, the electromagnetic and gravitational forces have the *finite* radius of action.

In addition, (in the MKS system) there are two fundamentally different electrostatic potentials:

a) $\phi = kq/r$ (with the dimension of speed);

b) $\varphi = kq^2/mr$ (with the dimension of the square of speed).

The potential of the first type, apparently, can be considered as a radiation potential and used to solve problems related to the process of energy emission by charged particles.

Potential of the second type is the motion potential that should be used to solve issues of mechanics, i.e., to study the movement of bodies (particles) and the interaction between them.

Conflicts of Interest: The author declares no competing interests.

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