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Dark-Matter and Gravitational-Force Details that Echo Familiar Physics and Explain Cosmic Data

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Dark-Matter and Gravitational-Force Details that Echo Familiar Physics and Explain Cosmic Data

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Abstract

We suggest details about dark matter and gravitational forces. The details explain cosmic data. The data pertain to galaxies, galaxy clusters, cosmic microwave background radiation, the expansion of the universe, and densities of the universe. Possibly, no other work explains the data. Our dark-matter specification reuses a set of known elementary particles. The gravitational-force details extend Newtonian-force concepts, extend a list of two-body gravitational properties of objects, and reuse Lorentz invariance.

Keywords: dark matter; gravity; galaxy formation; rate of expansion of the universe; elementary particles; catalogs of properties of objects

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1. Introduction

This unit provides context for our work, previews aspects of our work, and discusses data that our work seeks to explain.

1.1. Context

This unit discusses two 90-year-old challenges that provide context for our work.

Cosmology includes two 90-year-old challenges. Since the 1920s, people have discussed the extent to which large objects move away from each other [1,2]. Recently, people associate with such separation the term rate of expansion of the universe. People have not fully developed an explanation for the separation. For a mechanism that governs some aspects of the separation, people suggest the term dark energy. Since the 1930s, people have discussed phenomena that might not associate with the term ordinary-matter stuff [3,4]. People use the term dark matter when discussing some possible explanations for the phenomena. People have yet to settle on a preferred description of dark matter [5,6].

1.2. Preview of Our Work

This unit previews results and methods of our work.

Our work might offer insight and partial resolution regarding the two 90-year-old challenges. Our work has been ongoing for a decade [7,8]. Our work suggests the following. Dark matter exists. Dark-matter elementary particles are like ordinary-matter elementary particles. Some dark-matter stuff is like ordinary-matter stuff and comports with popular modeling notions of self-interacting dark matter. Most dark-matter stuff is unlike ordinary-matter stuff and comports with popular modeling notions of collisionless dark matter. A new use of multipole expansion mathematics helps describe two-body gravitation. Our techniques include means to catalog some interaction-properties of objects. Parallels exist between the combination of classical electrodynamics and Lorentz invariance and a combination of an expanded list of two-body gravitational properties of objects and gravitational multipole aspects. Uniting our dark-matter specification and our multipole gravitational expansions requires one new integer-based equation. Our approach offers candidate explanations for data. Our candidate explanations cover enough data for people to possibly find our explanations credible and compelling.

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1.3. Seemingly Otherwise Unexplained Cosmic Data

This unit discusses seemingly otherwise unexplained cosmic data that our work seeks to explain.

A goal for our work is to explain the following observed ratios of not-ordinary-matter effects to ordinary-matter effects.

- 1:0+ – Amounts of stuff in some individual galaxies [9–17].
- 0+:1 – Amounts of stuff in some individual galaxies. (Popular modeling associates the symbol z with redshift. Popular modeling associates redshifts of zero with the present universe. Popular modeling associates larger redshifts with earlier times in the history of the universe. Various online services associate popular modeling notions of time after a supposed so-called Big Bang and redshifts [18]. For example, $z = 7$ associates with approximately 0.76 billion years after the supposed Big Bang and with approximately 13 billion years before now.)
  - Redshifts of more than approximately seven [19,20].
  - Redshifts of approximately six [21].
  - Redshifts of less than six through redshifts of nearly zero [22–29].
- $\sim$ 4:1 – Amounts of stuff in some individual galaxies [30,31].
- 5+:1 – Amounts of stuff in many individual galaxies [9,32].
- 5+:1 – Amounts of stuff in many individual galaxy clusters [32–36].
- 5+:1 – Densities of the universe [37].
- 1:1 or 0:1 – Amounts of some depletion of cosmic microwave background radiation. (Ordinary-matter effects that associate with the depletion of cosmic microwave background radiation via hyperfine transitions in ordinary-matter hydrogen atoms might account for half of the observed depletion or for all the observed depletion.)
  - An observation [38–40] suggests 1:1.
  - Other pieces of research [41–44] suggest 0:1.

Another goal for our work is to explain the notion that the history of the universe includes one multibillion-year era of decreasing so-called rate of expansion of the universe and one subsequent multibillion-year era of increasing rate of expansion of the universe [45].

2. Methods

This unit discusses assumptions that our work makes and methods that our work develops and uses.

The following statements summarize aspects of our methods.

1. Our methods include an assumption that most dark matter associates with new elementary particles and that the new elementary particles are like elementary particles that underlie ordinary matter. Most dark matter associates with popular modeling notions of collisionless dark matter. Some dark matter associates with popular modeling notions of self-interacting dark matter.
2. Our methods include applications of multipole-expansion mathematics that describe electromagnetic fields and gravitational fields that associate with multiple properties, such as electromagnetic charge and electromagnetic magnetic moment or such as gravitational mass and gravitational effects of object-internal motions of the masses of sub-objects, of single objects. This contrasts with popular modeling multipole expansions that associate with spatial distributions of single properties such as charge or mass. Our multipole expansions underlie characterizations of two-body electromagnetic interactions and characterizations of two-body gravitational interactions. We suggest that attention to such two-body gravitational interactions can be key to explaining aspects regarding the formation of galaxies and to explaining eras in the rate of expansion of the universe.
3. Our methods include one new, with respect to popular modeling, integer-based equation. The equation applies once for each electromagnetic or gravitational property of objects. For each such application, one of four arithmetically-possible solutions to the equation pertains. We suggest a set of solutions that enables explaining cosmic data regarding galaxies, galaxy clusters,

cosmic microwave background radiation, the expansion of the universe, and densities of the universe. The breadth of the data that we explain might associate with credibility for our methods and explanations.

2.1. Dark-Matter Specification

This unit discusses the candidate specification for dark matter that our work assumes and uses.

We assume that nature includes dark matter. We assume that all or most not-ordinary-matter effects are effects of dark matter.

We consider the set of all known elementary particles other than the photon. Based on a ratio of five-plus to one for dark-matter density of the universe to ordinary-matter density of the universe, we suggest that nature includes five dark-matter isomers of the set of all known non-photon elementary particles. Here, as in other areas of physics, the word isomer can associate with variations with respect to might-be symmetries. Here, possible symmetries or broken symmetries might associate with the handedness of elementary particles and with the non-alignment between neutrino flavour eigenstates and neutrino mass eigenstates [46]. We suggest that one symmetry associates with the handedness of elementary particles (such as fermion elementary particles) that exhibit handedness. We suggest that one might-be symmetry associates with matches between charged-lepton flavours and charged-lepton masses.

Table 1 discusses a numbering scheme for the suggested six isomers of all known elementary particles except the photon, specifications for the one ordinary-matter isomer and the five dark-matter isomers, and aspects of the stuff that associates with each isomer.

Table 1. A numbering scheme for the suggested six isomers of all known elementary particles except the photon, specifications for the one ordinary-matter isomer and the five dark-matter isomers, and aspects of the stuff that associates with each isomer. The symbol l_{isomer} denotes the isomer number. The symbol $l_{isomer-pair}$ denotes the isomer-pair number. The masses of counterpart elementary particles are, across the isomers, the same. Handedness associates with whether the relevant handed elementary particles are left-handed or right-handed. For each row, the quark generations column assigns the three generation numbers in the order of increasing geometric-mean mass, with the geometric mean associating with the masses for the two quarks that are relevant to the generation. (The following pertain for the ordinary-matter isomer. Generation-1 associates with the up quark and the down quark. Generation-2 associates with the charm quark and the strange quark. Generation-3 associates with the top quark and the bottom quark.) For each row, the lepton flavours column assigns the three flavor numbers in the order of increasing mass for the one charged lepton that is relevant to the flavour. (The following pertain for the ordinary-matter isomer. Flavour-1 associates with the electron. Flavour-2 associates with the muon. Flavour-3 associates with the tau.) The stuff column identifies stuff made from the isomer as OM, as in ordinary matter, or DM, as in dark matter. The acronym SEA abbreviates our term significantly-electromagnetically-active. The stuff that associates with DM (SEA) interacts electromagnetically with itself on a par with OM stuff interacting electromagnetically with OM stuff. The acronym MEA abbreviates our term marginally-electromagnetically-active. The stuff that associates with MEA interacts electromagnetically with itself marginally, perhaps mostly via the magnetic moments of zero-net-charge objects. PMN abbreviates the two-element term popular-modeling notion. We suggest that some dark matter associates with popular modeling notions of self-interacting dark matter. The acronym SIDM abbreviates the popular modeling term self-interacting dark matter. We suggest that some dark matter associates with popular modeling notions of CDM. The acronym CDM abbreviates the popular modeling term collisionless dark matter.

l_{isomer}	$l_{isomer-pair}$	Handedness	Quark generations	Lepton flavours	Stuff	PMN
0	0	Left	1, 2, 3	1, 2, 3	OM (SEA)	OM
3	0	Right	1, 2, 3	1, 2, 3	DM (SEA)	SIDM
1	1	Left	1, 2, 3	3, 1, 2	DM (MEA)	CDM
4	1	Right	1, 2, 3	3, 1, 2	DM (MEA)	CDM
2	2	Left	1, 2, 3	2, 3, 1	DM (MEA)	CDM
5	2	Right	1, 2, 3	2, 3, 1	DM (MEA)	CDM

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We use the one-element term MEA-isomer to designate an isomer other than isomer-0 and isomer-3. We suggest that the fermion flavour-and-mass pairings for the MEA isomers (that is, isomer-1, isomer-2, isomer-4, and isomer-5) led to stuff that associates with the MEA isomers forming stable counterparts to isomer-0-stuff neutrons and to stuff that associates with the MEA isomers not forming significant numbers of counterparts to isomer-0-stuff atoms. The following notions underlie the suggestion that features stable counterparts to isomer-0-stuff neutrons. For each one of the six isomers, a ground-state singly-charged baryon that includes exactly three generation-3 quarks would be more massive than the counterpart, within the same isomer, ground-state zero-charge baryon that includes exactly three generation-3 quarks. For example, for isomer-0, a ground-state nonzero-charge baryon that includes just two tops and one bottom would have a larger mass than would a ground-state zero-charge baryon that includes just one top and two bottoms. Popular modeling suggests that, for isomer-0, W bosons play key roles regarding the decay of generation-3 baryons, such as possible generation-3 baryons to which the previous sentence alludes, into ground-state generation-1 baryons, namely the neutron and the proton [47]. Per Table 1, MEA-isomer flavour-3 charged leptons would be less massive than isomer-0 flavour-3 charged leptons. When generation-3 quark states are much populated, the stuff that associates with an MEA-isomer would convert more charged baryons to zero-charge baryons than would the stuff that associates with isomer-0. Eventually, regarding the stuff that associates with the MEA-isomer, interactions that entangle multiple MEA-isomer W bosons would result in the stuff that associates with the MEA-isomer having more counterparts to isomer-0 neutrons and fewer counterparts to isomer-0 protons than does the stuff that associates with isomer-0. The sum of the mass of an MEA-isomer-counterpart-to-isomer-0 proton and the mass of an MEA-isomer flavour-1 charged lepton would exceed the mass of a counterpart-to-isomer-0 neutron. Compared to isomer-0 neutrons, MEA-isomer neutrons would scarcely decay.

Regarding DM (SEA), we note that some observational results [48–50] suggest that some dark matter might comport with popular modeling notions of self-interacting dark matter [5,51]. Some popular modeling results [52–55] point to possible benefits of considering that some dark matter is self-interacting dark matter.

We note, as an aside, that similarities between isomer-0 and isomer-3 might provide a basis for popular modeling to better, than now, come to terms with the popular modeling notion of matter-antimatter asymmetry (which is also known as baryon asymmetry) [56].

2.2. Two-Body Gravitational Interactions and Gravitational Properties of Objects

This unit discusses gravitational multipole expansions that our work develops and uses. This unit discusses gravitational properties, of objects, that our work suggests and uses.

We assume that gravitational multipole expansions can help explain data about the rate of expansion of the universe [45]. For example, the following two notions might pertain. Quadrupole components of gravity might associate with attraction between objects and with the onset of the multibillion-year era of decreasing rate of expansion of the universe. In this case, the objects might be neighboring galaxies or galaxy clusters. Dipole components of gravity might associate with repulsion between objects and with the onset of the multibillion-year era of increasing rate of expansion of the universe. In this case, the objects might likely be neighboring galaxy clusters.

We anticipate discussing two types of multipole expansions.

- Popular modeling multipole expansions tend to feature spatial distributions of one property. For gravitational expansions, the property is mass. For electromagnetic expansions, the property is charge. We explore some aspects of popular modeling multipole gravitational expansions that feature spatially distributed mass.
- Our work features a (perhaps new) type of multipole expansion that features multiple properties of an object that models as pointlike. For gravitational modeling, the properties include mass and possible gravitational analogs to electromagnetic properties (other than charge), such as magnetic moment, of objects.

The following preview statements pertain regarding our applications of multipole expansions that feature the notions of multiple properties and pointlike objects.

- We anticipate suggesting a means for cataloging two-body electromagnetic properties of objects and two-body gravitational properties of objects. We anticipate suggesting new two-body gravitational properties of objects. We note, as an aside, that Tables 2 and 3 summarize key notions.
- We anticipate the notions that monopole, quadrupole, and hexadecapole aspects of gravitational interactions can associate with attraction and that dipole and octupole aspects of gravitational interactions can associate with repulsion. We note, as an aside, that equation (14), equation (15), and Table 4 summarize key notions.
- We anticipate suggesting that, for some circumstances, gravitational effects that associate with one body repel another body. We note, as an aside, that discussion related to Table 4 summarizes key notions.

We note, as an aside, that people can choose the extents to which to embrace each one of the following two statements.

1. Roles of Table 2, Table 3, equation (14), equation (15), and Table 4 in helping to explain data provide credibility for Table 2, Table 3, equation (14), equation (15), and Table 4.
2. Our discussion that develops Table 2, Table 3, equation (14), equation (15), and Table 4 provides credibility for Table 2, Table 3, equation (14), equation (15), and Table 4.

2.2.1. Perspective Regarding Our Developing Our Notions of Multipole Expansions

This unit establishes bases for perspective about our developing modeling regarding multipole expansions that feature the notions of multiple properties and pointlike objects.

We discuss interactions between an object-A and an object-P. The A in object-A associates with the two-word term active properties. Popular modeling associates active properties with the notion of properties about which fields, such as electromagnetic fields and gravitational fields, convey information. The P in object-P associates with the two-word term passive properties. Popular modeling associates passive properties with interactions, by object-P, with fields that associate with objects, such as object-A, other than object-P.

The following themes associate with this discussion. Expand the list of two-body gravitational properties of objects to include properties other than mass. For each one of some two-body gravitational properties of object-A, discuss the extent to which two-body gravitational interactions between that property of object-A and the mass of object-P associate with attraction (or, pull) of object-P toward object-A or associate with repulsion (or, push) of object-P away from object-A. Point to situations in which the total (across two-body gravitational properties of object-A) gravitational push on object-P can exceed the total (across two-body gravitational properties of object-A) gravitational pull on object-P.

Throughout this discussion, the symbol F_{oP} associates with the popular modeling notion of the rate of change, with respect to time, of the momentum of object-P. F_{oP} is a 3-vector.

Throughout our discussion of gravity and electromagnetism, we assume that the gravitational mass of object-P equals the inertial mass of object-P. Regarding gravity, we generally assume that the only adequately relevant property of object-P is mass. Regarding electromagnetism, we generally assume that the only adequately relevant properties of object-P are (the electromagnetic property of) charge and (the inertial property of) mass. We generally de-emphasize discussing the notion that object-P can experience a torque based on its interactions with object-A.

For much of our discussion of gravity and electromagnetism, we de-emphasize the notion that each one of object-A and object-P might change, for example via radiation, its internal state.

Throughout our discussion of gravity and electromagnetism, we de-emphasize the notion that object-A and object-P might collide with each other.

Throughout our discussion of gravity and electromagnetism, we exclude from the list of possible objects-A and from the list of possible objects-P popular modeling zero-mass objects (such as photons and gluons). Throughout our discussion of gravity and electromagnetism, we exclude from the list of possible objects-A and from the list of possible objects-P popular modeling objects (such as quarks and gluons) that popular modeling models as not existing individually.

Our work might have some parallels to work that popular modeling associates with the word gravitoelectromagnetism [57–60].

2.2.2. Two-Body Gravity and Seventeenth Century Modeling

This unit reviews aspects of Newtonian gravity.

Equations (1), (2), and (3) describe aspects regarding the motion of object-P [61]. G is the gravitational constant. m_{oA} is the mass of object-A. Mass is a scalar property. m_{oP} is the mass of object-P. r is the 3-vector distance that object-P is away from object-A. $\bigtriangledown$ is the gradient operator. $\bigtriangledown$ produces a 3-vector field from a scalar field. F_{oP} is the force that object-P feels. Object-P might sense effects of that force via an accelerometer that associates with object-P. In equations such as equation (1), V is a scalar field. Popular modeling associates with V the word potential. In equations such as equation (2), r^{n_r} denotes the n_r-th power of the magnitude of the 3-vector r.

$$Gm_{oA}m_{oP}(-\nabla V) = F_{oP} \tag{1}$$

$$V = -1/r^{n_r} \tag{2}$$

$$n_r = 1 \tag{3}$$

2.2.3. Two-Body Gravity and Aspects of Popular Modeling Multipole Expansions

This unit indicates that, regarding two-body gravitational interactions, popular modeling can point toward possibilities for dipole repulsion between two objects.

We discuss a thought experiment.

We assume that object-A consists of two equal-mass sub-objects. We assume that none of object-A and either of the two sub-objects moves relative to object-P. Equations (1), (2), and (3) pertain.

Via the notion of equal mass, the mass $m_{so1,oA}$ of one sub-object (sub-object-1) equals the mass $m_{so2,oA}$ of the other sub-object (sub-object-2). Also, $m_{oA} = m_{so1,oA} + m_{so2,oA}$ pertains.

We use the symbol $r_{so1,oA}$ to denote the distance 3-vector from the center of mass of object-A to sub-object-1. We use the symbol $r_{so2,oA}$ to denote the distance 3-vector from the center of mass of object-A to sub-object-2. $r_{so2,oA} = -r_{so1,oA}$ pertains.

We use the symbol $r_{oA,rel.oP}$ to denote the distance 3-vector from object-A to object-P.

Equation (4) provides dimensionless ratios of lengths. Here, in a subscript, K can be either 1 or 2.

$$\delta_{soK} = |r_{soK,oA}|/|r_{oA,rel.oP}| \tag{4}$$

Equation (5) defines the symbol δ_{so}.

$$\delta_{so} \equiv \delta_{so1} = \delta_{so2} \tag{5}$$

We assume that $0 < \delta_{so} \ll 1$.

Per equations (1), (2), and (3), the magnitude of a gravitational force scales, regarding distance, as the distance to the minus two power.

We discuss two cases.

For the first case, one of the two $r_{soK,oA}$ is parallel to $r_{oA,rel.oP}$ and the other one of the two $r_{soK,oA}$ is antiparallel to $r_{oA,rel.oP}$. Equation (6) compares the magnitude of the force, that affects object-P, calculated based on dipole-related assumptions, to the magnitude of the force, that affects object-P, calculated based on monopole-related assumptions.

$$|F_{oP,dipole}|/|F_{oP,monopole}| \approx (1/2)\cdot(1/(1-\delta_{so})^2) + (1/2)\cdot(1/(1+\delta_{so})^2) \approx 1 + (\delta_{so})^2 > 1 \tag{6}$$

Equation (6) provides an example of the notion that gravitational dipole effects, based on a non-pointlike distribution of mass, can augment gravitational monopole effects. One might say that, compared to monopole gravitational attraction (or, pull), dipole effects associate with additional gravitational attraction (or, pull).

For the second case, we assume that each one of $r_{so1,oA}$ and $r_{so2,oA}$ is perpendicular to $r_{oA,rel.oP}$.

Compared to the (monopole-related) magnitude of force that associates with m_{oA}, the (dipole-related) magnitude of force that associates with each sub-object-K associates (via the Pythagorean theorem) with $(1/2) \cdot m_{oA} \cdot (1/(1+(\delta_{soK})^2))$. The sub-object components of force that contribute to perceived object-A pull on object-P are further diluted because the force components that are perpendicular to $r_{oA,rel.oP}$ cancel each other. Equation (7) compares the magnitude of the force, that affects object-P, calculated based on dipole-related assumptions, to the magnitude of the force, that affects object-P, calculated based on monopole-related assumptions. In equation (7), the factor $1-(1/2)(\delta_{so})^2$ approximates, for small δ_{so}, the cosine of the angle between each $r_{soK,oA}$ and $r_{oA,rel.oP}$ (or, the cosine of the angle for which the arctangent is δ_{so}).

$$|F_{oP,dipole}|/|F_{oP,monopole}| \approx (1/(1+(\delta_{so})^2)) \cdot (1-(1/2)\cdot(\delta_{so})^2) \approx 1-(3/2)\cdot(\delta_{so})^2 < 1 \quad (7)$$

Equation (7) provides an example of the notion that gravitational dipole effects, based on a non-pointlike distribution of mass, can dilute gravitational monopole effects. One might say that, compared to monopole gravitational pull, dipole effects associate with gravitational push.

We note, as an aside, that the notion that dipole effects can associate with either gravitational push or gravitational pull might associate with a notion that, for applications of general relativity, effects of pressure can detract from or augment effects of energy density. We anticipate suggesting (below) possible associations between monopole two-body gravitational aspects and energy density and between dipole two-body gravitational effects and pressure.

Equation (7) might suggest that, for cosmological modeling based on distributions of mass, gravitational dipole push can be relevant. However, the significance of such push and other related notions might not be adequate to explain data regarding the rate of expansion of the universe. (We note, as an aside, that equation (7) does not consider popular modeling notions of non-rest-mass energy that might associate with a possible lack of gravitational collapse within object-A.)

Popular modeling for electromagnetism can feature properties, of objects, other than charge. We anticipate learning from popular modeling for electromagnetism and then developing modeling, that includes notions of gravitational properties other than mass, regarding gravitation.

2.2.4. Two-Body Electromagnetism and Eighteenth Century Modeling

This unit reviews aspects of eighteenth century two-body electromagnetism.

Equation (8) is an aspect of eighteenth century two-body electromagnetism [62,63]. ε_0 denotes the vacuum electric permittivity. q_{oA} is the charge of object-A. Charge is a scalar property. q_{oP} is the charge of object-P. Equations (2), and (3) pertain.

$$-(1/(4\pi\varepsilon_0))q_{oA}q_{oP}(-\nabla(V)) = F_{oP} \quad (8)$$

Equation (8) has similarities to equation (1).

2.2.5. Two-Body Electromagnetism and Nineteenth Century Modeling

This unit reviews aspects of nineteenth century two-body electromagnetism.

Compared to equation (8), popular modeling added two two-body electromagnetic properties, magnetic moment and charge current. Popular modeling supplanted the scalar field V with a 4-vector potential (ϕ, A). ϕ is a scalar field. A is a 3-vector field. Regarding popular modeling that accurately features only the property of charge and only equations (2), (3), and (8), $V = \phi$.

Compared to equation (8), popular modeling also added the notion that the perceived values of the properties of an object can vary based on a choice of observer that perceives the values. Popular modeling uses (as an adjective) the word rest to denote values that pertain when the object and the observer do not move relative to each other. The two-word term rest charge provides an example.

Popular modeling provides equation (9) as an observer-invariant substitute for the $\cdots = F_{oP}$ force equation (8).

$$q_{oP}(E_{oA} + (v_{oP} \times B_{oA})) = F_{oP} \tag{9}$$

The following notions pertain. E_{oA} denotes the electric field that the observer associates with contributions, to the electromagnetic field, that associate with object-A. v_{oP} denotes the velocity of object-P in the frame of reference that associates with the observer. B_{oA} denotes the magnetic field that the observer associates with contributions, to the electromagnetic field, that associate with object-A. Each one of E_{oA}, v_{oP}, and B_{oA} is a 3-vector.

Popular modeling provides equations (10) and (11). $\partial .../\partial t$ denotes a partial derivative with respect to the temporal coordinate t, which associates with the frame of reference that associates with the observer. ϕ_{oA} denotes the electromagnetic scalar potential that the observer associates with contributions to the electromagnetic field that associate with object-A. A_{oA} denotes the electromagnetic 3-vector vector potential that the observer associates with contributions to the electromagnetic field that associates with object-A. In popular modeling, ϕ_{oA} and A_{oA} combine to form a Lorentz-invariant 4-vector.

$$E_{oA} = -\nabla\phi_{oA} - \partial A_{oA}/\partial t \tag{10}$$

$$B_{oA} = \nabla \times A_{oA} \tag{11}$$

Equations (9), (10), and (11) are invariant with respect to a choice of a frame of reference. Values for a variable in an equation can vary, based on the choice of a frame of reference.

Popular modeling suggests that the following equations pertain. I_{oA} is the charge current that the observer associates with object-A. I_{oA} is a 3-vector. $A_{oA,I}$ is a 3-vector. $A_{oA,I}$ contributes to A_{oA}.

$$\phi_{oA} \propto q_{oA}/r^{n_r}, \text{ with } n_r = 1 \tag{12}$$

$$A_{oA,I} \propto I_{oA}/r^{n_r}, \text{ with } n_r = 1 \tag{13}$$

In popular modeling, q_{oA} and I_{oA} combine to form a Lorentz-invariant 4-vector. Based on Lorentz invariance [64], the perceived values of some object-A properties, including charge, can vary based on a choice of an observer.

We note that distances r may pertain to present times for object-P and earlier times for object-A. The popular modeling notion of retarded time would pertain. We suggest that, for the purposes of this paper, time delays regarding the propagation of information about object-A are not necessarily adequately important to warrant careful attention to the delays.

2.2.6. Some Suggestions Regarding Cataloging Some Electromagnetic Properties of Objects

This unit suggests a way to catalog some electromagnetic properties of objects.

We propose a way to catalog two-body electromagnetic properties of objects. The way features two integers. One integer is n_r, as in equations (2) and (3). The other integer is n_3. n_3 denotes a number of so-called threesomes that appear directly or indirectly in a term that appears in the left-hand sides of $\cdots = F_{oP}$ equations such as equation (1) and equation (8). A threesome might be a 3-vector.

Table 2 lists some two-body electromagnetic properties that an observer might associate with an object.

Table 2. Some two-body electromagnetic properties that an observer might associate with an object. In each of the $n_3 = n_r$ column and the $n_3 = n_r + 1$ column, the table lists a name for a property of the object. Popular modeling suggests the names that do not appear in parentheses. We suggest the properties and the names for the items that appear in parentheses. Throughout this table, n_3 counts sets of three parameters. Regarding charge, one set of three independent parameters associates with position. Regarding magnetic moment, one set of three independent parameters associates with position. Another set of three independent parameters associates with the three components of the magnetic moment 3-vector. The Earth exhibits nonzero self-precessing magnetic moment. For the Earth, the axis that associates with rotation does not equal the axis that associates with the magnetic moment. Self-precessing magnetic moment differs from Larmor precession, which associates with interactions between an object and magnetic fields that associate with bodies other than the object. This table omits referring directly to some properties, such as an $n_3 = n_r = 2$ property of electric dipole moment. Each $n_3 = n_r + 1$ property associates with adding, compared to the counterpart $n_3 = n_r$ property, one set of three independent parameters that associate with the linear velocity, of the object, as perceived by the observer. Because of the notion that the object is an object, one position is common to all properties. One velocity is common to all properties. For at least $n_r = 1$ and $n_r = 2$, popular modeling suggests that, for each nonzero-valued property, there is a contribution (to an overall potential) that has radial characteristics that associate with $(\phi, A) \propto -r^{-n_r}$. (ϕ, A) denotes a ϕ-and-A 4-vector. r denotes the distance from the object. For a property for which $n_3 > 1$, non-constant angular-dependence pertains regarding that property's contribution to an overall (ϕ, A). Uses of the words monopole, dipole, and so forth associate with values of n_r. For $n_r = 1$ and $n_r = 2$, uses of the words monopole and dipole echo popular modeling.

n_r	$n_3 = n_r$	$n_3 = n_r + 1$	**Potential**
1	Charge	Charge current	Monopole
2	Magnetic moment	(Magnetic-moment current)	Dipole
3	(Self-precessing magnetic moment)	(Self-precessing-magnetic-moment current)	Quadrupole

Popular modeling includes the notion that the magnetic moment of an object can associate with the motions of nonzero-charge sub-objects of the object. Each sub-object can associate with three 3-vectors. One 3-vector is the position of the center of charge of the object. One 3-vector is the position of the sub-object relative to the center of charge of the object. One 3-vector is the linear velocity of the charge of the sub-object. The contribution of the sub-object to the magnetic moment of the object associates with the cross product of the relative position vector and the linear velocity (or, a charge-current-like velocity) vector. Regarding the contribution of the sub-object to the magnetic moment of the object, $n_3 = 2$ associates with two threesomes. One threesome associates with the position of the object. One threesome associates with the contribution of the sub-object to the magnetic moment of the object. In terms of dimensions, the relative position vector associates with one of the two r^{-1} that associate with $n_r = 2$ and associates with the applicability of $n_r = 2$, as opposed to the charge-related $n_r = 1$. We note, as an aside, that aspects relating to magnetic-moment current (or, $n_3 = n_r + 1 = 3$) associate with, in effect, undoing otherwise possible miscounting (such as double counting), regarding aspects related to the motions of sub-object charges, by aspects related to charge current (or, $n_3 = n_r + 1 = 2$) and aspects related to magnetic moment (or, $n_3 = n_r = 2$).

We note, as an aside, that we do not explore popular modeling notions that there might be more than one popular modeling definition of magnetic moment and that the definitions of magnetic moment might not be equivalent regarding Lorentz-invariant transformations [65].

Popular modeling includes, for objects that model as having charged sub-objects, the notion of a charge dipole moment. Each sub-object can associate with two threesomes. One threesome is the 3-vector position of the center of charge of the object. One threesome is the 3-vector position of the sub-object relative to the center of charge of the object. $n_3 = 2$ pertains for the contribution of the sub-object to the charge dipole moment for the object. For the object, the charge dipole moment associates with $n_3 = 2$. In terms of dimensions, the relative position vector associates with one of the two r^{-1} that associate with $n_r = 2$ and associates with the applicability of $n_r = 2$, as opposed to the charge-related $n_r = 1$.

Popular modeling regarding an object does not necessarily consider structure-related energies that might pertain within the object. Popular modeling regarding sub-objects does not necessarily consider structure-related energies that might pertain within the relevant object.

2.2.7. Some Suggestions Regarding Cataloging Some Gravitational Properties of Objects

This unit suggests and catalogs some two-body gravitational properties, of objects, that might extend aspects of popular modeling.

We assume that, for the purposes of our work, the property of mass provides a gravitational analog to the electromagnetic property of charge.

Table 3 lists some two-body gravitational properties that an observer might associate with an object.

Table 3. Some two-body gravitational properties that an observer might associate with an object. Throughout this table, n_3 counts sets of three parameters. Regarding $n_3 = n_r = 1$ (or, mass), one set of three independent parameters associates with the position of the object. Regarding $n_3 = n_r = 2$ and sub-object-mass motions, for each sub-object, one set of three independent parameters associates with the position of the object. Another set of three independent parameters associates with the three components of the 3-vector that is the sub-object-mass-weighted sum, across sub-objects, of the cross product of the distance (of the sub-object from the center of mass of the object) 3-vector and the velocity (of the sub-object with respect to the center of mass of the object) 3-vector. Angular momentum can be either a special case of $n_3 = n_r = 2$ sub-object-mass motions or a sub-case of $n_3 = n_r = 2$ sub-object-mass motions. Regarding $n_3 = n_r = 3$ and moments of inertia, one set of three independent parameters associates with the position of the object. Another set of three independent parameters associates with the three on-diagonal components of the 3-by-3 symmetric moments-of-inertia tensor. Another set of three independent parameters associates with three off-diagonal components of the 3-by-3 symmetric moments-of-inertia tensor. The $n_3 = n_r = 4$ moments-of-inertia rotation property adds, compared to the case of moments of inertia, one set of three parameters that associate with the angular velocity 3-vector that associates with the rotation of the object. Each $n_3 = n_r + 1$ property associates with adding, compared to the counterpart $n_3 = n_r$ property, one set of three independent parameters that associate with linear velocity of the object. Because of the notion that the object is an object, one position for the object is common to all properties. One velocity of the object is common to all properties. Regarding at least $n_3 = n_r = 3$ and $n_3 = n_r = 4$, depending on the situation, properties other than the property that this table names might be at least as important as the property that this table names. For example, this table omits referring directly to some properties, such as an $n_3 = n_r = 3$ self-precessing angular momentum gravitational analog to the electromagnetic property of self-precessing magnetic moment. TBD abbreviates the three-word phrase to be determined. Uses of the words monopole, dipole, and so forth associate with values of n_r and can echo popular modeling uses of the words.

n_r	$n_3 = n_r$	$n_3 = n_r + 1$	Potential
1	Mass.	$n_3 = n_r = 1$ current	Monopole
2	Angular momentum. Other sub-object-mass motions.	$n_3 = n_r = 2$ current	Dipole
3	Moments of inertia.	$n_3 = n_r = 3$ current	Quadrupole
4	Moments-of-inertia rotation.	$n_3 = n_r = 4$ current	Octupole
5	TBD.	$n_3 = n_r = 5$ current	Hexadecapole

We note, as an aside, that an n_3 that exceeds by two the counterpart n_r might associate with a nonzero acceleration and with the notion that, from the perspective of object-P, object-A would model as part of a system, of objects, for which the system does not necessarily include object-P.

2.2.8. Two-Body Gravity and Suggested Twenty-First Century Modeling

This unit discusses aspects for suggested twenty-first century two-body gravitation.

We suggest that Table 3 associates with a new or extended type of gravitational multipole expansion. In each of popular modeling multipole expansions and our multipole expansions, an expansion features a series of terms. Each term contributes to a notion of an overall spatial potential. In popular modeling, a multipole expansion tends to have a basis in a spatial distribution of one property

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such as charge or mass. Our gravitational multipole expansions feature one object that can model as pointlike and as associating with some popular modeling spatial multipole distribution aspects and as associating with other aspects such as aspects related to the motions of sub-objects.

Each one of many of the gravitational $n_3 = n_r$ properties and each one of many of the gravitational $n_3 = n_r + 1$ properties might associate with a positive contribution toward an overall energy that an object-P might associate with an object-A. For example, object-P might associate perceived object-A $n_3 = n_r + 1 = 2$ (or, $n_3 = n_r = 1$ current) as associating with an object-A kinetic energy. Or, some $n_3 = n_r = 2$ sub-object-mass motions might associate with an energy that associates with adding angular momentum to an otherwise not-spinning object-A. Or, other $n_3 = n_r = 2$ sub-object-mass motions might associate with an energy that associates with heating object-A and, thereby, increasing the speeds of sub-objects of object-A.

$\cdots = F_{oP}$ equations associate directly with changes in object-P momentum. Items in the left-hand sides of gravitational $\cdots = F_{oP}$ equations might work, in the sense of pull of object-P toward object-A or push of object-P away from object-A, against each other.

We discuss two examples. For each example, we assume that the only significantly relevant property of object-P is mass.

For one example, the only relevant properties of object-A associate with $n_3 = n_r = 1$ and $n_3 = n_r + 1 = 2$. If the velocity of object-A is the same as the velocity of object-P, object-P associates object-A gravitational $n_3 = n_r = 1$ with object-A rest mass and object-P associates object-A gravitational $n_3 = n_r + 1 = 2$ with a value of zero. If the velocity of object-A is not the same as the velocity of object-P, object-P associates object-A gravitational $n_3 = n_r = 1$ with more mass than the object-A rest mass and object-P associates object-A gravitational $n_3 = n_r + 1 = 2$ with a nonzero value. The strength of object-A gravitational field at the location of object-P does not change based on the relative velocity. For nonzero relative velocity compared to zero relative velocity, we suggest that object-P associates the perceived value of object-A $n_3 = n_r = 1$ pull on object-P to be larger than for zero relative velocity. (The amount of the increase associates with the magnitude of the relative velocity. The amount of the increase does not vary based on the direction of the relative velocity.) For nonzero relative velocity compared to zero relative velocity, we suggest that object-P associates the perceived value object-A gravitational $n_3 = n_r + 1 = 2$ with a push that, in effect, restores overall interaction to one that associates with object-A rest mass. This example exemplifies the notion that, for interactions with object-P mass, perceived object-A gravitational $n_3 = 2$ push detracts from perceived object-A gravitational $n_3 = 1$ pull.

For the second example, the only relevant properties of object-A associate with $n_3 = n_r = 1$ and $n_3 = n_r = 2$. Also, the only relevant aspect related to $n_3 = n_r = 2$ is the motion of sub-objects of object-A. (For example, the dipole component of the distribution of mass is either not relevant or is zero.) The velocity of object-A is the same as the velocity of object-P. If the velocity of each sub-object of object-A is the same as the velocity of object-A, object-P associates object-A gravitational $n_3 = n_r = 1$ with object-A rest mass and object-P associates object-A gravitational $n_3 = n_r = 2$ effects with a value of zero. If the velocity of (at least) one sub-object of object-A is not the same as the velocity of object-A (and, thus, is not the same as the velocity of object-P), object-P associates object-A gravitational $n_3 = n_r = 1$ with more mass than object-A rest mass and object-P associates object-A gravitational $n_3 = n_r = 2$ with a nonzero value. For nonzero relative velocity (for at least one sub-object of object-A, relative to object-P) compared to zero relative velocity (for all sub-objects of object-A, relative to object-P), we suggest that object-P associates the perceived value of object-A $n_3 = n_r = 1$ pull on object-P to be larger. For nonzero relative velocity (for at least one sub-object of object-A, relative to object-P) compared to zero relative velocity (for all sub-objects of object-A, relative to object-P), we suggest that object-P associates the perceived value object-A gravitational $n_3 = n_r = 2$ with push. We suggest considering, based on the notion that one can consider that (object-A internal) angular momentum to be a gravitational analog to magnetic moment, the following two cases. For the first case, gravitational $n_3 = n_r = 2$ associates precisely with object-A angular momentum. For

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this case we suggest that gravitational $n_3 = n_r = 2$ push exactly balances the excess (above the pull that associates with object-A rest mass) pull that object-P senses regarding object-A. For the second case, gravitational $n_3 = n_r = 2$ associates with object-A angular momentum (which might be zero) and other effects. For this case we suggest that gravitational $n_3 = n_r = 2$ push exactly balances the excess (above object-A rest mass) mass pull that object-P senses regarding object-A angular momentum. For this case, we suggest that the other (than angular momentum) gravitational $n_3 = n_r = 2$ effects associate with an $n_3 = n_r = 2$ push for which there is no balancing pull. This example exemplifies the notion that, for interactions with object-P mass, perceived object-A gravitational $n_3 = 2$ push detracts from perceived object-A gravitational $n_3 = 1$ pull.

We note, as an aside, that discussion below related to equation (17) notes that, for applications of general relativity, pressure, which might associate with $n_3 = n_r = 2$, can work, in the sense of pull or push, against the pull that associates with energy density, which might associate with $n_3 = n_r = 1$.

Popular modeling suggests that the object-property of mass is always nonnegative. In the context of two-body interactions, the contribution that associates with the mass of object-A interacting with the mass of object-P associates with a pull component of force. Pull associates with notions of attraction of object-P toward object-A. More generally, we suggest that equations (14) and (15) pertain for two-body gravitation. $n_{3,oA}$ denotes an n_3 for a property of object-A. $n_{3,oP}$ denotes an n_3 for a property of object-P. Push associates with notions of repulsion of object-P away from object-A.

$$(n_{3,oA} + n_{3,oP}) \text{ is even } \leftrightarrow \text{ pull} \tag{14}$$

$$(n_{3,oA} + n_{3,oP}) \text{ is odd } \leftrightarrow \text{ push} \tag{15}$$

Table 4 lists some contributions, by an object-A, to pull-or-push aspects of gravitational forces, as perceived by an object-P for which the only relevant property is mass. Table 4 extends Table 3.

Table 4. Some contributions, by an object-A, to pull-or-push aspects of gravitational forces, as perceived by an object-P for which the only relevant property is mass. Pull associates with notions of attraction of object-P toward object-A. Push associates with notions of repulsion of object-P away from object-A. RSD abbreviates the three-word term radial spatial dependence, as in the radial spatial dependence of the relevant component of force. The values of the exponents for RSD items echo the popular modeling notion that a force can associate with the gradient of a potential. For each row, the RSD has the form $r^{-n_{RSD}}$, in which $n_{RSD} = n_{r,oA} + n_{r,oP}$ with $n_{r,oA}$ denoting the n_r for the object-A property and $n_{r,oP}$ denoting the n_r for the object-P property. For items this table lists, $n_{r,oP} = 1$. Type associates with relevant $n_3 = n_r$ properties. Notions of monopole, dipole, and so forth echo popular modeling use of such terms. Regarding at least $n_3 = n_r = 3$ and $n_3 = n_r = 4$, depending on the situation, properties other than the property that this table names might be at least as important as the property that this table names. For example, this table omits referring directly to some properties, such as an $n_3 = n_r = 3$ self-precessing angular momentum gravitational analog to the electromagnetic property of self-precessing magnetic moment.

Object-A property	Force	RSD	Type
$n_3 = n_r = 1$; Mass.	Pull	r^{-2}	Monopole
$n_3 = n_r + 1 = 2$; $n_3 = n_r = 1$ current.	Push	r^{-2}	Monopole
$n_3 = n_r = 2$; Angular momentum. Other sub-object-mass motions.	Push	r^{-3}	Dipole
$n_3 = n_r + 1 = 3$; $n_3 = n_r = 2$ current.	Pull	r^{-3}	Dipole
$n_3 = n_r = 3$; Moments of inertia.	Pull	r^{-4}	Quadrupole
$n_3 = n_r + 1 = 4$; $n_3 = n_r = 3$ current.	Push	r^{-4}	Quadrupole
$n_3 = n_r = 4$; Moments-of-inertia rotation.	Push	r^{-5}	Octupole
$n_3 = n_r + 1 = 5$; $n_3 = n_r = 4$ current.	Pull	r^{-5}	Octupole

For a pair of Table 4 rows that associate with the same RSD, we suggest that the pull (or push) that associates with an object-A $n_3 = n_r$ property dominates the push (or, respectively, pull) that associates with the counterpart object-A $n_3 = n_r + 1$ property.

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For a pair of Table 4 rows that associate with two different object-A $n_3 = n_r$ such that one row associates with pull and the other row associates with push, we suggest that dominance with respect to pull or push can depend on the magnitude $|r_{oA,rel.oP}|$ of the 3-vector distance $r_{oA,rel.oP}$ between object-A and object-P. For example, consider the object-A properties of $n_3 = n_r = 1$ mass and $n_3 = n_r = 2$ other sub-object-mass motions. For adequately large values of $|r_{oA,rel.oP}|$, pull dominates. For lesser values of $|r_{oA,rel.oP}|$, push can dominate. We note, as an aside, that for yet lesser values of $|r_{oA,rel.oP}|$, the notion that the objects are not colliding might not pertain.

We note, as an aside, that the object-A $n_3 = n_r = 2$ property that associates with the motions of sub-objects of object-A might be large compared to object-A angular momentum. For example, for a galaxy cluster, the object-A $n_3 = n_r = 2$ property can include contributions that associate with (thermal or other) motions of individual IGM (or, intergalactic medium) atoms or ions and contributions that associate with the motions of components of individual galaxies.

We note the following statements. The notion of energies that might be necessary to keep object-A structurally intact is not necessarily relevant. While notions that popular modeling might associate with potentials (or with potential energies) appear in discussions above, notions that popular modeling might associate with kinetic energy do not necessarily appear explicitly in discussions above. From the perspective of object-P, object-P perceived masses add across sub-objects of object-A. For one value of n_r, $n_3 = n_r + 1$ effects cannot dominate the object-P push or pull sense that associates with $n_3 = n_r$ effects. For one value of n_g, $n_3 = n_r = n_g + 1$ effects can dominate the object-P push or pull sense that associates with $n_3 = n_r = n_g$ effects if object-A and object-P are adequately close to each other and are not colliding with each other.

2.3. Cosmic Clumping of Stuff

This unit suggests that the formation and evolution of many smaller cosmic objects occurred earlier and more quickly than did the formation and evolution of many larger cosmic objects.

We assume, for discussion purposes, that, at some time in the evolution of the universe, stuff (ordinary matter and dark matter) had a nearly uniform spatial distribution. We assume that, at that time, bunches of stuff were moving away from each other. We de-emphasize notions related to properties that associate with the word current.

We consider gravitational $n_3 = n_r$ notions. We consider two spatially non-overlapping, similarly-sized spherical regions of stuff. The amount of stuff in each region scales as the cube of the radius of the region. The monopole contribution to the gravitational force that one region exerts on the other region scales inversely as the square of the distance between the centers of the two regions. If one scales the two radii and the one distance similarly, the monopole aspect of gravitational force scales as $r^3r^3r^{-2} = r^4$. Dipole aspects scale as $r^3r^3r^{-3} = r^3$. Quadrupole aspects scale as $r^3r^3r^{-4} = r^2$. Octupole aspects scale as $r^3r^3r^{-5} = r^1$. Hexadecapole aspects scale as $r^3r^3r^{-6} = r^0$.

We suggest notions that might associate with observed and possible clumping, at various scale sizes, in the history of the universe. Hexadecapole attraction might be scale-invariant and might not necessarily be adequately relevant for this discussion of clumping. Across a range of sizes, octupole repulsion was dominant at some time. Many smaller similar-sized clumps transited a sequence, of octupole repulsion to quadrupole attraction and onward, faster than did many larger similar-sized clumps. Many pairs of neighboring solar-system-sized clumps transited to dominance by monopole attraction before many pairs of neighboring galaxy-sized clumps transited to dominance by monopole attraction. Many pairs of neighboring galaxy-sized clumps transited to dominance by monopole attraction before many pairs of neighboring galaxy-cluster-sized clumps did or might transit to dominance by monopole attraction.

2.4. Instances of Properties of Objects, Plus Reaches per Instance of Contributions to Interactions Between Objects

This unit introduces a notion of instances of electromagnetic properties and of two-body gravitational properties and a notion of reach per instance. This unit suggests, for some key two-body

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gravitational properties of objects and some key electromagnetic properties of objects, instances and reaches per instance. This unit suggests that numeric values of instances and reaches per instance can be key to explaining some cosmic data.

Popular modeling suggests that ordinary-matter stuff scarcely, if at all, sees dark-matter stuff. We suggest that nature includes six instances of the electromagnetic property of charge. We suggest that each isomer associates with its own instance of charge. We say that, for each one of the six instances of charge, the reach per instance is one isomer. Similarly, we suggest (based on the notion that ordinary matter does not see dark matter stars) that each isomer associates with its own instance of the electromagnetic property of blackbody temperature and that the reach per instance for blackbody temperature is one isomer.

Popular modeling suggests that each nonzero-mass object can interact gravitationally with all other nonzero-mass objects. We say that nature includes one instance of the gravitational property of mass. We say that the reach per instance for the one instance of the property of mass is six isomers.

We use the symbol n_{in} to denote the number of instances of a property. We use the symbol $R_{/in}$ to denote the reach of an instance of the property. The reach is a number of isomers. Each one of n_{in} and $R_{/in}$ is a positive integer.

We suggest that, for each electromagnetic property and for each gravitational property, equation (16) pertains.

$$n_{in} \cdot R_{/in} = 6 \tag{16}$$

Equation (16) pertains regarding properties, such as properties that Table 3 lists, of individual objects.

For a solution for which $n_{in} = 3$ and $R_{/in} = 2$, we assume that each one of the three instances associates with an isomer-pair and that the reach of two isomers associates with the two isomers that associate with the isomer-pair. We assume that the $n_{in} = 2$ and $R_{/in} = 3$ solution is not relevant for our work.

Table 5 suggests instances and reaches per instance for some two-body gravitational properties of objects and for some electromagnetic properties of objects.

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Table 5. Suggested instances and reaches per instance for some two-body gravitational properties of objects and for some electromagnetic properties of objects. Each item in the property or application column is a property that associates with single objects or is an application for large-object gravitational interaction properties that associate with at least some of the formation of many solar systems, the formation of many galaxies, the interactions between the two galaxy clusters within many pairs of neighboring non-colliding galaxy clusters, and (possibly) the interactions between the two galaxies within many pairs of neighboring non-colliding galaxies. The properties that this table lists do not include $n_3 = n_r + 1$ properties, such as mass current or charge current. NNR denotes that a value is not necessarily relevant for this study. Properties that associate with NNR do not associate with the notion of no change of the internal state of the relevant object. G2BFC denotes gravitational two-body force component. NR denotes that the item is not relevant because the property is not a gravitational property. n_{in} denotes the number of instances of the property. $R_{/in}$ denotes the interaction reach, in number of isomers, per instance. The G2BFC information, gravitational instances and gravitational reaches per instance pertain for interactions with the mass of a second object. TBD denotes to be determined. We suggest that the reach per instance for magnetic moment might be one. The choice of a reach per instance of one isomer for blackbody temperature associates with the notion that people do not observe thermal radiation from stars other than ordinary-matter stars. If the 0:1 ratio pertains regarding observations of some depletion of cosmic microwave background radiation, the reach per instance regarding hyperfine interactiveness is one. If the 1:1 ratio pertains regarding observations of some depletion of cosmic microwave background radiation, we suggest that a practical reach per instance regarding hyperfine interactiveness is two. (Because we suggest that isomer-1, isomer-2, isomer-4, and isomer-5 do not underlie many hydrogen-like atoms, we note that a theoretical reach per instance regarding hyperfine interactiveness for the 1:1 case might be six instead of two.)

Type of property	Property or application	$n_3 = n_r$	G2BFC	n_{in}	$R_{/in}$
Gravitational	Mass	1	Pull	1	6
Gravitational	Large-object interactions	2	Push	3	2
Gravitational	Large-object interactions	3	Pull	6	1
Electromagnetic	Charge	1	NR	6	1
Electromagnetic	Magnetic moment	2	NR	TBD (6)	TBD (1)
Electromagnetic	Blackbody temperature	NNR	NR	6	1
Electromagnetic	Hyperfine interactiveness	NNR	NR	TBD (6, 3, or 1)	TBD (1, 2, or 6)

For a component, such as a component that Table 4 lists, of a force that associates with a two-body interaction, we suggest that one needs to calculate the n_{in} and $R_{/in}$ based on the relevant $n_{in,property.of.oA}$, $R_{/in,property.of.oA}$, $n_{in,property.of.oP}$, and $R_{/in,property.of.oP}$. We suggest the following notions. If at least one of $R_{/in,property.of.oA}$ and $R_{/in,property.of.oP}$ is one, $n_{in} = 6$ and $R_{/in} = 1$. If one of $R_{/in,property.of.oA}$ and $R_{/in,property.of.oP}$ is two and the other one of $R_{/in,property.of.oA}$ and $R_{/in,property.of.oP}$ is at least two, $n_{in} = 3$ and $R_{/in} = 2$. For this case, each one of the three instances of $R_{/in} = 2$ associates with an isomer-pair. If each one of $R_{/in,property.of.oA}$ and $R_{/in,property.of.oP}$ is six, $n_{in} = 1$ and $R_{/in} = 6$.

We suggest that the instances and reaches per instance that Table 5 features seem to be compatible with all the data that this study seeks to explain.

3. Results

This unit suggests that Table 5 numeric values of instances and reaches per instance underlie steps forward regarding explaining observed ratios of presumed-dark-matter effects to ordinary-matter effects and regarding explaining eras in the rate of expansion of the universe.

3.1. Galaxy Formation and Galaxy Evolution

This unit suggests that our work provides candidate quantitative explanations for some observations regarding galaxy formation and evolution and that our work adds insight regarding galaxy formation and galaxy evolution.

The solar system that includes the Earth associates with the notion of a solar-system-size clump of stuff that associates essentially with just one isomer. Our solar system is not spatially spherical.

We suggest that many other such one-isomer, not-spatially-spherical, somewhat-solar-system-sized clumps exist and generally date to early in the history of the universe. We suggest that the

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formation of such clumps associated with nonzero quadrupole (or, $n_3 = n_r = 3$) gravitational properties (possibly such as moments of inertia). We suggest that, during some period early in the history of the universe, quadrupole gravitational attraction dominated regarding interactions between neighboring same-isomer, solar-system-sized clumps. Table 5 suggests that a reach per instance of one isomer pertains. We suggest that solar-system-sized clumps clumped to form the halos of galaxies.

We suggest that the discussion above explains 1:0+ ratios that pertain to some early galaxies and 0+:1 ratios that pertain to some early galaxies.

We suggest that some later 1:0+ galaxies and some later 0+:1 galaxies retain their ratios from early in the evolution of the universe.

For each of some $\sim$ 4:1 galaxies, we suggest the following scenario. The galaxy started as a 0+:1 galaxy. Table 5 suggests that a reach per instance of two isomers pertains for gravitational dipole interactions. Reach-2 dipole push contributions to gravity drove away some ordinary-matter stuff and the stuff that associated with one dark-matter isomer. That one dark-matter isomer is isomer-3. Table 5 suggests that a reach per instance of six isomers pertains for gravitational monopole interactions. Reach-6 monopole pull contributions to gravity attracted remaining nearby stuff. The galaxy evolved to a ratio of $\sim$ 4:1.

For each of some $\sim$ 4:1 galaxies, we suggest the following scenario. The galaxy started as a not-isomer-3 1:0+ galaxy. Table 5 suggests that a reach per instance of two isomers pertains for gravitational dipole interactions. Reach-2 dipole push contributions to gravity drove away some dark-matter stuff but essentially no ordinary-matter stuff. Table 5 suggests that a reach per instance of six isomers pertains for gravitational monopole interactions. Reach-6 monopole pull contributions to gravity attracted remaining nearby stuff. The galaxy evolved to a ratio of $\sim$ 4:1.

Many later galaxies are 5+:1 galaxies. We suggest that many 5+:1 galaxies resulted from mergers of smaller, previous galaxies. We suggest that such mergers associate with monopole gravitational attraction. Table 5 suggests that a reach per instance of six isomers pertains. We suggest that the earliest mergers that led to a 5+:1 galaxy could have been mergers that involved 1:0+ galaxies and 0+:1 galaxies.

3.2. The Fives in 5+:1 Ratios of Dark-Matter Effects to Ordinary-Matter Effects

This unit suggests that our work provides a candidate quantitative explanation for the fives in some observed 5+:1 ratios of dark-matter effects to ordinary-matter effects.

Popular modeling suggests that 5+:1 ratios of dark-matter effects to ordinary-matter effects pertain for many galaxies, many galaxy clusters, and for densities of the universe.

We suggest that the notion of five dark-matter isomers explains the fives in such 5+:1 ratios of dark-matter effects to ordinary-matter effects.

3.3. The Pluses in 5+:1 Ratios of Dark-Matter Effects to Ordinary-Matter Effects

This unit suggests that our work provides a candidate qualitative explanation for the pluses in some observed 5+:1 ratios of dark-matter effects to ordinary-matter effects.

Popular modeling suggests that 5+:1 ratios of dark-matter effects to ordinary-matter effects pertain for many galaxies, many galaxy clusters, and for densities of the universe.

We suggest that nature might include electromagnetic properties (of objects) that would associate with reach-6. Table 5 provides a possible example. Based on reach-6 properties, early in the evolution of the universe, more electromagnetic energy would flow from isomer-pair-0 stuff to isomer-pair-1 stuff than would flow from isomer-pair-1 stuff to isomer-pair-0 stuff. Similarly, based on reach-6 properties, early in the evolution of the universe, more electromagnetic energy would flow from isomer-pair-0 stuff to isomer-pair-2 stuff than would flow from isomer-pair-2 stuff to isomer-pair-0 stuff.

We suggest that the electromagnetic energy flow imbalances would result in the existence of more isomer-pair-1 stuff than isomer-pair-0 stuff and in the existence of more isomer-pair-2 stuff than isomer-pair-0 stuff. We suggest that the flows would not disturb a one-to-one ratio of presence of isomer-3 stuff to presence of isomer-0 stuff.

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We suggest that these energy flow imbalances might have produced (or, at least, contributed to) the pluses in the 5+:1 ratios of dark-matter presence to ordinary-matter presence.

Our work does not necessarily rule out the possibility that some portions of (or the entireties of) the pluses in the 5+:1 ratios of dark-matter presence to ordinary-matter presence associate with axions or with other popular modeling suggestions regarding the nature of dark matter.

3.4. Hyperfine Depletion of COSMIC microwave Background Radiation

This unit suggests that our work provides a candidate quantitative explanation for each of two possibilities regarding some depletion of cosmic microwave background radiation.

Regarding the depletion of cosmic microwave background radiation, popular modeling suggests that the second 1 in the possible 1:1 ratio or the only 1 in the possible 0:1 ratio associates with hyperfine effects of ordinary-matter hydrogen atoms.

Should popular modeling eventually settle on the 1:1 ratio, we suggest, in Table 5, that a reach per instance of at least two isomers pertains regarding hyperfine interactivity. We suggest that MEA-isomers do not underlie significant numbers of hydrogen-like atoms. We suggest that the first 1 in the 1:1 ratio associates with hyperfine effects of hydrogen-like atoms that associate with isomer-3.

Should popular modeling eventually settle on the 0:1 ratio, we suggest, in Table 5, that a reach per instance of one isomer pertains regarding hyperfine interactivity.

3.5. Eras in the Rate of Expansion of the Universe

This unit suggests that our work provides candidate qualitative explanations for some observations regarding the rate of expansion of the universe.

Popular modeling suggests two observed multibillion-year eras regarding the so-called rate of expansion of the universe [66–69]. Chronologically, the first multibillion-year era associates with a positive rate of expansion that decreases as time increases. The second multibillion-year era associates with a positive rate of expansion that increases as time increases.

Data and popular modeling might provide hints that the second multibillion-year era might be ending [70,71] and that a new era, which would associate with a positive rate of expansion that decreases as time increases, might be starting.

We suggest that those two or three eras associate with the moving apart from each other of neighboring, but not colliding, large objects. We suggest that above-discussed notions regarding cosmic clumping of stuff pertain.

We suggest that the rate of expansion of the universe associates with interactions between neighboring non-colliding large objects. We suggest that the start of the first multibillion-year era associates with a transition to dominance, regarding interactions between many neighboring non-colliding large objects, including galaxy clusters, by gravitational quadrupole attraction (or, pull). We suggest that the start of the second multibillion-year era associates with a transition to dominance, regarding interactions between many neighboring non-colliding large objects, including galaxy clusters, by gravitational dipole repulsion (or, push).

Table 6 summarizes possible explanations for some phenomena regarding the evolution of the universe.

Table 6. Possible explanations for some phenomena regarding the evolution of the universe. Observations suggest the two multibillion-year eras. We (and possibly some observations) suggest a possible period after the later multibillion-year era. ROE abbreviates the six-word phrase rate of expansion of the universe. The words pull and push refer to gravitational effects. NNCLO abbreviates the four-element term neighboring non-colliding large objects.

Phenomena	Explanation
Start of the earlier multibillion-year era (decreasing ROE)	Quadrupole pull (between NNCLO)
Start of the later multibillion-year era (increasing ROE)	Dipole push (between NNCLO)
After the later multibillion-year era (decreasing ROE)	Monopole pull (between NNCLO)

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3.6. Recap of How Our Methods Seem to Explain Otherwise Unexplained Cosmic Data

This unit recaps phenomena that our work seeks to and seems to help explain. This unit points to some aspects of our work that contribute to the seeming explanations.

The following items recap phenomena that our work seeks to and seems to help explain. Each item names phenomena, notes an approximate characterization of the explanation that we suggest, and points to one or two relevant bases for the explanation. DM:OM denotes ratios of dark-matter effects to ordinary-matter effects. CMB denotes cosmic microwave background radiation. Isomers refers to the notion that nature includes six similar, but not necessarily identical, instances of each one of most elementary particles. Gravitational-force details refers to notions that lead to and include Tables 3 and 4.

- Dark-matter elementary particles: Quantitative (Isomers).
- Dark-matter stuff: Quantitative (Isomers).
- Galaxy evolution and DM:OM regarding some galaxies: Quantitative (Isomers, Gravitational-force details).
- DM:OM regarding some galaxy clusters: Quantitative (Isomers).
- DM:OM densities of the universe: Quantitative (Isomers).
- DM:OM regarding some depletion of CMB: Quantitative (Isomers).
- Eras in the rate of expansion of the universe: Qualitative (Isomers, Gravitational-force details).
- Gravitational phenomena, including so-called dark-energy: Qualitative (Isomers, Gravitational-force details).

4. Discussion

This unit discusses relationships between our work and popular modeling. This unit suggests opportunities for observational work and for enhancing popular modeling.

4.1. Gravitational-Force Details, Isomers, and General Relativity

This unit suggests similarities and differences between aspects of our work and aspects of general relativity.

Our explanations for data do not use general relativity.

Our work above regarding gravitational-force details has bases in properties of individual objects. Popular modeling includes two properties, energy and momentum, of individual objects that our work above somewhat scarcely mentions. General relativity has bases in a stress-energy tensor that has bases in properties (such as energy density and momentum density) that associate with regions.

We suggest that popular modeling might want to consider the extent to which each one of the following four statements pertains regarding gravitation and general relativity. Mass (as in gravitational $n_r = n_3 = 1$) associates with general relativistic notions of energy density. Mass current (as in gravitational $n_r = 1$ and $n_3 = 2$) associates with general relativistic notions of energy flux and of momentum density. Gravitational $n_r = n_3 = 2$ associates with general relativistic notions of pressure. Gravitational $n_3 = n_r + 1 = 3$ associates with general relativistic notions of shear stress and momentum flux.

More generally, popular modeling might want to consider the extent to which the following four statements pertain regarding gravitation and general relativity. Regarding two-body gravitational properties, $n_3 = n_r \leftrightarrow odd$ associates with general relativistic notions of energy density. Regarding two-body gravitational properties, $n_3 = n_r + 1 \leftrightarrow even$ associates with general relativistic notions of energy flux and of momentum density. Regarding two-body gravitational properties, $n_3 = n_r \leftrightarrow even$ associates with general relativistic notions of pressure. Regarding two-body gravitational properties, $n_3 = n_r + 1 \leftrightarrow odd$ associates with general relativistic notions of shear stress and momentum flux.

To the extent that the possible associations between our suggested gravitational-force details and the general-relativity stress energy tensor pertain, the following statements might pertain. Our work suggests that general relativity might not be adequately accurate regarding circumstances for which

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at least one of the following sentences pertains. Non-gravitational effects, such as electromagnetic effects or chromodynamics effects, are significant. The isomeric composition of stuff varies significantly between regions. Significant (or dominant) effective reaches per instance vary with time.

Our suggested cautions about possible lacks of adequate accuracy regarding applications of general relativity might not pertain significantly regarding circumstances for which general relativity has passed so-called precision tests [72]. For one example, popular modeling regarding the perihelion precession of the orbit of the planet Mercury associates with just one isomer, namely isomer-0. For another example, popular modeling of the deflection of light by the Sun also associates with the ordinary-matter isomer and with no other isomers. Neither of the two examples associates with a variation of the isomeric composition of stuff between regions. Neither of the two examples associates with a temporal change regarding dominant effective reaches.

4.2. Some Possible So-Called Cosmic Tensions

This unit discusses examples, such as the so-called Hubble tension, of possible mismatches between cosmic data and popular modeling. This unit discusses possibilities that, assuming further data does not resolve the tensions, our work points qualitatively to sources of the tensions.

Popular modeling uses the word tension to describe some possible mismatches between popular modeling and cosmic data. Some popular modeling suggests that early-multibillion-year-era values for the Hubble constant are not compatible with later-multibillion-year-era values for the Hubble constant [73]. Popular modeling associates the two-word term Hubble tension with this possible mismatch. Some popular modeling suggests that popular modeling overestimates large-scale lumpiness of stuff [74–81]. Popular modeling sometimes associates the two-element phrase S8 tension with some such possible overestimates.

Recent observations suggest that some such gaps between data and popular modeling might be smaller than previously thought or might be essentially nonexistent. Some recent observations push back against the notion of a Hubble tension [82,83]. Some recent observations push back against the notion of an S8 tension [84].

Our work suggests qualitative explanations for such possible mismatches. We suggest that the explanations might point toward total or partial closings, but not toward wider openings, of the possible gaps between modeling and data. Lacking means to quantify the would-be explanations, we do not try to suggest the usefulness of the explanations and we do not try to estimate the extents to which might-be-tension-related data might tend to confirm or refute our work.

4.2.1. The Hubble Tension

This unit suggests that our work might provide a candidate qualitative explanation for possible mismatches, known as the Hubble tension, between data and popular modeling regarding some large-scale phenomena.

Popular modeling estimates for a Hubble constant that would associate with the early universe might suggest a Hubble constant that is significantly less than estimates for a Hubble constant that would associate with the recent universe. Estimates tend to associate [85] with applications of general relativity and with equation (17). c denotes the speed of light. ρ denotes energy density and is nonnegative. P denotes pressure and can be negative.

$$-(c\rho + 3P) \tag{17}$$

Popular modeling suggests that an increasing rate of expansion associates with equation (17) evaluating to a positive number. Popular modeling suggests that a decreasing rate of expansion associates with equation (17) evaluating to a negative number.

We note, as an aside, that equation (17) provides an example in which some elements (in this case, pressure) in a stress-energy tensor can work against other elements (in this case, energy density) in a stress-energy tensor.

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Table 6 suggests that interactions between neighboring non-colliding large objects provide a basis for the rate of expansion. Regarding around the beginning of the multibillion-year era of decreasing rate, we suggest that attraction (between neighboring non-colliding large objects) that associates with $n_3 = n_r = 3$ two-body gravitational pull dominates regarding changes in the rate of expansion. Regarding around the beginning of the multibillion-year era of increasing rate, we suggest that repulsion (between neighboring non-colliding large objects) that associates with $n_3 = n_r = 2$ two-body gravitational push dominates regarding changes in the rate of expansion.

Popular modeling suggests the possibility that, throughout essentially the entirety of the two multibillion-year eras, the pressure P might be negative. (The transition to the second multibillion-year era would associate with $c\rho + 3P$ becoming negative [86,87].)

Per discussion above, our notions of two-body-interaction reaches might apply to stress-energy-tensor components such as pressure. Here, our discussion assumes that notions of reaches pertain regarding pressure.

Table 7 suggests possibly dominant contributions to pressure, relevant to popular modeling rate-of-expansion calculations, at selected times in the evolution of the universe. In Table 7, each one of the two numerically specified pairs of one n_{in} and one $R_{/in}$ comports with Table 5 and with data about galaxy evolution.

Table 7. Possibly dominant contributions to pressure, relevant to popular modeling rate-of-expansion calculations, at selected times in the evolution of the universe. The information about dominant gravitational property, gravitational instances n_{in}, and reaches per instance $R_{/in}$ pertains for interactions with the mass of a second object.

Approximately at the start of ...	Dominant gravitational property	n_{in}	$R_{/in}$
The multibillion-year era of decreasing rate	$n_3 = n_r = 3$	6	1
The multibillion-year era of increasing rate	$n_3 = n_r = 2$	3	2

We suggest that a reach per instance of one for pressure might associate with the start of the multibillion-year era of decreasing rate of expansion of the universe. We suggest that a reach per instance of two for pressure might associate with the start of the multibillion-year era of increasing rate of expansion of the universe. Based on such an increasing-with-time-from-one-to-two effective reach, we suggest that extrapolating from so-called early-universe pressures might lead to underestimations of so-called later-universe pressures. Underestimations of later-universe pressures might associate with possible underestimations of later-universe values of the Hubble constant.

4.2.2. Lumpiness Tensions

This unit suggests that our work might provide a candidate qualitative explanation for some possible mismatches, including a so-called S8 tension, between data and popular modeling regarding some large-scale phenomena.

We suggest that large-scale lumpiness associates, at least in part, with transitions, from gravitational $n_3 = n_r = 3$ dominance to gravitational $n_3 = n_r = 2$ dominance, that are similar to transitions that we suggest above regarding galaxy evolution and regarding the rate of expansion of the universe. We suggest that popular modeling might underestimate repulsion that we associate with $n_3 = n_r = 2$. We suggest that the underestimate might associate with possible overestimates, by popular modeling, of large-scale lumpiness.

4.3. Relationships Among Our Work, Data, and Popular Modeling

This unit summarizes, in a context of present data and popular modeling, aspects of our work.

Compared to state-of-the-art popular modeling, our work seems to offer the following strengths. Our work explains some otherwise unexplained data. Our work offers additional insight into galaxy formation. Our work offers a better-defined candidate specification for dark matter. Our work offers

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a perhaps more-promising basic description of two-body gravity. Our work might better identify possible bases for possible tensions between data and popular modeling.

Compared to state-of-the-art popular modeling, our work might seem to exhibit the following weaknesses. People might yet to have developed means to perform calculations and simulations based on our work. Our work might not yet say quantitatively enough about contributions, to two-body gravitational interactions, for which neither property is mass. Our work does not yet include a many-body-physics analog to general relativity.

Our work seems to offer the following opportunities. Advance scientific understanding regarding dark matter, gravitation, and galaxy formation. Develop quantum gravitation notions that parallel quantum electrodynamics notions. Advance techniques regarding modeling and simulations.

4.4. Suggestions Regarding Cataloging Types of Cosmic Data that Physics Collects

This unit suggests means for characterizing and cataloging types of cosmic data that physics collects.

Tables 2 and 3 use n_r and n_3 to catalog some two-body electromagnetic properties of objects and some two-body gravitational properties of objects.

We suggest that our work points to possible advantages for using notions from characterization mathematics [88,89], along with or in place of using some popular modeling vocabulary, when discussing properties of objects. For example, the characterization gravitational property plus $n_3 = n_r = 2$ might be more useful for some circumstances than a phrase like object-internal angular momentum. Or, the characterization gravitational property plus $n_3 = n_r = 3$ might also (that is, in addition to moments of inertia) include a gravitational analog to self-precessing magnetic moment.

We suggest that popular modeling might want to consider using types of interactions, values of n_r, values of n_3, and values of $R_{/in}$ to characterize observations. Cosmological redshifts provide an example. Popular modeling suggests that the motion of a source of light relative to an observer of that light can pertain. One might characterize such so-called Doppler redshift effects as electromagnetic, with $n_3 = n_r + 1$ (or, velocity-related), with n_r not necessarily having relevance, and with $R_{/in} = 1$. Popular modeling suggests that interactions between light and gravitational fields can pertain. One might characterize such so-called gravitational redshift effects as gravitational, with $n_3 = n_r$ (or, not-necessarily velocity-related), with n_r having relevance, and with $R_{/in}$ possibly having relevance.

One might speculate regarding the possible relevance of object-properties for which $n_3 \geq n_r + 2$. $n_3 = n_r$ associates with relative spatial position. $n_3 = n_r + 1$ associates with relative spatial position and with relative velocity. By extrapolation, $n_3 = n_r + 2$ might associate with relative spatial position, with relative velocity, and with relative acceleration. Based on (for example) time-lapse observations or multi-messenger observations or multi-object simulations, an object-P might be able to infer an object-A retarded-time charge acceleration $a_{oA,rel.oP}$.

4.5. Possible Insight Regarding Possible Early-Universe Phenomena

This unit suggests that our work might add insight regarding possible early eras, in the evolution of the universe, for which there might not be much data.

There might not be much data that directly pertain to before the start of the earlier multibillion-year era in the rate of expansion of the universe.

Popular modeling proposes, for before the start of the earlier multibillion-year era, a so-called inflationary epoch [90–94].

Table 8 suggests possible explanations for some possible phenomena regarding the evolution of the universe.

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Table 8. Possible explanations for some possible phenomena regarding before the earlier multibillion-year era in the evolution of the universe. We suggest the possibilities that this table lists for before inflation. (Some popular modeling suggests notions of a so-called Big Crunch.) We suggest the possibilities that this table lists for the start of inflation. (Popular modeling suggests an inflationary epoch.) ROE abbreviates the six-word phrase rate of expansion of the universe. The words pull and push refer to gravitational effects. NNCLO abbreviates the four-element term neighboring non-colliding large objects.

Phenomena	Explanation
Before inflation	Hexadecapole pull
Start of inflation	Pauli-exclusion bounce
Early inflationary epoch ROE	Octupole push (between proto NNCLO)

Regarding dominance by hexadecapole gravitational attraction, one might need to suggest a non-gravitational mechanism that associates with a transition from a Big Crunch [95] to a Big Bang. Without a notion such as the notion of a bounce based on fermion particles and the Pauli-exclusion principle, hexadecapole pull might lead toward effects that would associate with 32-pole aspects. The word dotriacontapole and the word triacontadipole associate with 32-pole. Assuming that hexadecapole gravitational attraction pertained, we suggest that the reach per instance for each one of hexadecapole gravitational attraction and octupole gravitational repulsion might be six isomers. Consequences of those reaches being six isomers might associate with notions that, throughout the known evolution of the universe, large-scale densities of stuff tend to be, to a first approximation, spatially uniform [96].

Our notions regarding before inflation, early in the inflationary epoch, and after the later multibillion-year era might comport with some popular modeling notions of bounce cosmology [97,98].

4.6. Suggestions for Observational Work

This unit suggests some opportunities, to which our work points, for verifying or refuting aspects of our work and for pinpointing opportunities for observational or experimental work.

We suggest considering possible synergies between galaxy-evolution studies and our work. To what extent does our work comport with early-universe galaxy-formation and galaxy-evolution? Did nature form at least as many 1:0+ (or, dark-matter) galaxies as 0+:1 (or, ordinary-matter) galaxies? To what extent does it seem reasonable that some of today's 1:0+ galaxies and 0+:1 galaxies maintained those ratios from early in the evolution of the universe? To what extent do our suggested scenarios for the formation of ∼4:1 galaxies comport with nature?

We suggest considering observational work, regarding the aftermath of galaxy-cluster collisions, that could help verify or refute our notions of significantly-electromagnetically-active dark matter. To what extent does isomer-3 intergalactic medium exist? To what extent does isomer-3 intergalactic medium lag other dark-matter stuff? More specifically, our specification for dark matter might not necessarily be incompatible with data about collisions, such as the Bullet Cluster collision, of two galaxy clusters [99,100]. Popular modeling suggests two types of trajectories for stuff. Most dark matter, from either one of the clusters, exits the collision with trajectories that are consistent with having interacted just gravitationally with the other cluster. Ordinary-matter stars, from either cluster, exit the collision with trajectories that are consistent with having interacted just gravitationally with the other cluster. However, ordinary-matter intergalactic medium, from either cluster, lags the cluster's ordinary-matter stars and dark matter. Popular modeling suggests that the ordinary-matter intergalactic medium interacted electromagnetically with the other cluster's ordinary-matter intergalactic medium, as well as gravitationally with the other cluster. We suggest that the stuff that associates with four isomers, namely isomer-1, isomer-2, isomer-4, and isomer-5, does not form much electromagnetically active intergalactic medium. We suggest that further analysis of data might help determine the validity of our notion that the stuff that associates with isomer-3 forms electromagnetically interactive intergalactic medium.

We suggest considering observational (and perhaps even experimental) work that could help verify, refute, or extend a table such as Table 5. Which reach pertains for hyperfine phenomena? Which reach or reaches pertain for atomic transitions that are not hyperfine transitions? To what extent might data suggest a reach for magnetic moment? To what extent do the reaches for $n_3 = n_r + 1$ current properties comport with the reaches for the counterpart $n_3 = n_r$ properties?

We suggest considering observational work to try to associate rate of expansion data with interactions between various types of neighboring non-colliding large objects. Examples of such interactions include interactions between galaxy clusters, interactions between galaxies within a galaxy cluster, interactions between galaxy clusters and galaxies that do not associate with galaxy clusters, and interactions for which at least one object is bigger than a galaxy cluster.

We suggest considering experimental work that might detect dark matter of the types that we suggest.

We suggest considering work that would test general relativity for circumstances in which our work would suggest that dominant isomeric compositions change spatially. To what extent do gravitational-wave signatures from collisions differ between collisions of pairs of same-isomer-based neutron stars and collisions of pairs of different-isomer-based neutron stars?

We suggest considering work that would test general relativity for circumstances in which our work would suggest that dominant reaches change temporally or spatially. To what extent do gravitational-wave signatures from collisions differ among collisions of pairs of same-isomer-based neutron stars, collisions of pairs of different-isomer-based neutron stars, and collisions of pairs of supermassive black holes (for which each black hole might include roughly equal presences of stuff that associates with each one of the six isomers)?

4.7. *Suggestions for Enhancing Popular Modeling*

This unit suggests some opportunities, to which our work points, for enhancing popular modeling.

We suggest considering how to evolve popular modeling to embrace notions that, while much dark matter might qualify as collisionless dark matter, significant amounts of dark matter might not qualify as collisionless dark matter.

We suggest considering how to incorporate notions regarding instances and reaches per instance into popular modeling and into numerical simulations.

We suggest considering the extent to which popular modeling might benefit by using, at least in the context of two-body modeling, the series gravitational scalar potential, gravitational vector potential, gravitational two-tensor potential, and so forth.

We suggest considering details regarding how to implement multipole expansions that feature potentials that associate with multiple properties, such as charge and magnetic moment or such as mass and sub-object-mass motions. We note, as an aside, that equation (18) and data might point to some relationships between energies that might associate with four properties (spin, or gravitational $n_3 = n_r = 2$; mass, or gravitational $n_3 = n_r = 1$; charge, or electromagnetic $n_3 = n_r = 1$; and magnetic moment, or electromagnetic $n_3 = n_r = 2$) of the three known nonzero-mass boson elementary particles [7,8,101]. S denotes the spin (or, object-internal angular momentum) of a particle divided by the spin of the Z boson or by the spin of the W boson. M' denotes the mass of a particle divided by one-third the mass of the Z boson. Q denotes the charge of a particle divided by the magnitude of the charge of a W boson. μ denotes the magnetic moment of a particle divided by the magnitude of the magnetic moment of a W boson. We note, as a second-level aside, that equation (18) has a solution for which $S = 2$, $M' = 1$ (that is, a mass of about 30.396 GeV$/c^2$), $Q = 0$, and $\mu = 0$.

$$(4 - S)^2 + 1 = (M')^2 + S^2 + Q^2 + \mu^2 \tag{18}$$

We suggest considering how to integrate into popular modeling (for electromagnetism and for gravitation) methods that we use to develop and apply our suggested gravitational-force details. We suggest considering how to extend such methods to apply to torques (as well as to pull-or-push aspects of forces).

5. Conclusion

This unit summarizes key aspects of this study.

We suggest details about dark matter and gravity.

The details associate with suggested reuses of and suggested extensions to familiar popular modeling. For example, regarding dark matter, we reuse, with variations regarding details, a list of known elementary particles. Also, regarding gravity, we reuse, with variations, notions of multipole expansions.

Our work quantitatively explains dark-matter phenomena that seemingly no other work explains. Our work suggests insight about galaxy formation and about the large-scale evolution of the universe.

The data that we explain pertain to galaxies, galaxy clusters, cosmic microwave background radiation, the expansion of the universe, and densities of the universe. Based in part on the breadth of the data that we explain, people might find our explanations credible and compelling.

We suggest opportunities for new uses of extant data, for new observational work, and for upgrades to popular modeling.

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