Short Note

On Quadratic Polynomials Rich in Prime Numbers

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Article

On Quadratic Polynomials Rich in Prime Numbers

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Abstract: Prime numbers and methods of their generation have attracted mathematicians for centuries and, in the digital age, have found their applications in cryptography, signal processing and data compression, secure communications, hashing algorithms, cybersecurity, quantum computing algorithms, blockchain technology, and other areas. Prime numbers and prime generating polynomials were studied in [1-20]. There are many prime generating polynomials of different degrees [18] found so far; the most famous of them is $x^2 + x + 41$ found by Euler in 1772 and $x^2 - x + 41$ found by Legendre in 1798. Researchers are curious to find such a polynomial $P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ that produces more consecutive prime numbers for given integer values of x. This work is no exception, but the reader will not find any mathematical formulas or theorems as in mathematical works; instead, we want to show the result of our computational experiments in the programming language Julia, which in particular led to the discovery of quadratic polynomials that, similarly to Euler's prime generating polynomial, generate 40 consecutive primes. We also show that some of the currently known polynomials are not the richest in terms of the percentage of primes appearing in larger intervals, i.e., they produce fewer primes. For a better and more systematic understanding of what happens in prime number research, we demonstrate a video with network visualisation of keyword co-occurrence and co-authorship based on data from 7548 documents indexed in the Scopus database. Readers of this work are welcome to send me comments, suggestions, or proposals for collaborative research on prime numbers and their applications in science and engineering.

Keywords: prime numbers; quadratic polynomial; Julia programming language

MSC: Primary 11N05, Secondary 11N13, 11Y11.

1. Networks of Keywords and Co-Authors of Scopus Documents on Prime Numbers

The query TITLE-ABS-KEY ("prime number") was used in Scopus to find the related documents. Using the software VOSviewer[1], we analysed the keywords of 7548 documents, and the visualisation of the keyword and co-authorship network is available at https://youtu.be/L3zmETYtJGI.

Our analysis showed that the most popular keywords are cryptography, number theory, algorithms, public key cryptography, polynomials, finite fields, codes, data security, factorisation, matrix algebra, network security, RSA[2], computational complexity, geometry, algebra, graph theory, computational theory, encryption, RSA algorithms, theorem proving, authentication, elliptic curve, and others.

2. Julia Code for Checking Prime Numbers

The computational experiments were carried out in the Julia[3] v. 1.11.2 programming language on a computer with an Intel® Core™ i5-10500 CPU at 3.10 GHz, 8.00 GB of RAM, 64-bit architecture, and Windows 11 operating system.

1 https://www.vosviewer.com/

2 RSA (Rivest-Shamir-Adleman) is a public key cryptosystem, one of the oldest and most widely used for secure data transmission.

3 https://julialang.org/

 doi:10.20944/preprints202502.2214.v1


```
using Primes  # Import the Primes package

function compute_prime_stats()
    # Define the polynomial function
    f(x) = x^2 - x + 41

    total_count = 10^7
    prime_count = 0
    max_streak = 0
    current_streak = 0

    # Loop through values of x from 0 to total_count
    for x in 0:total_count
        value = f(x)
        if isprime(value)
            prime_count += 1
            current_streak += 1
        else
            max_streak = max(max_streak, current_streak)
            current_streak = 0
        end
        println("f($x) = $value -> Prime: ", isprime(value))
    end

    # Check in case the last numbers were primes
    max_streak = max(max_streak, current_streak)

    prime_percentage = (prime_count / total_count) * 100
    println("Percentage of prime numbers: $prime_percentage%")
    println("Maximum number of consecutive prime values: $max_streak")
end

# Execute the function
@time compute_prime_stats()
```

For $x^2 - x + 41$ prime generating polynomial for $x_{max} = 1 \times 10^7$, in 27.230569 seconds (23 allocations: 1008 bytes), the program determined that 22.08% of the values were prime and recorded a maximum streak of 40 consecutive prime numbers.

3. Results of the Computational Experiment

In the following tables, a, b, c are the coefficients of a quadratic polynomial, and the other columns show how many primes are found in a given range.

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Table 1. Prime-generating polynomials with positive a, b, c that produce 30 consecutive prime numbers (some of them can produce even more).

No.	a	b	c	Number of primes	Percentage
1	1	1	41	30	100.0
2	1	3	43	30	100.0
3	1	5	47	30	100.0
4	1	7	53	30	100.0
5	1	9	61	30	100.0
6	1	11	71	30	100.0
7	1	13	83	30	100.0
8	1	15	97	30	100.0
9	1	17	113	30	100.0
10	1	19	131	30	100.0
11	1	21	151	30	100.0

Table 2. Prime-generating polynomials that produce 40 consecutive prime numbers (note that some polynomials are likely to produce more primes and can be independently checked using the Julia code). Euler's prime-generating polynomial and some others can also be found among the results.

No.	a	b	c	Number of primes	Percentage
1	1	-61	971	40	100.0
2	1	-59	911	40	100.0
3	1	-57	853	40	100.0
4	1	-55	797	40	100.0
5	1	-53	743	40	100.0
6	1	-51	691	40	100.0
7	1	-49	641	40	100.0
8	1	-47	593	40	100.0
9	1	-45	547	40	100.0
10	1	-43	503	40	100.0
11	1	-41	461	40	100.0
12	1	-39	421	40	100.0
13	1	-37	383	40	100.0
14	1	-35	347	40	100.0
15	1	-33	313	40	100.0
16	1	-31	281	40	100.0
17	1	-29	251	40	100.0
18	1	-27	223	40	100.0
19	1	-25	197	40	100.0
20	1	-23	173	40	100.0
21	1	-21	151	40	100.0
22	1	-19	131	40	100.0
23	1	-17	113	40	100.0

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 doi:10.20944/preprints202502.2214.v1

Table 2 – continued from previous page

No.	a	b	c	Number of primes	Percentage
24	1	-15	97	40	100.0
25	1	-13	83	40	100.0
26	1	-11	71	40	100.0
27	1	-9	61	40	100.0
28	1	-7	53	40	100.0
29	1	-5	47	40	100.0
30	1	-3	43	40	100.0
31	1	-1	41	40	100.0
32	1	1	41	40	100.0
33	2	-88	997	40	100.0
34	2	-84	911	40	100.0
35	2	-80	829	40	100.0
36	2	-76	751	40	100.0
37	2	-72	677	40	100.0
38	2	-68	607	40	100.0
39	2	-64	541	40	100.0
40	2	-60	479	40	100.0
41	2	-56	421	40	100.0
42	2	-52	367	40	100.0
43	2	-48	317	40	100.0
44	2	-44	271	40	100.0
45	3	-105	941	40	100.0
46	6	-150	967	40	100.0
47	6	-138	823	40	100.0
48	6	-126	691	40	100.0

Table 3. One hundred prime generating polynomials with positive a, b, c and thirteen quadratic polynomials from [18] for values of x up to 10.000. Polynomials 9 and 10 from the bottom of the table can also be found in Table 3 (polynomials 56 and 21, respectively).

No.	a	b	c	Number of primes	Percentage
1	2	44	43	4366	43.6556
2	2	40	1	4365	43.6456
3	1	1	41	4149	41.4859
4	1	3	43	4149	41.4859
5	1	5	47	4148	41.4759
6	6	6	31	3859	38.5861
7	6	18	43	3858	38.5761
8	4	2	41	3836	38.3562

Continued on next page

 doi:10.20944/preprints202502.2214.v1

Table 3 – continued from previous page

No.	a	b	c	Number of primes	Percentage
9	4	10	47	3835	38.3462
10	2	40	19	3805	38.0462
11	4	6	43	3785	37.8462
12	1	35	23	3663	36.6263
13	9	9	43	3637	36.3664
14	9	3	41	3604	36.0364
15	9	15	47	3602	36.0164
16	24	12	31	3598	35.9764
17	2	34	31	3580	35.7964
18	24	36	43	3577	35.7664
19	16	4	41	3534	35.3365
20	16	20	47	3524	35.2365
21	3	39	37	3510	35.0965
22	3	33	1	3509	35.0865
23	7	49	41	3509	35.0865
24	10	20	29	3491	34.9065
25	22	22	17	3484	34.8365
26	2	4	31	3483	34.8265
27	2	8	37	3483	34.8265
28	2	12	47	3482	34.8165
29	16	12	43	3468	34.6765
30	25	25	47	3467	34.6665
31	1	23	23	3446	34.4566
32	1	25	47	3446	34.4566
33	1	21	1	3445	34.4466
34	25	5	41	3412	34.1166
35	7	7	17	3409	34.0866
36	7	21	31	3409	34.0866
37	1	37	29	3405	34.0466
38	38	40	1	3404	34.0366
39	33	15	11	3392	33.9166
40	36	18	43	3388	33.8766
41	36	30	47	3388	33.8766
42	35	35	19	3387	33.8666
43	36	6	41	3381	33.8066
44	25	15	43	3375	33.7466
45	43	3	1	3374	33.7366
46	49	35	47	3360	33.5966
47	47	5	1	3356	33.5566

Continued on next page

 doi:10.20944/preprints202502.2214.v1

Table 3 – continued from previous page

No.	a	b	c	Number of primes	Percentage
48	41	1	1	3355	33.5466
49	32	18	47	3344	33.4367
50	20	20	43	3342	33.4167
51	49	21	43	3339	33.3867
52	49	7	41	3314	33.1367
53	34	46	23	3306	33.0567
54	1	27	13	3305	33.0467
55	1	29	41	3305	33.0467
56	3	3	23	3299	32.9867
57	3	9	29	3298	32.9767
58	3	15	41	3298	32.9767
59	34	22	11	3287	32.8667
60	40	40	29	3267	32.6667
61	8	8	31	3263	32.6267
62	8	24	47	3263	32.6267
63	8	16	37	3243	32.4268
64	4	46	23	3242	32.4168
65	23	35	1	3222	32.2168
66	14	6	29	3214	32.1368
67	14	34	49	3214	32.1368
68	2	30	29	3211	32.1068
69	2	26	1	3210	32.0968
70	26	32	43	3206	32.0568
71	46	46	17	3203	32.0268
72	28	42	31	3185	31.8468
73	33	33	41	3172	31.7168
74	28	14	17	3154	31.5368
75	14	22	37	3146	31.4569
76	18	12	31	3142	31.4169
77	18	36	47	3133	31.3269
78	42	30	29	3131	31.3069
79	22	14	11	3123	31.2269
80	15	45	1	3117	31.1669
81	26	20	37	3109	31.0869
82	4	42	1	3107	31.0669
83	4	50	47	3107	31.0669
84	15	15	17	3102	31.0169
85	15	45	47	3101	31.0069
86	7	49	23	3096	30.9569

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 doi:10.20944/preprints202502.2214.v1

Table 3 – continued from previous page

No.	a	b	c	Number of primes	Percentage
87	18	24	37	3089	30.8869
88	13	27	1	3086	30.8569
89	46	20	47	3082	30.8169
90	11	9	11	3080	30.7969
91	11	31	31	3079	30.7869
92	11	13	13	3075	30.7469
93	11	35	37	3074	30.7369
94	25	17	17	3073	30.7269
95	33	15	19	3068	30.6769
96	32	32	37	3065	30.6469
97	1	19	17	3061	30.6069
98	22	30	19	3061	30.6069
99	1	21	37	3060	30.5969
100	26	26	37	3060	30.5969
1	36	-810	2753	3981	39.81
2	47	-1701	10181	3758	37.58
3	103	-4707	50383	3874	38.74
4	43	-537	2971	3808	38.08
5	8	-488	7243	4048	40.48
6	6	-342	4903	3874	38.74
7	2	0	29	3484	34.84
8	7	-371	4871	3526	35.26
9	3	3	23	3299	32.99
10	3	39	37	3510	35.10
11	1	1	17	2628	26.28
12	4	4	59	3408	34.08
13	2	0	11	2080	20.8

4. Conclusions

Using the Julia programming language, we have carried out a computational experiment and found quadratic polynomials rich in primes in a given range. Some of these polynomials may have already been found by others, so the reader can check them more carefully using the provided Julia code. The results can be updated and extended by increasing the range of x and a, b, c, which will, of course, increase the computational cost.

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