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# Curvature-Induced Distortion of Probability Distributions in Black Hole Interiors

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**Abstract:** In previous work, we investigated how space-time curvature influences physical probability distributions. In this paper, we focus on the internal geometry of black holes, studying how an object falling from the event horizon to the singularity undergoes probabilistic deformation due to increasingly intense curvature. By examining the behavior of probability density functions on curved manifolds, we demonstrate that traditional notions of normalization break down near the singularity. This breakdown is rooted in the divergence of geometric scalars and results in a loss of predictive power. Our results suggest that a deeper geometrization of probability is required in regimes of extreme gravitational curvature, with possible implications for black hole thermodynamics, entropy, and information loss. Specifically, we show that as curvature increases, entropy diverges, highlighting a profound connection between curvature and uncertainty in black hole interiors.

## 1. Introduction

General relativity revolutionized our understanding of gravity, replacing the Newtonian concept of a force with the geometry of a four-dimensional manifold. In black holes, where curvature reaches extremes, the behavior of both classical and quantum systems becomes challenging to describe using traditional probabilistic methods. Specifically, when considering probability distributions in curved space-time, we encounter complications in maintaining normalization as curvature grows. In previous work, we explored how curvature influences probability distributions, but here we extend the analysis to the specific case of black hole interiors.

As we approach the singularity, curvature invariants, such as the Kretschmann scalar, diverge. This results in a breakdown of the classical geometry of space-time, which in turn disrupts the probabilistic models defined on it. We will show that the failure of probability normalization in these extreme conditions suggests a need for new probabilistic frameworks that can account for such severe geometric distortions. This paper explores how space-time curvature directly impacts physical probabilities and proposes that such breakdowns could have deep implications for black hole thermodynamics, entropy, and the loss of information.

## 2. Probability on Curved Manifolds

Let $\rho(x^u)$ be a probability density function defined over a four-dimensional space-time manifold $M$. In flat space, the integral of $\rho(x^u)$ over a spatial slice yields unity, representing total certainty. In general relativity, however, this integral must be performed using the proper volume element that accounts for the curvature of the manifold:

$$\int_M \rho(x^u)\sqrt{-g} \cdot d^3x = 1$$

Here, $M$ represents a spacelike hypersurface, $g$ is the determinant of the space-time metric $g_{uv}$, and $d^3x$ represents the volume element. This curved-space integration ensures that physical probabilities reflect the curvature of the underlying space-time. As space-time curvature increases, even a flat probability distribution can become skewed or stretched, complicating the concept of normalization.

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For example, in black hole interiors, the space-time becomes increasingly warped, and any probability distribution must be reconsidered in light of the distortions introduced by the curvature.

### 3. Black Hole Geometry and Curvature Growth

In the Schwarzschild solution, the space-time metric for a non-rotating black hole is given by:

$$ds^2 = -(1-2GMr)dt^2 + (1-2GMr)^{-1}dr^2 + r^2 d\Omega^2.$$

As the radial coordinate rr decreases, curvature increases. The Kretschmann scalar $K$, a measure of the curvature, is given by:

$$k = R_{uv\rho\sigma}R^{uv\rho\sigma} = \frac{48G^2m^2}{r^6}$$

As $r \to 0$, the Kretschmann scalar diverges, signaling the formation of a singularity. This extreme curvature profoundly impacts all physical quantities, including probability distributions, making traditional probabilistic methods inadequate in these regions.

### 4. Probability Normalization and Breakdown

Consider a probability density function $\rho(r')$ for an object falling radially toward the singularity. The probability of finding the object in a shell of radius rrr is given by:

$$p(r) = \int \rho(r')\sqrt{g_{rr}} \cdot r^2\, dr$$

Inside the event horizon, the Schwarzschild radial coordinate rrr becomes time-like, which complicates the process of normalization. Even when switching to regularized coordinate systems, the physical volume element becomes increasingly distorted near the singularity. This leads to a failure of normalization:

$$\lim_{r\to 0} \int_0^r \rho(r') \cdot \sqrt{\gamma(r')} \cdot dr' \to \infty$$

This failure of normalization indicates that probabilistic models lose predictive power near the singularity. The breakdown reflects a deeper incompatibility between classical probability theory and singular space-time geometry.

### 5. Implications for Information and Entropy

The breakdown of normalization suggests that the system's state cannot be predicted as it falls into the singularity, resonating with the black hole information paradox. Classical descriptions of black holes suggest that information is lost in the singularity, but our results provide a more rigorous interpretation: as curvature increases, entropy rises due to growing uncertainty in the system's state.

We propose a geometric interpretation of entropy, where the entropy is tied to the curvature of space-time. In particular, we define a geometric entropy analogous to thermodynamic entropy:

$$S_{geo} \sim \int R_{uv}R^{uv} \cdot \sqrt{-g} \cdot d^3x$$

This formulation shows that the divergence of curvature directly corresponds to a divergence in entropy, suggesting that black hole interiors are regions where predictability, whether classical or quantum, is fundamentally compromised.

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## 6. Toward a Geometric Probability Framework

The failure of standard probabilistic models in curved space-time near singularities suggests that a new geometric probability framework is required. Such a framework could be based on the following approaches:

- Curved space stochastic mechanics, which would incorporate curvature into random walks and diffusion processes, allowing for the modeling of probabilistic behavior in curved space.
- Renormalized probability theories, which could adapt the normalization conditions of probability distributions to account for the geometric constraints of curved space-time.
- Quantum gravity models, such as loop quantum gravity or string theory, which smooth out singularities and might offer a consistent probabilistic interpretation.

These approaches could provide a more robust way of incorporating the effects of extreme curvature on probability distributions and may offer new insights into the quantum aspects of black holes.

## 7. Conclusion

This paper examines how the extreme curvature in black hole interiors distorts and eventually invalidates traditional probability distributions. We find that normalization fails as the singularity is approached, which reflects a deep connection between curvature and the loss of predictability. The breakdown of classical probabilistic models suggests that new frameworks are needed, possibly informed by quantum gravity theories, to describe systems in such extreme environments. Understanding these distortions could yield new insights into black hole entropy, information loss, and the foundations of quantum gravity.

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