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Article

Timelike Vacuum Interfaces and Entropic Completion of the Stress–Energy Tensor in General Relativity

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Abstract

This study develops a coarse-grained description of timelike vacuum interfaces in classical general relativity and explores how such interfaces can support an effective dust-like dark contribution without modifying Einstein's equations. The starting point is the thin-shell formalism: a timelike hypersurface Σ separating two vacuum or cosmological-constant regions is endowed with an induced metric $h_{\mu\nu}$, extrinsic curvature $K_{\mu\nu}$ and a surface stress tensor $S_{\mu\nu}$ fixed by the Israel junction conditions. To this purely geometric structure an area-based entropy $S_{\Sigma} = k_B A_{\Sigma} / (4\ell_p^2)$ is assigned to spacelike cross-sections of Σ , motivated by the Bekenstein–Hawking area law and the area scaling of entanglement entropy, with the patch number $N_{\Sigma} = A_{\Sigma} / (4\ell_p^2)$ serving as a geometric control parameter for the entropic loading of the interface. After coarse graining over many interface events, the shell stress–energy acquires an entropic contribution $T_{\text{ent}}^{\mu\nu} \simeq \rho_{\text{ent}} u^{\mu} u^{\nu}$ that is well approximated by a pressureless component on large scales. In homogeneous FLRW backgrounds the entropic density obeys $\dot{\rho}_{\text{ent}} + 3H\rho_{\text{ent}} = 0$ and thus follows the standard cold-matter scaling $\rho_{\text{ent}} \propto a^{-3}$, providing an effective dark contribution in the Friedmann equations. In the stationary, weak-field regime the logarithmic temperature potential $\theta = \ln T_{\text{grav}}$ satisfies a Poisson-type equation $\nabla^2 \theta = -(4\pi G/c^2)(\rho_{\text{vis}} + \rho_{\text{ent}})$ and yields the gravitational field via $\mathbf{g} = c^2 \nabla \theta$, so that ρ_{ent} appears as an apparent halo component in clusters and galaxies. The framework organises familiar dark-matter phenomenology in terms of timelike vacuum interfaces and their entropic state, providing a classical arena for studying coarse-grained gravitational entropy on timelike surfaces and its connections to entanglement- and holography-inspired ideas, while leaving fine-grained microphysical interpretations to future work.

Keywords: timelike thin shell; vacuum interface; entropic completion; effective dark matter; general relativity; Bekenstein–Hawking entropy; Tolman relation; stationary gravity

1. Introduction

General relativity provides an accurate description of gravitational phenomena over a wide range of scales, from laboratory experiments to compact objects and gravitational waves. On cosmological and galactic scales, however, observations are usually interpreted in terms of additional dark components in the stress–energy tensor, modelled as cold dark matter and dark energy in the standard Λ CDM picture. Alternative approaches modify the gravitational field equations in the low-acceleration regime or introduce new long-range degrees of freedom, often with the aim of reproducing dark-matter phenomenology without invoking new particle species.

In parallel, a large body of work has explored gravitational entropy and holographic ideas at the interface between gravity, quantum field theory and thermodynamics. Black-hole and de Sitter horizons obey an area–entropy law, and semiclassical considerations assign an entropy proportional to the horizon area, suggesting that part of the gravitational information content is encoded on codimension-one surfaces [1–3]. Entanglement entropy of quantum fields across spatial surfaces also exhibits an area scaling [4], supporting the idea that coarse-grained measures of information in gravitational systems are naturally tied to areas. Building on these observations, Jacobson showed that the Einstein equations can be interpreted as an equation of state obtained from the Clausius relation on

local Rindler horizons [5], while other authors have proposed entropic interpretations of Newtonian and relativistic gravity in terms of holographic screens [6].

The present work is motivated by this interplay between geometry, entropy and coarse-grained information, but takes a deliberately conservative stance with respect to the underlying dynamics. We keep Einstein's equations unchanged and work strictly within classical general relativity. In earlier work a timelike thin shell was used to describe a vacuum interface in gravitational collapse and to relate its surface stress–energy to a quasilocal energy and gravitational temperature in a purely geometric setting [7]. Here we extend this interface perspective from the collapse setting to more general timelike vacuum interfaces and focus on their entropic completion and large-scale phenomenology. This entropy is then used to define an *entropic completion* of the shell stress–energy tensor: in addition to contributions from visible matter and gravitational waves, we introduce an entropic contribution associated with changes of the area-based entropy on the interface.

On sufficiently large scales the entropic contribution can be modelled as an effective pressureless fluid confined to the interface. From the bulk point of view this fluid is gravitationally indistinguishable from an additional cold, collisionless matter component in Einstein's equations. In this sense the entropic completion provides a purely geometric and thermodynamic mechanism within classical general relativity that can account for familiar dark-matter phenomenology without modifying the field equations or postulating a new fundamental dark particle. The construction is coarse-grained: we do not attempt to derive the area law from a specific microphysical model, nor do we address fine-grained microstates. Instead, we focus on macroscopic properties of timelike vacuum interfaces and their role in organising the stress–energy budget.

1.1. Goals and Scope

The present work is concerned with coarse-grained aspects of gravitational entropy on timelike interfaces and with their potential role in effective dark components. In horizon thermodynamics, the Bekenstein–Hawking area law $S_{\text{BH}} = k_B A / (4\ell_p^2)$ admits several complementary interpretations: as a semiclassical property of black-hole horizons, as an entanglement entropy across causal surfaces, or as a holographic bound on bulk information. More fine-grained approaches model Planck-scale patches on the horizon as effective detectors for quantum field modes and reproduce the area law in an explicitly information-theoretic bookkeeping, as in horizon-as-apparatus constructions where each $4\ell_p^2$ patch acts as a measurement cell [4,8]. The focus here is different and complementary: the same area scaling is adopted as a macroscopic constitutive input for timelike vacuum interfaces in classical general relativity, without specifying a particular apparatus or microstate model.

The main goal of this article is to construct a geometric and thermodynamic framework in which timelike vacuum interfaces carry an effective dust-like contribution that is entirely encoded in the classical shell geometry and junction conditions. Concretely, a timelike hypersurface separating two vacuum or cosmological-constant regions is described by its induced metric $h_{\mu\nu}$, extrinsic curvature $K_{\mu\nu}$ and surface stress tensor $S_{\mu\nu}$ determined by the Israel junction conditions. Spacelike cross-sections of this interface are endowed with an area-based entropy $S_\Sigma = k_B A_\Sigma / (4\ell_p^2)$ and a patch number $N_\Sigma = A_\Sigma / (4\ell_p^2)$, which plays the role of a geometric control parameter for the entropic loading of the interface. After coarse graining over many interface events, the shell stress–energy is reorganised into a visible part, an entropic contribution $T_{\text{ent}}^{\mu\nu}$ that is approximately pressureless on large scales, and an effective gravitational-wave contribution.

The analysis is deliberately conservative in its dynamical content. Einstein's equations are kept unchanged; no new fundamental fields are introduced, and the area–entropy relation is used purely as a constitutive law for timelike interfaces rather than as a derived microphysical entropy. The scope of the article is to show that, under these assumptions, timelike vacuum interfaces admit a natural entropic completion whose coarse-grained contribution behaves as an effective cold component in homogeneous cosmology and as an apparent halo contribution in clusters and galaxies. Fine-grained microscopic interpretations of the patch number—for instance in terms of detector-like Planck cells or

underlying entanglement structure, as in more detailed horizon models [8]—are left open and are only invoked as motivation for the robustness of the $A/(4\ell_p^2)$ scaling.

1.2. Structure of the Paper

Section 2 introduces timelike vacuum interfaces in general relativity. We review the thin-shell formalism and Israel junction conditions, and recall how the induced metric and extrinsic curvature encode the surface stress tensor $S_{\mu\nu}$. We then summarise the notion of gravitational temperature for stationary configurations via the Tolman relation and introduce a logarithmic temperature potential θ that combines gravitational redshift and field strength in the stationary weak-field regime.

In Section 3 we endow spacelike cross-sections of the interface with an area-based entropy using the Bekenstein–Hawking scaling as a constitutive relation, motivated by horizon thermodynamics and entanglement entropy of quantum fields [4]. This leads to a dimensionless patch number $N_\Sigma = A_\Sigma/(4\ell_p^2)$, which measures the entropic loading of the interface and sets a natural curvature scale in Planck units. For spherical cross-sections we relate N_Σ to the Gaussian curvature and interpret it as a geometric control parameter.

Section 4 introduces the entropic completion of the shell stress–energy tensor. We decompose the interface contribution into visible, entropic and gravitational-wave parts and express stress–energy conservation in terms of exchange currents between these contributions. On large scales the entropic contribution can be written as an effective fluid, which in many situations behaves as a pressureless component and thus acts as an apparent dark contribution in Einstein’s equations.

In Section 5 we discuss three settings in which the entropic contribution reproduces standard dark-matter phenomenology: cosmological background evolution, where the entropic density follows a cold-dark-matter scaling in the absence of exchange; galaxy clusters, where an effectively collisionless entropic contribution participates in lensing; and galactic rotation curves, where entropic density profiles act as apparent halos in the stationary θ -formulation.

Section 6 provides a discussion and interpretation of the interface picture in terms of quiet and excited regions on the shell, and comments on the resulting global coupling between local interface dynamics, vacuum temperature and large-scale gravitational fields. In Section 7 we outline directions for quantitative tests, possible acceleration-dependent constitutive laws, and a more microscopic understanding of interface events. Technical details of the stationary θ -formulation and the FLRW background with an entropic contribution are collected in Appendices B and A, respectively.

2. Timelike Vacuum Interfaces in General Relativity

In this section we collect the geometric ingredients needed to describe timelike vacuum interfaces in general relativity. The setting is that of a timelike hypersurface Σ separating two spacetime regions that each satisfy Einstein’s equations with their own bulk stress–energy content. The hypersurface carries a distributional stress–energy tensor determined by the jump in extrinsic curvature via the Israel junction conditions, and in stationary situations admits a natural notion of gravitational temperature encoded in a scalar potential θ .

2.1. Thin-Shell Geometry and Junction Conditions

Let Σ be a timelike hypersurface that divides the spacetime manifold $\mathcal{M}$ into two regions $\mathcal{M}^+$ and $\mathcal{M}^-$. Each region is equipped with a metric $g_{\mu\nu}^{(\pm)}$ solving

$$G_{\mu\nu}^{(\pm)} + \Lambda^{(\pm)} g_{\mu\nu}^{(\pm)} = 8\pi G T_{\mu\nu}^{(\pm)}. \quad (1)$$

Across Σ the metric is continuous, so that the interface inherits an induced metric $h_{\mu\nu}$, while first derivatives may have a finite jump.

The hypersurface is specified by a unit normal n^μ with $n^\mu n_\mu = +1$ and induced metric

$$h_{\mu\nu} = g_{\mu\nu} - n_\mu n_\nu. \quad (2)$$

Its extrinsic curvature is defined as

$$K_{\mu\nu} = h_{\mu}^{\alpha} h_{\nu}^{\beta} \nabla_{\alpha} n_{\beta}, \quad (3)$$

where ∇_{α} is the covariant derivative compatible with $g_{\mu\nu}$. In general $K_{\mu\nu}$ takes different values when computed from the two sides of the interface, $K_{\mu\nu}^{(+)}$ and $K_{\mu\nu}^{(-)}$, with jump

$$[K_{\mu\nu}] := K_{\mu\nu}^{(+)} - K_{\mu\nu}^{(-)}, \quad [K] := h^{\mu\nu} [K_{\mu\nu}]. \quad (4)$$

The distributional stress–energy of the shell is encoded in a surface stress tensor $S_{\mu\nu}$ living on Σ , related to the jump in extrinsic curvature by the Israel junction conditions [7,9,10]:

$$8\pi G S_{\mu\nu} = -([K_{\mu\nu}] - h_{\mu\nu} [K]). \quad (5)$$

In a local orthonormal frame adapted to Σ , $S_{\mu\nu}$ can be decomposed into an energy surface density σ , surface pressure or tension p , and possible anisotropic or dissipative terms. For spherically symmetric shells one obtains

$$S_{\hat{\mu}\hat{\nu}} = \text{diag}(-\sigma, p, p) \quad (6)$$

in an orthonormal basis $\{\hat{t}, \hat{\theta}, \hat{\phi}\}$ tangent to the interface. Equation (5) shows that $S_{\mu\nu}$ is completely determined by the embedding of Σ into the bulk spacetime via the jump in extrinsic curvature.

2.2. Gravitational Temperature and the θ -Potential

If the spacetime admits a timelike Killing vector ξ^{μ} tangent to Σ , static or stationary observers on the interface follow worldlines aligned with ξ^{μ} . The norm $\xi^2 = \xi^{\mu} \xi_{\mu} < 0$ defines a redshift factor, and thermal equilibrium in a static gravitational field is governed by the Tolman relation [11,12]:

$$T_{\text{grav}}(\vec{x}) \sqrt{-\xi^{\mu} \xi_{\mu}(\vec{x})} = T_{\infty}, \quad (7)$$

where $T_{\text{grav}}(\vec{x})$ is the local temperature measured by static observers and T_{∞} is a constant reference temperature. In the Newtonian weak-field limit one may write

$$\xi^{\mu} \xi_{\mu} \simeq -e^{2\Phi(\vec{x})}, \quad T_{\text{grav}}(\vec{x}) = T_{\infty} e^{-\Phi(\vec{x})}, \quad (8)$$

with $\Phi(\vec{x})$ the Newtonian potential.

We introduce the logarithmic temperature field

$$\theta(\vec{x}) := \ln T_{\text{grav}}(\vec{x}), \quad (9)$$

which encodes the gravitational redshift through Equation (8). In the stationary weak-field regime, Einstein's equations reduce to Poisson's equation

$$\nabla^2 \Phi = 4\pi G \rho, \quad (10)$$

with ρ the matter density. Combining Equations (8) and (10) gives, to leading order,

$$g = -\nabla \Phi = c^2 \nabla \theta, \quad (11)$$

and θ satisfies

$$\nabla^2 \theta = -\frac{4\pi G}{c^2} \rho. \quad (12)$$

Thus, in the stationary weak-field limit, gravitational redshift and field strength can be expressed in terms of the single scalar potential θ .

For timelike vacuum interfaces embedded in such stationary backgrounds, the induced geometry $(h_{\mu\nu}, K_{\mu\nu})$ and the field θ characterise, respectively, the surface stress–energy content via Equation (5)

and the local gravitational temperature via Equation (7). In the next section we equip spacelike cross-sections of Σ with an area-based entropy and introduce a dimensionless patch number that will serve as a geometric control parameter for the entropic completion of the shell stress–energy tensor.

3. Area Quantization and Entropic Patch Number

The geometric data $(h_{\mu\nu}, K_{\mu\nu})$ and the gravitational temperature T_{grav} characterise a timelike vacuum interface as a purely classical object. To introduce an entropic description, this structure is supplemented by an area-based entropy assignment inspired by the Bekenstein–Hawking area law and by the area scaling of entanglement entropy in quantum field theory. This leads to a natural dimensionless patch number $N_\Sigma = A_\Sigma / (4\ell_p^2)$ for spacelike cross-sections of Σ , which is used as a control parameter for the entropic loading and curvature scale of the interface.

3.1. Bekenstein–Hawking Area Law as a Constitutive Relation

For causal horizons in general relativity, the Bekenstein–Hawking entropy takes the form [1–3,5]

$$S_{\text{BH}} = \frac{k_B c^3}{4G\hbar} A = k_B \frac{A}{4\ell_p^2}, \quad \ell_p^2 = \frac{G\hbar}{c^3}, \quad (13)$$

where A is the area of a horizon cross-section and ℓ_p is the Planck length. Independently, entanglement entropy of quantum fields across spatial surfaces also exhibits an area scaling, reinforcing the role of area as a natural measure of coarse-grained information in gravitational systems [4]. More fine-grained constructions go further and treat Planck-scale patches of size $4\ell_p^2$ on a horizon as effective detectors for field modes, so that the Bekenstein–Hawking entropy arises from an explicit counting of such measurement cells [8].

In the present work Equation (13) is adopted as an *effective constitutive relation* for timelike vacuum interfaces. Spacelike two-surfaces obtained as cross-sections of Σ are assigned an entropy

$$S_\Sigma = k_B \frac{A_\Sigma}{4\ell_p^2}, \quad (14)$$

with A_Σ the area of the cross-section. This extension of the area–entropy proportionality beyond null horizons is not derived from a specific microscopic model; it is a coarse-grained modelling choice motivated by the robustness of area scaling in horizon thermodynamics, entanglement entropy and patch-based horizon models. Throughout this article Equation (14) is used only as a macroscopic constitutive law, and Einstein’s equations are kept unchanged.

It is convenient to define the dimensionless *patch number*

$$N_\Sigma := \frac{A_\Sigma}{4\ell_p^2}, \quad (15)$$

so that Equation (14) becomes

$$S_\Sigma = k_B N_\Sigma. \quad (16)$$

An increment $\Delta S_\Sigma = k_B$ corresponds to an area change $\Delta A_\Sigma = 4\ell_p^2$, interpreted as the addition or removal of a single effective entropy patch on the interface. In microscopic horizon models such patches can be associated with detector-like Planck cells [8]; here N_Σ is used more modestly as a coarse-grained counting parameter that organises the entropic loading of timelike interfaces without committing to a specific microstate or apparatus picture.

3.2. Patch Number and Curvature Scale

For a spherical cross-section S of the interface with areal radius R and area $A_\Sigma = 4\pi R^2$, the intrinsic Gaussian curvature is

$$K = \frac{1}{R^2} = \frac{4\pi}{A_\Sigma}. \quad (17)$$

Combining Equations (15) and (17) gives a simple relation between curvature and patch number in Planck units,

$$K \ell_p^2 = \frac{\pi}{N_\Sigma}. \quad (18)$$

Thus N_Σ provides a geometric control parameter: large patch numbers $N_\Sigma \gg 1$ correspond to weakly curved, semiclassical interfaces, while $N_\Sigma \sim \mathcal{O}(1)$ signals curvature approaching the Planck scale.

For more general (not necessarily spherical) cross-sections one may similarly interpret N_Σ as the total number of effective $4\ell_p^2$ patches covering the surface, with the local curvature scale set by the ratio of typical patch size to geometric features of the interface. In this sense the coarse-grained entropy $S_\Sigma = k_B N_\Sigma$ and the patch number N_Σ quantify how strongly the timelike vacuum interface is populated by area–entropy degrees of freedom, while the induced metric $h_{\mu\nu}$ and extrinsic curvature $K_{\mu\nu}$ describe its embedding into the bulk spacetime.

In Section 4 these geometric and entropic quantities are combined into an entropic completion of the shell stress–energy tensor, where the interface contribution is decomposed into visible, entropic and gravitational-wave parts, and the patch number N_Σ plays the role of a natural counting parameter for coarse-grained interface events.

4. Entropic Completion of the Shell Stress–Energy Tensor

We now combine the geometric data of the timelike vacuum interface with the area-based entropy assignment to define an entropic completion of the shell stress–energy tensor. The key step is to separate the shell contribution into visible, entropic and gravitational-wave parts, and to describe their mutual exchange in terms of covariantly conserved currents. On sufficiently large scales the entropic contribution can be modelled as an effective pressureless fluid.

4.1. Shell contribution and decomposition

The surface stress tensor $S_{\mu\nu}$ on the interface encodes a distributional contribution to the bulk stress–energy tensor, supported on the hypersurface Σ and determined by the Israel junction conditions (5). For bookkeeping purposes it is convenient to split this shell contribution into three parts,

$$T_{\text{shell}}^{\mu\nu} = T_{\text{vis}}^{\mu\nu} + T_{\text{ent}}^{\mu\nu} + T_{\text{GW}}^{\mu\nu}, \quad (19)$$

with the following interpretation:

- $T_{\text{vis}}^{\mu\nu}$ collects contributions that can be traced back to ordinary matter fields or classical sources localised on the interface (if present).
- $T_{\text{ent}}^{\mu\nu}$ is the *entropic contribution*, associated, at a coarse-grained level, with changes of the area-based entropy S_Σ on spacelike cross-sections of Σ .
- $T_{\text{GW}}^{\mu\nu}$ represents the effective stress–energy carried by gravitational waves interacting with the interface, in the sense of the Isaacson approximation for high-frequency gravitational radiation [13–15].

The sum in Equation (19) does not introduce new degrees of freedom beyond those already encoded in the shell geometry. It repackages the same distributional contribution that appears in the Israel junction conditions, but organises it into three pieces that are useful for discussing energy exchange and coarse-grained phenomenology.

4.2. Entropic Surface Energy and Patch Events

The area-based entropy assignment (14) and the patch number N_Σ introduced in Equation (15) allow one to associate an energy scale to changes of the interface entropy. For a spacelike cross-section of Σ with entropy $S_\Sigma = k_B N_\Sigma$, a small change of the entropy is related to the energy of the interface by

$$\delta E_\Sigma = T_{\text{grav}} \delta S_\Sigma, \quad (20)$$

where T_{grav} is the local gravitational temperature on the interface defined by the Tolman relation (7). Using $S_{\Sigma} = k_B N_{\Sigma}$, one finds

$$\delta E_{\Sigma} = k_B T_{\text{grav}} \delta N_{\Sigma}. \quad (21)$$

In particular, an increment by a single patch, $\Delta N_{\Sigma} = 1$, corresponds to an energy change of order

$$\Delta E_{\Sigma} \sim k_B T_{\text{grav}}, \quad (22)$$

up to factors of order unity that depend on the detailed dynamics of interface events. Equation (22) identifies $k_B T_{\text{grav}}$ as the characteristic energy scale associated with the creation or annihilation of an effective $4\ell_p^2$ entropy patch on the interface.

At the microscopic level, one may think of changes in N_{Σ} as arising from individual “interface events” in which patches are created, destroyed or rearranged. In the present work we do not specify such a microscopic picture. Instead, we treat their net effect on scales large compared to the patch size and microscopic interaction lengths. After coarse graining, the cumulative contribution of many patch events in a given region of the interface is encoded in an entropic surface energy density and flux, which together define the entropic contribution $T_{\text{ent}}^{\mu\nu}$ in Equation (19).

4.3. Exchange Currents and Effective Fluid Description

The total stress–energy tensor, including the shell contribution, satisfies the covariant conservation law $\nabla_{\mu} T_{\text{tot}}^{\mu\nu} = 0$. Using the decomposition (19) it is natural to describe exchanges between the visible, entropic and gravitational-wave contributions in terms of currents J_{vis}^{ν} , J_{ent}^{ν} and J_{GW}^{ν} defined by

$$\nabla_{\mu} T_{\text{vis}}^{\mu\nu} = J_{\text{vis}}^{\nu}, \quad (23)$$

$$\nabla_{\mu} T_{\text{ent}}^{\mu\nu} = J_{\text{ent}}^{\nu}, \quad (24)$$

$$\nabla_{\mu} T_{\text{GW}}^{\mu\nu} = J_{\text{GW}}^{\nu}, \quad (25)$$

with the constraint

$$J_{\text{vis}}^{\nu} + J_{\text{ent}}^{\nu} + J_{\text{GW}}^{\nu} = 0. \quad (26)$$

Equation (26) expresses the fact that energy and momentum can flow between visible matter, entropic interface degrees of freedom and gravitational waves, while the total remains conserved. In particular, dissipative processes in the visible component and gravitational radiation can feed the entropic contribution through J_{ent}^{ν} and J_{GW}^{ν} , changing the patch number N_{Σ} over time.

On scales large compared to the patch size and microscopic mean free paths, the entropic contribution can be modelled as an effective fluid. Denoting by u^{μ} the four-velocity of comoving observers on the interface and by ρ_{ent} and p_{ent} the effective energy density and pressure of the entropic contribution, one may write

$$T_{\text{ent}}^{\mu\nu} = \rho_{\text{ent}} u^{\mu} u^{\nu} + p_{\text{ent}} h^{\mu\nu} + \dots, \quad (27)$$

where $h^{\mu\nu}$ is the induced metric on Σ and the dots denote possible anisotropic or dissipative terms generated by interface dynamics. In many situations of interest, and in particular in the cosmological and galactic applications considered below, the entropic contribution can be approximated as pressureless on the relevant scales, so that

$$T_{\text{ent}}^{\mu\nu} \simeq \rho_{\text{ent}} u^{\mu} u^{\nu}, \quad (28)$$

which is gravitationally indistinguishable from a cold, collisionless matter component in Einstein’s equations.

In summary, the entropic completion consists of three ingredients: (i) an area-based entropy for spacelike cross-sections of the interface, encoded in the patch number N_{Σ} ; (ii) an entropic contribution $T_{\text{ent}}^{\mu\nu}$ that accounts, after coarse graining, for the energy and momentum carried by patch events on the interface; and (iii) exchange currents (26) that describe how visible matter and gravitational waves can feed the entropic contribution over time. The next section illustrates how, in simple symmetric

settings, this entropic contribution behaves as an effective dust-like component and gives rise to familiar dark-matter phenomenology.

5. Applications: Apparent Dark Matter from Entropic Shells

The entropic completion of the shell stress–energy tensor introduced in Section 4 provides, on large scales, an effective dust-like contribution ρ_{ent} confined to timelike vacuum interfaces. From the bulk point of view this contribution is gravitationally indistinguishable from an additional cold, collisionless matter component in Einstein’s equations. In this section we illustrate how such an entropic contribution can account for familiar dark-matter phenomenology in three standard arenas: cosmological background evolution, galaxy cluster lensing, and galactic rotation curves.

5.1. Cosmological Background Evolution

We consider a spatially homogeneous and isotropic universe with FLRW metric

$$ds^2 = -dt^2 + a(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right], \quad (29)$$

where $a(t)$ is the scale factor and $k = 0, \pm 1$ encodes the spatial curvature. The energy content is split into visible matter and radiation with density ρ_{vis} and pressure p_{vis} , an entropic contribution with density ρ_{ent} and effective pressure p_{ent} , and a cosmological-constant term with density ρ_{Λ} .

The Friedmann equations take the usual form

$$H^2 = \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} (\rho_{\text{vis}} + \rho_{\text{ent}} + \rho_{\text{rad}} + \rho_{\Lambda}) - \frac{k}{a^2}, \quad (30)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho_{\text{vis}} + \rho_{\text{ent}} + \rho_{\text{rad}} + \rho_{\Lambda} + 3p_{\text{vis}} + 3p_{\text{ent}} + 3p_{\text{rad}} - 3\rho_{\Lambda}), \quad (31)$$

with $H = \dot{a}/a$ the Hubble parameter and $\rho_{\text{rad}}, p_{\text{rad}}$ the radiation density and pressure. The conservation equation $\nabla_{\mu} T_{\text{tot}}^{\mu\nu} = 0$ can be split into coupled continuity equations for ρ_{vis} and ρ_{ent} , using the exchange currents defined in Equation (26). At the background level this yields

$$\dot{\rho}_{\text{vis}} + 3H(\rho_{\text{vis}} + p_{\text{vis}}) = -Q(t), \quad (32)$$

$$\dot{\rho}_{\text{ent}} + 3H(\rho_{\text{ent}} + p_{\text{ent}}) = +Q(t), \quad (33)$$

where $Q(t)$ encodes the net exchange of energy density between the visible and entropic contributions. Radiation and the cosmological constant satisfy their usual continuity equations,

$$\dot{\rho}_{\text{rad}} + 4H\rho_{\text{rad}} = 0, \quad \dot{\rho}_{\Lambda} = 0. \quad (34)$$

In the regime where the coarse-grained entropic contribution is effectively pressureless, $p_{\text{ent}} \simeq 0$ as in Equation (28), and the exchange term $Q(t)$ varies slowly on a Hubble timescale, the solution of Equation (33) approaches the standard cold-matter scaling,

$$\rho_{\text{ent}}(t) \propto a(t)^{-3}. \quad (35)$$

In this case the entropic contribution reproduces, at the background level, the same evolution as a cold, collisionless matter component usually attributed to dark matter. The detailed time-dependence of $Q(t)$ is determined by the history of interface events and the exchange currents in Equation (26), but at the level of homogeneous cosmology the entropic completion is indistinguishable from a CDM contribution in the Friedmann equations.

5.2. Cluster Mergers and Lensing

Galaxy clusters provide a second arena in which an effective dust-like contribution behaves as apparent dark matter. In merging systems, gravitational lensing maps and X-ray observations reveal a separation between the dominant lensing mass and the bulk of the baryonic gas. In the present framework, such configurations can be interpreted in terms of an entropic contribution that is effectively collisionless and tied primarily to the gravitational potential rather than to the visible gas.

Qualitatively, consider two clusters whose interaction generates strong shocks, turbulence and dissipative processes in the intracluster medium. On the interface these processes are represented by a non-vanishing exchange current J_{ent}^{ν} in Equation (26), which increases the patch number N_{Σ} and builds up an entropic energy density ρ_{ent} associated with the shared gravitational potential of the system. Each effective patch event carries an energy of order $k_B T_{\text{grav}}$ as in Equation (22), so that sustained dissipation can accumulate a macroscopic entropic contribution on the interface. The entropic component does not participate in hydrodynamic drag and remains approximately aligned with the underlying potential wells during the merger.

For distant observers, the entropic density ρ_{ent} enters the lensing potential on the same footing as any additional collisionless mass. Lensing reconstructions therefore trace the combined density $\rho_{\text{vis}} + \rho_{\text{ent}}$, while the X-ray emission traces only the visible gas ρ_{vis} . An offset between lensing peaks and the brightest baryonic component is then a natural outcome of an interface that has accumulated an entropic contribution through past exchange processes. No modification of the gravitational field equations is required; the effect arises from the entropic completion of the shell stress–energy tensor.

5.3. Galactic Rotation Curves and the Stationary Limit

In the stationary, weak-field regime relevant for galactic dynamics, the θ -potential introduced in Equation (9) obeys the Poisson-type equation (12) with total density

$$\rho = \rho_{\text{vis}} + \rho_{\text{ent}}. \quad (36)$$

For a spherically symmetric mass distribution one has $\theta = \theta(r)$ and

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d\theta}{dr} \right) = -\frac{4\pi G}{c^2} (\rho_{\text{vis}}(r) + \rho_{\text{ent}}(r)). \quad (37)$$

The gravitational field is

$$g(r) = c^2 \frac{d\theta}{dr}, \quad (38)$$

and the circular velocity of a test body on a circular orbit of radius r satisfies

$$v_c^2(r) = r g(r) = c^2 r \frac{d\theta}{dr}. \quad (39)$$

Given a visible density profile $\rho_{\text{vis}}(r)$ determined by the luminous matter in a galaxy, the entropic contribution $\rho_{\text{ent}}(r)$ modifies $\theta(r)$ and hence $v_c(r)$. For instance, an approximately isothermal entropic profile $\rho_{\text{ent}}(r) \propto 1/r^2$ at large radii leads to

$$v_c(r) \simeq \text{const}. \quad (40)$$

over a broad radial range, reproducing the familiar flat or slowly declining rotation curves of spiral galaxies. More general entropic profiles can emulate standard dark-matter halo models, such as Navarro–Frenk–White or cored profiles, by an appropriate choice of $\rho_{\text{ent}}(r)$.

In this stationary setting the entropic completion manifests itself only through the additional source term ρ_{ent} in Equation (37). The field equations retain their standard form, and the gravitational acceleration is obtained from θ by Equation (38). The effective entropic density thus acts as an apparent

dark halo for galactic rotation curves, while remaining a derived contribution to the interface stress–energy tensor rather than a new fundamental matter species.

6. Discussion and Interpretation

The entropic completion of the shell stress–energy tensor provides a geometric framework in which timelike vacuum interfaces carry an effective dust-like contribution without modifying Einstein’s equations. The construction combines three elements: the thin-shell formalism and Israel junction conditions, an area-based entropy assigned to spacelike cross-sections of the interface, and a coarse-grained entropic contribution $T_{\text{ent}}^{\mu\nu}$ that behaves as a pressureless component on large scales. In cosmological backgrounds this contribution reproduces the evolution of cold matter, while on cluster and galactic scales it appears as additional lensing mass and as an apparent halo contribution ρ_{ent} in the gravitational field.

In this section we discuss how the interface perspective organises the underlying physics in terms of local activity on the shell, how accelerated and distant observers describe the same situations in complementary ways, and how the entropic contribution encodes a coarse-grained record of past interface events. We distinguish between quiet and excited regions on the interface, comment on the resulting global coupling between local dynamics and large-scale gravitational fields, and summarise the main assumptions and limitations of the framework.

6.1. Quiet and Excited Regions on the Interface

The entropic completion suggests a natural classification of timelike vacuum interfaces in terms of their local activity. Quiet regions are near-equilibrium patches with slowly varying gravitational temperature and negligible exchange currents; excited regions are domains in which strong accelerations, dissipation and gravitational waves drive significant interface events at the $4\ell_p^2$ scale. The entropic contribution $T_{\text{ent}}^{\mu\nu}$ can then be viewed as the macroscopic imprint of accumulated interface events in excited regions.

6.1.1. Quiet Regions: Near-Equilibrium Vacuum Interfaces

A region of the interface is quiet if the induced geometry $(h_{\mu\nu}, K_{\mu\nu})$ and the gravitational temperature T_{grav} vary only slowly along Σ , and if the exchange currents J_{ent}^ν and J_{GW}^ν in Equation (26) are negligible. In such a regime the entropy associated with spacelike cross-sections, $S_\Sigma = k_B N_\Sigma$ with $N_\Sigma = A_\Sigma / (4\ell_p^2)$, is approximately conserved and the patch number N_Σ can be treated as constant over the timescales of interest.

On quiet segments the entropic contribution $T_{\text{ent}}^{\mu\nu}$ behaves as a passive background: its density ρ_{ent} is determined by past interface events, but there is essentially no ongoing exchange with visible matter or gravitational waves. For comoving observers on the interface the local geometry appears stationary, with a time-independent Tolman temperature satisfying Equation (7). In this limit the timelike vacuum interface reduces to a near-equilibrium membrane whose entropic loading and curvature scale are captured by the fixed patch number N_Σ introduced in Section 3.

6.1.2. Excited Regions: Acceleration, Unruh-Type Temperature and Entropic Dark Components

Excited regions on the interface are characterised by strong accelerations, significant baryonic dissipation, and non-negligible gravitational-wave activity. Examples include shock fronts, turbulent zones in the interstellar or intracluster medium, and regions of intense structure formation. In such environments local observers following accelerated worldlines on or near Σ experience an effective vacuum temperature of Unruh type, with magnitude set by their proper acceleration [16,17],

$$k_B T_{\text{Unruh}} \sim \frac{\hbar |a|}{2\pi c}, \quad (41)$$

where a denotes the magnitude of the four-acceleration. At the same time, distant observers describe the same situation in terms of an additional gravitational field $|a|$ encoded in the potential θ through Equation (11).

From the interface perspective, excited regions are those in which the exchange currents J_{ent}^ν and J_{GW}^ν in Equation (26) are sizeable. Baryonic dissipation, shock heating and turbulent transport feed J_{ent}^ν and increase the patch number N_Σ over time, while gravitational radiation contributes to J_{GW}^ν . Each effective patch increment $\Delta N_\Sigma = 1$ corresponds to an entropy change $\Delta S_\Sigma = k_B$ and an energy transfer of order $\Delta E_\Sigma \sim k_B T_{\text{grav}}$ at the local gravitational temperature, as in Equation (22). The cumulative effect of many such interface events is a build-up of an entropic density ρ_{ent} , which appears in $T_{\text{ent}}^{\mu\nu}$ and is gravitationally equivalent, on large scales, to a cold, collisionless matter contribution [Equation (28)].

Locally accelerated observers interpret excited regions in terms of a non-zero vacuum temperature and ongoing entropy production, while distant observers infer an additional gravitational field and an effective dark contribution ρ_{ent} from the same underlying geometry. The entropic completion translates between these two descriptions by attributing the additional field to the stress–energy of accumulated interface events.

6.1.3. Global Coupling and Effective Mach-Type Behaviour

The entropic completion highlights how local dynamics on the interface feed into the global gravitational field. Through Equation (26), any process that changes the patch number N_Σ or redistributes interface stress–energy modifies $T_{\text{ent}}^{\mu\nu}$ and hence the total source term in Einstein’s equations. In this sense the effective inertial and gravitational properties of test bodies moving in the bulk are influenced by the entropic state of the vacuum interface, which encodes the integrated history of accelerations, dissipation and gravitational waves.

On cosmological scales, this picture suggests an interpretation in which the homogeneous entropic density $\rho_{\text{ent}}(t)$ entering Equation (30) summarises the cumulative effect of past interface events in the Universe, while quiet regions at late times correspond to an almost frozen entropic background behaving as cold matter [Equation (35)]. On cluster and galactic scales, spatial variations of ρ_{ent} inherited from excited regions act as apparent halos, as discussed in Sections 5.2 and 5.3.

The resulting picture is that of a timelike vacuum interface whose geometric and entropic degrees of freedom link local accelerations, vacuum temperature and large-scale gravitational fields. Quiet regions describe near-equilibrium patches with almost fixed patch number N_Σ , while excited regions accumulate entropic stress–energy through interface events driven by visible dynamics and gravitational radiation. The entropic contribution $T_{\text{ent}}^{\mu\nu}$ is the coarse-grained record of this activity and appears, in the bulk gravitational field, as an effective dark contribution within unmodified general relativity.

6.2. Assumptions and Limitations

The framework developed here relies on a small number of structural assumptions that go beyond standard textbook general relativity but do not modify the Einstein equations.

First, the area–entropy relation $S_\Sigma \propto A_\Sigma$ for spacelike cross-sections of timelike interfaces is adopted as an effective constitutive law. In semiclassical gravity the Bekenstein–Hawking area law is derived for null horizons and is commonly interpreted in terms of entanglement entropy and holographic bounds. Extending the same scaling to timelike vacuum interfaces is therefore a modelling choice inspired by the robustness of area scaling in horizon thermodynamics and entanglement-based approaches. In the present work we use this input only at a coarse-grained level and do not attempt a microscopic derivation.

Second, the logarithmic temperature potential $\theta = \ln T_{\text{grav}}$ is employed in the stationary weak-field regime, where the Tolman relation and the Newtonian limit apply. Outside this regime the θ -description is not assumed to hold, and fully dynamical configurations would require a more general treatment.

Third, the entropic contribution $T_{\text{ent}}^{\mu\nu}$ is a coarse-grained effective description obtained by averaging many interface events over scales large compared to the patch size and microscopic interaction

lengths. When we say that it “behaves like” a pressureless dark component, we mean that, on such scales, it is gravitationally equivalent to an additional cold matter contribution in Einstein’s equations, not that it represents a new fundamental particle species.

Finally, small-scale dynamics of the interfaces, including stability analyses based on an effective potential for the shell radius, lie beyond the scope of this work and are left for future study. The present analysis is restricted to regimes in which timelike vacuum interfaces can be treated as slowly evolving carriers of area-based entropy and effective stress–energy on macroscopic scales.

7. Outlook

The entropic completion of timelike vacuum interfaces developed in this work provides a geometric mechanism by which an effective dust-like dark contribution emerges from the coarse-grained stress–energy on the interface. The construction remains entirely within classical general relativity and uses only thin-shell geometry, the Tolman relation and an area-based entropy assignment motivated by horizon thermodynamics and entanglement entropy. Several directions for further investigation are natural.

A first direction is quantitative confrontation with cosmological and astrophysical data. Here the entropic density ρ_{ent} has been discussed at a qualitative level and it has been illustrated how it can reproduce the background evolution of cold matter, cluster lensing signals and flat rotation curves. A next step is to specify concrete forms of the exchange term $Q(t)$ in Equations (32) and (33) and of the spatial profiles $\rho_{\text{ent}}(r)$ in Equation (37), and to test whether the resulting models can match cosmological and galactic observables without fine-tuning. This would clarify to what extent the entropic completion can stand as a viable effective description of dark-matter phenomenology within unmodified general relativity.

A second direction is the study of acceleration- and curvature-dependent constitutive laws for the exchange currents J_{ent}^ν . In the stationary limit, the θ -formulation, Equations (12) and (38), expresses the gravitational field directly in terms of θ . Allowing J_{ent}^ν to depend on local invariants built from the acceleration and curvature on the interface could lead, after coarse graining, to effective interpolation laws between the field strength g and the visible density ρ_{vis} . This raises the question under which conditions MOND-like behaviours might appear as macroscopic limits of the entropic completion, and when such behaviours remain compatible with large-scale constraints on gravitational dynamics.

A third line of work concerns the dynamical response of the entropic contribution to strongly non-linear processes and gravitational waves. The present framework already isolates an effective gravitational-wave stress–energy $T_{\text{GW}}^{\mu\nu}$ and an exchange current J_{GW}^ν on the interface. Quantifying how mergers, violent structure-formation events or stochastic gravitational-wave backgrounds feed into ρ_{ent} over cosmic time would clarify possible correlations between effective dark contributions, entropy production and gravitational radiation in a purely geometric setting.

Finally, the entropic completion points toward a more microscopic, fine-grained description of interface events. In this work the area–entropy relation is used as a constitutive law, without specifying microstates. It is conceivable that a more detailed model of the interface degrees of freedom, for example in terms of families of null directions or related internal structures of the gravitational field, could link the patch number N_Σ to entanglement patterns across the interface and interpret certain “dark” or non-recoverable paths as gravitational work stored in the entropic contribution. Horizon-as-apparatus models, in which Planck-scale patches of area $4\ell_p^2$ act as effective measurement cells and reproduce the Bekenstein–Hawking law in an explicitly information-theoretic bookkeeping [8], provide an example of such a fine-grained implementation on null surfaces; an analogous apparatus-type description for timelike vacuum interfaces could supply a microscopic underpinning for the coarse-grained patch number used here. Developing such a microscopic picture would move from the coarse-grained description adopted in this article toward a fine-grained account of gravitational entropy on timelike interfaces, and would connect the present construction more directly to entanglement-based and holographic approaches to gravitational entropy.

Overall, timelike vacuum interfaces with an entropic completion offer a flexible framework for organising dark-matter phenomenology within unmodified general relativity and for exploring coarse-grained aspects of gravitational entropy beyond null horizons. The qualitative mechanisms identified here can be sharpened and tested quantitatively once specific exchange laws and constitutive relations are chosen, and they provide a classical arena in which the relation between geometric, thermodynamic and informational descriptions of gravity can be studied in more detail.

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Appendix A. FLRW Background with an Entropic Component

This appendix collects the standard relations for a homogeneous and isotropic FLRW background in the presence of an additional entropic contribution, as used in Section 5.1.

Appendix A.1. Friedmann Equations and Continuity Relations

We consider the FLRW metric (29) with scale factor $a(t)$ and spatial curvature parameter $k = 0, \pm 1$. The total stress–energy tensor is written as a sum of visible matter and radiation, an entropic contribution, and a cosmological-constant term,

$$T_{\text{tot}}^{\mu\nu} = T_{\text{vis}}^{\mu\nu} + T_{\text{ent}}^{\mu\nu} + T_{\text{rad}}^{\mu\nu} + T_{\Lambda}^{\mu\nu}. \quad (\text{A1})$$

At the background level each component is modelled as a perfect fluid with density ρ_i and pressure p_i ,

$$T_i^{\mu\nu} = (\rho_i + p_i)u^\mu u^\nu + p_i g^{\mu\nu}, \quad (\text{A2})$$

where u^μ is the comoving four-velocity. The cosmological constant corresponds to $\rho_\Lambda = \Lambda/(8\pi G)$ and $p_\Lambda = -\rho_\Lambda$.

Einstein's equations reduce to the Friedmann equations,

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho_{\text{vis}} + \rho_{\text{ent}} + \rho_{\text{rad}} + \rho_\Lambda) - \frac{k}{a^2}, \quad (\text{A3})$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho_{\text{vis}} + \rho_{\text{ent}} + \rho_{\text{rad}} + \rho_\Lambda + 3p_{\text{vis}} + 3p_{\text{ent}} + 3p_{\text{rad}} - 3\rho_\Lambda), \quad (\text{A4})$$

which reproduce Equations (30) and (31) in the main text.

Covariant conservation $\nabla_\mu T_{\text{tot}}^{\mu\nu} = 0$ leads to coupled continuity equations. Using the exchange-current decomposition introduced in Equation (26), the background evolution of visible and entropic components can be written as

$$\dot{\rho}_{\text{vis}} + 3H(\rho_{\text{vis}} + p_{\text{vis}}) = -Q(t), \quad (\text{A5})$$

$$\dot{\rho}_{\text{ent}} + 3H(\rho_{\text{ent}} + p_{\text{ent}}) = +Q(t), \quad (\text{A6})$$

where $Q(t)$ parametrises the net energy transfer between the visible and entropic contributions. Radiation and the cosmological constant satisfy

$$\dot{\rho}_{\text{rad}} + 4H\rho_{\text{rad}} = 0, \quad \dot{\rho}_{\Lambda} = 0, \quad (\text{A7})$$

consistent with Equation (34).

Appendix A.2. Scaling Behaviour of the Entropic Contribution

The effective equation of state of the entropic contribution is defined by

$$w_{\text{ent}} := \frac{p_{\text{ent}}}{\rho_{\text{ent}}}. \quad (\text{A8})$$

In the coarse-grained regime of interest, the entropic contribution is approximately pressureless on large scales, $w_{\text{ent}} \simeq 0$, as in Equation (28). Neglecting exchange with visible matter on Hubble timescales, $Q(t) \simeq 0$, the continuity equation (A6) reduces to

$$\dot{\rho}_{\text{ent}} + 3H\rho_{\text{ent}} = 0, \quad (\text{A9})$$

so that

$$\rho_{\text{ent}}(a) \propto a^{-3}, \quad (\text{A10})$$

as in Equation (35). In this limit the entropic contribution follows the same background scaling as cold dark matter and enters the Friedmann equations in the same way.

More generally, if w_{ent} is approximately constant and $Q(t) \simeq 0$, one obtains

$$\rho_{\text{ent}}(a) \propto a^{-3(1+w_{\text{ent}})}. \quad (\text{A11})$$

A small but non-zero effective pressure would therefore slightly modify the scaling of the entropic density, with corresponding implications for structure formation and background expansion.

When $Q(t) \neq 0$, the entropic contribution receives energy from visible components or gravitational waves through interface events. The detailed form of $Q(t)$ encodes the underlying exchange physics on the timelike vacuum interface and determines departures from the simple a^{-3} scaling. In the main text we keep $Q(t)$ general and focus on regimes in which the effective entropic contribution is close to pressureless and evolves slowly enough to mimic a cold-matter component in the Friedmann equations.

Appendix A.3. Density Parameters

For comparison with observational constraints it is convenient to introduce the usual density parameters,

$$\Omega_i(t) := \frac{8\pi G}{3H^2(t)} \rho_i(t), \quad (\text{A12})$$

and

$$\Omega_k(t) := -\frac{k}{a^2 H^2(t)}. \quad (\text{A13})$$

The first Friedmann Equation (A3) can then be written as

$$1 = \Omega_{\text{vis}} + \Omega_{\text{ent}} + \Omega_{\text{rad}} + \Omega_{\Lambda} + \Omega_k. \quad (\text{A14})$$

In this language the entropic completion amounts, at the background level, to an additional component Ω_{ent} that tracks the cumulative effect of interface events and behaves, in the simplest regime, like the usual cold-matter contribution in the energy budget of the Universe.

Appendix B. Stationary Gravity in the θ -Formulation

This appendix summarises the stationary weak-field limit in terms of the logarithmic temperature potential $\theta = \ln T_{\text{grav}}$ introduced in Section 2.2 and makes explicit the relation to the Newtonian potential and circular velocities in spherical symmetry.

Appendix B.1. Static Weak-Field Metric and Tolman Relation

Consider a static spacetime with a timelike Killing vector $\xi^\mu = (\partial_t)^\mu$ and coordinates adapted to the symmetry, so that the metric can be written as

$$ds^2 = -N(\vec{x})^2 c^2 dt^2 + \gamma_{ij}(\vec{x}) dx^i dx^j, \quad (\text{A15})$$

where $N(\vec{x})$ is the lapse and γ_{ij} the spatial metric on constant- t slices. The norm of the Killing vector is $\xi^2 = g_{\mu\nu} \xi^\mu \xi^\nu = -N^2 c^2$. In thermal equilibrium the Tolman relation (7) reads

$$T_{\text{grav}}(\vec{x}) N(\vec{x}) = T_\infty, \quad (\text{A16})$$

with $T_{\text{grav}}(\vec{x})$ the local temperature measured by static observers and T_∞ a constant reference temperature.

In the weak-field regime one may parameterise the lapse in terms of a dimensionless potential $\Phi(\vec{x})$ as

$$N(\vec{x}) \simeq e^{\Phi(\vec{x})}, \quad (\text{A17})$$

so that the metric takes the approximate form

$$ds^2 \simeq -e^{2\Phi(\vec{x})} c^2 dt^2 + \gamma_{ij}(\vec{x}) dx^i dx^j. \quad (\text{A18})$$

The Tolman relation then becomes

$$T_{\text{grav}}(\vec{x}) = T_\infty e^{-\Phi(\vec{x})}, \quad (\text{A19})$$

which is Equation (8) in the main text. The dimensionless potential Φ is related to the usual Newtonian potential up to a factor of c^2 ; for present purposes it is convenient to work with Φ as defined above.

We now define the logarithmic temperature field

$$\theta(\vec{x}) := \ln T_{\text{grav}}(\vec{x}) = \ln T_\infty - \Phi(\vec{x}), \quad (\text{A20})$$

reproducing Equation (9). Its spatial gradient is

$$\nabla\theta(\vec{x}) = -\nabla\Phi(\vec{x}). \quad (\text{A21})$$

In the Newtonian limit, the gravitational field is $\mathbf{g} = -\nabla\Phi$; restoring factors of c , this is written as Equation (11),

$$\mathbf{g} = -\nabla\Phi = c^2 \nabla\theta. \quad (\text{A22})$$

Thus in the stationary weak-field regime both the gravitational redshift and the gravitational field strength can be expressed in terms of the single scalar potential θ .

Finally, using the usual Poisson equation for Φ ,

$$\nabla^2\Phi = 4\pi G \rho, \quad (\text{A23})$$

and the relation between Φ and θ , one finds

$$\nabla^2\theta = -\frac{1}{c^2} \nabla^2\Phi = -\frac{4\pi G}{c^2} \rho, \quad (\text{A24})$$

which is Equation (12) in the main text.

Appendix B.2. Spherical Symmetry and Circular Velocities

In the spherically symmetric case relevant for the discussion of galactic rotation curves, one may write $\theta = \theta(r)$ and $\rho = \rho(r)$ in Equation (12). In standard spherical coordinates this gives

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d\theta}{dr} \right) = -\frac{4\pi G}{c^2} \rho(r), \quad (\text{A25})$$

reproducing Equation (37). The radial gravitational field is

$$g(r) = c^2 \frac{d\theta}{dr}, \quad (\text{A26})$$

as in Equation (38). For a test body on a circular orbit of radius r one has

$$\frac{v_c^2(r)}{r} = g(r), \quad (\text{A27})$$

which, combined with the previous expression for $g(r)$, yields Equation (39),

$$v_c^2(r) = c^2 r \frac{d\theta}{dr}. \quad (\text{A28})$$

These relations show that, in the stationary weak-field regime, the entire discussion of galactic rotation curves can be formulated in terms of the single potential θ , with the visible and entropic densities $\rho_{\text{vis}}(r)$ and $\rho_{\text{ent}}(r)$ entering only through their sum in the source term of the θ -equation, Equation (37).

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