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The Null Cone is Enough: Geometric Unification of Massless Fields

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Abstract

We prove that the null cone is enough: at every event in Minkowski spacetime, the null cone carries a two-dimensional conformal field theory with spectrum $\Delta_\ell = \ell + 1$, unifying all massless fields of spin $\ell = 0, \frac{1}{2}, 1, \frac{3}{2}, 2$ through pure geometry. From two postulates—four-dimensional Minkowski spacetime and the Isometric Sampling Condition—the unique Lorentz-invariant propagator is $G(x, y) = \mathrm{sinc}(\Omega\sqrt{-\sigma^2 - i\varepsilon})$, where the Feynman $i\varepsilon$ prescription selects the unique L^2 branch in the spacelike region. The RKHS normalisation $K(x, x) = 1$ forces $G = 1$ on the null cone, and the full two-point function is controlled entirely by a 2D CFT on the transverse S^2, yielding $\Delta_\ell = \ell + 1$. Fermionic statistics arise from the $\mathbb{Z}_2$ holonomy of an $\mathrm{SL}(2, \mathbb{C})$ fibre bundle without any additional postulate. We provide the microscopic foundation for Jacobson's thermodynamic Einstein equation and Verlinde's entropic gravity. We extend the framework to $\mathrm{SU}(N)$ gauge theories and derive asymptotic freedom: $b_\ell = (12\ell^2 - 1)C_2(G)/(12\pi) > 0$ for all $\ell \geq 1$, reproducing the Gross–Wilczek–Politzer result for $\ell = 1$. We develop the structural correspondence between the sinc kernel and the Riemann zeta function, construct a mathematical bridge to the non-trivial zeros of $\zeta(s)$, and identify the precise step separating the ISC framework from a proof of the Riemann Hypothesis.

Keywords: null cone; conformal field theory; reproducing kernel Hilbert space; massless fields; asymptotic freedom; Riemann hypothesis

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1. Introduction

1.1. The Null Cone is Enough

The central result of this paper is the following: *the null cone is enough*. At every event in Minkowski spacetime, the null cone cross-section carries a two-dimensional conformal field theory whose spectrum $\Delta_\ell = \ell + 1$ is uniquely determined by the local geometry. No asymptotic boundaries, no Planck-scale physics, no extra dimensions are needed.

This is not an asymptotic statement about scattering amplitudes at null infinity, nor a holographic duality between a bulk gravitational theory and a boundary CFT. It is a *local geometric fact*. Between null-separated events, the propagator is forced to be $G = 1$ by the RKHS normalisation. All energy dependence freezes out on null geodesics, coupling constants flow to a common geometric fixed point, and the distinction between scalars, spinors, photons, gravitinos, and gravitons reduces to the representation label ℓ of the same $\mathrm{SL}(2, \mathbb{C})$ structure group.

The construction assembles into a smooth fibre bundle $S^2 \hookrightarrow E \xrightarrow{\pi} M$ with structure group $\mathrm{SL}(2, \mathbb{C})$ acting by Möbius transformations. Spacetime is itself a CFT fibre bundle.

The sole inputs are: (1) Four-dimensional Minkowski spacetime $(M, \eta_{\mu\nu})$. (2) The Isometric Sampling Condition (Definition 2).

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1.2. Physical Postulates

Definition 1 (Postulate 1)**.** *The spacetime of classical field theory is four-dimensional Minkowski spacetime* $(M, \eta_{\mu\nu})$ *with* $\eta = \mathrm{diag}(-1, +1, +1, +1)$.

Definition 2 (Postulate 2 — Isometric Sampling Condition)**.** *There exists a Hilbert space* $\mathcal{H}$ *of fields on* M*, and a countable sampling lattice* $\{x_n\}$ *in* M*, such that the sampling map* $S : \mathcal{H} \to \ell^2$ *is a unitary isomorphism.*

Every theorem below is a mathematical consequence of Postulates 1 and 2.

1.3. Physical Motivation for the ISC

Quantum field theory is formulated on a separable Hilbert space, which possesses a countable orthonormal basis $\{e_n\}_{n\in\mathbb{N}}$. Any field $\phi \in \mathcal{H}$ can be expanded as $\phi = \sum_n c_n e_n$ with $\sum_n |c_n|^2 < \infty$. The ISC elevates this abstract basis to a physical sampling lattice: it requires that point evaluations $\phi \mapsto \phi(x_n)$ at lattice sites constitute a unitary map to ℓ^2. This is the content of the Whittaker–Shannon–Kotel'nikov sampling theorem when $\mathcal{H}$ is a Paley–Wiener space of bandlimited functions.

The Butzer–Splettsstösser–Stens theorem [1] provides the rigorous mathematical foundation: the sampling map is not merely injective but *unitary*, preserving the full Hilbert space structure.

1.4. Structure of the Derivation

The logical chain is: (Step 1, §3) BSS unitarity ⇒ sinc is the unique RKHS kernel. (Step 2, §4) Global isometry + ISC ⇒ $G = \mathrm{sinc}(\Omega\sqrt{-\sigma^2 - i\varepsilon})$. (Step 3, §5) $K(x, x) = 1$ ⇒ null cone is natural boundary. (Step 4, §6–7) $G = 1$ on null cone ⇒ $\Delta_\ell = \ell + 1$. (Step 5, §9) $\mathbb{Z}_2$ holonomy ⇒ Fermi–Dirac statistics. (Step 6, §10) Jacobson's argument ⇒ Einstein equation. (Step 7, §11) Verlinde ⇒ Newton's law. (Step 8, §12) Matrixification ⇒ asymptotic freedom. (Step 9, §13) Sinc–zeta structural correspondence. (Step 10, §17) Toward the Riemann Hypothesis.

2. Notation and Preliminaries

2.1. Minkowski Spacetime

Four-dimensional Minkowski spacetime is $(M, \eta_{\mu\nu})$ with $M \cong \mathbb{R}^4$ and $\eta_{\mu\nu} = \mathrm{diag}(-1, +1, +1, +1)$. The Synge world function is

$$\sigma^2(x, y) := \eta_{\mu\nu}\, \Delta x^\mu\, \Delta x^\nu\,. \tag{1}$$

The null cone at y is $\Lambda_y := \{x \in M : \sigma^2(x, y) = 0\}$.

Proposition 3. *In Minkowski spacetime,* $\exp_p(v) = p + v$ *is a global diffeomorphism and isometry.*

Proof. The differential $d\exp_p = \mathrm{id}$ since $\exp_p$ is affine. Flatness $R^\mu{}_{\nu\rho\sigma} = 0$ implies geodesics are straight lines, so $\exp_p$ is bijective. For any vectors $u, w \in T_pM$:

$$g_{\exp_p(v)}(d\exp_p(u), d\exp_p(w)) = \eta_{\mu\nu} u^\mu w^\nu = g_p(u, w)\,, \tag{2}$$

confirming the isometry. □

This ensures the sampling lattice extends globally and isometrically.

2.2. Reproducing Kernel Hilbert Spaces

A Hilbert space $(\mathcal{H}, \langle\cdot|\cdot\rangle)$ of functions on X is an RKHS if for every $x \in X$ the evaluation $\mathrm{ev}_x : f \mapsto f(x)$ is bounded. By Riesz, there exists $K_x \in \mathcal{H}$ with $f(x) = \langle f|K_x\rangle$. The reproducing kernel $K(x, y) = K_x(y)$ is uniquely determined by $\mathcal{H}$ (Aronszajn [4]). Key properties: $K(x, x) = \|K_x\|^2 \geq 0$, and when normalised, $K(x, x) = 1$.

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2.3. Paley–Wiener Space

The Paley–Wiener space $\mathrm{PW}_\Omega := \{f \in L^2(\mathbb{R}) : \operatorname{supp} \hat{f}_e \subseteq [-\Omega, \Omega]\}$ has reproducing kernel $K_\Omega(x,y) = (\Omega/\pi)\operatorname{sinc}(\Omega(x-y))$ where $\operatorname{sinc}(z) := \sin z/z$, $\operatorname{sinc}(0) := 1$. The normalised kernel $\tilde{K}(x,y) = \operatorname{sinc}(\Omega(x-y))$ satisfies $\tilde{K}(x,x) = 1$.

Verification: For $f \in \mathrm{PW}_\Omega$ with Fourier transform $\hat{f}_e$ supported in $[-\Omega, \Omega]$:

$$\begin{aligned} \langle f | K_\Omega(x,\cdot)\rangle_{L^2} &= \int_{-\infty}^{\infty} f(t) \frac{\Omega}{\pi} \frac{\sin \Omega(x-t)}{\Omega(x-t)}\, dt \\ &= \frac{1}{2\pi} \int_{-\Omega}^{\Omega} \hat{f}_e(\omega)\, e^{i\omega x}\, d\omega = f(x)\,, \end{aligned} \tag{3}$$

where the second equality uses the convolution theorem and the fact that the Fourier transform of $\operatorname{sinc}(\Omega t)$ is $(\pi/\Omega) \cdot \mathbf{1}_{[-\Omega,\Omega]}(\omega)$.

3. The Sampling–RKHS Bridge

3.1. The BSS Theorem

Theorem 4 (Butzer–Splettsstösser–Stens [1]). *Let $\Omega > 0$ and $x_n = n\pi/\Omega$ (Nyquist lattice). The map $W : \mathrm{PW}_\Omega \to \ell^2(\mathbb{Z})$, $(Wf)_n := \sqrt{\pi/\Omega}\, f(x_n)$, is a unitary isomorphism with inverse*

$$f(x) = \sum_{n\in\mathbb{Z}} f(x_n)\, \operatorname{sinc}\big(\Omega(x - x_n)\big)\,. \tag{4}$$

Detailed verification of unitarity. We must show $\langle Wf | Wg\rangle_{\ell^2} = \langle f|g\rangle_{L^2}$ for all $f, g \in \mathrm{PW}_\Omega$. The left side is:

$$\sum_{n\in\mathbb{Z}} \frac{\pi}{\Omega} \overline{f(x_n)}\, g(x_n)\,. \tag{5}$$

By Parseval's formula for bandlimited functions:

$$\int_{-\infty}^{\infty} \overline{f(t)}\, g(t)\, dt = \frac{1}{2\pi} \int_{-\Omega}^{\Omega} \overline{\hat{f}_e(\omega)}\, \hat{g}_e(\omega)\, d\omega\,. \tag{6}$$

Poisson's summation formula applied to $h(t) = \overline{f(t)}\, g(t)$, which has Fourier support in $[-2\Omega, 2\Omega]$, gives:

$$\sum_{n\in\mathbb{Z}} h(x_n) = \frac{\Omega}{\pi} \int_{-\infty}^{\infty} h(t)\, dt\,, \tag{7}$$

since the only term in the Poisson sum that survives for bandwidth $\le 2\Omega$ and sampling period π/Ω is the $k = 0$ term. Therefore:

$$\frac{\pi}{\Omega} \sum_n \overline{f(x_n)}\, g(x_n) = \int_{-\infty}^{\infty} \overline{f(t)}\, g(t)\, dt\,, \tag{8}$$

confirming unitarity. The reconstruction formula (4) follows from the inverse.

3.2. From unitarity to the reproducing kernel

Theorem 5. *The unitarity of W uniquely determines the reproducing kernel $K(x,y) = (\Omega/\pi)\operatorname{sinc}(\Omega(x-y))$.*

Proof. Set $\kappa_x(t) := (\Omega/\pi)\operatorname{sinc}(\Omega(x-t))$. From the reconstruction formula:

$$\begin{aligned} f(x) &= \sum_n f(x_n) \operatorname{sinc}(\Omega(x - x_n)) \\ &= \sum_n \underbrace{\sqrt{\tfrac{\pi}{\Omega}} f(x_n)}_{(Wf)_n} \underbrace{\sqrt{\tfrac{\Omega}{\pi}} \operatorname{sinc}(\Omega(x - x_n))}_{(W\kappa_x)_n} \\ &= \langle Wf | W\kappa_x\rangle_{\ell^2} \overset{\text{unitarity}}{=} \langle f|\kappa_x\rangle_{L^2}\,. \end{aligned} \tag{9}$$

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Uniqueness follows from Aronszajn's theorem [4]: if K' were another reproducing kernel for the same RKHS, then $\langle f | K_x - K'_x \rangle = 0$ for all f, implying $K_x = K'_x$. □

The sinc kernel is *derived*, not postulated.

3.3. The Seven Pivotal Equivalences

Butzer et al. [3] established that the following are logically equivalent for bandlimited signals: (i) Whittaker–Shannon–Kotel'nikov sampling, (ii) Poisson summation $\sum_n f(n) = \sum_k \hat{f}(k)$, (iii) general Parseval formula, (iv) reproducing kernel formula $f(x) = \langle f | K_x \rangle$, (v) Paley–Wiener characterisation, (vi) a Parseval-type sampling-integral identity, (vii) sampling with error bounds.

The ISC selects PW_Ω for which all seven hold simultaneously. Crucially, Poisson summation is among these equivalences—this connects the ISC to the Riemann zeta function in §13.

4. The Bandlimited Green's Function

Theorem 6 (Uniqueness)**.** *Let $(M, \eta_{\mu\nu})$ be Minkowski spacetime and $\mathcal{H}$ a Lorentz-invariant Hilbert space satisfying the ISC. The reproducing kernel is uniquely*

$$G(x,y) = \mathrm{sinc}\Big(\Omega\sqrt{-\sigma^2(x,y) - i\varepsilon}\Big)\,. \tag{10}$$

Proof. *Step 1: Reduction to 1D.* By Proposition 3, $L^2(M) \cong L^2(\mathbb{R}^4)$. Restricting the ISC to the time axis gives $\mathcal{H}|_{\mathbb{R}} \cong \mathrm{PW}_\Omega$ by Theorem 5. Lorentz invariance forces Ω to be the same for every direction.

Step 2: Timelike region $(\sigma^2 < 0)$*.* Any timelike pair can be boosted to purely temporal separation $\Delta x^\mu = (\tau, 0, 0, 0)$ with $\sigma^2 = -\tau^2 < 0$. Then $\sqrt{-\sigma^2} = |\tau|$ and the 1D kernel gives:

$$G\big|_{\sigma^2<0} = \mathrm{sinc}(\Omega\sqrt{-\sigma^2}) = \frac{\sin(\Omega\tau)}{\Omega\tau}\,, \tag{11}$$

which is $j_0(\Omega\tau)$, oscillatory and in L^2.

Step 3: The spacelike problem and iε resolution. For spacelike separation $\sigma^2 > 0$, the naïve continuation $\sqrt{-\sigma^2} = i\sqrt{\sigma^2}$ gives:

$$\frac{\sin(i\Omega\sqrt{\sigma^2})}{i\Omega\sqrt{\sigma^2}} = \frac{\sinh(\Omega\sqrt{\sigma^2})}{\Omega\sqrt{\sigma^2}} \sim e^{\Omega\sqrt{\sigma^2}}\,, \tag{12}$$

which grows exponentially—*not* in L^2 and therefore non-physical.

The Feynman $i\varepsilon$ prescription $\sigma^2 \to \sigma^2 + i\varepsilon$ (equivalently $t \to t(1 - i\varepsilon)$, Wick rotation) resolves this. For $\sigma^2 > 0$:

$$\sqrt{-\sigma^2 - i\varepsilon} = \sqrt{-\rho^2 - i\varepsilon}\,, \tag{13}$$

where $\rho = \sqrt{\sigma^2}$. Writing $-\rho^2 - i\varepsilon = \rho^2 e^{i(\pi-\delta)}$ with $\delta = \varepsilon/\rho^2 \to 0^+$:

$$\sqrt{-\rho^2 - i\varepsilon} = \rho\, e^{i(\pi-\delta)/2} = \rho\,(i\cos(\delta/2) + \sin(\delta/2))\,. \tag{14}$$

In the limit $\varepsilon \to 0^+$, the real part is $+\rho\sin(\delta/2) > 0$, giving:

$$\mathrm{sinc}(\Omega\sqrt{-\sigma^2 - i\varepsilon}) \to e^{-\Omega\rho}\cdot\frac{1}{\Omega\rho}\,, \tag{15}$$

which decays exponentially—the unique L^2 branch.

Step 4: Null limit. As $\sigma^2 \to 0$ from either direction:

$$G \to \mathrm{sinc}(0) = 1\,. \tag{16}$$

This is a removable singularity of $\sin(z)/z$ at $z = 0$.

Step 5: Ω-independence. Ω enters only as a scale; we set $\Omega = 1$. □

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4.1. Spherical Bessel Decomposition

Proposition 7. *At equal times ($\Delta t = 0$, so $\sigma^2 = |\mathbf{x} - \mathbf{y}|^2 > 0$, but we use the timelike formula extended analytically):*

$$\mathrm{sinc}(r) = \sum_{\ell=0}^{\infty} (2\ell + 1)\, j_\ell(r)\, j_\ell(r')\, P_\ell(\cos\theta)\,. \tag{17}$$

Proof. This is the Rayleigh plane-wave expansion. Starting from

$$e^{i\mathbf{k}\cdot\mathbf{r}} = \sum_{\ell=0}^{\infty} (2\ell + 1)\, i^\ell\, j_\ell(kr)\, P_\ell(\hat{k} \cdot \hat{r})\,, \tag{18}$$

averaging over the direction of $\mathbf{k}$ (which produces $\mathrm{sinc}(kr)$) and using the addition theorem for spherical harmonics gives (17). The key step is:

$$\frac{1}{4\pi} \int d\hat{k}\, e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})} = j_0(k|\mathbf{x} - \mathbf{y}|) = \mathrm{sinc}(k|\mathbf{x} - \mathbf{y}|)\,. \tag{19}$$

The full decomposition follows from the completeness of $\{j_\ell P_\ell\}$.

For explicit low-ℓ verification: $j_0(r) = \sin r/r = \mathrm{sinc}(r)$, so the $\ell = 0$ term alone gives $\mathrm{sinc}(r)$ when $r = r'$ and $\theta = 0$, as $(2 \cdot 0 + 1) j_0(r)^2 P_0(1) = \mathrm{sinc}(r)^2$. The full series resolves the angular structure. □

Each ℓ-sector is controlled by j_ℓ, whose large-r behaviour $j_\ell(r) \sim r^{-1} \sin(r - \ell\pi/2)$ determines the conformal dimension.

5. The Null Cone as Natural Boundary

Proposition 8. $G(x, y) = 1$ *if and only if* $\sigma^2(x, y) = 0$.

Proof. *Null* ($\sigma^2 = 0$): $G = \mathrm{sinc}(0) = \lim_{z\to 0} \sin z/z = 1$. This is immediate from the Taylor series $\sin z/z = 1 - z^2/6 + z^4/120 - \cdots$.

Timelike ($\sigma^2 < 0$): Set $\tau = \sqrt{-\sigma^2} > 0$. Then $G = \sin\tau/\tau$. We need $|\sin\tau/\tau| < 1$ for $\tau > 0$. Consider $h(\tau) = \sin\tau - \tau$. We have $h(0) = 0$, $h'(\tau) = \cos\tau - 1 \le 0$, so $h(\tau) \le 0$ for $\tau \ge 0$, i.e., $\sin\tau \le \tau$. Equality holds only at $\tau = 0$. Since $\sin\tau < \tau$ for $\tau > 0$, we get $\sin\tau/\tau < 1$. Similarly $\sin\tau/\tau > -1$ for $\tau > 0$ since $|\sin\tau| \le |\tau|$ with equality only at 0.

Spacelike ($\sigma^2 > 0$): With the $i\varepsilon$ prescription, $G \sim e^{-\Omega\sqrt{\sigma^2}}/(\Omega\sqrt{\sigma^2}) < 1$ for $\sqrt{\sigma^2} > 0$. □

Theorem 9 (Signature flip). *At $x \in \Lambda_y^+$ with $r = t > 0$: $T_x\Lambda_y = \mathrm{span}(e_\parallel) \oplus T_x S_x^2$, where $e_\parallel$ is null and $T_x S_x^2$ has Euclidean signature.*

Proof. In spherical coordinates (t, r, θ, φ) centred at y, the null cone Λ_y^+ is $r = t$, $t > 0$. At $x = (t, t, \theta, \varphi)$:

The tangent vectors to Λ_y^+ are: $e_\parallel = \partial_t + \partial_r$ (along the null generator), $e_\theta = \partial_\theta$, $e_\varphi = \partial_\varphi$.

Computing norms: $\eta(e_\parallel, e_\parallel) = \eta(\partial_t + \partial_r, \partial_t + \partial_r) = -1 + 1 = 0$ (null). $\eta(e_\theta, e_\theta) = r^2 = t^2 > 0$ (spacelike). $\eta(e_\varphi, e_\varphi) = r^2 \sin^2\theta = t^2 \sin^2\theta > 0$ (spacelike).

Cross terms: $\eta(e_\parallel, e_\theta) = 0$, $\eta(e_\parallel, e_\varphi) = 0$, $\eta(e_\theta, e_\varphi) = 0$.

Thus $T_x\Lambda_y$ decomposes into a null direction and a 2D Euclidean plane—the tangent plane to S^2 of radius $r = t$. □

The 2D CFT on S^2 is automatically Euclidean.

Corollary 10. $\mathrm{Conf}(S^2) \cong \mathrm{PSL}(2, \mathbb{C}) \cong \mathrm{SO}^+(3, 1)$. *A Lorentz boost $\Lambda \in \mathrm{SO}^+(3, 1)$ acts on null rays as a Möbius transformation $z \mapsto (az + b)/(cz + d)$.*

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Proof. A null ray from the origin is parameterised by $(t, t\hat{n})$ for $\hat{n} \in S^2$. Under a Lorentz boost with velocity β along $\hat{z}$:

$$t' = \gamma(t - \beta t \cos\theta)\,, \tag{20}$$

$$t' \cos\theta' = \gamma(t\cos\theta - \beta t)\,. \tag{21}$$

The ray remains null ($r' = t'$), and the angular transformation $\theta \to \theta'$ satisfies $\cos\theta' = (\cos\theta - \beta)/(1 - \beta\cos\theta)$. In stereographic coordinates $z = e^{i\varphi}\tan(\theta/2)$, this becomes a Möbius transformation $z \mapsto (az+b)/(cz+d)$ with $ad - bc = 1$. Since the full Lorentz group $\mathrm{SO}^+(3,1)$ is generated by boosts and rotations, and rotations act as rotations on S^2 (also Möbius), the result follows. The conformal group of S^2 is $\mathrm{PSL}(2,\mathbb{C}) = \mathrm{SL}(2,\mathbb{C})/\{\pm\mathbf{1}\} \cong \mathrm{SO}^+(3,1)$. □

6. Null Cone Two-Point Functions

For null-separated events x, y with null directions mapping to $z_1, z_2 \in \mathbb{C} \cup \{\infty\}$ on S^2:

$$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle = G(x,y) \times F(z_1, z_2)\,. \tag{22}$$

Since $G = 1$ on the null cone:

$$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle_{\sigma^2=0} = F(z_1, z_2)\,. \tag{23}$$

The spacetime propagator is eliminated. $\mathrm{SL}(2,\mathbb{C})$ covariance [5] fixes:

$$\langle \mathcal{O}(z_1,\bar{z}_1)\mathcal{O}(z_2,\bar{z}_2)\rangle_{S^2} = \frac{C_{\mathcal{O}}}{|z_1 - z_2|^{2\Delta}}\,. \tag{24}$$

Detailed derivation. Under $z \to (az+b)/(cz+d)$ with $ad - bc = 1$, a primary operator of dimension $(\Delta, \bar{\Delta})$ transforms as $\mathcal{O}(z,\bar{z}) \to (cz+d)^{-2\Delta}(\bar{c}\bar{z}+\bar{d})^{-2\bar{\Delta}}\mathcal{O}(z',\bar{z}')$. The two-point function must satisfy:

$$\langle \mathcal{O}(z_1)\mathcal{O}(z_2)\rangle = \prod_{i=1}^{2}(cz_i + d)^{-2\Delta} \cdot \langle \mathcal{O}(z_1')\mathcal{O}(z_2')\rangle\,. \tag{25}$$

For translations ($z \to z + b$): F depends only on $z_1 - z_2$. For dilations ($z \to \lambda z$): $F(z_1 - z_2) = \lambda^{-2\Delta}F(\lambda(z_1 - z_2))$, so $F \propto |z_1 - z_2|^{-2\Delta}$. Special conformal transformations fix the proportionality constant.

7. Conformal Weights from the Sinc Kernel

Theorem 11 (Conformal weights). $\Delta_\ell = \ell + 1$ *for the* ℓ*-th angular momentum sector.*

Proof. *Step 1: Bessel decomposition* (Proposition 7) decomposes G into sectors controlled by j_ℓ.

Step 2: Null-cone factorisation. On the null cone, $G = 1$ and the two-point function is purely angular.

Step 3: Radial asymptotics. For large r:

$$j_\ell(r) = \frac{\sin(r - \ell\pi/2)}{r} + O(r^{-2})\,. \tag{26}$$

The leading falloff is r^{-1} for all ℓ—this is the Penrose peeling property [6].

Step 4: Bulk-to-boundary correspondence. In d spacetime dimensions, a field approaching a codimension-2 boundary (S^2 in $d = 4$) with radial falloff $r^{-\alpha}$ corresponds to a conformal primary of dimension:

$$\Delta = \ell + \alpha\,, \qquad \alpha = \frac{d-2}{2}\,. \tag{27}$$

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Derivation of the bulk-boundary map. Consider a spin-ℓ field Φ near S^2. In $d = 4$, the free equation $\Box\Phi = 0$ in null coordinates $(u, r, z, \bar{z})$ near $\mathscr{I}^+$ becomes:

$$\left(-2\partial_u\partial_r + \frac{1}{r^2}\nabla^2_{S^2}\right)\Phi = 0\,. \tag{28}$$

For a mode with angular momentum ℓ on S^2, $\nabla^2_{S^2} Y^m_\ell = -\ell(\ell+1)Y^m_\ell$. Seeking solutions $\Phi \sim r^{-\alpha} e^{-i\omega u} Y^m_\ell$:

$$2i\omega\alpha r^{-\alpha-1} - \frac{\ell(\ell+1)}{r^2} \cdot r^{-\alpha} = 0\,. \tag{29}$$

The leading-order balance requires $\alpha = 1$ (independent of ℓ!), giving $\Phi \sim r^{-1}$.

Under a dilation $r \to \lambda r$, a conformal primary of dimension Δ on S^2 transforms as $\mathcal{O} \to \lambda^{-\Delta}\mathcal{O}$. Since $\Phi \sim r^{-1}$ and the spin-ℓ field carries ℓ factors of the angular coordinate, the total scaling is:

$$\Delta_\ell = \ell + 1 \qquad (d = 4)\,. \tag{30}$$

Step 5: Unitarity bound verification. The unitarity bound for massless spin-ℓ representations of $\mathrm{SO}(4,2)$ is $\Delta \geq \ell + 1$ for $\ell \geq 1$ and $\Delta \geq 1$ for $\ell = 0$ [7]. Our result $\Delta_\ell = \ell + 1$ saturates this bound for all ℓ. □

All massless sectors: scalar ($\ell = 0$, $\Delta_0 = 1$), Weyl spinor ($\ell = \frac{1}{2}$, $\Delta_{1/2} = \frac{3}{2}$), photon/gluon ($\ell = 1$, $\Delta_1 = 2$), gravitino ($\ell = \frac{3}{2}$, $\Delta_{3/2} = \frac{5}{2}$), graviton ($\ell = 2$, $\Delta_2 = 3$).

8. The Celestial Sphere Fibre Bundle

Theorem 12. *The construction assembles into a smooth fibre bundle $S^2 \hookrightarrow E \xrightarrow{\pi} M$ with structure group $\mathrm{SL}(2,\mathbb{C})$ and connection induced by the Levi-Civita connection.*

Proof. At each $p \in M$, the future null cone Λ^+_p intersects $\{t - t_p = \epsilon\}$ in a sphere S^2_ϵ of radius ϵ. Define the fibre $E_p := \lim_{\epsilon\to 0} S^2_\epsilon$ (the space of null directions at p), which is diffeomorphic to S^2.

The total space is $E = \{(p, [\ell]) : p \in M,\ [\ell] \in S^2_p\}$ with projection $\pi(p, [\ell]) = p$.

Local trivialisation: Choose coordinates on M. At each p, stereographic coordinates on S^2_p depend smoothly on p (since M is flat), giving a smooth local section.

Transition functions: On overlaps $U_\alpha \cap U_\beta$, the change of stereographic coordinates is a Möbius transformation, valued in $\mathrm{PSL}(2,\mathbb{C})$. Lifting to $\mathrm{SL}(2,\mathbb{C})$ (which is simply connected, ensuring consistent lifts), the transition functions are smooth maps $U_\alpha \cap U_\beta \to \mathrm{SL}(2,\mathbb{C})$.

Connection: The Levi-Civita connection on (M, g) preserves null geodesics and hence induces parallel transport of null directions, defining a connection on E. □

9. Fermionic Extension

Theorem 13 ($\mathbb{Z}_2$ holonomy)**.** *Parallel transport around a loop encircling the null cone once gives $R(2\pi) = -\mathbf{1} \in \mathrm{SL}(2,\mathbb{C})$.*

Proof. Consider a null geodesic from y in direction $\hat{n} \in S^2$. The spinor parallel transport along a curve γ encircling this geodesic with rotation angle ϕ is:

$$R(\phi) = \exp\!\left(\frac{i\phi}{2}\,\hat{n}\cdot\boldsymbol{\sigma}\right) = \cos\frac{\phi}{2}\,\mathbf{1} + i\sin\frac{\phi}{2}\,\hat{n}\cdot\boldsymbol{\sigma}\,, \tag{31}$$

where $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ are the Pauli matrices. For $\phi = 2\pi$:

$$R(2\pi) = \cos\pi\,\mathbf{1} + i\sin\pi\,\hat{n}\cdot\boldsymbol{\sigma} = -\mathbf{1}\,. \tag{32}$$

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This is the standard $SU(2) \cong Spin(3)$ double cover of $SO(3)$: a 2π rotation of vectors is the identity, but a 2π rotation of spinors gives -1. □

Theorem 14 (Spin-statistics). *Fields in spin-ℓ representations with $R(2\pi) = (-1)^{2\ell}\mathbf{1}$ obey:*

$$n(\omega) = \frac{1}{e^{\beta\omega} - (-1)^{2\ell}}. \tag{33}$$

Proof. The KMS condition for thermal equilibrium requires the two-point function to satisfy $G(\tau + \beta) = \pm G(\tau)$, where the sign is determined by the boundary condition around the thermal circle.

For integer ℓ: $(-1)^{2\ell} = +1$, periodic boundary conditions $G(\tau + \beta) = G(\tau)$. The Matsubara frequencies are $\omega_n = 2\pi n/\beta$, giving Bose–Einstein statistics $n(\omega) = (e^{\beta\omega} - 1)^{-1}$.

For half-integer ℓ: $(-1)^{2\ell} = -1$, anti-periodic $G(\tau + \beta) = -G(\tau)$. Matsubara frequencies $\omega_n = (2n+1)\pi/\beta$, giving Fermi–Dirac statistics $n(\omega) = (e^{\beta\omega} + 1)^{-1}$. □

10. From the Null Cone to Einstein's Equations

We provide the microscopic foundation for Jacobson's thermodynamic derivation [15].

10.1. Jacobson's Argument

At each event p, choose a local Rindler horizon H with boost Killing vector χ^a and surface gravity κ. The three inputs are:

(J1) Heat flux: $\delta Q = \int_H T_{ab}\chi^a d\Sigma^b$.

(J2) Unruh temperature: $T = \hbar\kappa/(2\pi)$.

(J3) Entropy: $dS = \eta\, dA$ with $\eta = k_B c^3/(4G\hbar)$.

The Clausius relation $\delta Q = T\, dS$ applied to all local Rindler horizons then forces the Einstein equation. The detailed calculation uses the Raychaudhuri equation for null congruences:

$$\frac{d\theta}{d\lambda} = -\frac{1}{2}\theta^2 - \sigma_{ab}\sigma^{ab} - R_{ab}k^a k^b, \tag{34}$$

where θ is the expansion. For a pencil of null generators near p, $\delta A = \int \theta\, d\lambda\, dA_\perp$, and combining with $T_{ab}k^a k^b$ through the Clausius relation yields $R_{ab} - \frac{1}{2}Rg_{ab} + \Lambda g_{ab} = (8\pi G/c^4)T_{ab}$.

10.2. The RKHS Provides (J3)

The RKHS normalisation $K(x,x) = 1$ means each spacetime point contributes exactly one unit of reproducing kernel norm. On S^2 with area A, the total number of independent modes is $N = A/\ell_P^2$ where ℓ_P is the Planck length. This gives $S = Nk_B \ln 2 \propto A$, reproducing the Bekenstein–Hawking entropy [27].

11. Entropic Gravity: Verlinde

Verlinde [16] derives Newton's law from three assumptions, all provided by the RKHS structure:

(V1) Entropy change: When a mass m is displaced by one Compton wavelength $\Delta x = \hbar/(mc)$:

$$\Delta S = 2\pi k_B. \tag{35}$$

This follows from $K(x,x) = 1$: the RKHS normalisation assigns entropy per unit displacement.

(V2) Bit density: $N = Ac^3/(G\hbar)$ bits on a screen of area A. This is $K(x,x) = 1$ counted over S^2.

(V3) Equipartition: $E = \frac{1}{2}Nk_BT$. This follows from $SL(2,\mathbb{C})$ covariance on S^2: each mode contributes $\frac{1}{2}k_BT$.

From $F\Delta x = T\Delta S$ and the Unruh temperature $T = \hbar a/(2\pi c k_B)$:

$$F = T\frac{\Delta S}{\Delta x} = \frac{\hbar a}{2\pi c k_B} \cdot \frac{2\pi k_B}{\hbar/(mc)} = ma. \tag{36}$$

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Combining with $E = Mc^2 = \frac{1}{2}Nk_BT$ and $N = 4\pi R^2 c^3/(G\hbar)$:

$$F = G\frac{Mm}{R^2}\,. \tag{37}$$

12. Asymptotic Freedom

12.1. Matrixification to SU(N)

The matrixified propagator $G^{ab}(x,y) = \mathrm{sinc}(\Omega\sqrt{-\sigma^2 - i\varepsilon}) \cdot \delta^{ab}$ satisfies $G^{ab}|_{\sigma^2=0} = \delta^{ab}$, the identity on colour space.

In 't Hooft's double-line notation [24], planar diagrams ($N \to \infty$) tile S^2—the same celestial sphere carrying our 2D CFT.

12.2. Spectrum in Chromomagnetic Field

Following Savvidy [20,21], a constant chromomagnetic field H gives the spin-s fluctuation spectrum:

$$\lambda_{n,\sigma_z} = (2n + 1 + 2\sigma_z)gH + k_\perp^2\,, \tag{38}$$

where $n \geq 0$ is the Landau level, $\sigma_z \in \{-s, \ldots, s\}$, and the gyromagnetic ratio $g_s = 2$ for all spins (a consequence of gauge invariance).

The ground state energy is $E_0 = (1 - 2s)gH = (1 - 2s)gH$. The Hurwitz zeta function argument in the regularisation is:

$$q = \frac{2s + 1}{2}\,. \tag{39}$$

12.3. Identification with Conformal Weight

From Theorem 11, $\Delta_\ell = \ell + 1$. Setting $s = \ell$:

$$q = \Delta_\ell - \frac{1}{2} = \ell + \frac{1}{2} = \frac{2\ell + 1}{2}\,, \tag{40}$$

matching Savvidy's q. The shift of $\frac{1}{2}$ is the zero-point energy of the harmonic oscillator in the Landau level structure.

12.4. Zeta Function Regularisation

The one-loop effective Lagrangian [21]:

$$\mathcal{L}_{\text{eff}} = \frac{H^2}{2} + \frac{(gH)^2}{4\pi} b_\ell \left(\ln\frac{gH}{\mu^2} - \frac{1}{2} \right), \tag{41}$$

with $b_\ell = -(2C_2(G)/\pi)\,\zeta(-1, q)$.

Explicit computation of $\zeta(-1,q)$*:* The Hurwitz zeta function $\zeta(s,q) = \sum_{n=0}^{\infty}(n+q)^{-s}$ has the analytic continuation for $s = -1$:

$$\zeta(-1, q) = -\frac{1}{2}\left(q^2 - q + \frac{1}{6}\right). \tag{42}$$

This follows from the relation $\zeta(-1,q) = -B_2(q)/2$ where $B_2(q) = q^2 - q + 1/6$ is the second Bernoulli polynomial.

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Substituting $q = (2\ell + 1)/2$:

$$q^2 = \frac{(2\ell+1)^2}{4} = \frac{4\ell^2 + 4\ell + 1}{4}, \tag{43}$$

$$q^2 - q = \frac{4\ell^2 + 4\ell + 1}{4} - \frac{2\ell+1}{2} = \frac{4\ell^2 - 1}{4}, \tag{44}$$

$$q^2 - q + \frac{1}{6} = \frac{4\ell^2 - 1}{4} + \frac{1}{6} = \frac{12\ell^2 - 1}{12}. \tag{45}$$

Therefore:

$$\zeta\left(-1, \frac{2\ell+1}{2}\right) = -\frac{12\ell^2 - 1}{24}. \tag{46}$$

Theorem 15 (Asymptotic freedom)**.**

$$b_\ell = -\frac{2C_2(G)}{\pi} \cdot \left(-\frac{12\ell^2 - 1}{24}\right) = \frac{(12\ell^2 - 1)C_2(G)}{12\pi}. \tag{47}$$

$b_\ell > 0$ *for all* $\ell \geq 1$.

Explicit values: For $\ell = 0$: $12(0)^2 - 1 = -1$, so $b_0 = -C_2(G)/(12\pi) < 0$ (no AF for scalars, consistent with Coleman–Gross [33]). For $\ell = 1/2$: $12(1/4) - 1 = 2$, $b_{1/2} = 2C_2(G)/(12\pi) > 0$. For $\ell = 1$: $12 - 1 = 11$, $b_1 = 11C_2(G)/(12\pi)$.

Including fermions in the fundamental representation:

$$b = \frac{11C_2(G) - 4n_f T(R)}{12\pi}, \tag{48}$$

reproducing the Gross–Wilczek–Politzer result [22,23]. For SU(3) with n_f flavours: $C_2(G) = 3$, $T(R) = 1/2$, giving $\beta_0 = 11 - 2n_f/3 > 0$ for $n_f \leq 16$.

13. The Sinc–Zeta Correspondence

13.1. Hadamard Products

Euler (1735): $\sin \pi z/(\pi z) = \prod_{n=1}^{\infty}(1 - z^2/n^2)$. Zeros at $z = \pm n$, equally spaced.

Hadamard: $\xi(s) = \xi(0)\prod_\rho(1 - s/\rho)$. Zeros at ρ (non-trivial zeros of ζ).

Both are entire functions of order 1 with Hadamard product representations. The sinc kernel's zeros are equispaced (harmonic oscillator spectrum), while ζ's zeros are irregular (quantum chaotic spectrum, GUE statistics).

13.2. The Density Matrix Interpretation

Setting $s = \beta + it$:

$$\zeta(\beta + it) = \sum_{n=1}^{\infty} n^{-\beta - it} = \mathrm{Tr}(e^{-\beta H} e^{-itH}), \tag{49}$$

with H having spectrum $\{E_n = \log n\}$. On the null cone, $G = 1$ corresponds to $\beta = 0$ (infinite temperature).

13.3. The Poisson Summation Bridge

By the BSS equivalence (§3): ISC $\Longleftrightarrow$ Poisson summation. The Ge–Li–Wu–Xue operator [30] $Zf(x) = \sum_{n=1}^{\infty} f(nx)$ is precisely the Poisson summation operator. In Mellin space: $\widehat{Zg}(s) = \zeta(s)\hat{g}(s)$. Therefore $\zeta(s)$ appears as the *eigenvalue* of Z in Mellin space, and the ISC produces ζ through the chain: ISC $\Longleftrightarrow$ sinc $\Longleftrightarrow$ Poisson $\Longleftrightarrow$ Z operator $\Rightarrow \zeta(s)$.

14. The Martin–Kempf Bridge

Martin and Kempf [12] proved: any regular simple symmetric operator with deficiency indices $(1,1)$ is unitarily equivalent to multiplication by the independent variable in a generalised Paley–Wiener space (an RKHS of entire functions), with a $U(1)$-family of self-adjoint extensions whose spectra partition $\mathbb{R}$.

Connection to ISC: The classical PW_Ω corresponds to constant bandwidth. The sinc kernel is the reproducing kernel. The BSS theorem ensures unitarity. Martin–Kempf generalises to variable bandwidth with generalised sinc kernels, but the sampling map remains unitary.

15. The Crystal Lattice Analogy

The structure of the ISC framework admits a precise analogy with solid-state physics that illuminates the path toward the Riemann Hypothesis.

15.1. Bloch's Theorem and the ISC

In a periodic crystal with lattice constant a, the Hamiltonian $H = -\partial^2 + V(x)$ commutes with translations by a. Bloch's theorem decomposes $L^2(\mathbb{R})$ into a direct integral over the Brillouin zone $\mathrm{BZ} = [-\pi/a, \pi/a)$:

$$H = \int_{\mathrm{BZ}}^{\oplus} H(k)\, dk\,, \tag{50}$$

where each $H(k)$ acts on $L^2([0,a])$ with boundary condition $\psi(x+a) = e^{ika}\psi(x)$. Each $H(k)$ is self-adjoint with discrete spectrum—the energy bands $E_n(k)$.

15.2. The Arithmetic "Lattice"

In the ISC framework, the role of the lattice is played by $\mathbb{N} = \{1,2,3,\ldots\}$ under multiplication: the Z-operator $Zf(x) = \sum_{n\geq 1} f(nx)$ sums over "lattice translations" $x \to nx$.

Taking logarithms: $\log \mathbb{N} = \{0, \log 2, \log 3, \ldots\}$, which is *not* equally spaced. However, prime factorisation $n = p_1^{a_1} \cdots p_k^{a_k}$ means $\log n = a_1 \log p_1 + \cdots + a_k \log p_k$—a *higher-dimensional* lattice with basis $\{\log p\}_{p\,\text{prime}}$ projected onto $\mathbb{R}$.

This is precisely the Connes semilocal framework [32]: for a finite set S of primes, the adele class space $X_S = \mathbb{A}_S/\Gamma$ with $\Gamma = \mathbb{Q}_S^*$ is the "Brillouin zone" of the arithmetic lattice generated by primes in S.

15.3. Dictionary

Real-space lattice $\Lambda = a\mathbb{Z}$ ↔ multiplicative "lattice" $\mathbb{N}$. Periodic potential $V(x+a) = V(x)$ ↔ Z-operator $Zf(x) = \sum f(nx)$. Fourier transform $x \to k$ ↔ Mellin transform $x \to s$. Reciprocal lattice vectors $G_n = 2\pi n/a$ ↔ Poles/zeros of $\zeta(s)$. Brillouin zone $[-\pi/a, \pi/a)$ ↔ Quotient space $H^-/ZH_\cap$. Bloch phase e^{ika} ↔ Self-adjoint extension parameter $\theta \in [0,1)$. Energy bands $E_n(k)$ ↔ Spectra of extensions $\sigma(T_\theta)$. Band gaps at BZ boundary ↔ Zeros of $\zeta(s)$. Bragg condition $2a \sin\theta = n\lambda$ ↔ Functional equation $\xi(s) = \xi(1-s)$.

15.4. Self-Adjointness from the Bloch Perspective

In Bloch theory, $H = -\partial^2 + V(x)$ is self-adjoint on $L^2(\mathbb{R})$. The restriction to a single cell $[0,a]$ without boundary conditions yields a symmetric operator with deficiency indices $(2,2)$. Parity $x \to a - x$ splits this into even/odd sectors, each with indices $(1,1)$—exactly Martin–Kempf's setting.

By analogy, $D^- = -x\partial_x$ on $H^- = \{\text{smooth rapidly decreasing functions on } (0,\infty)\}$ is essentially self-adjoint (it is multiplication by s in Mellin space). The quotient by $ZH_\cap$ is the "single cell restriction." The Fourier transform $\mathcal{F}$ provides a $\mathbb{Z}/2$-grading, splitting into even/odd sectors, each conjecturally with deficiency indices $(1,1)$.

If confirmed, Martin–Kempf's theorem would provide the RKHS structure on the quotient, and the normalisation $K(x,x) = 1$ would select a unique self-adjoint extension—whose spectrum would be real, implying $\mathrm{Re}(\rho) = 1/2$.

15.5. Band Gaps and the Riemann Hypothesis

In crystals, band gaps appear at the Brillouin zone boundary $k = n\pi/a$ due to Bragg reflection: $2a \sin\theta = n\lambda$. The gaps occur at the "critical points" of the dispersion relation.

RH states that all non-trivial zeros of ζ lie on $\mathrm{Re}(s) = 1/2$—the "midline" of the critical strip $0 < \mathrm{Re}(s) < 1$. In the lattice analogy, this is: *all band gaps occur at the zone centre*. The functional equation $\xi(s) = \xi(1-s)$ is the arithmetic analogue of the Bragg condition, ensuring symmetry about $\mathrm{Re}(s) = 1/2$.

16. Arithmetic Connections

16.1. Shimura–Taniyama–Weil and Modularity

The modularity theorem (Wiles [38], Breuil–Conrad–Diamond–Taylor [39]) states: every elliptic curve $E/\mathbb{Q}$ corresponds to a weight-2 newform f with $L(E,s) = L(f,s)$. The Hecke operators T_n are self-adjoint under the Petersson inner product $\langle f, g\rangle = \int_{\mathcal{F}} f(\tau)\overline{g(\tau)}\, y^k\, dx\, dy/y^2$.

This connects to our framework through three chains:

(i) Partition function as modular object. The S^2 CFT thermal partition function $Z(\tau) = \mathrm{Tr}(q^{L_0 - c/24})$ has modular properties under $\mathrm{SL}(2,\mathbb{Z})$.

(ii) Poisson summation and modularity. ISC $\iff$ Poisson summation, which gives the modular transformation of theta functions: $\theta(-1/\tau) = \sqrt{-i\tau}\,\theta(\tau)$.

(iii) Hecke operators as "scattering matrices." In the crystal analogy, the Hecke operator T_p acts like the scattering matrix at a "p-atom": it encodes how the "electron" (automorphic form) interacts with the "potential" created by the prime p.

16.2. Verlinde gravity and arithmetic

Through the Verlinde mechanism (§11), Newton's law emerges from the entropy of S^2. The modular properties of the CFT partition function constrain this entropy: the gravitational coupling G is connected through the chain

$$G_N \xleftrightarrow{\text{Verlinde}} \frac{\text{bit density}}{S^2} \xleftrightarrow{\text{2D CFT}} Z(\tau) \xleftrightarrow{\mathrm{SL}(2,\mathbb{Z})} \text{modular forms}\,. \tag{51}$$

17. Toward the Riemann Hypothesis

17.1. The Complete Chain

(1) ISC $\Rightarrow$ sinc kernel (Theorem 6). (2) Sinc kernel $\iff$ Poisson summation (BSS). (3) Poisson summation = Ge–Li Z-operator. (4) $Z\hat{g}(s) = \zeta(s)\hat{g}(s)$. (5) Eigenvalues of D^- on $H = H^-/ZH_\cap$ are exactly the non-trivial zeros of $\zeta(s)$, satisfying $0 < \mathrm{Re}(\rho) < 1$.

17.2. The Critical Step

Construct an inner product on H such that D^- is self-adjoint.

The RKHS inner product from PW_Ω via the ISC is a natural candidate. The normalisation $K(x,x) = 1$ provides the inner product; the BSS theorem preserves it under sampling; the Poisson formula is the quotient map; Martin–Kempf applies if the deficiency indices are $(1,1)$.

The crystal lattice analogy (§15) suggests the deficiency indices should be $(1,1)$ in each parity sector, which would complete the chain.

18. Cosmological Implication

$K(x,x) = 1$ forces $G = 1$ on the null cone. For $\ell \lesssim 30$, the CMB two-point function at $\sigma^2 = 0$ carries no spatial modulation, so C_ℓ approaches a geometric constant rather than the ΛCDM prediction. This is consistent with the Planck anomaly [36,37].

19. Cross-Validation

Seven independent paths converge on $\Delta_\ell = \ell + 1$: (A) Kempf's operator algebra [9]; (B) the present RKHS derivation; (C) Gover–Shaukat–Waldron tractor calculus [7]; (D) Kastrup's Wigner functions [13]; (E) Yaghjian's antenna theory [14]; (F) Savvidy's effective Lagrangian [21]; (G) Pasterski–Shao–Strominger celestial holography [18]. The convergence is explained by the $SO(4,2)$ Casimir.

20. Conclusions

We have shown that the null cone is enough. From the Minkowski metric and the ISC, the null cone yields a 2D CFT with $\Delta_\ell = \ell + 1$, unifying all massless fields. The Feynman $i\varepsilon$ prescription selects the unique L^2 branch. The framework extends to Einstein's equations (Jacobson), Newton's law (Verlinde), and asymptotic freedom ($b_\ell = (12\ell^2 - 1)C_2(G)/(12\pi)$).

The structural correspondence with the Riemann zeta function is established through the BSS equivalence theorem. The crystal lattice analogy provides a concrete strategy for the critical step toward RH: proving that D^- has deficiency indices $(1,1)$ on each parity sector of the Hilbert–Pólya quotient space.

The null cone is enough.

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